CHAPTER VI

ON THE DEFINITION AND EXISTENCE OF LORENTZ INVARIANT CLOCKS*

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6.1 INTRODUCTION

This chapter deals with the indepth study of the question of the existence of Lorentz - Invariant - Clocks (LIC) in a relativistic world. This is prompted by some recent claims and counter claims (Schlegel 1973,1975,1977; Rodrigues 1985) regarding LIC. Here we have reexamined the definition of a Lorentz - Invariant (LI) clock in the light of the Conventionality of distant Simultaneity thesis (C-S thesis). The main purpose of the present text is to first present a synchrony independent definition of a LI clock and then to prove generally that the existence of an invariant clock is indeed incompatible with Special Relativity (SR). We hope that the present exposition will also help readers grasp and appreciate once more the importance of the C-S thesis in understanding the foundational questions of relativity.

In an article Schlegel (1973,1975,1977) claimed that it is possible to construct theoretically a Lorentz Invariant (LI) clock whose rate does not depend on its state of motion. The author further stated that the Principle of Relativity (PR) does not come in the way in conceiving such a clock. To advance his thesis, Schlegel theoretically built his gedanken clock (ball and track clock) and analysed its functioning to show that the clock rate was indeed Lorentz invariant.

Rodrigues (1985) correctly replies to Schlegel's paper by pointing out explicitly the fault in Schlegel's analysis. However, we find it rather difficult to understand his "general proof" that the theory of relativity forbids the existence of a LI clock. Rodrigues's "general proof" is unsatisfactory on two counts. First, his definition of LI clock seems to be ambiguous. For, as we shall see later that the authors' definition itself presupposes a preferred inertial frame and therefore any effort to prove further that PR forbids the existence of a LI clock becomes meaningless. In our opinion a careful definition of a LI clock is no less crucial than the issue of its existence. In section 6.2 we shall elaborate on what is wrong with Rodrigues' definition and then in section 6.3 we shall seek a more profound definition of an invariant clock.

The second drawback of the paper concerns the author's indifference to the Conventionality of distant Simultaneity thesis (C-S thesis) of special relativity (Reichenbach 1958; Grünbaum 1963; Winnie 1970; Sjödin 1979; Ghosal et al 1991a, 1991b; Mansouri & Sexl 1977). The c-s thesis observes that Einstein's formulation of SR rests on a special synchronization convention and since a convention is devoid of any empirical content, one is free to choose any value (with certain restrictions) to be assigned to the One-Way Speed (OWS) of light in a given inertial frame.

However we take this occasion to comment here once again that the above observation by no means imply that the Relativistic Physics (RP) is empirically empty and on the contrary, as we have observed in a recent paper (Ghosal et al 1991b), we believe that the validity of Einstein's second relativity postulate, when reinterpreted in the light of the C-S thesis, distinguishes RP from other non-relativistic transformations.

Now Rodrigues argues in his article that if LI clocks (as defined by the author) exist it will be possible to synchronize standard clocks in any inertial frame S with the help of a master LI clock and the resulting synchrony will not be equivalent to the standard Einstein synchrony or the so-called slow transport synchrony. This would immediately imply that the velocity of light will not remain isotropic in S. The author then argues that the principle of relativity as if forbids such a situation to arise! Obviously this argument is contrary to the spirit of the C-S thesis. It is evident that when Rodrigues speaks about the velocity of light he refers to the OWS alone and as its value, according to the C-S thesis, can be assigned arbitrarily, any conclusion based on the OWS of light definitely falls through.

Now our aim is to first present a synchrony independent definition of a LI clock and then to prove generally that the existence of such an invariant clock is indeed incompatible with special relativity. The other sections of this text will discuss the problem in details.

6.2 LI CLOCK : A DEFINITIONAL PROBLEM

In trying to refute Schlegel's "claim" that the existence of a Lorentz invariant clock is consistent with special relativity Rodrigues defines his LI clock in the following way :

(A) It is a clock such that when in motion relative to an inertial frame S_0, does not lag behind relative to a series of clocks synchronized according to Einstein in S_0.

Rodrigues' analysis is based on the above definition; therefore before one proceeds it is necessary first to examine its content carefully. Note that the statement (A) refers to a particular inertial frame S_0 ; the set of standard clocks belonging to S_0 are used as reference clocks with which the LI clock (non-standard clock) rate is to be compared. Now the question is whether the inertial frame S_0 is a preferred one or not. Obviously it should be a tacit stipulation that S_0 be *any* inertial frame, otherwise the whole exercise of refuting Schlegel's "claim" becomes meaningless. Unfortunately we shall see in a moment that the frame S_0 is a preferred one by definition. But before giving the formal proof to this effect let us first point out another feature of the definition (A), the cognizance of which itself will provide us some preparedness for the journey to the next section. In the definition (A) the rate of a LI clock in motion is compared with that of standard clocks at rest in S_0. It is evident therefore that any such comparison automatically calls for a particular procedure for distant clock synchrony in S_0. Since the experimental clock moves from one point to another in S_0 the rate of the single LI clock is actually compared with the difference of readings of the two spatially separated clocks in S_0 which are to be synchronized beforehand. Obviously the result of such a comparison depends on the adopted synchronization convention for the separated clocks in S_0. The above fact thus makes the definition (A) convention dependent. This is another undesirable feature of the definition.

Now consider an inertial frame S in motion relative to S_0. The proper period $\Delta\tau$ of a standard clock C_s comoving with S is given by

$$\Delta\tau = (1-u^2/c^2)^{1/2}\Delta t \tag{6.1}$$

where Δt is the period of C_s as observed from S_0 and u is the relative speed between the reference frames in question. Let us suppose that an Intelligent Observer (IO) sitting on S is able to change the time rate of C_s with the help of a rate adjuster. If, by using the adjuster the rate of C_s is continuously changed in some arbitrary way depending on the state of motion of S with respect to S_0, C_s can no longer be considered to be a standard one. If in particular the adjusted time rate $\Delta\tau'$ is given by

$$\Delta\tau' = (1-u^2/c^2)^{1/2}\Delta\tau \tag{6.2}$$

equations (6.1) and (6.2) yield

$$\Delta\tau' = \Delta t \quad . \tag{6.3}$$

The above relation will continue to hold as long as the observer "knows" the state of motion of S relative to S_0 and adjusts the clock rate according to equation (6.2). Equation (6.3) now implies that the clock C_s behaves like a LI clock according to Rodrigues' definition.

If we now consider another inertial frame S_1 moving with respect to S_0 with a relative speed v along their common x-axis, we obtain for the proper period of a standard clock in S

$$\Delta\tau = (1-w^2/c^2)^{1/2}\Delta t_1 \tag{6.4}$$

where Δt_1 is the period of the same clock as measured from S_1 and w is given by the well known velocity transformation formula

$$w = -\frac{v - u}{1 - uv/c^2} \tag{6.5}$$

Inserting (6.2) in (6.4) we obtain

$$\Delta\tau' = [(1-w^2/c^2)/(1-u^2/c^2)]^{1/2}\,\Delta t_1 \tag{6.6}$$

which implies that the clock C_{Ξ}', even though invariant from the point of view of S_0, now differs in its rate with respect to the series of clocks synchronized a' la Einstein in S_1 unless v=0 i.e when S_1 and S_0 coincide. This means, in other words that as if the frame S_0 has some special significance!

The above fact thus makes the definition (A) uninteresting if not totally meaningless, since now the existence of the so-called LI clock of Rodrigues, the definition of which itself gives a preferential status to S_0 can no longer be cited as an evidence for or against PR.

6.3 INVARIANT CLOCKS AND TRANSFORMATION LAWS

From the last section it is clear that a logically consistent definition of an invariant clock should not refer to any particular reference frame or to any particular synchronization convention. Keeping this in mind let us proceed to present our definition (Definition B) : A clock is such that when initially compared and adjusted (rate and phase) with an inertial standard clock at any given point in space does not differ its phase with the standard clock when they meet after the former performs a round-trip. Note that the above definition makes no reference to any particular inertial frame and the fact that the comparison of clocks are performed between the same pair of clocks (at the same space point) does away with any need for synchronization of distant clocks.

We shall now first look for the transformation laws between inertial frames if standard clocks are replaced by the invariant ones (according to definition B). In order to do so we shall take the so-called conventionalists' view point of special relativity and follow Sjödin's approach in particular (Sjödin 1979). According to Sjödin, relativistic physics can be described consistently in terms of real length contraction and time dilatation of moving rods and standard clocks with respect to a given inertial frame S_0 and it is held following the C-S thesis (vide sec.6.1) that a plethora of transformation equations between inertial frames with different synchronization parameters may describe the same physical reality. Sjödin derived from some simple considerations the general form of the transformation equations between two inertial frames S_0 and S' (when the latter moves with a velocity u with respect to S_0 along their common x-axis) as

$$\begin{aligned} x' &= \phi_u^{-1}\,(x-ut) \\ y' &= y \\ t' &= Ax + (\Omega_u - uA)t \end{aligned} \tag{6.7}$$

where the values for the length contraction factor ϕ and time dilatation factor Ω depend on the world that the above equations represent and A is the synchronization parameter which according to C-S thesis' can be assigned arbitrarily.

For the relativistic world (Sjödin 1979; Ghosal et al 1991a, 1991b)

$$\phi_u = \gamma_u^{-1} = \Omega_u \qquad (6.8)$$

where as usual $\gamma_u = (1-u^2/c^2)^{-1/2}$.

For the Galilean world (Sjödin 1979) on the other hand

$$\phi_u = \Omega_u = 1 \qquad (6.9).$$

It may appear at a first sight that since equations (6.7) are not formally symmetric between S_0 and S', S_0 might have been given a preferred status; but that is not so. For example for a relativistic world ($\phi = \Omega = \gamma^{-1}$) one may choose the standard synchrony so that (6.7) would represent Lorentz Transformations (LT) in which case S_0 is just *any* inertial frame.

One may now also write down the transformations if standard clocks (but not the rods) are replaced by non-standard ones. In this case the value of $\phi_u = \gamma_u^{-1}$ is retained but Ω_u remains arbitrary in (6.7) :

$$\begin{aligned} x' &= \gamma_u (x-ut) \\ y' &= y \\ t' &= Ax + (\Omega_u - uA)t \end{aligned} \qquad (6.10).$$

We may now define a "Semi Relativistic World" (SRW) when the non-standard clocks of (6.10) are the invariant clocks in particular.

One may easily find out the value of Ω_u in this case by explicitly using definition (B) given at the begining of this section for invariant clocks. However for the present analysis, in order to obtain a better insight into the present problem we first intuitively postulate the transformations for the SRW and hence prove afterwards that the clocks of the said world are indeed invariant (according to definition B).

The proposed transformations are

$$
\begin{aligned}
x' &= \gamma_u \ (x-ut) \\
y' &= y \\
t' &= Ax + (1-uA)t
\end{aligned}
\qquad (6.11).
$$

Note that we have taken the value of ϕ_u from (6.8) and that of Ω_u from (6.9) and inserted these values in (6.10).

6.4 Standard Synchrony in SRW.- Velocity transformation equations that follow from (6.11) are given by

$$v_x' = \frac{\gamma_u (v_x - u)}{1+A(v_x - u)} \qquad (6.12)$$

$$v_y' = \frac{v_y}{1+A(v_x - u)} \qquad (6.13)$$

where v_x and v_y are the components of velocity of a particle (for instance) as observed from S_0 and the corresponding quantities with prime denote the same as observed from S'.

Now, since our definition of the invariant clock is independent of clock synchrony one is free to adopt any convention for distant synchrony in SRW without any loss of generality. We shall use for the following analysis the so-called Standard Synchrony (SS) (Winnie 1970; Sjödin 1979; Ghosal et al 1991a; Mansouri and Sexl 1977), according to which one way speed of light is independent of direction in any inertial frame. We can explicitly use the above definition of SS to obtain the value of A as follows. Consider a light signal travels in the x-direction. Now if one puts, for the "one way speeds" of light in S_0, $u=\pm c$ in (6.12), according to the SS the corresponding speeds of light v_x' (in S′) will also be equal in magnitude i.e

$$\frac{\gamma_u (c - u)}{1 + A(c - u)} = \frac{\gamma_u (c + u)}{1 - A(c + u)} \qquad (6.14)$$

which gives

$$A = -(u/c^2)\gamma_u^{\,2} \qquad (6.15).$$

Inserting this value of the parameter for synchrony in (6.11) we obtain the transformations with standard synchrony in SRW :

$$x' = \frac{x - ut}{(1-u^2/c^2)^{1/2}}$$
$$y' = y \qquad (6.16).$$
$$t' = \frac{t - ux/c^2}{1 - u^2/c^2}$$

Now we shall proceed to prove that the clocks of SRW as represented by the transformations (6.16) are indeed invariant (according to definition B). From (6.16) we can write down the transformations between *any* two frames S' and S'' for example, where the later moves with a relative velocity v with respect to the first frame S_0. They are obtained as

$$x'' = \gamma_v[\gamma_u(1-uv/c^2)x'-(v-u)t'] \tag{6.17}$$

and

$$t'' = \gamma_v^2[(1-uv/c^2)t'-(\gamma_u/c^2)(v-u)x']$$

and also inverting the above equations we obtain

$$x' = \gamma_u[\gamma_v(1-uv/c^2)x''+(v-u)t''] \tag{6.18a}$$

and

$$t' = \gamma_u^2[(1-uv/c^2)t''+(\gamma_v/c^2)(v-u)x''] \tag{6.18b}.$$

Suppose now a clock C_1 of SRW is placed at the origin ($x''=0$) of S''. From eqn. (6.18b) the observed period $\Delta t'$ of C_1 from S' may be obtained in terms of its "proper" period (i.e the period as observed from the clock's own rest frame) as

$$\Delta t' = \gamma_u^2 \; (1-uv/c^2) \; \Delta\tau'' \tag{6.19}.$$

Note that in order to arrive at (6.19) we have put $\Delta x''=0$ & $\Delta t''=\Delta\tau''$ in (6.18b).

Observe that the time dilatation factor in (6.19) differs from unity in general. If for the moment we anticipate that the clocks under question are invariant, the time dilatation effect (eqn. 6.19) appears to be a bit surprising. However, there is little to be surprised since here the invariance of the clock period is clearly masked by the process of distant clock synchrony in S'. This is the precise reason why it is necessary to define invariant clock in a synchrony independent way.

Now suppose C_1 completes its journey in the following way. It first starts from the origin of S' at $t=0$, travels with a uniform speed v with respect to S_0 a distance x' of S' and after a brief period of deceleration g, returns to the origin of S', with a velocity $-v$ with respect to S_0. Now if there is another clock C at the origin of S', one is able to measure the time required for the round-trip journey of C_1 by C and as well as by C_1. Note that the time measurements in this case are independent of the process of distant clock synchrony in the concerned inertial frames. We rewrite (6.19) with a change of notations

$$\Delta t'_f = \gamma_u^2 \ (1-uv/c^2)\Delta\tau''_f \tag{6.20}$$

and

$$\Delta t'_r = \gamma_u^2 \ (1+uv/c^2)\Delta\tau''_r \tag{6.21}$$

where the indices f and r represent the respective quantities for the forward and return journey of C_1 respectively. Note that the last equation is obtained by replacing v by $-v$ for the return journey in (6.19).

For the round-trip journey of C_1, C time and C_1 time are given by

$$\Delta\tau'' = \Delta\tau''_f + \Delta\tau''_r \tag{6.22}$$

and $$\Delta t' = \Delta t'_f + \Delta t'_r \tag{6.23}$$

respectively.

In the above calculations we have considered only uniform motion for C_1 both for the forward and for the return jouney and omitted any effect of its retardation at the end of the forward journey, because the retardation effect in time dilatation (in this case apparent) calculations can be made arbitrarily small as the retardation $g \to \infty$ at constant v (vide clock paradox calculations due to Moler (1972), for example).

To prove that C_1 represents an invariant clock we shall have to prove $\Delta\tau'' = \Delta t'$ i.e

$$\Delta t'_f + \Delta t'_r = \Delta\tau''_f + \Delta\tau''_r \tag{6.24}$$

or, in other words, using (6.20) and (6.21)

$$\Delta t'_f + \Delta t'_r = \gamma_u^{-2}[\Delta t'_f/(1-uv/c^2) + \Delta t'_r/(1+uv/c^2)] \tag{6.25}$$.

The above identity may easily be verified by noting that

$$\omega_f \Delta t'_f = \omega_r \Delta t'_r = x' \quad \text{or,} \quad \Delta t'_r = (\omega_f/\omega_r)\Delta t'_f \tag{6.26}$$

where ω_f and ω_r are the speeds of C_1 with respect to S' for the forward and the return journies respectively which, from (6.17) are given by

$$\omega_f = (v-u)/\gamma_u(1-uv/c^2) \tag{6.27}$$

and $\omega_r = (v+u)/\gamma_u(1+uv/c^2)$ (6.28).

By virtue of (6.26), the $\Delta t'_f$ cancels from both sides of (6.25) giving

$$1 + (\omega_f/\omega_r) = \gamma_u^{-2}[1/(1-uv/c^2) + (\omega_f/\omega_r)(1+uv/c^2)^{-1}] \quad (6.29).$$

If one now makes use of (6.27) and (6.28), the verification of the above identity becomes a trivial exercise.

Observe that the identity (6.24) or (6.25) thus established is not generally valid since similar calculations with Lorentz transformations for example, unlike the case in hand, would display real time dilatation effect (Moller 1972).

6.5 <u>SRW and its incompatibility with PR.</u>- In section 6.1 we observed that Rodrigues in his "general proof" did not take into account the C-S thesis properly and on the contrary he equated the possible existence of a LI clock with the possibility of a particular non-standard synchronization giving rise to anisotropy of OWS of light in a general frame. We shall however work with the "Two-Way-Speed" (TWS) of light the definition of which does not call for distant synchrony (Ghosal et al 1991a & 1991b). The following steps will provide the proof that the invariant clocks are really incompatible with PR.

Consider that the light speed in S_0 is c and is the same in all directions. Recall the velocity transformation formula (6.12)

$$v'_x = \frac{\gamma_u(v_x - u)}{1+A(v_x - u)} \quad (6.12)$$

and assume in particular that in S_0, $v_x = -c$ for the return journey (obtained by reflection) of a light signal in the x-direction of S_0. The corresponding speeds of light as observed for S′ will be given (using 6.12) by

$$c'_f = \frac{\gamma_u (c - u)}{1 + A(c-u)} \qquad (6.30)$$

for the forward journey of light and for the return journey

$$c'_r = \frac{\gamma_u (c + u)}{1 - A(c+u)} \qquad (6.31).$$

The harmonic mean of last two speeds give the TWS of light as observed from S′ which is given by

$$c' = \frac{1}{2} \left(\frac{1}{c'_f} + \frac{1}{c'_r} \right)^{-1} = c(1 - v^2/c^2)^{1/2}. \qquad (6.32).$$

Note that the synchronization parameter A has no role in the last expression. This is expected since TWS requires a single clock for its measurement and therefore its value ought to be independent of synchrony. Similarly (Ghosal et al 1991b) one may obtain the value of TWS in the y-direction or any direction for that matter and verify that the observed round-trip speed is independent of direction. But it is evident that while in a general frame S′ the isotropy of light speed (TWS) is maintained, its magnitude differs and therefore in violation of PR one can tell one frame from the other just by performing experiments to determine TWS of light in different inertial frames.

However in tune with Rodrigues' final remark (1985) we also hold that the above proof cannot forbid invariant clocks to exist in nature but for that, only PR has to be discarded once and for all.

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