Appendix A

Conventionality of Simultaneity: A Review

A.1 Light speed and Synchronization

For unambiguous comparison of the temporal relations between two spatially separated arbitrary (unrelated) events in a **given** frame of reference, two synchronized clocks located at the points are necessary. In SR, synchronization of clocks is achieved using light signal whose velocity must be known a priori. But to measure the velocity of the signal two presynchronized clocks are necessary. And herein lies the logical circularity that the measurement of the velocity of light from one point to another (henceforth will be called the *one-way velocity/speed*(OWS)) needs two pre-synchronized clocks. The measurement of round-trip speed or the *two-way speed* (TWS) of light however poses no logical circularity since it needs only one clock. Einstein recognized this ‘problem’ of measurement of TWS of light and also identified the definitional character of simultaneity of two spatially separated events. In his 1905 paper [1, 2] he writes:

> *If there is a clock at point A in space, then an observer located at A can evaluate the time of events in the immediate vicinity of A by finding the positions of the hands of the clock that are simultaneous with these events. If there is another clock at point B that in all respects resembles the one at A, then the time of events in the immediate vicinity of B can be evaluated by an observer at B. But it is not possible to compare the time of an event at A with one at B without a further stipulation. So far we have defined only an “A-time” and a “B-time”, but not a common “time” for A and B. The latter can now be determined by establishing by definition that the “time” required for light to travel from A to B is equal to the “time” it requires to travel from B to A. For, suppose a ray of light leaves*

from A for B at "A-time" t_A, is reflected from B toward A at "B-time" t_B, and arrives back at A at "A-time" t'_A. The two clocks are synchronous by definition if

$$t_B - t_A = t'_A - t_B.$$

We assume that it is possible for this definition of synchronism to be free of contradictions, and to be so for arbitrarily many points, and therefore that the following relations are generally valid:

1. *If the clock at B runs synchronously with the clock at A, the clock at A runs synchronously with the clock at B.*
2. *If the clock at A runs synchronously with the clock at B as well as with the clock at C, then the clocks at B and C also run synchronously relative to each other.*

By means of certain (imagined) physical experiments, we have established what is to be understood by synchronous clocks at rest relative to each other and located at diferent places, and thereby obviously arrived at definitions of "synchronous" and "time". The "time" of and event is the reading obtained simultaneously from a clock at rest that is located at the place of the event, which for all time determinations runs synchronously with a specified clock at rest, and indeed with the specified clock.

Also, in his popular exposition of relativity, Einstein wrote "that light requires the same time to traverse the path AM ... as the path BM [M being the midpoint of the line AB] is in reality *neither a supposition nor a hypothesis* about the physical nature of light, but a *stipulation* which I can make of my own *free will*" [3].

It should now be quite clear that Einstein was well-aware of the *assumptions, definitions or stipulations* he had to make in regard to the OWS of light. It is interesting that textbooks considered these *stipulations* to be *truth.* But, truth must have some empirical content, which the measurement of OWS of light has not.

A.2 Reichenbach Formulation

It is therefore clear that in Einstein's formulation of SR, the synchronization procedure for spatially distant clocks in any inertial frame contains an element of convention (free stipulation) which is devoid of any empirical content. That the procedure for distant clock synchrony in SR has an element of convention other than that adopted by Einstein is known as the conventionality of simultaneity (CS) thesis. The possibility of using a synchronization other than that adopted by Einstein was first discussed by Reichenbach [4] in 1928. Reichenbach observes that the synchronization procedure adopted by Einstein fixes the simultaneity condition for two spatially separated events in a particular way. He comments that "this definition is *essential for the special theory of relativity but it is not epistemologically necessary*"[4, p127]. Einstein's definition is just one possible definition. One can follow an arbitrary rule restricted only to the form

$$t_B = t_A + \epsilon(t'_A - t_A) \qquad 0 < \epsilon < 1$$

and it would likewise be adequate and could not be termed false. Special relativity prefers the value of ϵ (often called "the *Reichenbach parameter*) to be $\frac{1}{2}$, then "*it does so on the ground that this definition leads to simpler relations*". This 'simplicity' is merely 'descriptive', and has no empirical content. The OWS of light is thus conventional. The arbitrariness in the choice of the value of ϵ and the choice of OWS of light is restricted through the following conditions.

1. ϵ must lie within 0 and 1 to preserve causality (light will not reach B before it is emitted from A)

2. The TWS of light is *empirically c.*

Note that the conventionality of synchronization leads to the conventionality in the OWS of any signal including that of light. Thus for any value for $\epsilon(\neq 1/2)$ the OWS' of light in the positive and negative $x-$direction, c_+ and c_- respectively (say) will be different. However the round-trip speed c will be given by the expression

$$\frac{2}{c} = \frac{1}{c_+} + \frac{1}{c_-}. \tag{A.2.1}$$

One is free to choose the OWS of light subject to *this* condition. At the same time, this condition sets the lower limit of OWS to be $c/2$. Note that for this choice, the OWS of return will be infinitely large. Thus, only the TWS of light c has objective status in relativity while the concept of the OWS of light is subjective – just a matter of *convention. Choosing a particular convention for the OWS of light means choosing a related definition of simultaneity of distant events.* To summarize, according to the CS thesis, therefore, there is a plethora of possibilities in assigning the value for the OWS of light. In appendix B, we shall show that the choice of different conventions for the OWS of light (*i.e.* choice of different values of ϵ) leads to different relativistic transformations. Although different synchronizations will be discussed therein, below we briefly mention source of the important synchronies used in the main chapters.

A.3 Einstein Synchrony and Relativity of Simultaneity

So far we have discussed the CS in a *given* inertial frame, without any reference to other inertial frames. Whatever may be the convention, if the *same* convention is adopted in different inertial frames, the spacetime coordinates of these frames will be connected by some transformation equation. According to Einstein's convention for synchrony, for light TWS=OWS in *any* frame. This stipulation is known as the *Einstein synchrony* or the *standard synchrony*, the adoption of which leads to the Lorentz transformation. a corollary of which is the so called "relativity of simultaneity".[1]

The assumption of isotropy of the OWS of light by Einstein made the transformation law (Lorentz transformation (LT)) symmetric and simpler. This alone made this synchronization procedure sacred to several authors. This simple and symmetric nature for the transformation equations however is only structural. According to the CS thesis the physics will remain the same if other synchronization conventions are used. Indeed "the relativity of simultaneity" which is often regarded as the new physical import is devoid of any empirical content in the light of the CS thesis. Unfortunately many authors fail to recognize this point. Some authors (see Ref. [5]) maintain that the relativity of simultaneity is the *actual* cause for the relativistic change of length and time. However others, for example Sjödin [6], contradicting this observes that "there are even physicists who maintain that there are no real length contraction and time dilation effects and that these effects are due solely to the process of synchronization and/or the properties of the space-time continuum ...." One may however favour the standard synchrony over the other because of its simplicity and symmetry but that

[1]The famous *train embankment experiment* qualitatively shows the role of standard synchrony in the train and the platform in arriving at the notion of relativity of simultaneity in relativity theory.

is altogether a different issue. Indeed if one considers synchronization on rotating frames one may give a value judgement (vide Sec. 5.6) in favour of absolute synchrony which we discuss in the next section.

A.4 Absolute synchrony

One may now ask if there exist a synchronization procedure which preserve simultaneity relation between two spatially separated events in all inertial frames, leading to different sets of transformation equations predicting relativistic effects such as Lorentz-Fitzerald contraction and time dilation. One such synchronization procedure is the so called *absolute synchronization.*[2] Here, first an inertial frame is *singled out arbitrarily* where the clocks are synchronized by Einstein method. In this sense, this is *preferred.* The arbitrariness in its choice is follows from the first relativity principle. The clocks in all other inertial frames are synchronized with those of the first frame when they fly past each of the latter ones (see Sec. B.4.2 below for details). As the same correspondence exits among the clocks in each inertial frame and the *preferred* frame, the spatially separated events simultaneous in the *preferred* frame will also be simultaneous in other inertial frames. The transformation arising from this synchrony procedure, the Tangherlini transformation (TT) (vide page 204) corroborates this results with the absence of space part in the time transformation. The Lorentz-Fitzerald contraction and time dilation effects are also predicted with reference to the *preferred* frame.[3] As this synchrony in an arbitrary frame is achieved with

[2] One must not be confused with the usage of the word *absolute*. Here it is meant that congruence of distant events between inertial frames related by this synchronization procedure is preserved.

[3] The simultaneity relation between events occurring in two arbitrary reference frames in this transformation set depends on the *addresses* of the two frames where *address of a frame* is certain function specifying the relation between an arbitrary reference frame and the preferred frame [7].

reference to the preferred frame, the synchrony is sometimes called *system external synchrony* [8]. The Einstein synchronization of clocks in each frame on the other hand is performed without drawing reference to any other reference frame. Hence the synchrony is called *system internal synchrony.* Further discussion in relation to the derivation of the TT is provided in Sec. B.4.2.

A.5 Synchronization in Rotating Ring

The problem of synchronization of clocks on a rotating frame was considered in connection with the Selleri paradox (vide Sec. 5.6). We therefore discuss this issue in somewhat more detail.

The Einstein method of synchrony works just fine in inertial frames. But the main disadvantage of the synchrony is that in accelerated frames the clocks go out of synchrony. Furthermore, in rotating frame, which is a special case of accelerated frame, a clock goes out synchrony with itself [9]![4] Absolute synchronization has the advantage that the clocks thus synchronized in a rotating frame (and, for that matter, in accelerated frames) never go out of synchrony. Cranor et. al. [10] suggested four synchronization schemes. In two of the methods, clocks on the given frame are synchronized before the frame is set into motion. In the other two, the clocks are synchronized when the frame is already in motion. We summarize the methods below.

Method 1. All other clocks on the ring are synchronized with an arbitrarily chosen clock on the ring in the *Einstein method of synchronization, i.e.* exchanging light signals, when the ring is at rest in the laboratory frame. The clocks now appear to be synchronized in an "incontrovertibly correct way" [10]. At this point the ring is

[4]This issue, in view of the present study has been discussed in Sec. 5.6.

uniformly (*i.e.* "all the points on the ring are treated identically") set into rotation.

Method 2. The clocks on the rings are synchronized with the laboratory clock stationed at the centre of the ring when the ring is at rest. The clocks on the ring are set to read $t = 0$ when they receive a signal from the lab clock at the centre. Note that this method is essentially equivalent to Einstein method of synchronization. After the synchronization process is complete the ring is gradually and uniformly put into rotation.

Method 3. All the clocks on the ring already in rotation are set to read $t = 0$ when they receive a flash of light signal from the lab clock stationed at the center of the ring, *i.e.* at rest in the laboratory frame. This method is essentially the same as described in the previous one with the difference that in this case at the time of the synchronization the ring is already in motion.

Method 4. The clocks are synchronized in the Einstein method (Method 1) but now the ring is in rotational motion.

Note that in *Method* 2 each clock on the rotating frame receives the light signal at the same time with the clock situated at the same space point on the inertial frame because in the later frame light wave is spherical. This amounts to synchronize the clock as described in Sec. A.4, *i.e.* this is a functional method of achieving *absolute synchronization in rotating frame.*

Is it possible to achieve absolute synchrony without any reference to the underlying inertial frame? A possible way is suggested in Chap. 5. We avoid repeating it and refer the reader to page 128.

A.6 Criticism of the CS Thesis: Arguments and Counter Arguments

Though the CS thesis stands on very firm logical ground of thesis of empiricism, there is no dearth of counter arguments. Textbooks, except a few (e.g. Ref. [11]) maintain a silence about the thesis and accept the standard synchrony as a proposition having a physical character (see, for example, Ref. [12]). The objections mainly circle around the following arguments:

CA1. Arbitrariness in the value of the OWS of light destroys the isotropy of space concept.

CA2. It is not necessary to put two clocks to measure the OWS of light and jump into the logical circularity. Maxwell's equations for electromagnetic waves set OWS of light at $(\epsilon_0\mu_0)^{-1/2}$.

CA3. It is possible to *arrive at* standard synchrony from the relation of symmetric causal connectibility structure of spacetime [13, 14]. Essentially the idea is first to show that standard Einstein synchrony is equivalent to the orthogonality of the time axis of the reference frame in Minkowski space and then to demonstrate that this orthogonality of Minkowski spacetime structure is definable from the causal structure of the spacetime, the lightcone structure. They claim that this approach does not adopt any convention about the OWS of light [15].

CA4. It is possible to bypass the *logical circularity* and thereby devise experiments to measure the OWS of light. Numerous such experiments over the years have been proposed by several authors [16, 17] which have been claimed to allow for an empirical test which might distinguish among the admissible synchrony conventions and thus refute the conventionalists' thesis that all admissible conventions are empirically equivalent. Also, it has been claimed that it is possible to synchronize clocks by other methods (without sending any signals to a distant point) unambiguously. One of the main contender is slow clock transport (SCT) synchrony.

CA5. The anisotropic effects arising from the adoption of nonstandard synchrony is associated with the loss of simplicity of the theory.

All of these counter arguments have been refuted by the conventionalists. We put them together below.

CC1. Isotropy of space is actually *defined* by one-way speed of light [18, 19] making the criticism incorrect. It is possible to define isotropy of space in terms of TWS of light (an empirically verifiable quantity). Fizeau's definition [18, p355:footnote 7] of isotropy of space contains time taken by light to perform round-trip journey in two opposite directions along the same path – equality rendering isotropy of the space. (Rotation destroys this isotropy as evident from the Sagnac effect [20]).

CC2. The simplistic point of view that the OWS of light may be "measured" using Maxwell's equation fails to appreciate the fact that Maxwell's equations use velocity of a charged particle. The adoption of the convention is included in the measurement procedure of velocity which needs two pre-synchronized clocks. Thus the magnitude of velocity of anything is *conventional.*

CC3. This approach, often referred to as Robb-Malament thesis, was considered to settle the issue of the CS thesis for the non-conventionalists [21]. But several authors refuted the claim (see, for example, Refs. [22–25]). We refrain from discussing the matter further. For a comprehensive review the reader is referred to Ref. [22].

CC4. There have been several attempts to devise experiments which can determine the OWS of light uniquely bypassing the *logical circularity*. Reichenbach discussed several experimental propositions (alongwith counter arguments) which *apparently* counters the CS thesis. Erlichson [26] discussed several possibilities and finally concludes that "distant absolute synchronization[5] is impossible and that distant synchronization must employ a convention". In response to the article by Erlichson, Brehme [12] tried to devise yet another method of synchronization without the use of light signals. However Ungar [27] rightly remarks that Brehme's method uses movement of clocks and does not "take into account anisotropic time dilation effects that ... affect the reading of moving clocks in such a way that anisotropy in one-way motion cannot be detected" thus upholding Erlichson's contention.

It was argued by many that it is possible to synchronize clocks by the method of slow transport of clocks without considering any convention whatsoever. A brief account of this debate is given in Sec. A.7 below.

CC5. This counter argument follows the "Occam's dictum that physical

[5]The term *absolute synchronization* used by Erlichson has a meaning different from that we have adopted throughout the present text. According to our definition of the term, a scheme of synchronization which gives 'non-relativity' of simultaneity between inertial frames moving with non-zero relative velocity is called absolute synchrony. The term *'absolute'* here is not to be confused with *'not conventional'*. But according to Erlichson the term *'absolute'* has been used to mean *'as opposed to convention'*. In other words when Erlichson says *absolute synchronization is impossible*, he means that *synchronization is conventional.*

descriptions ought to seek the simplest form" [12]. The description of physical theories becomes anisotropic with the adoption of nonstandard synchronies with anisotropic OWS of light. The transformation equations arising from nonstandard synchronies are also not as symmetric as the LT (see App. B). Brehme, in his response to Erlichson's article [12] commented that "it [nonstandard synchrony] can be done, but is so artificial as to jar our sense of fitness". He also commented there that "introducing an arbitrary convention is surely accompanied by the loss of simplicitly and sense". On this, Debs and Redhead rightly comment that "*... the conventionality thesis is an issue, not about simplicity, but about what is factual and what is conventional in the foundations of special relativity*" [15].

A.7 Slow Clock Transport

Poincaré is considered by many to be one of the first to discuss simultaneity, clock synchrony and their convention. He, in an essay in 1898 mentioned the distinction between the concept of the simultaneity of events that occurred at the same place and those that occurred at distant places [28]. The absence of access to a universal time to order distant events, according to him, compels one to decide on their simultaneity (or otherwise) on the basis of a convention.

In his 1891 essay Poincaré put forward the use of transported clocks to determine time differences of events occurring at distant places. These ideas were pre-relativistic and thus did not include the notion that the rate of a clock is affected by movement. Einstein remarks [1, 2] that where two separated clocks are synchronized with each other (using, obviously, standard synchronization), synchronization is lost if one of them is moved. But, according to a study by

Anderson, Bilger and Stedman [29, p.115], Einstein did not discuss, in that paper or elsewhere, the procedure of synchronization using the transport of clocks for the determination of simultaneity.

Eddington [30] suggested a different method of synchronization of clocks. In this method clocks are synchronized when they are at the same space point. Then they are moved away from each other to be placed at different space point of the reference frame. The fact of retardation of moving clocks are taken care of by limiting their velocities to zero. He notes that it leads to the same results as those obtained by the use of light, *i.e.* Einstein synchrony. He then comments

> *We can scarcely consider that either of these methods of comparing time at different places is an essential part of our primitive notion of time in the same way that measurement at one place by cyclic mechanism is; therefore they are best regarded as conventional.*

It is clear that his claim is that although these statements are conventions, they are empirically related, and are not independent. This all suggest that an empirical test of the equivalence would mean a test of "fundamental hypothesis" and of the directional independence (and observer independence) of the round trip speed of light [29].

One objection to the use of the SCT scheme of synchronization of clocks thus proposed is that there is no way of measuring the OWS of the transported clock until the clocks are synchronized – calling up again the logical circularity. Also, infinitesimal slowness of travel requires infinite arrival time which means that to have a set of synchronized clocks, one should start from infinite past. Bridgeman [31] suggests a way out. He modifies the Eddington synchronization in such a way that one need not start from infinite past. Instead of transporting single clock to a single point, he proposes to send a number of clocks (say from A to B) at various velocities. The readings of clocks at B differs and the one with minimum reading is singled out (the so-called self measured velocity). Comparison of readings of this clocks to that of other clocks will be plotted and then extrapolated to zero. Bridgeman remarks that "... Einstein's remark is by no means invalidated."

The method of synchronization of clocks described by Ellis and Bowman [32] is different but equivalent [23]. Two clocks C_A and C_B are placed at two spatially different points A and B. A third clock C_C is first synchronized with the clock C_A after placing it at A and then moved to B with "intervening 'velocity'" (their terminology) and the time interval is measured. This procedure is repeated with decreasing velocities. The result is extrapolated to find difference between the readings of C_B and C_C with the later's velocity limiting to zero. Finally, this difference is introduced in C_B.

A.8 SCT And The CS Thesis

Winnie [33, 34], in a series of two papers where he reformulated STR "without one-way velocity assumption" shows explicitly that synchrony by SCT agrees with Einstein synchrony when both are described in terms of an arbitrary value of ϵ (within the prescribed limit $0 < \epsilon < 1$). It is argued that in the arguments of EB, $\epsilon = 1/2$ is implicitly assumed. Winnie mentioned that "it is not possible that the method of slow-transport, or any other synchrony method, could, within the frameworks of the *non-conventional* ingredients of the Special Theory, result in fixing *any* particular value of ϵ to the exclusion of any other particular values."

Mansouri and Sexl [8, 35, 36] contends that standard synchrony *in general* differs from SCT synchrony. The equality, they claim, is "neither trivial nor logically cogent". Their findings supports the claim of Winnie that SCT synchrony and Einstein synchrony "agree if and only if the time dilation factor is given exactly by the special relativistic value $(1 - v^2)^{-1/2}$" $[c = 1]$. Both Winnie and Mansouri and Sexl accept SCT as just another convention.

The paper by Ellis and Bowman became prolegomenous to a panel discussion by Grünbaum, Salmon, Fraasen and Janis [37]. They criticized Ellis and Bowman from different angles. Grünbaum contends that "...in the STR, synchronism by slow clock transport neither refutes nor trivializes the ingredients of a convention in that theory's distant simultaneity." Fraasen concludes that "...Ellis and Bowman have not proved that the standard simultaneity relation is conventional, which it is not, but have succeded in exhibiting some *alternative conventions* which also yield that simultaneity relation."

There are other objections about SCT too. For example Erlichson remarks that to have a correct synchronization, clocks should be moved very slowly, and the synchronization will be acceptable in the limit $v \to 0$. In this limit the clock won't move at all and once it reaches the distant point it will show infinite time. Indeed for any arbitrary preassigned value for v one can choose the distance to be arbitrarily large. In practical sense this is unacceptable [26], although the present author believes that the question of practicality is beside the point.

A.9 Discussions

The basic idea behind the CS thesis is that the precise empirical determination of the OWS of light is impossible. It is the logical circularity that debars us to design any experiment. The logical circularity comes from the fact that to measure OWS of light one needs two spatially separated pre-synchronized clocks while to synchronize these clocks one needs to know the OWS of light. On the contrary, to measure the TWS or round trip speed of light, one needs only one clock. Thus the TWS of light is an empirically verifiable physical quantity. One can choose the OWS of light in one direction to be anything. But then the OWS of light in the opposite direction must be adjusted such that the round trip value (TWS) comes out to be c[6] thereby setting a limit to the choice of OWS.

Einstein first recognized this fact and chose light speed to be c in all directions at his 'free will' (as a convention). He then proposed to synchronize all the clocks in a frame with that *chosen* value of OWS of light with a clock at an arbitrarily chosen origin. In fact the "relativity of distant simultaneity' is an outcome of Einstein's convention which is often erroneously construed as a new philosophical import.

Reichenbach first recognized that it was possible to choose other values of the OWS of light still preserving the basic tenet of relativity. In that case the transformation equations will not be the Lorentz one. Tangherlini first showed that this is really possible. It was followed by Winnie, Mansouri and Sexl, Sjödin and Selleri, all of whom derived (general) relativistic transformation in some form. Ghosal, Mukhopadhyay and Chakraborty [39] derived a transformation, called Dolphin transformation, using nonluminal signals as synchronizing agent. General transformations that preserve the basics physics of SR will be discussed in appendix B.

Although many articles have been written on the CS thesis, most texts on relativity, except a few [11], do not discuss this topic at length. The fact that the CS thesis has not yet gained appropriate attention, among the physicists may be attributed to the fact that there is a tendency to regard the CS thesis as an antithesis of SR and anything that seems contrary to the standard formulation of relativity

[6]That this round trip value is independent of the inertial frame of reference chosen is the (reinterpreted) second relativity postulate [38]

is viewed with skepticism [40]. Several claims have been made about finding a method (thought experiment or performable experiment) to determine the OWS of light. Several round abouts have been suggested. To comment on these however, it is enough to say that every such test proposed can be shown to involve, in its analyses and assumptions, propositions logically equivalent to the adoption of the standard synchrony. And this amounts to a simple begging of the question rather than an independent empirical test [41]. In this context we quote a remark by Debs and Redhead [15]

> *...on the grounds that any method that establishes standard synchrony in amoving frame will automatically define nonstandard synchrony in a stationary frame, so the conventional element is restored in specifying simultaneity in the stationary frame ....*

Indeed in our opinion the CS thesis complements the SR and the understanding of the former helps clear out confusions that some times occur in the SR. As we have pointed out, the claim that the relativity of distant simultaneity is a new non-classical philosophical import is one example of various such confusions. In spite of the SR being one of the most simple physical theories, it is the most prolific in giving birth to fallacies, riddles, confusions and misconceptions. Overlooking of the CS thesis and misconstruing of the subtleties of the CS thesis are two of the major reasons for that. For a details analysis vide Ref. [38, 40, 42]. In this dissertation, the CS thesis is *used as a tool* precisely to understand some paradoxes in relativity theory.

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Appendix B

CS Thesis and General Transformation Equations

B.1 Introduction

We have already discussed that the CS thesis contends that there can be several possible choices on the OWS of light, of which Einstein convention is just one. In the relativistic world the set of transformation equations (TE) that arises out of this choice is the Lorentz transformation. Different choice of OWS of light will lead to different sets of transformation equations which, although different structurally, will predict the kinematical worlds. We have already seen in previous chapters that consideration of these TE's helps us in giving insights to many conceptual issues including some interesting paradoxes in SR. This appendix will supplement those chapters by discussing these TE's that can be obtained using different conventions regarding OWS of light in different worlds.

Winnie [1, 2] in 1970 first studied the consequences of SR when no assumption regarding the OWS of light was made and then developed a set of transformation equations, referred to in the literature as *ϵ-Lorentz transformation*, adopting non-Einstein OWS assumption or non-standard synchronization convention in general. In developing the ϵ-Lorentz transformation Winnie assumed a principle called the *Principle of equal Passage time*. This was used in addition to the *Linearity Principle* and the *Round-Trip light Principle*. These principles were then shown to be independent of OWS assumptions and thus may form the basis of a SR where no stipulation regarding distant simultaneity is made. In fact Winnie's theory was one dimensional. Ungar [3] extended Winnies's idea by considering a generalized Lorentz transformation group that does not embody Einstein's isotropy condition. The approach seems to be well suited for establishing the results of Winnie as well as some new results. However, these discussions were confined to one dimensional case only. But at least a two dimensional analysis is a must. Otherwise the isotropy of the TWS of light which follows from the reinterpreted second relativity postulate cannot be used and therefore some subtleties and richness of the relativistic physics have to be sacrificed[4, 5].

In a series of important papers Mansouri and Sexl [6, 7] developed a test theory of SR and investigated the role of convention in various definitions of clock synchronization and simultaneity in a given frame. They considered two principal methods of synchronization and called them *system internal* and *system external* synchronization. The *system internal synchronization* is the one where the clocks in a given reference frame are synchronized without any reference to any clock outside the frame. The *system external synchrony* on the contrary is done with reference to clocks residing outside the frame. Synchronization of clocks by the Einstein procedure (using the light signal) and that by slow clock transport (to correct all clocks at a given locality and then place them at all space points of a given reference frame) turn out to be equivalent if and only if the time dilation factor is given by the Einstein result $(1 - v^2/c^2)^{1/2}$. The authors also constructed an 'ether' theory with an external synchronization schemes that maintains absolute simultaneity and is shown to be kinematically equivalent to SR. This particular synchrony has already been discussed in the last appendix.

The next clarification came in 1979 from Sjödin. He developed and consolidated the CS thesis in an interesting paper by considering the whole issue more generally and also by assuming the role of synchronization in SR and some related theories. He presented all logically possible linear transformations between inertial frames depending on physical behaviour of scales and clocks in motion with respect to physical vacuum (ether) and then examined LT in the light of *true* length contraction and time dilation. In his article Sjödin tried to separate the *true effects* and the effects due to *synchronization convention*. For this the author considered two special cases:

- **The Newtonian world**, without any contraction of moving bodies and slowing down of moving clocks.
- **The Lorentzian world**, with longitudinal contraction of moving bodies and slowing down of clocks.

He then used standard synchrony (later called pseudo-standard synchrony by Ghosal, Mukhopadhyay and Chakraborti [8]) in Newtonian world and obtained the transformations derived by Zahar [9]. These transformations show some apparent relativistic effects which are only due to the choice of the special synchrony in this world. But when Sjödin used absolute synchronization in the Lorentzian world, the relevant transformations were due to Tangherlini [10] which shows the real effects. In this way Sjödin came to the conclusion that the confusion regarding the existence of the ether and the reality of the length contraction/time dilation effects is mainly due to the non-separation of the effects due to synchronization and the real contraction of moving bodies and retardation of moving clocks.

In this chapter, we present a derivation of a set of general transformation equations. We keep a free parameter (called the *synchronization parameter*) which depends on the method of synchronization (*i.e.* the adoption of the convention for the OWS of light). Choice of different OWS of light sets different values of the parameter giving different TEs. We use the matrix algebra because of its mathematical advantages in finding composite transformations, inversion, velocity addition foumula etc. We mention the relation of this TE, which relies heavily on the derivation by Sjödin [11] with those derived by Winnie and by Selleri. Then we discuss a TE due to Ghosal, Mukhopadhyay and Chakraborti [8] where the synchronization is performed, with nonluminal signal (assumed to propagate in a substratum) and show that all known relativistic (along with some non-relativistic) TEs can be obtained easily from this, choosing the appropriate synchronization convention. Indeed, in this set of TE, the role of light as a physical constant and as a synchronizing agent in SR are clearly separated.

B.2 The Transformation Matrix

We shall now find the transformation equations between two inertial frames. Let us choose an inertial frame S_0 arbitrarily (all inertial frames are equivalent). For definiteness we choose the convention that *in this frame* the OWS of light is the same in all directions and is equal to the TWS of light, *i.e.* c. (In the Galilean world however there is just one frame (ether frame) where the TWS of light will be isotropic). In other words in this frame we synchronize the clocks with the Einstein synchronization procedure. This frame will be a *preferred* one if the Einstein convention of isotropic OWS is chosen *only* in this frame. One should not confuse this notion of preferred frame with the pre-relativistic concept of an absolute space. Also, this choice does not violate the principle of relativity of SR because the frame is arbitrarily chosen, *any frame can be given the preferred status*. Following Sjödin we start from the assumptions below.

Assumption 1 *The dimensions of bodies and the time intervals measured by clocks depend on there velocities with respect to an inertial frame but do not depend on their position or acceleration. This is known as* Length and clock hypothesis.

Assumption 2 *The change of dimension of a body in the direction of motion is given by ϕ and that in the transverse direction by ψ. The change in the rate of* *clock is given by* Ω.[1] *According to assumption 5,* ϕ, ψ, Ω *are functions of velocity.*

Assumption 3 *Clocks are synchronized in the transverse direction in the same way as we do in the direction of motion.*

The last one is not an essential assumption, but inclusion of this simplifies calculation. The exclusion of the assumption will not give any new physical insight.

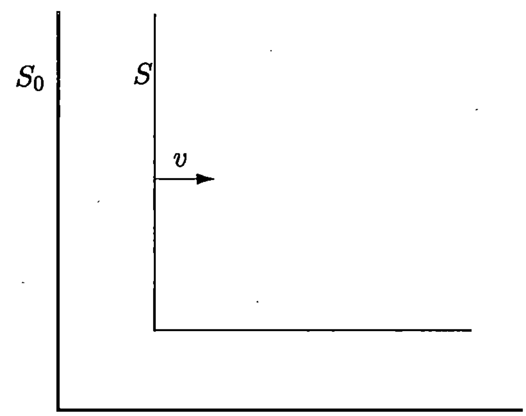

Figure B.1: Here S_0 is the preferred frame

Let us assume that an inertial frame S is moving with velocity[2] v, with respect to the preferred frame S_0. The motion of S in S_0 is along their common x-axis. Initially their origins coincided, *i.e.* ,

$$\text{at } t_0 = t = 0, \;\; x_0 = x = 0.$$

[1]We use the notations as in Ref. [11]

[2]There is, as we have mentioned earlier, an element of convention to measure velocity of any moving body. From now on, whenever we say *velocity*, it will mean that it is measured in the preferred frame *i.e.* $\epsilon = 1/2$, unless otherwise explicitly mentioned.

The homogeneity of spacetime allows us to choose the transformation equations to be linear.[3] Thus the most general transformation ($S_0 \to S$) is given by, in matrix form

$$\mathbf{X} = \mathbf{T}\mathbf{X}_0, \tag{B.2.1}$$

where (suppressing the z-coordinate)

$$\mathbf{X} = \begin{pmatrix} t \\ x \\ y \end{pmatrix}, \quad \mathbf{X}_0 = \begin{pmatrix} t_0 \\ x_0 \\ y_0 \end{pmatrix},$$

and

$$\mathbf{T} = \begin{pmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \\ a_{20} & a_{21} & a_{22} \end{pmatrix}.$$

In a more compact notation

$$\mathbf{T} = \{a_{\mu\nu}\}, \quad (\mu, \nu = 0, 2).$$

Note that here the time co-ordinate is t and not ct, as found in usual four-vector formulations. $\mathbf{T}$ is the transformation matrix. Thus to determine transformation equations, $\mathbf{T}$ is to be found out. Inverse transformation is given by

$$\mathbf{X}_0 = \mathbf{T}^{-1}\mathbf{X}. \tag{B.2.2}$$

According to how we constructed the problem, $x-$axis always coincides with the x_0-axis. Consequently,

$$a_{2\nu} = 0 \qquad \nu = 0, 1. \tag{B.2.3}$$

[3]However very complicated synchronization schemes are clearly possible so that the homogeneity of space will not be explicit in the transformation equations.

Assuming the isotropy of space we accept that clocks placed symmetrically in y axis, about the $x-$axis should agree.This will be true provided

$$a_{02} = 0. \tag{B.2.4}$$

As S is moving along common $x-$axis with a velocity v

$$x = 0 \Longrightarrow x_0 = vt_0.$$

This gives

$$a_{10} = -va_{11},$$
$$a_{12} = 0.$$

Thus the transformation matrix becomes

$$\mathbf{T} = \begin{pmatrix} a_{00} & a_{01} & 0 \\ -va_{11} & a_{11} & 0 \\ 0 & 0 & a_{22} \end{pmatrix}.$$

The parameter a_{22} must carry the information about the change of space in y direction respectively. If we write the transverse contraction factor (according to assumption 2)

$$a_{22} = \psi^{-1},$$

the transformation matrix becomes

$$\mathbf{T} = \begin{pmatrix} a_{00} & a_{01} & 0 \\ -va_{11} & a_{11} & 0 \\ 0 & 0 & \psi^{-1} \end{pmatrix}. \tag{B.2.5}$$

Let us consider now a rod of length L at rest on the $x-$axis. We may set $L = x_2 - x_1$. Also $t_{01} = t_{02}$ because to measure length, both the points should be measured simultaneously. This gives $L = a_{11}(x_{02} - x_{01})$. But $x_{02} - x_{01}$ is the length of the rod as measured in S_0, *i.e.* $x_{02} - x_{01} = \phi L$. Hence,

$$a_{11} = \phi^{-1}.$$

Let us now consider a clock at rest in S. Say its position is $x = x_1 = x_2$. Say two events occurred in S at time co-ordinates t_1 and t_2 $(t_2 > t_1)$.

In S_0 frame, the space co-ordinates of the clock is x_{01} and x_{02} and the times are t_{01} and t_{02} giving

$$x_{02} - x_{01} = v(t_{02} - t_{01}).$$

Thus we find,

$$t_2 - t_1 = (va_{01} + a_{00})(t_{02} - t_{01}).$$

Using *assumption* 1, we can write $t_2 - t_1 = \Omega(t_{02} - t_{01})$, and hence

$$a_{00} = \Omega - va_{01}.$$

Now inserting all the values in the transformation matrix (B.2.5) and writing $a = a_{01}$, we obtain

$$\mathbf{T} = \begin{pmatrix} \Omega - va & a & 0 \\ -v\phi^{-1} & \phi^{-1} & 0 \\ 0 & 0 & \psi^{-1} \end{pmatrix}. \qquad \text{(B.2.6)}$$

In Eq. (B.2.6), 'a' will depend on the synchronization that we make. Let us call it the *synchronization parameter.* Our main task is to find the factor 'a'. Note that exclusion of *Assumption 2* invalidates Eq. (B.2.4). The term a_{02} will then be present in the transformation matrix (B.2.6) [12].

B.3 Determination of a

The value of 'a' carries the synchronization convention. This means that a must be a function of the OWS of light

To find this function, let us imagine a rod of length L at rest in S [11]. Say, the OWS of light at an angle θ (in S) is given by $c(\theta)$.

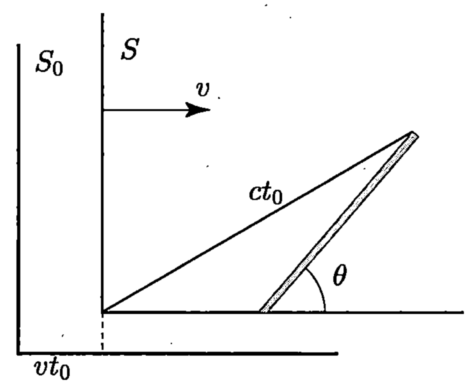

Figure B.2: Detivation of the OWS of light in an inertial frame, figure from Sjödin

In S_0, the light sphere relation is

$$x_0^2 + y_0^2 = c^2 t_0^2 \tag{B.3.1}$$

When measured in S_0, the length L suffers a change by a factor of ϕ in x direction and ψ in y and z directions. Using cylindrical co-ordinate system (r, θ, z) for L in S, we may write (see Fig. B.2)

$$\begin{aligned} x_0 &= vt_0 + \phi L \cos\theta \\ y_0 &= \psi L \sin\theta \end{aligned} \tag{B.3.2}$$

Putting all these in Eq. (B.3.1) and solving for t_0, we obtain

$$t_0 = \gamma^2 c^{-1} L\phi \left[\beta\cos\theta \pm \left\{1 - \left(1 - \gamma^{-2}\phi^{-2}\psi^2\right)\sin^2\theta\right\}^{\frac{1}{2}}\right].$$

In S, the OWS of light at an angle θ with respect to x axis is $c(\theta)$. Thus $L = c(\theta)\, t$. Using the transformation (B.2.6) we find

$$c(\theta) = \left[(1-\xi)\beta\cos\theta \pm \left\{1 - (1-\rho)\sin^2\theta +\right\}^{1/2}\right]^{-1} \chi, \tag{B.3.3}$$

where

$$\xi = -a\,\Omega^{-1}\gamma^{-2}\beta^{-1}c,$$
$$\rho = \gamma^{-2}\phi^{-2}\psi^{2},$$
$$\chi = \gamma^{-2}\Omega^{-1}\phi^{-1}c,$$
$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}.$$

Also,

$$a = (c(\theta)\phi\cos\theta)^{-1}\left(1 - \Omega c(\theta)\Sigma(\theta)\right), \tag{B.3.4}$$

where

$$\Sigma(\theta) = \gamma^{-2}c^{-1}\phi\left[\beta\cos\theta \pm \left\{1 - (1 - \rho)\sin^2\theta\right\}^{1/2}\right].$$

According to the CS thesis the OWS of light in a given reference frame can be fixed according to one's choice and convenience, while TWS of light is an empirically verifiable constant quantity. In our derivation of generalized relativistic transformation, we found that one parameter of the transformation matrix depends on the OWS. Surely this depends on the synchronization. The OWS of light is given by, as a function of the angle with $x-$axis,

$$c(\theta) = \left[(1 - \xi)\beta\cos\theta \pm \left\{1 - (1 - \rho)\sin^2\theta\right\}^{1/2}\right]^{-1}\chi. \tag{B.3.3}$$

The $\pm$ sign denotes two possible values of $c(\theta)$. We can choose any one for our discussion and let us choose the positive sign.

$$c(\theta) = \left[(1 - \xi)\beta\cos\theta + \left\{1 - (1 - \rho)\sin^2\theta\right\}^{1/2}\right]^{-1}\chi. \tag{B.3.5}$$

We avoided unnecessary complications choosing the same synchronization for x and y axes. This is evident here as we find, for $y-$axis,

$$c(\pi/2) = c(-\pi/2).$$

Also note that for $\xi = 1$, $c(\theta) = c(\theta + \pi)$ which means that OWS is same for the forward and reverse journey. This corresponds to the *Einstein convention.*

B.4 Transformation in Different Worlds

B.4.1 Classical World

In classical world, a signal with infinite speed is possible. Thus one can synchronize clocks with this signal and *empirically* find the OWS of light. Also since the readings of clocks do not change due to their motion, they can be synchronized in an absolute way. However different synchronization conventions can be chosen in this world too and hence different transformations are possible.

Zahar Transformation

In classical world, one may synchronize the clocks with light adopting the Einstein convention of OWS=TWS. Thus we obtain

- $\Omega = \phi = 1$ for classical world.
- $c(0) = c(\theta) = c(1 - \beta^2)$.
- $\psi = 1$, for classical world.

These will give the value of the synchronization parameter a.

$$a = -\gamma^2 \frac{v}{c^2}$$

Putting all this values in the transformation matrix (B.2.6) we obtain

$$\mathbf{T} = \begin{pmatrix} 1 + \beta^2\gamma^2 & -\frac{\beta}{c}\gamma^2 & 0 \\ -v & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{B.4.1}$$

This is the transformation matrix for Zahar transformation. For a detailed derivation and analysis of this transformation reader is referred to the original work of E. Zahar [9]. In this dissertation, this transformation has been used as an important tool to resolve two paradoxes, namely the Tippe top paradox (Chap. 4) and the Selleri paradox (Chap. 5).

Approximate Zahar Transformation

Sufficiently small v/c will render $\beta^2 \to 0$ and hence $\gamma \simeq 1$. The TE is know as approximate Zahar transformation and is given by

$$\mathbf{T} = \begin{pmatrix} 1 & -\frac{\beta}{c} & 0 \\ -v & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{B.4.2}$$

Note that this the same as the LT under the same approximation(discussed below)!

Galilean Transformation

In Zahar transformation, the synchronizing signal is light (with velocity c in the preferred frame). In classical world, we can synchronize with a signal whose velocity is infinite. So, we may put $c \to \infty$. This gives $a = 0$, and the transformation matrix (B.2.6) is given by

$$\mathbf{T} = \begin{pmatrix} 1 & 0 & 0 \\ -v & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{B.4.3}$$

B.4.2 Relativistic World

Lorentz Transformation

In relativistic world, the value of OWS=TWS=c in any reference frame if one uses the standard synchrony and it is the highest speed of signal transmission. This synchrony, by virtue of the second relativity postulate makes the OWS isotropic, that is, independent of θ. The value of ψ is also unity [13, pages 36-37].

Summarising

- $c(\theta) = c(\theta + \pi) = c$
- $\xi = 1$, as $c(\theta) = c(\theta + \pi)$
- $\psi = 1$

From these one may easily find out the values of all the parameters

$$\phi = \Omega = \gamma^{-1}$$
$$a = -\gamma\frac{\beta}{c}.$$

Putting these value in Eq. (B.2.6) we obtain the transformations matrix

$$\mathbf{T} = \begin{pmatrix} \gamma & -\gamma\frac{\beta}{c} & 0 \\ -\gamma v & \gamma & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{B.4.4}$$

This is the Lorentz transformation matrix.

Approximate Lorentz Transformation

If the velocity becomes sufficiently small with respect to the velocity of light ($v/c \ll 1$) then $\gamma \simeq 1$ and the transformation is known as approximate Lorentz transformation [14, page 11]

$$\mathbf{T} = \begin{pmatrix} 1 & -\frac{\beta}{c} & 0 \\ -v & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{B.4.5}$$

Note that here $\mathbf{T}_{01}$ has not been put zero (that would correspond to GT). This term appears in the TE multiplied by the space coordinate x. For a sufficiently large

x, βx may be comparable to c^2 and thus will not vanish. Also observe that under the small velocity approximation LT does not go over to the GT since a mere approximation cannot alter the synchronization convention. For this reason, here AZT=ALT since both uses the standard synchrony. This has been first pointed out by Ghosal, Nandi and Chakraborty [15].

Tangherlini Transformation

In standard relativity the clocks in a given frame are synchronized by light signals having isotropic one way velocity. All the clocks in any of the frames are connected to each other by light signals. No reference of clocks in another frame is required here. This method of synchrony is some time referred to as *system internal synchronization* [6].

As the synchronization of clocks in a given reference frame in Einstein method makes no reference to clocks outside the frame, the concept of simultaneity of two distant events becomes frame specific – two distance events simultaneous in one frame are not simultaneous in any other frame.

Is it possible to synchronize clocks in such a way that simultaneity relation between two distant events in two frames within relativistic domain is preserved? One method is described below:

We choose one inertial frame arbitrarily and synchronize the clocks by Einstein procedure. This is our *preferred* frame S_0. All other systems moving past S_0 will synchronize their clocks by adjusting these clocks to $t = 0$ whenever they fly past a clock in S_0 which reads $t_0 = 0$. To find the transformation matrix in this case we have to determine a, the synchronization parameter.

Recall that a is a rename of a_{01} of the original general transformation matrix. a_{01} is related to the zeroth and first coordinate, *i.e.* t and x. Clearly we can infer that there should not be any inter-connection between t and x if the clocks are synchronized in this manner. Thus $a_{01} = a = 0$. This gives $\xi = 0$. Putting this value in $c(0)$ (or $c(\pi)$) we obtain $\Omega = \gamma^{-1}$. Also, $\rho = 1$, giving $\phi = \gamma^{-1}$.

Thus we obtain the transformation matrix as

$$\mathbf{T} = \begin{pmatrix} \gamma^{-1} & 0 & 0 \\ -v\gamma & \gamma & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{B.4.6}$$

$\{\mathbf{T}\}_{01} = 0$ manifests the fact that the time transformation does not contain a space component, as stated.

This transformation was first derived by F. Tangherlini [10] and thus referred to in the literature as Tangherlini transformation (TT).

B.4.3 Discussion on TT

A study of TT shows that while the space transformation equation is similar to the LT, the time transformation equation does not contain a space part. The basic features of the TT are:

1. $\Delta t_0 = 0 \Longrightarrow \Delta t = 0$. This in turns implies that the simultaneity is *absolute*. Two spatially separated events simultaneous in the preferred frame are simultaneous in other frames. One can also find transformation from any frame to *any* (not necessarily from the preferred frame) other frame (see Sec. B.6 below) and satisfy oneself that the statement is also true for this case also. That is why this synchronization is called *absolute synchrony*.

2. With respect to the preferred frame we obtain the usual length contraction and time dilation effect. Thus it predicts *relativistic effects* but the

simultaneity is absolute! One sometimes interprets these as effects which are *real*, inherent in relativity theory. The preferred frame is often identified with that of 'ether' or a cosmologically preferred one [11, 16]. However the physical status of the arbitrarily singled-out frame in unimportant for the present work.

Absoluteness in simultaneity is possible if we are in classical world with the GT where the synchronization is achieved by signals moving with infinite speed.[4] But no confusion should arise because though both GT and TT give absolute simultaneity, GT, unlike TT, does not predict time dilation (as well as length contraction) and thus is not a relativistic transformation. [5]

Let us now find the OWS of light in the frame where *external synchronization* (or here, *the absolute synchronization*) was achieved. OWS of light at an angle θ with respect to the X-axis ($c(0)$ is the positive x-direction) is given by (B.3.3)

$$c(\theta) = \left[(1-\xi)\beta\cos\theta - \{1-(1-\rho)\sin^2\theta\}^{1/2}\right]^{-1}\chi.$$

Putting the parameter expressions suitable for TT,we obtain,

$$c(\theta) = \frac{c}{1+\beta\cos\theta}. \tag{B.4.7}$$

Thus the OWS' are given by

$$\left.\begin{aligned} c(0) &= \frac{c}{1+\beta} \\ c(\pi) &= \frac{c}{1-\beta} \end{aligned}\right\}. \tag{B.4.8}$$

Thus the OWS' of light in frames where the clocks are synchronized in absolute way are not c. We have already made use of this result in a Chap. 5.

[4]Note, however, that using a finite speed signal as light and making use of Einstein-like (pseudo-standard) synchrony leads to Zahar transformation in classical world.

[5]ZT however predicts an apparent length contraction and time dilation effects which is nothing but an artefact of the pseudo-standard synchrony.

Another important feature of TT is that a moving frame of reference is always related to the "preferred" frame in some way, even if the TT represents transformation between any two arbitrary (see B.6) reference frames. Also this particular form of TT is valid only between frames one of which must be the preferred one. Thus the transformations are not reciprocal like the LT. Consequently while a rod at rest in the moving frame with respect to the preferred frame, the rod gets contracted with respect to an observer in the preferred frame. On the contrary, a rod at rest in the "preferred" frame is elongated with respect to the moving frame by the same relativistic factor. The reason lies in the fact that definition of measurement of length of a moving rod depends on the definition of simultaneity which in turn depends on the clock synchronization.

This non-reciprocity occurs for time measurement too. Clocks moving with respect to the preferred frame runs slower but a clock at rest in the preferred frame runs faster (by the usual relativistic factor) with respect to the *moving frame*. This feature is generally ascribed to the fact that we have singled out (though arbitrarily) one system with a unique convention of OWS of light.

The spacetime diagram in this case is given in Fig. B.3 [6].

X_0 and T_0 represents space and time axes in the preferred frame. In the moving frame, like that in SR, x and t are not orthogonal. However the space axis X remains parallel to X_0 in this case. The figure also shows the fact that in the moving frame OWS' of light are not equal.

It is observed that one can use TT to handle non-inertial frames straightforwardly [12]. This feature of TT has already been discussed and used in earlier chapters.

TT was arrived at by different authors by different methods [6, 8, 10, 11, 17]. Our treatment in finding the general transformation matrix closely follows that of

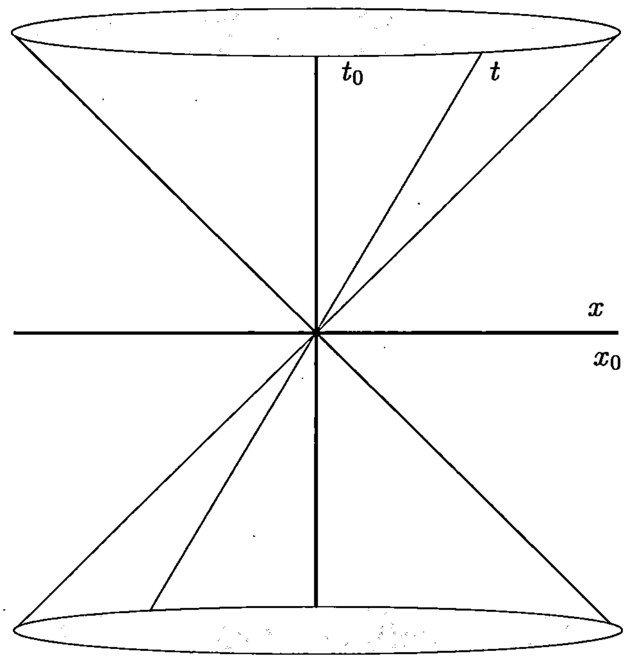

Figure B.3: Spacetime diagram for TT

Sjödin [11]. While Tangherlini himself obtained this by adjusting (among other things) the properties of line element, Ghosal, Mukhopdhyay and Chakraborty [8] obtained the same by studying the Dolphin transformations for arbitrarily large speed of the synchronizing signal.

Selleri [12] showed that while LT gives null result in the derivation of Sagnac effect from rotating frame perspective, TT gives the *correct* result. TT, however, has been obtained by him using his own inertial transformations. On commenting this, he termed TT (and *absolute synchronization*) as the *nature's choice* of synchrony.

B.4.4 OWS of light in Relativistic world

The OWS of light in any direction is given by Eq. (B.3.5)

$$c(\theta) = \left[(1-\xi)\beta\cos\theta + \left\{1-(1-\rho)\sin^2\theta\right\}^{1/2}\right]^{-1}\chi. \tag{B.4.9}$$

Following our previous discussion we can set, for the relativistic world

$$\Omega = \phi = \gamma^{-1}.$$

This gives

$$\xi = c, \qquad \xi = -a\frac{c}{\beta}\gamma^{-1}.$$

Putting these values in Eq. (B.4.9), we obtain

$$\frac{1}{c(\theta)} = \frac{1}{c} + \left[\frac{\beta}{c} + a\gamma^{-1}\right]\cos\theta. \tag{B.4.10}$$

B.5 The Reichenbach Parameter ϵ

If we denote the OWS of light in positive x direction to be $\overrightarrow{c}$ then, $\theta = 0$ and we may write

$$\begin{aligned}\overrightarrow{c} &= [(1-\xi)+1]^{-1}\xi \\ &= (2-\xi)^{-1}\gamma^{-2}\Omega^{-1}\phi^{-1}c.\end{aligned}$$

If one finds out $\overrightarrow{c}$ with the Reichenbach parameter then [1]

$$\overrightarrow{c} = \frac{c}{2\epsilon}$$

Thus

$$\epsilon = (1-\xi/2)\gamma^2\Omega\phi. \tag{B.5.1}$$

This is the relation between ϵ and ξ. One may easily check that for relativistic world where $\Omega = \phi = \gamma^{-1}$, and $\xi = 1$, $\epsilon = 1/2$, the value of the Reichenbach parameter for the standard synchrony [18].

B.6 Composite Transformation Matrix

So far we have derived the transformation matrices from the 'preferred' frame S_0 to an inertial frame S. What will be the transformation matrix from one inertial to another inertial frame where none is chosen to be the 'preferred' one?

The answer is straightforward. Say we have two inertial frames S_1 and S_2 moving with velocities v_1 and v_2 respectively with respect to the ether frame. Then the transformation equations ($S_0 \to S_1$ and $S_0 \to S_2$) can be written as

$$\mathbf{X_1} = \mathbf{T_1 X_0}, \qquad \mathbf{X_2} = \mathbf{T_2 X_0}.$$

From this we find that the transformation $S_1 \to S_2$ is given by

$$\mathbf{X_2} = \mathbf{T_2 T_1^{-1} X_1} = \mathbf{T_{12} X_1}$$

where

$$\mathbf{T_{12}} = \mathbf{T_2 T_1^{-1}}.$$

The $\mathbf{T_i}$, $(i = 1, 2)$ matrices are characterized by adding a subscript i $(i = 1, 2)$ to all of the parameters which should be different for different frames, *i.e.* v_i, A_i, Ω_i, ϕ_i and ψ_i. The general $\mathbf{T_{12}}$ is given by

$$\mathbf{T_{12}} = \begin{pmatrix} P & Q & 0 \\ R & S & 0 \\ 0 & 0 & \psi_2^{-1}\psi_1 \end{pmatrix}, \tag{B.6.1}$$

where

$$\begin{aligned}
P &= a_2 \Omega_2^{-1}(v_1 - v_2) \\
Q &= \phi_1 \Omega_2 \left(a_2 \Omega^{-1} a_1 \Omega_1^{-1}\right) - (v_2 - v_1) a_1 a_2 \Omega_1^{-1} \Omega_2^{-1} \\
R &= \phi_2^{-1} \Omega_1^{-1}(v_1 - v_2) \\
S &= \phi_1 \phi_2^{-1}(1 - a_1 \Omega_1^{-1}(v_1 - v_2)).
\end{aligned}$$

One must note here that a composite transformation is equivalent to a single transformation. Thus one can find the velocity transformation laws by comparing the equivalent terms in these two matrices [19].

B.7 Synchronization with Non-luminal Signal

In the earlier sections, different transformation have been discussed for different choices of the free parameter 'a'. For a given world, different choices of 'a' gives different OWS' of light, however the TWS remains the same since the latter is independent of the synchrony. Let us now take a look at the standard derivations of the LT. In the standard SR light has two roles to play. On the one hand it acts as a synchronizing agent, on the other hand it has invariant TWS in vacuum. (Indeed the relativistic world is defined by the existence of an invariant TWS). The second role has a basis in the empirically verifiable property, but the first one is purely prescriptive in origin. In the derivation of the LT in the standard SR, these two roles are mixed up. This inseparability contributes to several misconceptions and prejudices in relativity theory. In order to separate these roles one may introduce non-luminal signal to synchronize clocks and rederive transformation equations. This has been done by Ghosal, Mukhopadhyay and Chakraborty [8]. In their derivation of relativistic transformation equation, the authors considered reference frames submerged in a substrate. The clocks are synchronized by some signal mode characteristic of the substratum. Without any loss of generality one may assume it to be the acoustic signal (AS).

The authors first considered an acoustic wave generated at $t = 0$ at the common origin of the frames S_i and S_k. In all other frames except for the frame S_0 which is at rest relative to the substratum, the velocity of AS in the positive x-direction and negative x-direction will not be the same. However, according to the CS thesis, it is possible to define the synchronization of clocks so that these two velocities are equal in all the frames (although the values for these velocities are in general different in different frames). This synchrony is called (by the authors) the *pseudo-standard synchrony (PSS)* as opposed to the *Einstein (standard) synchrony.*

Now, according to pseudo-standard synchrony, the one dimensional wave front equation will be a pair of straight lines:

$$x_k^2 = a_{kx}^2\, t_k^2, \tag{B.7.1}$$

where x_k's are co-ordinates of a frame S_k which is moving with respect to S_0 frame which is fixed in the substrate and a_{kx} is the TWS of the AS in the x-direction.

The acoustic wave front will not be spherical in frames other than in S_0 frame, TWS of AS will not be the same in all direction. So, along the y-axis, say, one has to write

$$y_k^2 = a_{ky}^2\, t_k^2,$$

where, a_{ky} is the TWS of AS y-direction and may be different from a_{kx}.

B.7.1 The Derivation of Transformation Equation

In order to derive the TE, one first writes the TE in this usual linear form,

$$\begin{aligned} x_k &= \alpha(x_i - v_{ik}t_i), \\ y_k &= y_i, \\ t_k &= \xi_{ik}x_i + \beta_{ik}t_i. \end{aligned} \tag{B.7.2}$$

This is a set of transformation equations between two general inertial frame S_i and S_k. v_{ik} is the relative velocity between the two. α_{ik}, ξ_{ik} and β_{ik} are constant to be determined.

The observers within the substrate cannot use 3-sphere wave front of the AS wave as this will not be spherical in a general frame. But according to the chosen synchrony of the form (B.7.1) one can subject the TE to the condition

$$x_k^2 - a_{kx}^2 t_k^2 = \lambda_{ik}^2 (x_i^2 - a_{ix}^2 t_i^2), \tag{B.7.3}$$

where λ_{ik} is a scale factor and is independent of the space and time co-ordinates. One can now put Eq. (B.7.2) in Eq. (B.7.3) to obtain the transformation coefficients as

$$\begin{aligned} \alpha_{ik} &= \lambda_{ik}\gamma_{ik}, \\ \beta_{ik} &= \alpha_{ik}/\rho_{ik}, \\ \xi_{ik} &= -\frac{\alpha_{ik}/\rho_{ik}}{v_{ik}/a_{ix}^2}, \end{aligned} \tag{B.7.4}$$

where,

$$\begin{aligned} \rho_{ik} &= a_{kx}/a_{ix}, \\ \gamma_{ik} &= \left(1 - \frac{v_{ik}^2}{a_{ix}^2}\right)^{-1/2} \end{aligned} \tag{B.7.5}$$

and the TE for $S_i \to S_k$ can be written as

$$\begin{aligned} x_k &= \lambda_{ik}\gamma_{ik}\left(x_i - v_{ik}t_i\right), \\ t_k &= \frac{\lambda_{ik}}{\rho_{ik}}\gamma_{ik}\left(t_i - \frac{v_{ik}}{a_{ix}^2}x_i\right). \end{aligned} \tag{B.7.6}$$

The notable points about this set of equations are the following –

1. The TE contain TWS of synchronizing signal in the frames S_i and S_k.

2. The factor γ_{ik} resembles the relativistic γ factor with c replace by the velocity of the AS.

3. Simultaneity is relative.

All these are consequences of the PSS. Also, one can find

$$v_{ki} = -\rho_{ik} v_{ik}$$

which means that under this synchrony relative speeds are not symmetric as $\rho_{ik} \neq 1$ in general.

Note also that a mere change of subscripts give transformation between any two other inertial systems. Also one can easily work out the velocity transformation formula

$$v_{kl} = \frac{a_{kx}}{a_{ix}} \frac{v_{il} - v_{ik}}{1 - \frac{v_{ik} v_{il}}{a_{ix}^2}} \tag{B.7.7}$$

by comparing transformations $S_l \to S_k$ and $S_l \to S_i \to S_k$. This also gives a very important result

$$\lambda_{kl} = \frac{\lambda_{il}}{\lambda_{ik}}.$$

λ's are yet unknown quantities. Note that the TWS of AS is isotropic in the preferred frame S_o which is stationary with respect to the medium. But in a general frame S_k it will not be isotropic. If the isotropic signal speed is a_0, we may write

$$a_x^2 + a_y^2 = a_0^2.$$

The TWS in $x-$directions in S_k is given by

$$a_{kx} = \frac{\alpha_{0k} a_0 \left(1 - \frac{v_{0k}^2}{a_0^2}\right)}{\beta_{0k} + v_{0k}} \tag{B.7.8}$$

and,

$$a_{ky} = \frac{a_0 \left(1 - v_{0k}^2/a_0^2\right)^{1/2}}{\beta_{0k} + \xi_{0k} v_{0k}} \tag{B.7.9}$$

Also, the general TWS transformation laws for any *other* signal whose *isotropic*

TWS (equal to its OWS) in S_0 is a_0' (which may differ from a_0) can be written as

$$\text{TWS (longitudinal)} \quad a'_{kx} = \frac{\alpha_{0k} a'_0 \left(1 - v_{0k}^2/a_0'^2\right)}{\beta_{0k} + \xi_{0k} v_{0k}},$$
$$\text{TWS (transverse)} \quad a'_{ky} = \frac{a'_0 \left(1 - v_{0k}^2/a_0'^2\right)^{1/2}}{\beta_{0k} + \xi_{0k} v_{0k}}, \tag{B.7.10}$$

where a'_{kx} and a'_{ky} are the TWS's of the *other* signal as measure from in the longitudinal and the transverse directions respectively. It has been tacitly assumed here that in S_0 the PSS with AS and that with the *other* signal are equivalent.

From Eqs. (B.7.4), (B.7.9) and (B.7.8)

$$\lambda_{0k} = \frac{a_{kx}}{a_{ky}}. \tag{B.7.11}$$

Also

$$\lambda_{ik} = \frac{\lambda_{0k}}{\lambda_{0i}} = \frac{a_{kx}}{a_{ky}} \frac{a_{iy}}{a_{ix}} \tag{B.7.12}$$

Putting the values of the unknown parameter λ_{ik} , the complete transformation equations between any two inertial frames $S_i \to S_k$ become

$$x_k = \frac{a_{kx}}{a_{ky}} \frac{a_{iy}}{a_{ix}} \frac{x_i - v_{ik} t_i}{\left(1 - v_{ik}^2/a_{ix}^2\right)^{1/2}},$$
$$t_k = \frac{a_{ik}}{a_{ky}} \frac{t_i - (v_{ik}/a_{ix}^2) x_i}{\left(1 - v_{ik}^2/a_{ik}^2\right)}. \tag{B.7.13}$$

In the preferred frame S_0, $a_{0x} = a_{0y} = a_0$. The TE from S_0 to any other inertial frame S_k is given by

$$x_k = \frac{a_{kx}}{a_{ky}} \frac{x_0 - v_{0k} t_0}{\left(1 - v_{0k}^2/a_{0k}^2\right)^{1/2}},$$
$$t_k = \frac{a_0}{a_{ky}} \frac{t_0 - (v_{0k}/a_0^2) x_0}{\left(1 - v_{0k}^2/a_0^2\right)^{1/2}}. \tag{B.7.14}$$

The authors named this set of transformation equations *Dolphin transformation* (DT) in a lighter vein because this will be the TE found by some intelligent dolphins who are assumed to reside within the substratum of sea water (say).

Note that DT in the form presented above (Eqs. (B.7.13) and (B.7.14)) is usable provided one knows the TWS of AS in the two frames concerned. If now one chooses to use light signal (in vacuum) instead of AS, by virtue of CVL postulate of SR one would obtain the familiar LT because now

$$a_{ix} = a_{iy} = a_{kx} = a_{ky} = c.$$

It looks rather surprising that here c, *apparently*, is not playing any important role in absence of any communication with the outside world even though the dolphins live in the relativistic world! But c has a fundamental role to play in relativity that its TWS is constant in vacuum. Indeed in the DT, c will appear as a *physical constant* through a_{kx} and a_{ky}.

To make use of DT, the Dolphins will have to know the TWS of AS in S_k as function of v_{0k} and one can anticipate that they will eventually find that

$$\begin{aligned} a_{kx} &= a_{kx}(v_{0k}, c), \\ a_{ky} &= a_{ky}(v_{0k}, c), \end{aligned} \tag{B.7.15}$$

where c appears as some *physical constant*. If now the dolphins are able to communicate with the outside world and discover that their world admits an invariant speed c, they would set, in Eq. (B.7.10),

$$a'_{kx} = a'_{ky} = a'_0 = c.$$

One can now find, using Eq. (B.7.4)

$$\rho_{0k} = \frac{\left(1 - v_{0k}^2/a_0^2\right)}{\left(1 - v_{0k}^2/c^2\right)},$$

$$\lambda_{0k} = \frac{\left(1 - v_{0k}^2/a_0^2\right)^{1/2}}{\left(1 - v_{0k}^2/c^2\right)^{1/2}}. \tag{B.7.16}$$

Also, by using Eq. (B.7.5) and Eq. (B.7.11) one obtains

$$a_{kx} = a_0 \frac{\left(1 - v_{0k}^2/a_0^2\right)}{\left(1 - v_{0k}^2/c^2\right)},$$

$$a_{ky} = a_0 \frac{\left(1 - v_{0k}^2/a_0^2\right)^{1/2}}{\left(1 - v_{0k}^2/c^2\right)^{1/2}}. \tag{B.7.17}$$

Eq. (B.7.17) conforms to our previous assertion as expressed in Eq. (B.7.15). Now inserting Eq. (B.7.17) in Eq. (B.7.14) we find the DT in frames $S_0 \to S_k$ in the relativistic world

$$x_k = \frac{x_0 - v_{0k} t_0}{\left(1 - v_{0k}^2/c^2\right)^{1/2}},$$

$$t_k = \frac{\left(1 - v_{0k}^2/c^2\right)^{1/2}}{\left(1 - v_{0k}^2/a_0^2\right)} \left[t_0 - \frac{v_{0k}}{a_0^2} x_0 \right]. \tag{B.7.18}$$

From the above TE, the length contraction factor (LCF) and time dilation factor (TDF) come out to be

$$\mathrm{LCF} = \frac{\left(1 - v_{0k}^2/a_0^2\right)}{\left(1 - v_{0k}^2/c^2\right)^{1/2}},$$

$$\mathrm{TDF} = \frac{\left(1 - v_{0k}^2/c^2\right)^{1/2}}{\left(1 - v_{0k}^2/a_0^2\right)}. \tag{B.7.19}$$

Note that with respect to S_0, LCF and TDF will be the same as predicted by SR. This is not surprising because of the assertion that in S_0, the PSS and the Einstein synchrony coincide.

There are two very important consequences of DT and Eq. (B.7.19). These are the following –

1. There has been a considerable debate since the birth of relativity on reality of length contraction and time dilation. There are authors who believe that these effects are solely dependent on the *relativity of simultaneity* (for a discussion see also Ref. [11]). Earlier in this chapter we have shown that this is not so – different synchronization procedure may not have relativity of simultaneity but predict these effects. Eq. (B.7.19) further clarifies this. Observe that in Eq. (B.7.19), a_0 is the speed of AS, so it is *conventional.* c appears as a *physical constant* – the TWS of light – and is not based on any convention. The factor $\left(1 - v_{0k}^2/c^2\right)^{1/2}$ is due to real effects. The other factor, $\left(1 - v_{0k}^2/a_0^2\right)$ arises from the synchronization procedure which is evident from the presence of the term a_0.

2. As we have mentioned earlier that light has two roles to play in SR. One is that its TWS in vacuum is constant and the other is that it is the synchronization agent in SR. These two roles are mingled up beyond recognition in standard SR. In the derivation of DT we see that these two roles are clearly split up.

We shall show below that how some important transformation equations in relativistic and classical worlds can be obtained from DT by the choice and making use of the properties of the synchronization signal.

Relativistic world: Lorentz transformation

In standard synchrony the synchronization agent is light. Putting $a_0 = c$ in Eq. (B.7.18) one may obtain Lorentz transformation.

Relativistic world: Tangherlini transformation

If in the *preferred frame* the speed of synchonization signal $a_0 \to \infty$, then we obtain, (for $S_0 \to S_k$) the Tangherlini transformation.

Classical world: Zahar transformation

In classical world the velocity addition law is the Galilean one. Then the TWS of AS is obtained to be

$$a_{kx} = a_0\left(1 - v_{0k}^2/a_0^2\right),$$
$$a_{ky} = a_0\left(1 - v_{0k}^2/a_0^2\right)^{1/2}. \tag{B.7.20}$$

Inserting these in Eq. (B.7.14), we obtain DT ($S_0 \to S_k$) in the classical world.

$$x_k = x_0 - v_{0k}t_0,$$
$$t_k = \frac{t_0 - \left(v_{0k}/a_0^2\right)x_0}{1 - v_{0k}^2/a_0^2}. \tag{B.7.21}$$

This is Zahar transformation with PSS. If $a_0 = c$, *i.e.* standard synchrony is adopted, we obtain the Zahar transformation in its familiar form.

Classical world: Galilean transformation

If it is possible to synchronization with instantaneous signals, then we can put $a_0 \to \infty$ in Eq. (B.7.21) and the familiar from of GT is retrieved.

B.8 Epilogue

In his historic 1905 paper Einstein recognized the element of convention in the empirical measurement of the OWS of light. He chose the isotropy of OWS of light and considered a convenient value of OWS which equated the TWS of light, an experimentally measurable quantity. His synchronization procedure with light, which we have mentioned earlier as Einstein synchronization procedure, led to relativity of simultaneity. All this appeared so 'natural' that even today most of the textbook authors do not recognize the element of convention in the synchronization procedure. The first one who gave the conventionality its proper emphasis was Reichenbach in 1928. The whole CS thesis grew up to its present form due to work of Gruünbaum (and others) on the theory of Reichenbach.

Though a different transformation other than the LT for SR was first found out by Tangherlini in 1961, a general transformation which might incorporate all types of synchronization schemes was found first by Winnie in 1970. Later works were carried out by Mansouri and Sexl in 1977, Sjödin in 1979 and Selleri in 1995. They took different routes to reach the same goal. In 1977 Zahar did a remarkable job in our understanding of conventionality thesis by incorporating CS thesis in classical world to obtain a transformation where clocks are synchronized in Einstein procedure.

Our treatment of derivation of the TE (except DT) presented in the first few section of this appendix largely follows that of Sjödin. Though the approach is different, Selleri achieved the same transformation independently (inertial transformation) in a different form. It can be written as [12]

$$
\begin{aligned}
x &= \frac{x_0 - \beta c t_0}{R(\beta)}, \\
y &= y_0, \\
z &= z_0, \\
t &= R(\beta) t_0 + \epsilon(x_0 - \beta c t_0) + e(y_0 + z_0).
\end{aligned}
\tag{B.8.1}
$$

Here ϵ, e are two underdetermined functions of velocity v and $\beta = v/c$, and $R(\beta) = \sqrt{1-\beta^2}$ (we have changed some notations for consistency). In its final form (with $e = 0$, for rotational invariance around x axis) Selleri's ϵ parameter and our 'a' parameter is the same.

The ϵ in ϵ-Lorentz transformation derived by Winnie is the Reichenbach parameter. The transformation equations are given by [2]

$$
\begin{aligned}
x' &= \alpha^{-1}(x - \vec{v}_\epsilon t) \\
t' &= \alpha^{-1}\left[2\vec{v}_\epsilon c^{-1}(1 - \epsilon - \epsilon') + 1\right] - x c^{-2}\left[2c(\epsilon - \epsilon') + 4v_\epsilon(\epsilon)(1-\epsilon)\right],
\end{aligned}
\tag{B.8.2}
$$

where

$$
\alpha = \frac{\sqrt{(c - \vec{v}_\epsilon(2\epsilon - 1))^2 - \vec{v}_\epsilon^{\,2}}}{c},
$$

and ϵ and ϵ' are Reichenbach parameters in the two frames. The main difference between ϵ- LT and transformations derived by Sjödin and by Selleri is that ϵ-LT is derived in $(1+1)$ dimensions while the other two are derived in $(1+2)$ and $(1+3)$ dimensions respectively. Unless at least two space dimensions are taken into account, the concept of *isotropy* of TWS of light is devoid of any meaning and some subtleties of the relativistic physics have to be sacrificed [4, 5].

Ghosal Mukhopdhyay and Chakraborty first split up the two roles of light to find the dolphin transformation by introducing pseudo-standard synchrony. In this way they also identified the real and the synchrony dependent part in the length contraction and time dilation factors. Introduction of signal synchrony (luminal or non-luminal) give different transformation equation thus making this set of equations an interesting one by its ability to reveal true relativistic effects from apparent ones.

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