CHAPTER - V

CONVENTIONALITY OF DISTANT SIMULTANEITY AND LIGHT SPEED INVARIANCE

5.1 INTRODUCTION

In section 1.3 of chapter I we have introduced the *Conventionality of Simultaneity* thesis (C-S thesis) proposed by Reichenbach (1958) and Grünbaum (1963). It has been remarked that various misconceptions and prejudices that still prevail in Special Relativity (SR) arise mainly out of two reasons : (1) Misconstruing of the subtlities of the C-S thesis and (2) Overlooking of the C-S thesis. In chapter IV we have dicussed one common misconception which arises out of (2). Here we give an example of (1) i.e. we show here that confusions may occur which stem from misconstruing of the Reichenbach-Grünbaum thesis of *Conventionality of distant Simultaneity.*

Recently Cavalleri and Bernasconi (CB) (1989) have claimed that the two fundamental properties of Special Relativity (SR) viz the Constancy of Velocity of Light (CVL) and the relativity of Distant Simultaneity (DS) are not peculiar to Relativistic Physics (RP) alone and it is possible to formulate the prerelativistic or Galilean Physics (GP) which can demonstrate both CVL and relativity of DS. Conversely it has also been argued that the Relativistic Physics (RP) can be formulated in such a way that CVL be no longer valid and distant simultaneity be absolute. While we have nothing against the claims by CB regarding the question of relativity of DS in GP and RP, we hold that the remarks made about CVL in their works (1989) are rather unfortunate. The arguments of CB seem to have been based on the Reichenbach-Grünbaum (RG) thesis (1958,1963) of the conventionality of distant simultaneity in SR. According to this thesis it is held that in Einstein's formulation of SR, the synchronization procedure for spatially distant clocks in any inertial frame contains an element of convention which is obviously devoid of any empirical content. This observation inevitably leads to the possibility of other synchronyzation conventions different from that adopted by Einstein. This fact reveals itself in our ability to choose any value (with certain restrictions) to be assigned to the one-way speed of light in a given inertial frame. The CS thesis has later been upheld and clarified by several authors [Winnie 1970; Tangherlini 1961; Mansouri and Sexl 1977; Podlaha 1978] and has been further developed and consolidated in a beautiful paper by Sjödin (1979).

Now since the question of simultaneity of any two spatially separated events in a *given* inertial frame depends on the chosen synchronization convention the issue of relativity of DS, which is often considered as one of the most fundamental imports of SR, has little significance. Thus for example, one is able to synchronize clocks and formulate relativistic physics in such a way that the simultaneity remains absolute. In the same token it is possible to adopt a synchronization procedure in GP so that DS becomes relative. CB in their paper (1989), among other things, seem to have rediscovered this fact which is indeed correct. However the C-S thesis has been misconstrued by CB when they remark that the CVL in SR is "trivial".

To prove the triviality of CVL in SR, the authors (1989), in one hand, formulated a transformation between inertial systems in GP and hence claimed that for the chosen synchrony the signal speed remains invariant even in GP. On the other hand, CB have taken the Tangherlini transformations [Tangherlini 1961; Sjödin 1979; Ghosal et al 1991a] representing absolute synchrony in particular to show that relativistic physics also can demonstrate the non-invariance of light speed.

We shall show below that these claims as they should be are not quite correct.

5.2 GALILEAN PHYSICS AND SIGNAL SYNCHRONY

CB consider transformations of coordinates between two frames S and S′ where S is stationary with respect to a quiet sea representing the etherial fluid and S′ is moving with respect to the former with a non-zero relative speed v. The clocks of both the frames are synchronized with a submarine (signal) which has a constant speed c along any direction with respect to the sea water. The clocks in S′ frame are synchronized in such a way that the signal speed appears to be c in the direction away from the origin of S′. This is always permitted according to C-S thesis when applied to GP.

The transformation equations are

$$\mathbf{r}' = \mathbf{r} - \mathbf{v}t \qquad (5.1)$$

and

$$t = t' + \lambda r' \qquad (5.2)$$

where

$$\lambda = |c-v|^{-1} - c^{-1} \qquad (5.3).$$

For the discussions that will follow we shall assume the relative velocity v to be along the common x-axis of the S and S′. One may thus write the equations (5.1) and (5.2) explicitly in terms of the coordinates x and y as

$$x' = x - vt \qquad (5.4)$$

$$y' = y \qquad (5.5)$$

$$t = t' + \lambda(x^2+y^2)^{1/2} \qquad (5.6).$$

Since the last equation is non-linear, the general velocity transformation is expected to be somewhat complicated. However, the transformations of velocities for the longitudinal and transverse motion of particles passing through the origin at t=0 can be obtained with less difficulty from equations (5.4),(5.5) and (5.6) as

$$\omega_L' = \frac{(\omega_L - v)}{1 - \varepsilon_x\lambda_L(\omega_L - v)} \qquad (5.7)$$

$$\omega_T' = \frac{\omega_T}{1 - \varepsilon_y\lambda_T\omega_T} \qquad (5.8)$$

where ω_L and ω_T are the velocities in the longitudinal and transverse directions as measured from S and the corresponding quantities with prime denote the same as measured from S'. Note that the values of λ (λ_L and λ_T) are different for the above two cases. The values of ε_x (or ε_y) are ± 1 for x' (or y') $\gtrless 0$. The origin of these terms in (5.7) and (5.8) will be evident soon.

In the present case (when v is parallel to x-axis) λ can be written also as

$$\lambda = \left|c\frac{\mathbf{r}}{r} - \mathbf{v}\right|^{-1} - c^{-1} = [(c\cos\theta - v)^2 + c^2\sin^2\theta]^{-1/2} - c^{-1} \qquad (5.9)$$

where θ is given by $\tan\theta = y/x$.

Thus λ, being function of θ alone, remains constant for the rectilinear motion of a particle which passes through the origin of S and S′ at t=0. In order to derive equations (5.7) and (5.8) this constancy of λ has been assumed. Note that for the longitudinal and transverse motion, equation (5.6) takes the following two forms, respectively:

$$t = t' + \lambda_L |x'| = t' + \varepsilon_x \lambda_L x \tag{5.10a}$$

$$t = t' + \lambda_T |y'| = t' + \varepsilon_y \lambda_T y \tag{5.10b}$$

The above simple forms are guaranteed by the fact that the particle passes through the origin at t=0. The velocity transformation equations (5.7) and (5.8) then follow rather easily as usual from the transformation equations (5.4),(5.5) and (5.10).

For the longitudinal direction, for $x'>0$, λ can be calculated from (5.9) by putting $\theta=0$:

$$\lambda_L = \frac{v}{(c-v)c} \tag{5.11}$$

for $x'<0$, $\theta=180^0$, λ_L is given by

$$\lambda_L = -\frac{v}{(c+v)c} \tag{5.12}$$

However, unlike λ_L, λ_T does not depend on the spatial domain in S′. λ_T can be calculated directly from (5.3), by assuming

$$c = vi + uj \tag{5.13}$$

(i and j are unit vectors along x and y respectively) where u is given by the relation

$$c^2 = u^2 + v^2 \tag{5.14}$$

For the synchronization of clocks on the y′ axis the light (submarine) must travel along the y′ axis. To ensure that, it is necessary that the x component of c should be taken as v [equation (5.13)], i.e the speed of S′ with respect to S. This gives

$$\lambda_T = \frac{1}{c} [\{1 / (1 - \beta^2)^{1/2} \} - 1], \quad \beta = v/c \tag{5.15}$$

In table I we put these values of λ_L and λ_T in the 4th column against different spatial domains (2nd column) in S′.

TABLE I

Signal Direction	Domain in S′	Velocity Transformation Formula	λ	Submarine speed (outward) as measured from S′ (magnitude only)	Submarine speed (inward) as measured from S′ (magnitude only)	Round-trip average speed
Longitudinal	$x' > 0$	$\omega_L' = \frac{\omega_L - v}{1 - \lambda_L(\omega_L - v)}$	$\lambda_L = \frac{v}{c(c-v)}$	c	$\frac{c(1-\beta^2)}{(1+\beta^2)}$	$c(1-\beta^2)$
	$x' < 0$	$\omega_L' = \frac{\omega_L - v}{1 + \lambda_L(\omega_L - v)}$	$\lambda_L = \frac{-v}{c(c+v)}$	c	$\frac{c(1-\beta^2)}{(1+\beta^2)}$	$c(1-\beta^2)$
Transverse	$y' > 0$	$\omega_T' = \frac{\omega_T}{1 - \lambda_T \omega_T}$	$\lambda_T = \frac{1}{c}[(1-\beta^2)^{-1/2} - 1]$	c	$\frac{c(1-\beta^2)^{1/2}}{2-(1-\beta^2)^{1/2}}$	$c(1-\beta^2)^{1/2}$
	$y' < 0$	$\omega_T' = \frac{\omega_T}{1 + \lambda_T \omega_T}$	$\lambda_T = \frac{1}{c}[(1-\beta^2)^{-1/2} - 1]$	c	$\frac{c(1-\beta^2)^{1/2}}{2-(1-\beta^2)^{1/2}}$	$c(1-\beta^2)^{1/2}$

In order to understand the implications of CB's synchronization in GP it is necessary to calculate the signal (submarine) speed as observed from S′. Note that by assumption the submarine travels along any direction, with a constant speed c as observed from S. For the longitudinal direction the calculation is straightforward. If the signal travels along x-axis *away from the origin*, in order to obtain ω_L' we put

$$\omega_L = \pm c \ (\text{for } x' \gtrless 0) \tag{5.16}$$

in (5.7). On the other hand, if the submarine travels along y′ axis *away from the origin* we shall have to put using (5.14)

$$\omega_T = u = \pm c(1-\beta^2)^{1/2} \ (\text{for } y' \gtrless 0) \tag{5.17}$$

in (5.8) in order to compute ω_T'. The obtained ω_L' and ω_T', the velocities of the submarine for the longitudinal and transverse directions as observed from S′ and as shown in the fifth column of the table I, are found to be c. This is not surprising since this trivial result is the outcome of the assumed synchronization convention in S′. However, one may now enquire what happens if instead of going away from the origin the submarine comes towards the origin. To calculate ω_L' and ω_T', i.e the longitudinal and transverse speeds as observed from S′, for the case when the submarine comes towards the origin of S′ we shall have to put

$$\omega_L = \mp c \ (\text{for } x' \gtrless 0) \tag{5.18}$$

and

$$\omega_T = \mp c(1-\beta^2)^{1/2} \ (\text{for } y' \gtrless 0) \tag{5.19}$$

in (5.7) and (5.8) respectively. The results [signal speed (inward) in S'] are summarised in the sixth column and it can be seen that they are by no means equal to c. This clearly contradicts the claim by CB that the transformations (5.1) and (5.2) which represent GP keep the light speed invariant. The speed of light coming towards the origin of S' is not only different from c, but also its values depend on the direction (cf. ω_L' and ω_T' in column 6).

Indeed this is precisely what is expected of any transformation representing GP, since, one may calculate the round- trip average speed of the submarine (by taking row-wise harmonic average of columns (5) and (6)) and verify that they (represented in the last column : round -trip average speed) represent the to and fro average signal speeds that follow from Galilean transformations (Ghosal et al 1991a). Had the to and fro speeds of the signal been equal to c in all directions, the transformations (5.1) and (5.2) would not have represented GP. Note that the C-S thesis gives us option to choose any value to be assigned to the one-way speed of the synchronizing signal but there is no scope for any manipulation with regard to the two-way average speeds. This is because the two-way average speed of a signal can be measured with a single clock in a *given* inertial frame without any ambiguity and hence it should be convention independent.

5.3 SECOND RELATIVITY POSTULATE AND ITS EMPIRICAL CONTENT

The similar arguments can be given against the incorrect claim made by CB in their article (1989) that "SR can be formulated in such a way that c invariance be no longer valid". The implication of the above claim is very serious indeed, since it-questions the correctness of the second relativity postulate of Einstein which forms the very basis for the foundation of SR. In the light of C-S thesis however the CVL postulate of Einstein loses its conventional interpretation. According to the second postulate of SR light speed is independent of the inertial frame chosen and its value is c in all directions. Now the term "speed", if it means the one-way speed, has no empirical significance since according to C-S thesis it could be chosen arbitrary. However, if the term "speed" is interpreted as two-way average speed, there seems to be no problem because as we have already discussed in section (5.2) there is no conventionality ingredient in the measurement of round- trip speeds. The validity of the second relativity postulate, when interpreted in this fashion, clearly distinguishes RP from other non-relativistic transformations.

To advance their erroneous claim CB have chosen the Tangherlini transformations representing RP and shown that in the longitudinal direction light speed away from the origin in S′ is given by

$$\omega_L' \text{ (outward)} = c/(1+\beta) \tag{5.20}$$

and argue that CVL is not maintained! Clearly according to CB, CVL refers to one-way speed of light and according to C-S thesis this observation [equation (5.20)], though correct, is trivial. The last equation refers to the one-way speed of light moving away from the origin as observed from S′. If the light travels towards the origin, the transformed speed with respect to S' can be obtained by putting $u_x = -c$ in the equation preceding the equation (11) of the article by CB (1989) (there is a printimg mistake in this equation) and this is given by

$$\omega_L' \text{ (inward)} = c/(1-\beta) \tag{5.21}$$

Hence one can verify that the longitudinal round-trip speed obtained by taking the harmonic mean of ω_L'(outward) and ω_L'(inward) is clearly c. One can also check that the conclusion regarding the value of the round-trip speed of light holds for any direction in general. This fact reaffirms CVL (if interpreted correctly) in RP and invalidates CB's contrary claim in this regard.

5.4 AN INTERESTING FEATURE OF THE CB TRANSFORMATIONS

In Galilean Physics (GP) we assume that the distance between two points —— they may be particles ——— at a given time is quite independent of any particular frame of reference; that is we assume that we can construct rigid measuring rods whose length is independent of their state of motion. But in Special Relativity, every rigid body appears to be largest when at rest relatively to the observer. When it is not at rest, it appears contracted in the direction of its relative motion by the factor $(1-v^2/c^2)^{1/2}$, while its dimensions perpendicular to the direction of motion are unaffected.

Let us now investigate what happens if we consider the CB transformations i.e

$$x' = x - vt \tag{5.4}$$

and

$$t' = t - \lambda_\theta x \tag{5.6}$$

where λ_θ is θ dependent (vide eqn. (5.9)).

At a given time the equation (5.4) gives, for any spatial separation Δx of S (i.e for a rod of length Δx parallel to the x-axis)

$$\Delta x' = \Delta x \tag{5.22}.$$

This means that there is no length contraction if the moving rigid rod is measured from S.

Now the inverse transformations of (5.4) and (5.6) are given by

$$x = x'(1+v\lambda_\theta) + vt' \tag{5.4a}$$

and

$$t = t' + \lambda_\theta x' \tag{5.6a}$$

From these equations it is evident that a rod fixed with respect to S, when measured from the point of view of S′, will appear to have contracted. The amount of contraction however cannot be given in terms of a fixed proportionality factor (i.e a factor dependent on the relative velocity of S & S′ alone) because of the term λ_θ , which depends on θ. This is obvious since the end points of a rod parallel to x-axis will have different values of θ depending on the rod length. However if the rod is placed on the x-axis (i.e $y'=0$), one obtains

$$\Delta x = \Delta x'(1+v)\lambda_L \tag{5.23}$$

where λ_L is given by (5.11).

This equation (5.23) shows that there is a length contraction for the inverse transformations. This implies that for CB transformations, there is length contraction which is one-way! Similar thing happens for Zahar transformations also. However this issue will be discussed in some detail in the last chapter.

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