ON SOME FOUNDATIONAL QUESTIONS IN RELATIVITY AND GRAVITATION

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PAPIA CHAKRABORTY (MUKHOPADHYAY)
DEPARTMENT OF PHYSICS
University of North Bengal

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To

My Parents

Preface

The thesis is built up from the studies and observations made by me during the last five years in the Department of Physics, North Bengal University. It is a great pleasure to offer my sincere and deep gratitude to all those, whose favourable reaction and active interest provided continuous impetus for the preparation of this thesis.

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Papia Chakraborty (Mukhopadhyay).

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Papia Chakraborty (Mukhopadhyay)

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CHAPTER - I

GENERAL INTRODUCTION

1.1 Introduction

The present study addresses itself to broadly two interesting foundational questions of current interest in relativity and gravitation. One of them deals with the so-called *Conventionality thesis* of special relativity, its implications, misinterpretations and finally its pedagogical importance. The other concerns the enquiry on the possibility of consolidating special relativity and gravitation in order to obtain a logically viable Lorentz-covariant theory. The main text of the present study comprises of chapters II - VII in two parts which report our observations and results that I have obtained during my studies in the last few years. Some of these observations have either been published or been reported in the national and international fora. Sometimes we have made independent observations and in another time we have played the role of a watch-dog when commenting on some recent articles on relativity foundations.

The apparently assorted compilation of papers which form the main text have one central feature in common that they all deal with foundational questions in relativity and gravitation, and everything is discussed in context of "standard" rods and clocks unlike some recent papers which discuss transformation laws for non-standard instruments.

The whole volume is organised as follows: Part I will deal with gravitation (particularly in regard to its logical possibility) in the framework of standard formulation of Special Relativity (SR) while part II will discuss the foundational questions on the standard formulation of SR itself. These two parts can stand alone in their own rights, however a sort of logical connection between these two parts will be provided in the epilogue section of chapter III.

All the chapters of the main text are self-contained particularly for those who are somewhat acquainted with the issues discussed. However for others we provide, in the next two sections of this introductory chapter, a brief review of the previous works as a background (Background I & Background II). These sections will also provide the scope and objective of the present work. Finally in the last section of this chapter summaries of all the present findings are given topic-wise. This section will provide a glimpse in advance of what lies ahead and the reader may be at liberty to skip some of the topics he may find not quite interesting for his first reading, even though as an author I would recommend that this volume be read from front to back.

1.2 BACKGROUND - I

Einstein's theory of General Relativity (GR) draws a very keen interest among the Physicists mainly for two reasons:

1. The theory is able to explain the secular motion of Mercury's perihelion and it gives for it a value of about 43" per hundred years, which is in splendid accordance to the corresponding value ascertained by the astronomers.

2. The theory is predicting a deflection of star light by the Sun, giving a deviation of about 1.72" from the straight line, which is observed value.

Furthermore, there exists until now no other theory, which can explain these two effects on the basis of well known, generally accepted and fundamental theories in mechanics and electrodynamics. So it seems that the theory of General Relativity is an unavoidable part of modern Physics. But despite these attributes, it suffers from a serious drawback in that the experimental support of GR is very limited in number and for that too one has to make use of a rather cavalier approximation of GR. This raises the interesting possibility of other theories predicting the same results. There have been some attempts which try to explain some general relativistic effects by extending Newtonian Gravity (NG) in the light of special theory.

Historically NG could successfully explain Kepler's laws. However under close scrutiny it was found to be inadequate in accounting for the residual perihelion precession of Mercury's orbit ($\simeq$ 43"/century) around the Sun. In 1919 Arnold Sommerfeld showed the usual way to obtain the precessing electronic orbits of a Hydrogen atom and then transformed his formulae for the pathways of the electron in the H-atom to the case of the Kepler motion of Mercury around the Sun. For this he first modified the equation of NG, namely

$$\frac{d}{dt} m_0 \vec{v} = - \frac{GMm_0}{r^3} \vec{r} \tag{1.1}$$

where m_0 is the mass of Mercury, M is the mass of the Sun, r is the distance of Mercury from the Sun and G is the gravitational constant. Sommerfeld replaced $m_0\vec{v}$ in the L.H.S. of equation (1.1) by the relativistic momentum $m_0\vec{v}/(1-v^2/c^2)^{1/2}$. One may easily show however that the equation that results following the modification of (1.1)

$$\frac{d}{dt}\,[m_0\vec{v}/(1-\beta^2)^{1/2}] = -\frac{GMm_0}{r^3}\vec{r}\ , \qquad \beta=v/c, \qquad (1.2)$$

though qualitatively admitting precession, can account for only about one sixth of the observed effect. By this way Sommerfeld came to the conclusion that Special Relativity (SR) cannot explain Mercury's motion of the perihelion. Still today this is the generally accepted opinion.

In an article, Roxburgh (1977) demonstrated that the concept of space-time curvature in GR has the status of a mere convention. According to his article, one can look upon a flat-space-time with a field embedded in it as a curved-space-time with no field. This idea justifies the recent efforts to obtain the so-called general relativistic effects without the use of any space-time metric whatsoever. Some such efforts (Sjödin 1982, Nandi 1984) have proved successful but not all. For instance, Bagge (1981) suggested recently that one can obtain, solely within the framework of special relativistic dynamics, the correct precession of the perihelion of Mercury. To obtain this, Bagge made an interesting suggestion that equation (1.2) was to be modified further by replacing the rest mass m_0 of Mercury by the so-called 'relativistic' mass $m_0/(1-\beta^2)^{1/2}$ in the right-hand-side of (1.2). The implication of this suggestion is that the Equivalence Principle (EP) in the context of SR should be interpreted as

$$m_g = m_0/(1-\beta^2)^{1/2} \qquad (1.3)$$

where m_g is the gravitational mass. It was claimed that after this modification of (1.1), the resulting equation

$$\frac{d}{dt}[m_0\vec{v}/(1-\beta^2)^{1/2}] = [GMm_0 \ / \ r^3(1-\beta^2)^{1/2}]\vec{r} \tag{1.4}$$

for the central force problem of Mercury's motion brought a good agreement with the known facts. However, later the claim was refuted in an article (Ghosal et all 1987). The authors showed that Bagge's approach, though interesting, did "not" yield the claimed value. Solving equation (1.4) analytically, and also numerically, they found that it yielded a precession of only 14"/century and not 42.087"/century.

In a fairly recent paper (Phipps 1986) a similar suggestion was made and in order to improve (1.2) in the light of the new (unusual) version of the EP (equation (1.3)) Phipps started from the non-covariant special relativistic Lagrangian

$$L = - m_0c^2 \ (1-\beta^2)^{1/2} - GMm_g \ /r \tag{1.5}$$

the Euler-Lagrange equation of which yields equation (1.2) if m_g is assumed to be equal to m_0 (usual version of the EP). Hoping to obtain correct precession for planetary orbits, the author replaced m_g by $m_0/(1-\beta^2)^{1/2}$ in the potential energy term in equation (1.5) and claimed to have obtained the desired result. However, later it was shown (Peters 1987, Phipps 1987) that Phipps's calculations were in error too. One may also observe that things do not improve even if one starts from the energy integral that the equation (1.2) implies, and then makes use of the unusual version of the EP in the potential energy term (Bagge 1981, Ghosal et al 1987).

It may thus appear at this point that the efforts to consolidate Newtonian gravity and special theory make a history of failures. However, the possibility of obtaining a suitable Special Relativistic extension of Newtonian Gravity (SRNG) may not be as bleak as it appears from the foregoing examples. Indeed one may argue that if, in the efforts to extend NG in the framework of SR, the 'complete' EP (Strandberg 1986) is allowed to play its due role from the onset, things may improve. To understand this, note that the principle of equivalence (in its weaker form) for the motion of material particles, is embedded even in Newton's law since m_0 cancels from both sides of (1.1) so that the motion of particles becomes independent of the mass of the test particles. The same also applies to Bagge's equation (1.4) and also to the equations that follow from Phipps's Lagrangian (equation(1.5)) where the unusual version of EP had been used in order to modify equation (1.2). However, EP in its stronger form suggests that all non-local (i.e tidal-force-independent) gravitational effects (not restricted to the motion of material particles alone) are equivalent to the effects of a uniformly accelerated system and it is well known that NG together with the EP in its stronger form indicates gravitational retardation of clocks.

In an interesting paper Strandberg (1986) argued that not only time but also the radial length (in the spherically symmetric situation) should exhibit change once the full power of EP could be exploited. The author then demonstrated the heuristic power of SR to show how the above implications of strong EP (the 'complete' EP) could correctly explain the bending of light and the precession of Mercury's orbit around the Sun.

It is thus evident that the efforts to extend Newtonian gravity along the lines of special relativity (i.e SRNG) go broadly along two distinct directions: (1) modifying the Newtonian force law in the light of SR and (2) modifying the length and time scales as required by the full implications of EP postulated by Einstein. The paper of Strandberg and a number of other earlier works cited therein fall in the latter category. While, as we have seen, the former approach fails to predict observations the latter one, though somewhat successful in this respect, seems to mutilate the very fabric of special relativity since SR cannot afford to accommodate tampering of rigid lengths and standard clocks in a given inertial frame. In fact this was the precise reason why Einstein did not pause to consolidate his own EP into the SR and instead, in order to repair the breaks in the logical structure of SR that were created by the inclusion of EP, Einstein had to generalize his earlier work (Einstein 1923) to come out with his celebrated 1915 paper on GR.

Indeed in order to reconcile NG with SR not only the equation of motion has to be changed but also Newtonian field equation has to be modified. The previous examples only tried the former. Some authors sometimes do just the opposite. For example, Rawal and Narlikar (1982) modified the field equations of NG to include the gravitational energy as a source of gravitational potential. The modified theory, as expected was found to be self-coupled and non-linior which was derived from a Lorentz-invariant action principle. The authors considered the scalar field (ϕ) and the modified "Poisson equation" looked like

$$(1 + \phi/c^2)\ \nabla^2\phi - (1/2c^2)\ (\nabla\ \phi)^2 = -4\pi G\rho_m \qquad (1.6)$$

where ρ_m denotes the matter density. It was shown that the solution of the above equation (1.6) assuming spherical symmetry gives

$$\phi = \frac{GM}{r} + \frac{G^2M^2}{4c^2r^2} \qquad (1.7)$$

where M is the mass of the spherical object that produces the gravitational field. After the solution has been obtained the authors treated this potential in a purely non-relativistic way when they considered the equation of motion. In a subsequent paper (Narlikar & Padmanabhan 1985) though this drawback has been taken care of by introducing a suitable interaction term in the Lagrangian and even though the previous work has been generalised to all orders in which the feedback of gravitational energy on ϕ in turn modified the energy which further modified ϕ and so on, correct general relativistic results could not be predicted. The theory failed to explain correctly (i) the bending of light and (ii) the precession of planetary orbits. The bending of light in this theory was zero. It was suggested further that these inadequacies of the qusi-Newtonian framework call for more sophisticated approaches to gravity.

The consolidation of SR and NG has also been considered in a series of interesting articles by Petry (1976,1977,1979,1981,1982,1988,1990,1991). In his theory the Lagrangian for the gravitational field was constructed in analogy to the Hamiltonian for a particle in the gravitational field (g^{ij}) where the four-momentum (p_k) of the particle was replaced by the variation of the field in the direction of space and time (x_k), i.e, $\partial g^{ij}/\partial x_k$. The field equations were obtained by considering the energy -momentum tensor of matter and of gravitational field together to be the source for the field (g^{ij}). However, both physically and mathematically the ideas are more involved in our opinion and for the sake of heurism and pedagogical interest we keep our discussions confined to simple minded theories. However, the impressive list of references above on the attempts on flat-space-time theories of gravity will at least bear testimony to the existence of a general urge among scientific communities for understanding gravity in the framework of special relativity. In the second chapter we shall briefly outline the general reason for this. As a passing remark and for the sake of completeness we may also list as a motivation for "other" theories of gravitation, the well known foundational questions raised against GR by Logunov (1983):

(i) The GR has not, and cannot have, energy-momentum conservation laws when a gravitational field and matter are taken in conjunction.

(ii) The inertial mass defined in the GR has no physical meaning.

(iii) Einstein's quadrupole formula for gravitational radiation is not a corollary of the GR.

(iv) The idea that a double star system loses its energy by gravitational radiation, in principle, does not follow from the GR.

(v) The GR does not satisfy a fundamental physical principle, that is the correspondence principle since it does not have the classical Newtonian limit.

However, we are not here to discuss the controversial "defects" of GR. Nevertheless we shall maintain that GR as it stands now is entirely different from other physical fields and it is not a field in the Faradey Maxwell spirit (Logunov 1983). In the next two chapters we shall only contest the commonly held thesis that the concept of flat-space-time and gravitation logically cannot go together (Schield 1960).

In the main text of part I we shall focus on a simple minded SRNG due to Biswas (1988). He tried to develop a relativistic theory of gravitation purely from heuristic considerations. The approach of Biswas, was commendably different from that of others broadly in two respects. Firstly, it did not only incorporate some results of special relativity into the Newtonian *force law* alone but instead in a bid to modify NG, it kept the *full* Newtonian field theory in view. In other words instead of trying just to modify equation (1.1) of NG, Biswas tried to systematically introduce special relativity into the following equations of NG

$$\Delta^2\phi = 4\pi G\rho \tag{1.8a}$$

$$\frac{d}{dt}(mv) = - m\Delta\phi \tag{1.8b}$$

where ϕ and ρ represent the scalar potential of NG and the mass density respectively.

Secondly, in his SRNG approach, Biswas tacitly made use of only that version of EP which was directly suggested by NG and the implication of mass energy equivalence of SR and justifiably the author did not stretch its meaning any further. That would have tampered with length and time scales affecting the standard flat-space-time metric which was to be left untouched for any special relativistic theory. The importance of this aspect of Biswas's Paper (BP) will be discussed in the next chapter.

However, in obtaining the solutions of the field equations Biswas made a mistake and this has been pointed out recently by Peters (1990). Also for some strong reasons, BP needs certain modifications to the equation of motion part in order to make the theory more acceptable and beautiful. In part I of this volume (next chapter) we shall present heuristically a new version of a flat-space-time theory of gravitation consistent with the demands of special relativity and it will be shown that our version of SRNG will be able to duplicate all the major verifiable effects of GR.

1.3 BACKGROUND-II

In the last section we have reviewed some recent attempts to obtain a theory of gravitation in the frame-work of Special Relativity (SR). It has been pointed out that these efforts are not always problem free. In different chapters, which form the first part of this volume, we shall present our humble efforts to obtain a special relativistic theory of gravitation. In part II we shall show that everything is not right even in SR. The need for revisiting special relativity arises out of recognizing the role of conventionality in the standard formulation of SR.

It is well-known that Einstein started with two simply stated and intuitively satisfying postulates, viz., the Principle of Relativity (PR) and the Constancy of Velocity of Light (CVL). But that was not enough. In order to arrive at SR, Einstein added four operational definitions or conventions of measurements. According to Prokhovnick (1967) they are (i) How to use light signal to synchronize two separated stationary clocks. (ii) The time coordinate of an event is the reading on an adjucent stationary synchronized clocks. (iii) How to use a light signal to measure a space interval from an observer to a distant event and (iv) How to measure the velocity of an object relative to an observer.

Sherwin (1992) pointed out another convention:

(v) What instruments are chosen for all these measurements.

The role of conventionality regarding synchronization of spatially distant clocks are much discussed in the literature. Once the choice of instruments is made (convention v) and there is an agreement as to the operational definition of different instruments [conventions (ii) – (iv)], the first convention (regarding synchrony) stands out in prominence. The present section will discuss the meaning and the role of conventionality in the standard formulation of SR and below we shall review some existing important works in this direction:

The role of convention in the definition of the simultaneity of distant events is one of the most debated problems in special relativity. The source of this problem lies in the fact that in SR distant clocks in a *given* inertial frame are synchronized by light signals, the one-way speed of which has to be known beforehand for the purpose. To know the one-way speed of light on the other hand one requires to have presynchronized clocks and the whole process of synchronization ends up in a logical circularity which forces us to introduce a degree of arbitrariness in assigning the value for the one-way velocity of light. However, Einstein synchronized spatially distant clocks by assuming the equality of the velocity of light in two opposite directions. Einstein's procedure to synchronize clocks at different space points is but one of several possible alternative conventions and it is now known that many of the results he obtained depended on his special choice of synchrony. As an example the question of the discordant judgements of simultaneity by two inertial observers moving with respect to each other is also a matter of such a simultaneity convention.

Historically the role of convention in the synchronization of clocks has been advocated especially by Reichenbach (1958) and later by Grünbaum (1963). They claimed that the relation of simultaneity *within* an inertial frame of reference contains an ineradicable element of convention and the conventionality lies in the assumption regarding the one-way speed of light. To clarify this point further recall that Einstein originally proposed that the criterion for the synchrony of distant clocks be that the time of arrival and reflection of a light ray as determined at one clock be precisely halfway between the time of its departure and its return upon reflection as determined at the other clock. This criterion clearly presupposes that light has the same speed in all directions. Indeed, because the specification of a value for the one-way speed of light enables directly a simple light-signal procedure for the synchronization of distant clocks, any assumption of one-way speed values is equivalent to the assumption of a criterion for synchrony. It follows that the specifications of either distant simultaneity criterion or one-way light speeds will alike be referred to as synchrony conventions (Townsend 1983).

Einstein himself referred to the distant simultaneity criterion he proposed as a free stipulation of the empirical meaning of distant simultaneity (Einstein 1961), and at issue is whether other criteria leading to different distant simultaneity judgements and consequently different one-way speeds might not have been chosen without compromising the empirical success of the theory. The conventionalist thesis holds that a range of choices are possible, all fully equivalent with respect to experimental outcome. According to the conventionalist thesis, any synchrony convention will be admissible so long as it is consistent with the round-trip principle, the principle which holds that the average speed of a light ray over any closed path has a constant value (Reinterpreted second relativity postulate, vide sec.3 of chapter V). A convention within the SR must be consistent with the round-trip principle since this principle is a consequence of the theory prior to the adoption of any criterion for distant simultaneity and may in principle be tested with a single clock. This thesis which shall here be called the thesis of the *Conventionality of distant Simultaneity* (the C-S thesis). According to the C-S thesis the conventional ingredient of SR which logically cannot have any empirical content, gives rise to results that are often erroneously construed as the new philosophical imports of special relativity theory.

There has now been a substantial amount of clarification of the C-S thesis due to a number of authors. Possibility of using synchronization convention other than that adopted by Einstein has also been much discussed.

Winnie (1970) first studied the consequences of special theory when no assumption regarding the one-way speed of light was made and then developed the so-called ε-Lorentz transformations adopting non-Einstein one-way velocity assumption or non-standard synchronization convention in general. In developing the ε-Lorentz transformation Winnie assumed a principle called the "Principle of equal Passage time". This was used in addition to the "Linearity Principle" and the "Round-trip light Principle". These principles were then shown to be independent of one-way velocity assumptions and thus may form the basis of a SR without distant simultaneity assumptions. In fact Winnie's theory was one dimensional. Unger (1986) extended Winnie's idea by considering a generalised Lorentz transformation group that does not embody Einstein's isotropy convention. The approach seems to be well suited for establishing the results of Winnie as well as some new results. However, these discussions were confined to one-dimension only. Later it will be noted that at least a two-dimensional analyses is a must. Otherwise the isotropy of two-way speed of light which follows from the reinterpreted second relativity postulate (vide sec. 3 of chapter V) cannot be used and therefore some subtlities and richness of the relativistic Physics (Ghosal et al 1991b) have to be sacrificed.

In a series of important papers Mansouri and Sexl (1977) developed a test theory of SR and investigated the role of convention in various definitions of clock synchronization and simultaneity. They showed that two principal methods of synchronization could be considered: system internal and system external synchronization. Synchronization by the Einstein procedure (using the light signal) and by slow clock transport (to correct all clocks at a given locality and then place them at all space points of a given reference frame)* turn out to be equivalent if and only if the time dilatation factor is given by the Einstein result $(1-v^2/c^2)^{1/2}$. The authors constructed an ether theory that maintains absolute simultaneity and is kinematically equivalent to SR.

In 1979 Sjödin developed the C-S thesis and consolidated it in a beautiful paper by considering the whole issue more generally and also by assuming the role of synchronization in SR and some related theories. Sjödin presented all logically possible linear transformations between inertial frames depending on physical behaviour of scales and clocks in motion with respect to physical vacuum and then examined Lorentz Transformations (LT) in the light of true length contraction and time dilatation. In his article Sjödin tried to separate the true effects and the effects due to synchronization convention. For this the author considered two special cases. The Newtonian world, without any contraction of moving bodies and slowing down of moving clocks and the Lorentzian world, with longitudinal contraction of moving bodies and slowing down of clocks. The author then used standard synchrony in Newtonian world and got the transformations which were already derived by Zahar (1977). These transformations show that the

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*For some remarks on slow-transport synchrony vide Podlaha (1980). In the present volume we would not take up this issue.

relativistic effects are only due to choice of special synchrony. But when Sjödin used absolute synchronization in Lorentzian world, the relevant transformations were due to Tangherlini (1961) which shows the real effects. In this way Sjödin came to the conclusion that the confusion regarding the existence of the ether and the reality of the length contraction / time dilatation effects is mainly due to the non-separation of the effects due to synchronization and the real contraction of moving bodies and retardation of moving clocks.

Although many articles have been written on this thesis of Conventionality of Simultaneity, most Physics texts on relativity, except a few (Taylor & Wheeler 1963), do not bring this topic into question. The fact that the C-S thesis has not yet gained wide spread attention among Physicists may be attributed to the fact that there is a tendency to regard the C-S thesis as an antithesis of SR and anything that seems contrary to the standard formulation of relativity is viewed with skepticism. In fact there are non-believers of the C-S thesis too. Among the oponents, are the authors (for example vide Fung & Hsieh 1980; Nissim-Sabat 1982) who proposed numerous experiments over the years which have been claimed to allow for an empirical test which might distinguish among the admissible synchrony conventions and thus refute the conventionalists thesis that all admissible conventions are empirically equivalent. To comment on this however, it is enough to say that every such test proposed can be shown to involve in its analyses and assumptions logically equivalent to the adoption of the standard synchrony and thus to amount to a simple begging of the question rather than an independent empirical test

(Townsend 1983).

Indeed in our opinion the C-S thesis complements SR and the understanding of the former helps clear out confusions that sometimes occur in SR. As we have pointed out, the claim that the relativity of distant simultaneity is a new non-classical philosophical import is one example of various such confusions. In spite of SR, being one of the most simple physical theories, it is the most prolific in giving birth to falacies, riddles, confusions and misconceptions. We identify at least two reasons for them: (i) Overlooking of the C-S thesis and (ii) Misconstruing of the subtlities of the C-S thesis.

As an example of one of such misconception whose origin may be attributed to the reason no. (i) above is the common belief that relativistic transformations can be derived using the first relativity postulate alone. Indeed the literature is abundant with the so-called one-postulate derivation of Lorentz transformations (Ignatowasky 1910, Frank and Rothe 1911, Wiechest 1911, Pars L A 1921,Terletskii 1968, Sussmann 1969, Arzelies H 1966, Lee A R and Kalotas T M 1975, Levy-Leblond J M 1976, Srivastava A M 1981). In order to criticize the "overemphasized" role of the the second relativity postulate (round-trip light principle according to the C-S thesis) in the standard two postulate derivations of SR these authors have shown that the existence of a limiting universal speed is a necessary consequence of the First Relativity Postulate (FRP) alone. Apparently these approaches make the Second Relativity Postulate (SRP) superfluous, but practically that is not so because the authors obtained only the Lorentzian *form* of transformation equations where c (the velocity of light) is replaced by an unknown constant velocity parameter ['σ' according to Lee and Kalotas 1975] to be determined empirically. Thus unless a further empirical information is supplemented with, nothing prevents the transformation equations to become even Galilean (σ = ∞). In order to develop LT without the SRP, no reference of clock synchronization has been made: this is a feature which is common to all the articles mentioned above is the root of all confusions. We shall later (chapter VII) show how important it is to incorporate round-trip light principle in order to derive relativistic transformations.

There are other examples of common misconception that arises from overlooking of the role of conventionality ingredients of SR. It is a still prevailing belief among Physicists that SR goes over to Galilean Relativity (GR) for relative speeds that are very small compared to the speed of light in vacuum. This is not correct. In fact it can be shown that if the belief is taken to be true it would have led to an interesting fallacy which we shall discuss in chapter IV. It will be argued that Galilean synchrony and Einstein synchrony are different and we will show that small velocity approximation cannot alter the convention of distant simultaneity.

Misconstruction of the C-S thesis itself is also not uncommon. For example in a recent article (Cavalleri & Bernasconi 1989) it has been erroneously suggested that light speed invariance in SR is a trivial matter and as if, by virtue of the C-S thesis, even Galilean Physics can be reformulated so that light speed remains invariant. We shall show in chapter V that these claims are not quite correct and it will be presented that the above claims have their origin in the misconstruction of the Reichenbach-Grünbaum thesis of conventionality of distant simultaneity in SR.

There are other examples also. Schlegel (1973,1975,1977) in his papers claimed that it is possible to construct theoretically a Lorentz Invariant (LI) clock whose rate does not depend on its state of motion and also stated that the PR does not come in the way in conceiving such a clock. As a counter claim then Rodrigues (1985), in connection with the enquiry whether Lorentz Invariant (LI) clocks can exist without violating the PR, incorrectly remarked that the possibility of having absolute synchrony is an antithesis of the relativity principle! In chapter VI we will address ourselves to the task of clarifying these issues. The definition of a LI clock will be reexamined in the light of the C-S thesis.

Sometimes in connection with the C-S thesis, the debatable issue of ether (as a *hypothetical* substrate providing a preferred inertial frame) often crops up (Sjödin 1979; Mansouri & Sexl 1977; Cavalleri & Bernasconi 1989). But questions have been raised whether considerations of synchronization alone can distinguish an ether frame or not (Spinelli 1983; Cavalleri & Spinelli 1983; Stone 1991). As it stands now, as if the existence of a real physical ether as a preferred frame would have placed the C-S thesis on a stronger footing. In fact efforts are still on to give a physical support to this preferred frame of ether. We shall later see in chapter VII that for the understanding of the C-S thesis at least, one can bypass the debate concerning the existence of ether by introducing at the out-set a real physical substrate through which different inertial frames may be considered to be in relative motion.

Given this perspective of confusion, misconstruction and polemics regarding the C-S thesis or SR for that matter, we are led to conclude that everything of SR is still not well understood and we hope that all the chapters (IV, V, VI, VII) which form the part II of this volume will demonstrate that the C-S thesis, instead of being an adversary to SR may aid us indeed to resolve confusions in SR itself.

1.4 TOPIC - WISE SUMMARY OF THE PRESENT INVESTIGATIONS:

(I)

ON THE VIABILITY OF A FLAT-SPACE-TIME THEORY OF GRAVITATION

An improved version of the flat-space-time theory of gravitation has been presented. To obtain the equation of motion of test objects a covariant Lagrangian formulation has been developed heuristically and a non-general-relativistic tensorial field equations have also been proposed. It has been shown that with these results the present theory when applied to a matter-free spherically symmetric situation, produces correct values for the advance of the perihelion of Mercury's orbit and bending of light near the Sun. The energy integral for a particle trajectory has been calculated and this has been used to obtain the general red-shift formula.

PARTICLE TRAJECTORIES IN SRNG

In the light of a Lorentz covariant theory of gravitation, the radial and the circular trajectories of test objects have been studied in a static spherically symmetric situation. It has been found that the gravitational field is characterised by a characteristic radius $r_c \simeq 1.58 r_s$ (r_s = Schwarzschild radius) which defines the surface of infinite red-shift. For a radial free fall it has been shown that a particle coming from a large distance first gets accelerated towards the source. However, as the velocity increases and the particle penetrates deep into the field, the non-Newtonian features of gravity begin to show up. From some point along the radial trajectory, depending on the initial energy, the particle starts getting retarded and finally stops at r_c. It is therefore observed that the radial fall in general is characterised by a "terminal velocity" in the velocity field. Another non-Newtonian character of the present flat-space-time gravity concerns the question of existence of circular orbits. Calculations revealed that circular orbits cannot exist below a limiting radius which is approximately equal to 2.23 r_s.

The last section of the paper (chapter III) discusses some objections which may be raised against the consolidation of gravitation & special relativity and it is remarked that the content of part II of the present volume (particularly the chapter VII) may come out with the answer.

(II)

PASSAGE FROM EINSTEINIAN TO GALILEAN RELATIVITY AND CLOCK SYNCHRONY

There is a general belief that under small velocity approximation, Special Relativity goes over into Galilean Relativity. However, a misconception could easily arise that would stem from overlooking the role of conventionality ingredients at Special Relativity Theory. It is observed that the small velocity approximation cannot alter the convention of distant simultaneity. In order to exemplify this point further, the Lorentz transformations are critically compared, under the same approximation, with the two other space-time transformations, one of which represents an Einsteinian world with Galilean synchrony whereas the other describes a Galilean world with Einsteinian synchrony.

CONVENTIONALITY OF DISTANT SIMULTANEITY AND LIGHT SPEED INVARIANCE

A recent claim that it is possible to formulate Galilean Physics so that the light speed remains invariant and also that Special Relativity (SR) can be reformulated in such a way that the constancy of light speed is no longer maintained, has been refuted. Had it been correct, it would have rendered the second relativity postulate trivial! It is held that the above claim has its origin in the misconstruction of the Reichenbach - Grünbaum thesis of conventionality of distant simultaneity in SR.

ON THE DEFINITION AND EXISTENCE OF LORENTZ INVARIANT CLOCKS

In the light of the Conventionality of distant Simultaneity thesis (C-S thesis) the definition of a Lorentz Invariant (LI) clock has been reexamined. A recent definition of a LI clock has been found to be logically unsound. The C-S thesis has been clarified in order to obtain a synchrony independent definition of a LI clock. General transformations of coordinates between two inertial frames when standard clocks are replaced by LI clocks have been obtained in order to understand the question of incompatibility of LI clocks and relativity. It is held that contrary to some statements in the literature the theory of relativity *does* forbid the existence of LI clocks but a recent approach which tries to establish the above fact seems to be unsatisfactory.

THE ROLE OF c IN SPACE-TIME TRANSFORMATIONS : RELATIVITY IN A SUBSTRATE

Reichenbach - Grünbaum thesis of the Conventionality of distant Simultaneity of special relativity is clarified by developing relativity within a medium. Instead of light, spatially distant clocks are imagined to have been synchronized by "acoustic signal". Einstein's procedure for synchrony which assumes that the two-way-speed of the synchronizing signal along a given line is the same as its one-way-speed, has however been retained. It is shown that by deliberately opting for the non-luminal synchrony (but at the same time following Einstein's procedure for it) and hence by obtaining the transformation equations for the relativistic world one is able to visualize more clearly the conventionality ingredients in the standard formulation of special relativity. The Sjödin point of view of clock synchronization which requires the concept of a preferred inertial frame is seen to be more appropriate in the present context. First the transformations have been obtained generally and later some special cases have been investigated. The common confusions regarding the "real" and "apparent" effects in special relativity have been made clear by studying the transformation equations for the relativistic world and that for the Galilean world under the non-luminal synchrony. It is shown that γ-factors of special relativity *partly* originates from Einstein's procedure for clock synchrony.

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PART - I

CHAPTER - II

ON THE VIABILITY OF A FLAT-SPACE-TIME THEORY OF GRAVITATION

2.1 INTRODUCTION

In Special Relativity (SR) the laws of dynamics and the electromagnetic field equations refer to *global* inertial frames. On the other hand the assumption that all effects of a gravitational field are locally identical to the effects of uniform acceleration of the coordinate system led Einstein to believe in curved space time and hence to formulate his celebrated theory of gravitation commonly known as the General Theory of Relativity (GR). According to GR matter introduces non-flat space time, the metric of which being the measure of the produced gravitational field. Thus in presence of matter the concept of a global inertial frame is discarded once and for all. However from the pedagogical standpoint one may reasonably ask if it were absolutely necessary to deal with curved space-time in order to understand gravitation and whether it was not at all possible for certain to extend Newtonian Gravity (NG) in the frame work of global inertial frames to suit the requirements of special relativity (electromagnetism for example did not pose any such difficulty in this respect). In other words one may enquire whether historically all options had been explored in this regard before one was forced to discard such endeavours. It may sound highly unprofessional and out of culture to raise such questions after all these years, however, it will be rather interesting to note that nearly three quarters of a century have elapsed since the discovery of GR but still today we witness numerous articles which strive to understand gravity in the frame work of SR. We feel that no matter whether some specific efforts are rewarding or not they should not be lightly dismissed in general.

Apart from the pedagogical question there is still another reason which may prompt one to look for other theories of gravitation. It is true that the GR is very elegant and rich in its structure and is endowed with interesting philosophical imports. But despite these attributes,it suffers from a serious drawback in that the experimental support of GR is very limited in number and for that too one has to make use of a rather cavalier approximation of GR (Biswas,1988). But since the issue of Physics is distinct from that of the elegance of a formalism, the meagreness of empirical support for GR clearly leaves room for a plethora of other possible theories.

In a recent paper Biswas (1988) tried to introduce SR into the theory of NG in a systematic manner to obtain a flat space-time theory of gravitation which was claimed to have produced correct values for (1) the perihelion advance of Mercury's orbit, (2) the bending of light near the Sun and (3) the gravitational redshift. The earlier simple minded attempts (Bergmann 1969; Bagge 1981; Phipps 1986,1987) to fit SR into classical gravity in order to predict correct value for (1) did not succeed (Bergmann 1969; Peters 1987; Ghosal et al 1987). Some rigorous attempts are however available due to Thirring (1961), Feynmamm (1971) and Narlikar and others (1982,1985) but we leave them out of the present scope since in the present text we are more interested in a simple minded approach of Special Relativistic extension of Newtonian Gravity (SRNG). However, in the background I of chapter I (sec. 1.2) we discussed all the simple minded earlier attempts in detail. These earlier works dealt with the equation of motion alone and made no reference to any possible gravitational field equations and that they failed to give results is not surprising since the perihelion advance is commonly known to be the major observational test of the non-linear terms of the field equations (Peters 1987). In this perspective Biswas's Paper (BP) seems to be a bit more promising and therefore deserves close scrutiny.

The scheme of presentation is as follows. In section **2.2**, we discuss briefly some salient features of BP. In section **2.3**, we develop heuristically a covariant formulation for the equation of motion. Planetary precession and bending of light have been obtained in section 2.4. In section 2.5 we propose to develop heuristically the tensorial field equations and section 2.6 provides a guideline along which the redshift can consistently be treated in the framework of flat-space-time theories of gravitation in general. Section 2.7 contains some concluding remarks.

2.2 SOME SALIENT FEATURES OF BP

According to BP gravitational field is represented by a tensor potential $h_{\mu\nu}$ in a global pseudo Euclidean space-time. The proposed non-linear field equation is given by

$$\Box^2 h_{\mu\nu} = (2\pi G/c^2)\rho_{\mu\nu} + K\partial_\lambda h_{\mu\alpha}\partial^\lambda h^\alpha_\nu \qquad \text{----(2.1)}$$

where the second term on the r.h.s of (2.1) with undetermined factor K is added to include the contribution of the energy momentum tensor of the produced field. $\rho_{\mu\nu}$ denotes the energy momentum tensor for matter.

Applying eqn.(2.1) [with $\rho_{\mu\nu}$=0] to the static spherically symmetric situation, Biswas obtained the following pair of equations:

$$\nabla^2 h_{00} = -K\ (\nabla h_{00})^2 \qquad \text{-----(2.2)}$$

$$\nabla^2 h_{11} = +K\ (\nabla h_{11})^2 \qquad \text{-----(2.3)}$$

where h_{00} & h_{11}, which are functions of r alone, are the only non-zero components of $h_{\mu\nu}$ in spherical coordinates. The free field solutions of the above are taken to be

$$h_{00} = \frac{1}{K} \ln [1- \frac{2KGM}{r}] \qquad \text{-----(2.4)}$$

$$h_{11} = - \frac{1}{K} \ln [1+ \frac{2KGM}{r}] \qquad \text{-----(2.5)}$$

In order to obtain the equation of motion Biswas started from the free particle mass shell equation

$$\eta_{\mu\nu}\ p^{\mu}\ p^{\nu} + m^2c^2 = 0 \qquad \text{-----(2.6)}$$

where $\eta_{\mu\nu}$ is the Minkowski metric, p^{μ} & m are the four momenta and mass of the particle respectively. For a charged particle in an electromagnetic field (A^{μ}), p^{μ} in equation (2.6) is replaced by $p^{\mu}+A^{\mu}$. Similarly it is argued that for motion under gravity $\eta_{\mu\nu}$ is to be replaced by $a_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ in equation (2.6). The equation of motion then follows from the three Hamiltonian cp_0 given by

$$cp_0 = c\ [-(P_a\ a^{ab}\ P_b + m^2c^2)/a^{00}]^{1/2} = H \qquad \text{-----(2.7)}$$

It is thus demonstrated that the solutions (2.4) & (2.5) with K=2 when put in canonical equations for the Hamiltonian (2.7) correctly explains two major tests of GR, namely the perihelion precession of Mercury's orbit and the bending of light.

Unfortunately it has been recently demonstrated by Peters (1990) that while eqn. (2.2) is correct, equation (2.3) does not follow from the equation (2.1). The field equation (2.1) when written in a general coordinate system should read

$$h_{\mu\nu;\alpha}{}^{;\alpha} = \frac{2\pi G}{c^2}\, \rho_{\mu\nu} + K\, h_{\mu\alpha;\beta}\, h_{\nu}^{\alpha;\beta} \qquad \text{-----(2.8)}$$

and by correctly applying equation (2.8) [with $\rho_{\mu\nu}=0$] to case of tensors resolved in spherical coordinate system gives the unique solution for h_{11}

$$h_{11} = 0 \qquad \text{-----(2.9)}$$

which is different from Biswas' result [equation (2.5)]. However there is no change in equation (2.4). Since Biswas' results depend on (2.4) and (2.5) and the latter being incorrect, the claims of Biswas fall through.

However, if this is the only error of BP, it appears that some minor amendments to (2.8) could provide a viable flat-space-time theory of gravitation. In our opinion the error in BP as pointed out by Peters is not the only weak point in the theory. We observe that there are other drawbacks, which are to be removed before one hopes to obtain a viable relativistic theory of gravitation. Firstly, in order to obtain the equation of motion Biswas resorted to a non-covariant Hamilton's formulation. Equations are thus not manifestly covariant. One therefore needs to develop heuristically a fully covariant formulation for this purpose.

The other shortcoming of BP refers to the claim by Biswas that his theory correctly predicts red-shift. Though no supporting calculations have been given on grounds of simplicity! We feel that the matter of gravitational red-shift cannot be treated lightly in connection with any flat-space-time theory what-so-ever. Indeed, before one hopes to explain the observed gravitational red-shift according to any theory of gravitation in the framework of global inertial frames one will have to first contest the commonly held thesis that " the existence of gravitational red-shift shows that a consistent theory of gravity cannot be constructed within the framework of special relativity" (Misner 1973). Any attempt in this regard is clearly absent in BP. In section 2.6, we will provide a guideline along which the red-shift can consistently be treated in the framework of flat-space-time theories of gravitation in general.

2.3 EQUATIONS OF MOTION

We have already noted that Biswas' formulation of the equation of motion starts with the introduction of a 3-Hamiltonian, therefore the resulting canonical equations were not manifestly covariant. Besides, in view of the cherished parallelism between gravitation and electromagnetism in BP, it is not intelligible why in the mass shell equation (2.6) $\eta_{\mu\nu}$ and not p^{μ} would undergo change in presence of gravity.

In order to guess a correct equation of motion of a test object we shall rather resort to the covariant Lagrangian formulation. The covariant Lagrangian of a charged particle in an electromagnetic field is composed of two parts ($L = L_f + L_c$). The free particle Lagrangian L_f is proportional to $m_0\eta_{\mu\nu}u^\mu u^\nu$ and the coupling term L_c is proportional to $qA_\alpha u^\alpha$ where m_0, q, and u^μ are the inertial mass, charge and the 4-velocities of the test particle and A_μ are the components of the four vector potential of the electromagnetic field (Goldstein 1964). With this analogy, for a tensor potential $h_{\mu\nu}$ of the gravitational field one may reasonably guess a minimal coupling term to be $m_g h_{\mu\nu}u^\mu u^\nu$ where the gravitational charge m_g may be assumed to be $-m_0$ in view of equivalence principle and the attractive nature of gravitational force.

Thus the Lagrangian may be believed to be given by

$$L \propto m_0\, \eta_{\mu\nu}\, u^\mu u^\nu - m_0\, h_{\mu\nu}\, u^\mu u^\nu$$

It may be noted here that any other possible proportionality constant (including the sign) in the 2nd term above may be dumped into $h_{\mu\nu}$ without any consequence. One may thus assume

$$L = m_0(\eta_{\mu\nu} - h_{\mu\nu})\, u^\mu u^\nu$$

However, m_0 may be dropped and one may write

$$L = a_{\mu\nu}u^\mu u^\nu \qquad \text{------(2.10)}$$

where $$a_{\mu\nu} = \eta_{\mu\nu} - h_{\mu\nu} \qquad \text{------(2.11)}$$

The above arguments may thus provide some justification for the assumed transition $\eta_{\mu\nu} \to \eta_{\mu\nu} - h_{\mu\nu}$ in the free particle mass shell equation [equation (2.6)]. We also take this cue but instead of taking the Lagrangian form (2.10) we shall choose

$$L = (a_{\mu\nu}u^{\mu}u^{\nu})^{1/2} \qquad \text{-----(2.12)}$$

which also reduces to the free particle Lagrangian $L_f = (\eta_{\mu\nu}u^{\mu}u^{\nu})^{1/2}$ if $h_{\mu\nu}$ is assumed to be zero. The form (2.10) for the Lagrangian is rejected because one of the first integrals of motion which follows from L is

$$a_{\mu\nu}\frac{dx^{\mu}}{d\tau}\frac{dx^{\nu}}{d\tau} = \text{const.} \qquad \text{-----(2.13).}$$

This clearly contradicts the definition of proper time interval :

$$d\tau^2 = -\eta_{\mu\nu}dx^{\mu}dx^{\nu} \qquad \text{-----(2.14)}$$

No such problem arises out of the Lagrangian L given in equation (2.12).

To obtain the equation of motion we start with the Hamilton's principle. The action integral may be written as

$$\int L\, d\tau = \int\left(a_{\mu\nu}\frac{dx^{\mu}}{d\tau}\frac{dx^{\nu}}{d\tau}\right)^{1/2} d\tau = \int\left(a_{\mu\nu}\frac{dx^{\mu}}{ds}\frac{dx^{\nu}}{ds}\right)^{1/2} ds$$

where s may be any scalar parameter of motion. Therefore the Lagrangian may also be taken as

$$L' = \left(a_{\mu\nu}\bar{u}^{\mu}\bar{u}^{\nu}\right)^{1/2} \qquad \text{-----(2.15)}$$

$$\text{where } \bar{u}^{\mu} = \frac{dx^{\mu}}{ds}$$

The Euler-Lagrange equation corresponding to (2.15) becomes greatly simplified if the parameter 'ds' is taken to be

$$ds = (a_{\mu\nu}\, dx^{\mu}\, dx^{\nu})^{1/2} \qquad \text{------(2.16)}$$

In this case the equation of motion can conveniently be written as

$$\frac{1}{2}\,(a_{\mu\lambda,\nu} + a_{\nu\lambda,\mu} + a_{\mu\nu,\lambda})\,\frac{dx^{\mu}}{ds}\,\frac{dx^{\nu}}{ds} + a_{\mu\nu}\,\frac{d^2x^{\mu}}{ds^2} = 0 \qquad \text{---(2.17)}$$

The form (2.16) looks like the metric in GR. Indeed the equation of motion (2.17) as obtained above formally looks like that of a geodesic in a curved space-time defined by the line element (2.16). However this has nothing to do with geodesics since the present theory is in flat space-time and raising and lowering of indices is done by $\eta_{\mu\nu}$ (and not $a_{\mu\nu}$).

2.4 PLANETARY PRECESSION AND BENDING OF LIGHT

To obtain the orbit equation however we shall not use the equation (2.17) directly, instead it will be advantageous to start from the first integrals that are suggested by the symmetry of the problem. Note that here we are interested only in the static spherically symmectric fields. We first restrict ourselves to θ = $\pi/2$ plane as usual. We have two first integrals of motion at our disposal. Since $L^{/}$ does not contain (in a static spherically symmetric situation) $x^0(=t)$ and $x^3(=\phi)$ explicitly the corresponding generalized momenta are constants of motion. Therefore,

$$p_0 = \frac{\partial L^{/}}{\partial \bar{u}^0} = \frac{a_{00}\bar{u}_0}{(a_{\mu\nu}\bar{u}^{\mu}\,\bar{u}^{\nu})^{1/2}} = \varepsilon \qquad \text{-----(2.18)}$$

$$p_3 = \frac{\partial L^{/}}{\partial \bar{u}^3} = \frac{a_{33}\bar{u}_3}{(a_{\mu\nu}\bar{u}^{\mu}\,\bar{u}^{\nu})^{1/2}} = p_{\phi} \qquad \text{------(2.19)}$$

where ε & p_ϕ are constants.

However the equations (2.18) & (2.19) are supplemented with a constraint condition that follows from the definition of the parameter ds given by equation (2.16) :

$$a_{\mu\nu}\, \bar{u}^{\mu}\, \bar{u}^{\nu} = 1 \qquad \text{-----(2.20)}$$

Thus making use of (2.20) we can rewrite the two first integrals (2.18) & (2.19) as

$$a_{00}\, \bar{u}^{0} = \varepsilon \qquad \text{-----(2.21)}$$

$$a_{33}\, \bar{u}^{3} = p_\phi \qquad \text{-----(2.22)}$$

In spherical coordinate system both $\eta_{\mu\nu}$ & $h_{\mu\nu}$ are diagonal and for spherically symmetric $h_{\mu\nu}$ the only non-zero components of $h_{\mu\nu}$ can be h_{00} and h_{11} and so $a_{\mu\nu}$ takes the form (for the chosen orbit plane)

$$a_{\mu\nu} = \mathrm{diag}\ (a_{00}\ ,\ a_{11}\ ,\ r^2, r^2).$$

Equation (2.20) can be written explicitly as

$$a_{00}(\bar{u}^0)^2 + a_{11}(\bar{u}^1)^2 + r^2(\bar{u}^3)^2 = 1.$$

Using (2.21) and (2.22) we can write

$$\frac{\varepsilon^2}{a_{00}} + a_{11}\ (dr/ds)^2 + p_\phi^2\ /\ r^2 = 1 \qquad \text{-----(2.23)}$$

Now since

$$\frac{dr}{ds} = \frac{dr}{d\phi}\ \frac{d\phi}{ds} = \frac{dr}{d\phi}\ u^{-3}$$

the equation (2.23) reduces to the orbit equation

$$\frac{\varepsilon^2}{a_{00}} + a_{11}\ (dr/d\phi)^2\ (p_\phi^2\ /r^4) + p_\phi^2\ /\ r^2 = 1 \qquad \text{-----(2.24)}.$$

The usual substitution $u = \frac{1}{r}$ in equation (2.24) produces

$$\frac{\varepsilon^2}{a_{00}} + a_{11}\ (du/d\phi)^2\ p_\phi^2 + p_\phi^2\ u^2 = 1 \qquad \text{-----(2.25)}.$$

With the assumed solution of h_{00} & h_{11} of BP with K=1 [to obtain correct precession Biswas had to put K = 2] and using the flat metric in spherical coordinates as $\eta_{\mu\nu}$= dig $(-1,1,r^2,r^2\sin^2\theta)$

$$a_{00} = -\ [1+\ln(1-2GM/r)] \simeq -\ (1-2GM/r) \qquad \text{-----(2.26)}$$

$$a_{11} = \ [1+\ln(1+2GM/r)] \simeq \ (1+2GM/r) \qquad \text{-----(2.27)}$$

rearranging and differentiating (2.25) with respect to ϕ and using (2.26) and (2.27) the orbit equation takes the approximate form

$$\frac{d^2u}{d\phi^2} + u = 3GMu^2 + GM/S^2 \qquad \text{-----(2.28)}$$

where we have put $S^2 = -\ p_\phi^2$, S being the angular momentum of the test object per unit mass.

It is well known that the differential equation

$$d^2u\ /\ d\phi^2 + u = nGMu^2 + GM/S^2$$

gives precession

$\omega \simeq n \times 14''$ /century

for the planet Mercury. Equation (2.28) therefore, correctly gives precession of about $3 \times 14''$/century for Mercury.

In order to obtain the bending of starlight grazing the Sun one can also use the orbit equation (2.28) but in that case the last term must be dropped since S, the angular momentum per unit mass for a zero rest mass particle is infinite. This is because for a photon, for example, the linear momentum and hence the angular momentum (when its motion is not directed towards the origin of the coordinate system) are always finite. In equation (2.28) the term S was interpreted as the angular momentum per unit mass for a massive particle. Here we extend this meaning of S to the case of a massless photon as well.

The resulting equation

$$d^2u / d\phi^2 + u = 3GMu^2 \qquad \text{-----(2.29)}$$

gives correct values for the bending of light (Bose 1980).

2.5 THE FIELD EQUATION

The field equations of Biswas

$$\Box^2 \; h_{\mu\nu} = 2\pi G\rho_{\mu\nu} + k\partial_\lambda h_{\mu\alpha}\partial^\lambda h^\alpha_\nu \qquad \text{-----(2.1)}$$

written in covariant notation yields

$$h_{\mu\nu;\alpha}{}^{;\alpha} = 2\pi G\rho_{\mu\nu} + kh_{\mu\alpha;\beta}\, h^{\alpha}_{\nu}{}^{;\beta} \qquad \text{-----(2.8)}$$

or, $h_{\mu\nu;\alpha}{}^{;\alpha} = kh_{\mu\alpha;\beta}\, h_{\nu}^{\alpha;\beta}$ -----(2.30)

in absence of matter ($\rho_{\mu\nu} = 0$).

For the spherically symmetric static fields

$$h_{\mu\nu} = \mathrm{diag}\,[\,h_{00}(r),\ h_{11}(r),\ 0,\ 0\,].$$

From (2.30) Biswas erroneously arrived at the pair of equations

$$\nabla^2 h_{00} = -k(\nabla h_{00})^2 \qquad \text{-----(2.2)}$$

$$\nabla^2 h_{11} = +k(\nabla h_{11})^2 \qquad \text{-----(2.3).}$$

Peters (1990) has shown that the latter equation (2.3) is incorrect. It was further shown that if equation (2.30) is correctly applied in a spherical coordinate system the only consistent solution for h_{11} is

$$h_{11} = 0 \qquad \text{-----(2.9).}$$

However, h_{00} can be taken as given by Biswas

$$h_{00} = \frac{1}{k} \ln \left[1 - \frac{2kGM}{r}\right] \qquad \text{-----(2.4).}$$

It will be immediately seen that when these solutions [(2.4) and (2.9)] are used in equation (2.25) the orbit equation becomes approximately

$$d^2u \,/\, d\phi^2 + u = GMu^2 + GM\,(du/d\phi)^2 + GM/S^2 \qquad \text{-----(2.31).}$$

Surprisingly the above equation is also the path equation that follows from Bagge's ponderomotive force law of gravitation (Bagge 1981; Ghosal et al 1987). It was shown elsewhere (Ghosal et al 1987) that this equation yields only about $14''$/century for Mercury's precession.

Thus to have correct results we need to modify the field equations of Biswas. In order to do this we shall follow the following guidelines :

(1) The new equations should be such that it would formally look like equation (2.8). This is required by the fact that the typical non-linear form of Biswas' equations result from a simple and well considered heuristic arguments and we feel therefore that one should not disturb it.

(2) Since the field equations are expected to be of the second order, the solutions for each component will have two arbitrary constants in general. In the spherically symmetric case there will be four such constants in total.Two of them may be made to vanish from the requirement that h_{00} & h_{11} tend to zero as $r \to \infty$. The one remaining constant in h_{00} may be set by using the far field approximation and correlating the result with that of Newtonian Gravity. The question now is how to fix the other constant in h_{11}. For this, one needs to have an additional postulate. Biswas had to make such a postulate in order to fix the arbitrary constant in his erroneous solution of h_{11} [vide section (1v) of BP]. We will rather do it differently. Our field equation will be such that in the spherically symmetric situation it automatically would display one built-in constraint so that the problem of fixing the arbitrary constant in h_{11} will be done away with.

Now we give *our* version of the field equations after the following steps.

(1) Define a tensor D^{μ}_{ν} which is diagonal and has non-zero components $D^{1}_{1} = -D^{0}_{0} = 1$ in spherical polar coordinates.

(2) Construct a scalar

$$\psi = \frac{1}{2} D^{\mu}_{\nu} h^{\nu}_{\mu} \qquad \text{-----(2.32).}$$

If the r.h.s of (2.32) is resolved in spherical polar coordinates (Note that since ψ is a scalar, the value of the same is independent of the coordinate system) we obtain

$$\psi = \frac{1}{2} [h_{00}(r) + h_{11}(r)] \qquad \text{-----(2.33)}$$

where $h_{00}(r)$ & $h_{11}(r)$ are the values of the field components in the same coordinate system.

(3) Define another tensor $\Lambda_{\mu\nu}$ whose only non-zero component in spherical polar coordinate is $\Lambda_{11} = 1$.

Now we postulate the field equations

$$H_{\mu\nu;\alpha}{}^{;\alpha} = k\, H_{\mu\alpha;\beta}\, H^{\alpha\ ;\beta}_{\nu} + 2\pi G\rho_{\mu\nu} \qquad \text{-----(2.34)}$$

where

$$H_{\mu\nu} = h_{\mu\nu} - \psi\, \Lambda_{\mu\nu} = h_{\mu\nu} - \frac{1}{2} D^{\alpha}_{\beta} h^{\beta}_{\alpha} \Lambda_{\mu\nu} \qquad \text{-----(2.35).}$$

Though equation (2.34) contains the tensors $\Lambda_{\mu\nu}$ and $D_{\mu\nu}$ which have simple forms in spherical polar coordinates, it does not mean that the equation is adapted to spherical coordinates alone. Indeed, the equation (2.34) represents the general field equations which can be used in any coordinate system.

In the matter free region we have

$$H_{\mu\nu;\alpha}{}^{;\alpha} = k\, H_{\mu\alpha;\beta}\, H_{\nu}^{\alpha\ ;\beta} \qquad \text{-----(2.36).}$$

From equation (2.35) it is evident that in spherical polar coordinates and assuming spherical symmetry in $h_{\mu\nu}$, the only non-zero components of $H_{\mu\nu}$ are H_{00} and H_{11}. Therefore following Peters we have

$$H_{00} = \frac{1}{k} \ln \left[1- \frac{2kGM}{r}\right] \qquad \text{-----(2.37)}$$

and

$$H_{11} = 0 \qquad \text{-----(2.38).}$$

However, from equation (2.35),

$$H_{00} = h_{00}(r) \qquad \text{-----(2.39)}$$

and

$$H_{11} = h_{11}(r) - h_{00}(r) \qquad \text{-----(2.40).}$$

From (2.38) and (2.40) follows the constraint condition

$$h_{11}(r) = h_{00}(r)$$

Thus we have

$$h_{00} = \frac{1}{k} \ln \left[1- \frac{2kGM}{r}\right] \qquad \text{-----(2.41a)}$$

$$h_{11} = \frac{1}{k} \ln \left[1- \frac{2kGM}{r}\right] \qquad \text{-----(2.41b)}$$

One can verify that these solutions also give under approximations the same path equation that was obtained in section

(2.4) using Biswas' incorrect solutions [equations (2.4) and (2.5)]. The result of course is not surprising since the solutions (2.41b) and (2.5) are the same in their first order. Thus we see that after some modifications in the field equations and the ponderomotive force law of Biswas it is possible to obtain correct precession of Mercury's orbit and bending of light. It is also evident that our Lagrangian (2.12) representing the motion of a test object will also give the correct result for the radar eco delay experiment.

If for some reasons one wishes to adhere to Biswas' incorrect solutions for $h_{11}(r)$ [equation (2.5)] and is prepared to sacrifice the guidelines mentioned earlier in this section, there is another version of the field equation

$$h_{\mu\nu;\alpha}{}^{;\alpha} - \phi\, \Lambda_{\mu\nu;\alpha}{}^{;\alpha} = k\ (h_{\mu\alpha;\beta}\, h_{\nu}^{\alpha\ ;\beta} - \phi\, \Lambda_{\mu\alpha;\beta}\, \Lambda_{\nu}^{\alpha\ ;\beta}) \qquad \text{--(2.42)}$$

where the scalar,

$$\phi = \eta^{\alpha\beta}\, h_{\alpha\gamma}\, \Lambda_{\beta}^{\gamma}$$

One may resolve the equation in spherical polar coordinates and verify the following non trivial equations :

$$1/r^2 \frac{d}{dr} (r^2\, dh_{00}/dr) = -k\ (dh_{00}/dr)^2 \qquad \text{-----(2.43a)}$$

$$1/r^2 \frac{d}{dr} (r^2\, dh_{11}/dr) = k\ (dh_{11}/dr)^2 \qquad \text{-----(2.43b)}$$

Solutions of which (dropping the additive constants)

$$h_{00} = \frac{1}{k} \ln [1 - \frac{k}{Ar}] \qquad \text{-----(2.44a)}$$

$$h_{11} = -\frac{1}{k}\ln\left[1 - \frac{k}{Br}\right] \qquad \text{-----(2.44b)}$$

and

with the additional postulate $B = -A$ result in equations (2.4) and (2.5) used by Biswas.

2.6 ENERGY EQUATION AND REDSHIFT

Biswas claimed that the results of the gravitational red-shift [Pound-Rebka Experiment (1960)] matches with his theory . However, it was not worked out on the ground of simplicity. We thus have no means to understand what precisely was in the author's mind regarding the explanation of gravitational red-shift and we think that the matter of red-shift in the context of a flat-space-time theory requires some deliberations. Being essentially a special relativistic theory, BP deals with standard clocks and scales which should remain unaffected by the field. Therefore the common general relativistic interpretation that red-shift is the result of gravitational retardation of clocks (Weinberg 1972) is hence no longer valid in special relativistic theory of gravitation. There is another heuristic interpretation according to which energy of a photon must decrease when it climbs out of a gravitational field (Misner 1973; Einstein 1923). An important corollary that follows from the above interpretation is that the gravitational red-shift must imply the non-existence of global Lorentz frames (Misner 1973; schield 1960). Thus any effort to consolidate (globally) special relativity and gravitation seems to fall through unless red-shift is viewed from a different perspective. Below we shall see how gravitational field should bring in changes into the atomic (or neuclear) processes itself so that similar atoms (or neuclei) at two different spatial locations would behave differently. This fact will provide an explanation for the red-shift according to BP or any other flat-space-time theories for that matter.

To understand this let us first calculate the energy of a test object of rest mass m_0 undergoing radial motion.

We start from the equation (2.20). For radial motion we can write

$$a_{00}\ (\bar{u}^0)^2 + a_{11}\ (\bar{u}^1)^2 = 1$$

or $\quad a_{00}(dt/ds)^2 + a_{11}(dr/dt)^2(dt/ds)^2 = 1 \qquad$ -----(2.45).

Substituting $\bar{u}^0 = dt/ds = \varepsilon/a_{00}$ (vide 2.21) in equation (2.45) and multiplying it with m_0 on both sides we have

$$E = \frac{m_0\ (-a_{00})^{1/2}}{[\ 1 + (a_{11}/a_{00})v^2\ ]^{1/2}} \qquad \text{-----(2.46)}$$

where we have put $-m_0^2\varepsilon^2 = E^2$. The quantity E can be interpreted as the total energy of the particle in a gravitational field, since, in the far field the right hand side reduces to $m_0/(1-v^2)^{1/2}$ which is the so-called relativistic energy.

Consider that an atomic or neuclear transition (as an example one may consider the 14.4 kev transition of a Fe^{57} nucleus which was used in the Pound Rebka experiment) takes place between the two of its energy states E_1 and E_2 with the corresponding rest masses m_1 & m_2 of the atom / nucleus respectively [Note that according to SR an atom (or a nucleus) will have different rest masses for its different energy states]. Then from (2.46) we have

$$E_2 - E_1 = (m_2 - m_1)\ \frac{(-a_{00})^{1/2}}{[1 + (a_{11}/a_{00})v^2]^{1/2}} = h\nu \qquad \text{-----(2.47)}$$

where ν is the frequency of the emitted photon. If we wish to compare this transition with that of a similar atom with two energy states corresponding to the same rest masses m_1 & m_2 at infinity ($a_{00} \rightarrow -1$, $a_{11} \rightarrow 1$) we have similarly

$$E_2' - E_1' = (m_2 - m_1) \frac{1}{(1-v^2)^{1/2}} = h\nu' \qquad \text{-----(2.48)}$$

where the prime denotes the corresponding quantities at ∞ .

If the transition takes place when the atoms are at rest ($v=0$) at their respective positions,we have from equations (2.47) and (2.48) the frequencies of the emitted photons, given by

$$h\nu = \Delta m \ (-a_{00})^{1/2} = \Delta m \ [1 + \ln \ (1-2GM/r)]^{1/2} \qquad \text{-----(2.49)}$$

and $$h\nu' = \Delta m \qquad \text{-----(2.50)}$$

where $\Delta m = m_2 - m_1$

or combining (2.49) and (2.50)

$$\nu = \nu' \ [1 + \ln \ (1-2GM/r)]^{1/2} \qquad \text{-----(2.51).}$$

The equation (2.51) represents the general redshift formula according to our theory. Retaining the first term in the expansion of the logarithmic term in (2.51) we have

$$\nu \simeq \nu' \ (1 - 2GM/r)^{1/2} \qquad \text{-----(2.52).}$$

From the last expression (which is the typical general relativistic expression) it is evident that the present theory can correctly account for the observed red-shift.

2.7 SUMMARY

Among several attempts to obtain a Special Relativistic extension of Newtonian Gravity (SRNG), a recent one proposed by T.Biswas seems to be promising. However, there are several shortcomings, some modifications have been suggested in order to remove these shortcomings. In section 2.3 we have heuristically developed a covariant Lagrangian formulation for the equation of motion of test objects. Secondly in section 2.5 we have developed heuristically a tensorial field equations and the equations are found to be non-linear, the vacuum spherically symmetric solutions for which when put in the proposed equation of motion produces correct values for the advance of the perihelion of Mercury's orbit and bending of light near the Sun. The prospect of accommodating gravitational red-shift in context of flat-space-time theories in general is discussed in section 2.6.

All this has been done just to show that it is possible to obtain a Lorentz covariant theory of gravitation, similar to that of electromagnetism, which can duplicate all the verifiable general relativistic effects. Among other things this article discusses the pedagogical importance of the present flat-space-time theory of gravitation and in no way intends to project it as a viable alternative to GR.

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CHAPTER - III

PARTICLE TRAJECTORIES IN SRNG

3.1 INTRODUCTION

It has been pointed out in the previous chapter that SRNG is a simple minded approach of Special Relativistic extension of Newtonian Gravity. In chapter II an attempt was made to consolidate Special Relativity and gravitation in order to obtain a viable Lorentz covariant theory of gravitation. There we, in one hand, developed tensorial field equations for the gravitational potential $h_{\mu\nu}$ and on the other hand developed heuristically a covariant Lagrangian formulation for the equation of motion. The proposed covariant Lagrangian have been given by [equation (2.15) of chapter II]

$$L' = (a_{\mu\nu} \bar{u}^{\mu} \bar{u}^{\nu})^{1/2} \tag{3.1}$$

where $\bar{u}^{\mu} = dx^{\mu}/ds$ and s may be any scalar parameter of motion.

$$\text{Here,} \quad a_{\mu\nu} = \eta_{\mu\nu} - h_{\mu\nu} \tag{3.2}$$

where $\eta_{\mu\nu}$ is the flat metric which in spherical coordinate system is taken as $\eta_{\mu\nu}$= diag $(-1,1,r^2,r^2\sin^2\theta)$, and $h_{\mu\nu}$ is the tensor potential of the gravitational field which are obtained from the proposed field equations. The vacuum spherically symmetric solutions for $h_{\mu\nu}$ of the field equations (vide chapter II) when inserted in the Lagrangian [equation (3.1)] via equation (3.2) gives the equation of motion.

$$\frac{1}{2}\,(a_{\mu\lambda,\nu} + a_{\nu\lambda,\mu} + a_{\mu\nu,\lambda})\,\frac{dx^{\mu}}{ds}\,\frac{dx^{\nu}}{ds} + a_{\mu\nu}\,\frac{d^2x^{\mu}}{ds^2} = 0 \tag{3.3}$$

where the parameter 'ds' is taken to be

$$ds = (a_{\mu\nu}\, dx^{\mu}\, dx^{\nu})^{1/2} \tag{3.4}$$

Note once again that the present theory is a flat-space-time one and the last equation and that preceding it have nothing to do with the metric or geodesic of General Relativity (GR). Here raising and lowering of indices are still done by $\eta_{\mu\nu}$ (and not by $a_{\mu\nu}$). We have then shown how the precession of Mercury's orbit, light bending and also the gravitational red-shift can be explained by using explicit forms of $h_{\mu\nu}$. Our results match with the empirical observations.

It is however well known that all such empirical observations refer to the far field. In the asymtotic region the results of GR and that of Newtonian Gravity (NG) do not qualitatively differ much. For example, both bending of light and red-shift can be predicted (for the latter accurately) by NG. Introducing special relativistic force law in NG one can also explain qualitatively the precession of planetary orbits (Bagge 1981; Phipps 1986; Ghosal et al 1987). The real qualitative difference of these two theories of gravitation lies in the deep field. For example a Newtonian gravity does not predict a characteristic radius like Schwarzschild radius in GR. In GR the Schwarzschild radius defines a surface from which no particle even a photon can escape. (In NG however a characteristic radius can be calculated from the escape velocity considerations, but unlike in GR the term "escape" in NG refers to escape of a particle from the field and not from a characteristic surface). The non-Newtonian features of the gravitation may be best obtained by studying particle trajectories near a massive compact object. In an article Pandey and Gupta (1987) brought out the role of "repulsive force" in GR by studying the equation of motion of a test object in a radial trajectory:

$$\frac{d^2r}{dt^2} = -\,MG/r^2\,[1-(2MG/c^2r)]+3MG/c^2r^2[1-(2MG/c^2r)]^{-1}(dr/dt)^2 \quad (3.5)$$

Note that r and t in (3.5) refer to Schwarzschild coordinates and they do not refer to measurements by standard rods and clocks. Thus the above equation which involves coordinate velocity and acceleration does not *directly* give us insight into the true dynamic behaviours of test objects. [For example, due to the presence of "repulsive force" term in (3.5) particle motion is characterised by a terminal velocity; however "proper velocity" (Bose 1980) does not exhibit such an effect]. In SRNG theory there is no such coordinate problem. Since here, r and t in the equation of motion will refer to standard radial length and standard time. This fact will therefore provide a straight forward interpretation of the equation of motion and a lot of insight may be gained by bringing out the real qualitative difference between NG and SRNG in the deep field.

The purpose of the present text is to theoretically probe into the deep field by studying the radial and circular trajectories of a test particle and to study the role of the velocity dependent and repulsive force terms (which have no classical analogue) in SRNG trajectories and finally to compare qualitatively the results with that of GR (in Schwarzschild coordinate). It will be evident that some of the well known general relativistic results which are assumed to be purely general relativistic in origin are also true in SRNG.

We shall work with the solutions of three different field equations. One of which has been proposed by Biswas (1988) [correct solutions obtained by Peters (1990)] where the other two have been suggested in earlier chapter II. Hereafter we refer to these solutions as solutions I,II and III respectively. The non-trivial components of $h_{\mu\nu}$ according to I,II and III are given below

$$h_{00} = \frac{1}{K} \ln \left(1 - \frac{2KGM}{r}\right) \quad \& \quad h_{11} = 0 \qquad \mathrm{I}$$

$$h_{00} = \frac{1}{K} \ln \left(1 - \frac{2KGM}{r}\right) \quad \& \quad h_{11} = -\frac{1}{K} \ln \left(1 + \frac{2KGM}{r}\right) \qquad \mathrm{II}$$

$$h_{00} = h_{11} = \frac{1}{K} \ln \left(1 - \frac{2KGM}{r}\right) \qquad \mathrm{III}$$

where K is the undetermined factor in the non-linear term of the proposed field equations (Biswas 1988; Ghosal & Chakraborty 1991c; Ghosal & Chakraborty 1991d) and M is the mass of the spherically symmetric source. The field equations (in the absence of matter) are given by [eqns. (2.30), (2.42) & (2.36) of chapter II].

$$h_{\mu\nu;\alpha}{}^{;\alpha} = K\, h_{\mu\alpha;\beta}\, h^{\alpha\ ;\beta}_{\nu}$$

$$h_{\mu\nu;\alpha}{}^{;\alpha} - \phi\, \Lambda_{\mu\nu;\alpha}{}^{;\alpha} = K\, (h_{\mu\alpha;\beta}\, h^{\alpha\ ;\beta}_{\nu} - \phi\, \Lambda_{\mu\alpha;\beta}\, \Lambda^{\alpha\ ;\beta}_{\nu})$$

$$H_{\mu\nu;\alpha}{}^{;\alpha} = K\, H_{\mu\alpha;\beta}\, H^{\alpha\ ;\beta}_{\nu}$$

where $\phi = \eta^{\alpha\beta}\, h_{\alpha\gamma}\, \Lambda^{\gamma}_{\beta}$ and $H_{\mu\nu} = h_{\mu\nu} - \frac{1}{2} D^{\alpha}_{\beta}\, h^{\beta}_{\alpha}\, \Lambda_{\mu\nu}$
and the non-zero components of the tensors $\Lambda_{\mu\nu}$ and D^{μ}_{ν} in spherical polar coordinates are given by

$$\Lambda_{11} = 1, \qquad D^{1}_{1} = -D^{0}_{0} = 1.$$

Note that in chapter II we assumed K=1. Under the far field approximation one can verify that the known general relativistic results are *practically* independent of the sign and value of K. In the deep field also the conclusions regarding the qualitative features of a test particle motion will also not depend on the value of K. In absence of any empirical information in the deep field we assume for simplicity K=1.

Although the results and their interpretations can be obtained by studying the equation of motion numerically it is instructive to obtain the results analytically as far as possible. In section 3.3, in order to study the radial motion, solution III will be used for the analytical treatment and numerical computations for I and II will be given just for a quantitative comparison. However we shall see that for red-shift calculations (sec.3.2) and for the study of circular orbits (sec.3.4) all the solutions (I,II & III) can be treated on equal footing.

3.2 THE SURFACE OF INFINITE REDSHIFT

In GR Schwarzschild radius is also characterised by a Surface of Infinite Red-Shift (SIRS) [Bose 1980; Weinberg 1972]. A surface with the similar property is also predicted according to the present theory. However, the characteristic radius (r_c) in this case is expected to be different from the Schwarzschild radius (r_s). To obtain r_c (if it exists) corresponding to the present SRNG we proceed as follows.

We first recall our red-shift formula (Ghosal & Chakraborty 1991c) [vide eqn. (2.51) of chapter II].

$$\nu = \nu'[1+h_{00}]^{1/2} = \nu'[1+\ln(1-\frac{2GM}{r})]^{1/2} \tag{3.6}$$

where ν' denotes the frequency of the emitted photon when the emitter atom is at infinity, and ν is the redshifted frequency of the photon when the emitter atom is situated at r. Note that the above formula is independent of h_{11} and hence the equation (3.6) and the results that follow from it are equally valid for the solutions I, II & III mentioned in the earlier section.

To obtain infinite red-shift we put $\nu=0$ in equation (3.6). The corresponding characteristic radius r_c is given by

$$r_c = \frac{2GM}{1-e^{-1}} \qquad (3.7)$$

The positive definiteness of r_c provides the proof of existence of the SIRS which according to the present theory is given by equation (3.7). Observe that as an atom, which emits photon, approaches the above radius the red-shift becomes increasingly large and finally becomes infinite at this radius. This radius in the present case is directly measurable by rigid standard rods and $r=r_c$ here therefore defines a "physical" singular surface characterised by the impossibility of a photon being emitted from it. From (3.7), numerically r_c is given by $r_c \simeq 1.58\ r_s$. Loosely speaking the size of a SRNG Black-Hole is slightly more than that of the relativistic one.

3.3 Radial motion near SIRS

The essential non-Newtonian features of the SRNG field may be best understood by studying the radial motion of a test particle. Near SIRS the non-Newtonian character of the field is expected to be more pronounced.

We first write down the energy integral for the particle trajectory [eqn. (2.46) of chapter II].

$$E = \frac{m_0 (-a_{00})^{1/2}}{[1+(a_{11}/a_{00})v^2]^{1/2}} \tag{3.8}$$

Or, rearranging one obtains

$$g = -\frac{a_{00}}{a_{11}} (1+fa_{00}) \tag{3.9}$$

where $g=v^2$ & $f=m_0^2/E^2$. Note that the constant E can be interpreted as the total energy of the particle in the gravitational field (Ghosal & Chakraborty 1991c). After substituting the values of a_{00} and a_{11} (using solution III) equation (3.9) reduces

$$fY^2 - Y(1+g) + 2g = 0 \tag{3.10}$$

where $Y = 1+X$ and $X = \ln(1-\frac{2GM}{r})$

The last equation gives the dependence of velocity of the test particle as a function of its distance from the origin with energy as a parameter. One of the roots of the above quadratic equation is given by

$$Y = \frac{(1+g)-[(1+g)^2-8gf]^{1/2}}{2f} \tag{3.11}$$

With the aid of (3.11) we may now enquire whether we have a unique surface in the deep field (similar to that one finds in GR) where the velocity of a particle irrespective of its initial energy will attain in its free fall a value equal to zero (which corresponds to g=0).

First we consider the case when the test object starts from infinity with zero velocity. Clearly at infinity $a_{00}=-1$ and from equation (3.8) putting v=0, we get $f=m_0^2/E^2=1$. Then by putting g=0 in equation (3.11) one obtains the null velocity surface which comes out to be the same as the SIRS:

$$r = \frac{2GM}{1-e^{-1}} \tag{3.7}$$

The other root of equation (3.11) corresponds to $r=\infty$. This result is of course trivial by virtue of the chosen initial condition. Thus we see that a test particle initially at rest at infinity will come to rest again at the radius $r\simeq 1.58r_s$ which, as we have already seen, defines the surface of infinite redshift.

Now one may ask whether it is possible for a test particle to be able to cross this surface (i.e. $r\simeq 1.58r_s$) if its energy is increased arbitrarily. We shall see that this is not possible. To understand this, return again to (3.11) in order to determine the null-velocity-surface (characterised by g=0), but this time we do not put f=1. This gives again two values of r of which one is again $r\simeq 1.58r_s$ and is independent of energy (f). The other root is given by

$$r = \frac{2GM}{1-e^{n-1}} \tag{3.12}$$

where $n=1/f$.

Now note that r should be positive by definition. To ensure this, as it follows from (3.12), f must be greater than unity. Clearly this is impossible since the maximum value of f ($= m_0^2/E^2$) is unity. Thus the second root of r of (3.11) is to be discarded. It is therefore evident that even if the energy of the particle increases indefinitely it cannot cross the surface $r \simeq 1.58 r_s$. We experience in Schwarzschild coordinates a similar thing to happen in GR too. However, there is a subtle difference. In GR the singular surface characterised by the metric (field) and the surface marked by the peculiarity of the test particle motion coincide. But in SRNG, though the field singularity is still at $r=2GM$, it is bounded from above by an event-horizon-like-surface $r \simeq 1.58 r_s$.

Now we can have a picture of the radial free fall of a test particle towards a spherically symmetric source. As a particle falls from infinity the velocity is increased due to the gravitational attraction but as the velocity is increased the gravitational acceleration gets reduced and finally the particle gets decelerated and its velocity becomes zero at the SRNG horizon (i.e at $r=r_c$). It is therefore clear that the velocity field (v as a function of r) should display a "maximum" during the free fall.

In order to estimate the position of these maxima (as a function of particle energies) we return to the equation (3.10) and differentiate it with respect to Y.

$$\frac{dg}{dY} = \frac{2fY - (g+1)}{Y-2} \tag{3.13}$$

which should be equal to zero for the "maximum" to occur. Again from equation (3.10) one may write

$$g = \left[\frac{Y(fY-1)}{Y-2}\right] \tag{3.14}$$

which when inserted in (3.13) gives

$$\frac{dg}{dY} = \frac{fY^2-4fY+2}{Y-2} \tag{3.15}$$

Now putting dg/dY = 0 in (3.15) one obtains

$$fY^2 - 4fY + 2 = 0 \tag{3.16}$$

the physically meaningful root of which is given by

$$Y_m = \frac{2f-(4f^2-2f)^{1/2}}{f} \tag{3.17}$$

(The other root of (3.16) has been discarded since it corresponds to r<0). The value of Y_m obtained from (3.17) as function of f gives the radial position of the maxima under study. The value of Y_m , when substituted in equation (3.14) give the maximum velocity that a particle attains. The results of these calculations are given in table I.

TABLE I

$f(=m_0^2/E^2)$	The radial position (r_m) where the maximum velocity occurs (in unit of 2GM)	Maximum velocity (c=1)
0.60	-4.96	0.65
0.62	-7.83	0.62
0.64	-14.99	0.60
0.66	-65.00	0.58
0.67	129.58	0.57
0.77	5.98	0.51
0.87	3.78	0.46
1.00	2.98	0.41

From the table it is clear that a particle having energy more than a critical value E_c characterised by a value of f which lies between 0.66 & 0.67, does not exhibit velocity maxima in the region $r>0$. Physically this means that the gravitational force on a particle (thinking in Newtonian way) having energy more than E_c is always repulsive. For energies less than E_c however, the maxima begin to occur. From the table it is also evident that as the particle energy decreases the maxima get nearer to the singular surface ($r \simeq 1.58r_s$). However the value of the maximum velocity becomes lesser. Similar tables may be obtained for solutions I and II also. However, for the purpose of comparison it is better to study the (r−v) diagrams for all the solutions (Figs. I, II & III). In these diagrams r and v have been plotted numerically, with f as the parameter, by directly using eqn. (3.9). The figures I, II & III are basically of the same nature except some minor quantitative variations. The graphs indicate the general features of the test particle's free fall trajectories which may be summarised as follows:

A particle starting from a large distance with a vanishingly small velocity (energy $f=1$) will first be accelerated due to the gravitational attractive force (which is asymtotically Newtonian in nature) and as the velocity is increased the particle gets retarded because of a velocity dependent repulsive force and finally it comes to rest at a point $r=r_c \simeq 1.58r_s$. Note that this value of r_c not only is independent of energy but also is independent of the solutions considered. For increased particle energies the gravitational attraction gets gradually replaced by repulsion and complete deceleration takes over. This phenomenon is marked by the gradual flattening of (r−v) curves as f decreases.

We have already indicated that the above mentioned qualitative features of radial trajectories though akin to general relativistic predictions in Schwarzschild coordinates, have no classical analogue.

To bring out another non-classical feature let us study the question of existence of circular orbits.

3.4 The Existence of Circular Orbits

The first integrals that follow from the Lagrangian given in sec. 3.1 (eqn.(3.1)) are given by

$$a_{00}\,\bar{u}^{\,0} = \varepsilon \tag{3.18}$$

$$a_{33}\,\bar{u}^{\,3} = p_{\phi} \tag{3.19}$$

where the constants ε and p_{ϕ} are proportional to energy and angular momentum of the particle respectively. The other integral is provided by the definition of the parameter 'ds' (eqn.(3.4)) which after rearrangement gives

$$a_{00}(\bar{u}^{\,0})^2 + a_{11}(\bar{u}^{\,1})^2 + r^2(\bar{u}^{\,3})^2 = 1 \tag{3.20}$$

where we have assumed θ = constant as usual. The equationns (3.18), (3.19) & (3.20) together give

$$(dr/dt)^2 = (a_{00}^{\;2}/a_{11}\varepsilon^2) - (a_{00}/a_{11}) - (p_{\phi}^{\;2}a_{00}^{\;2}/a_{11}r^2\varepsilon^2) \tag{3.21}.$$

The last equation would represent circular orbit provided, the r.h.s and its first derivative with respect to r vanish (Bose 1980). They together give the following expression for the particle energy as a function of the orbit radius R

$$- \frac{1}{\varepsilon^2} = \frac{R}{2} \frac{a'_{00}(R)}{a_{00}^2(R)} - \frac{1}{a_{00}(R)} - \frac{m_0^2}{E^2} \tag{3.22}$$

where we have put $- m_0^2 \varepsilon^2 = E^2$ (Ghosal & Chakraborty 1991c).

Note that the above condition is also independent of h_{11} and therefore is valid for all the solutions of $h_{\mu\nu}$ mentioned in sec. 3.1.

Written explicitly the equation (3.22) takes the following form

$$\frac{m_0^2}{E^2} = \frac{2R\ \{1-(1/R)\}\ [1 + \ln\ \{1-(1/R)\}] - 1}{2R\ \{1-(1/R)\}\ [1 + \ln\ \{1-(1/R)\}]^2} \tag{3.23}$$

The existence of circular orbit will be ensured as long as E is real. Numerically the limiting radius turns out to be approximately equal to $2.23r_s$ below which, circular orbits are not possible. This limiting radius is slightly higher than that one would have obtained for GR (Narlikar 1978).

3.5 SUMMARY

In the light of a recent Lorentz covariant theory of gravitation, the radial and the circular trajectories of test objects have been studied in a static spherically symmetric situation. It has been found that the gravitational field is characterised by a characteristic radius $r_c = 1.58r_s$ (r_s= Schwarzschild radius) which defines the surface of infinite red-shift. For a radial free fall it has been shown that a particle coming from a large distance first gets accelerated

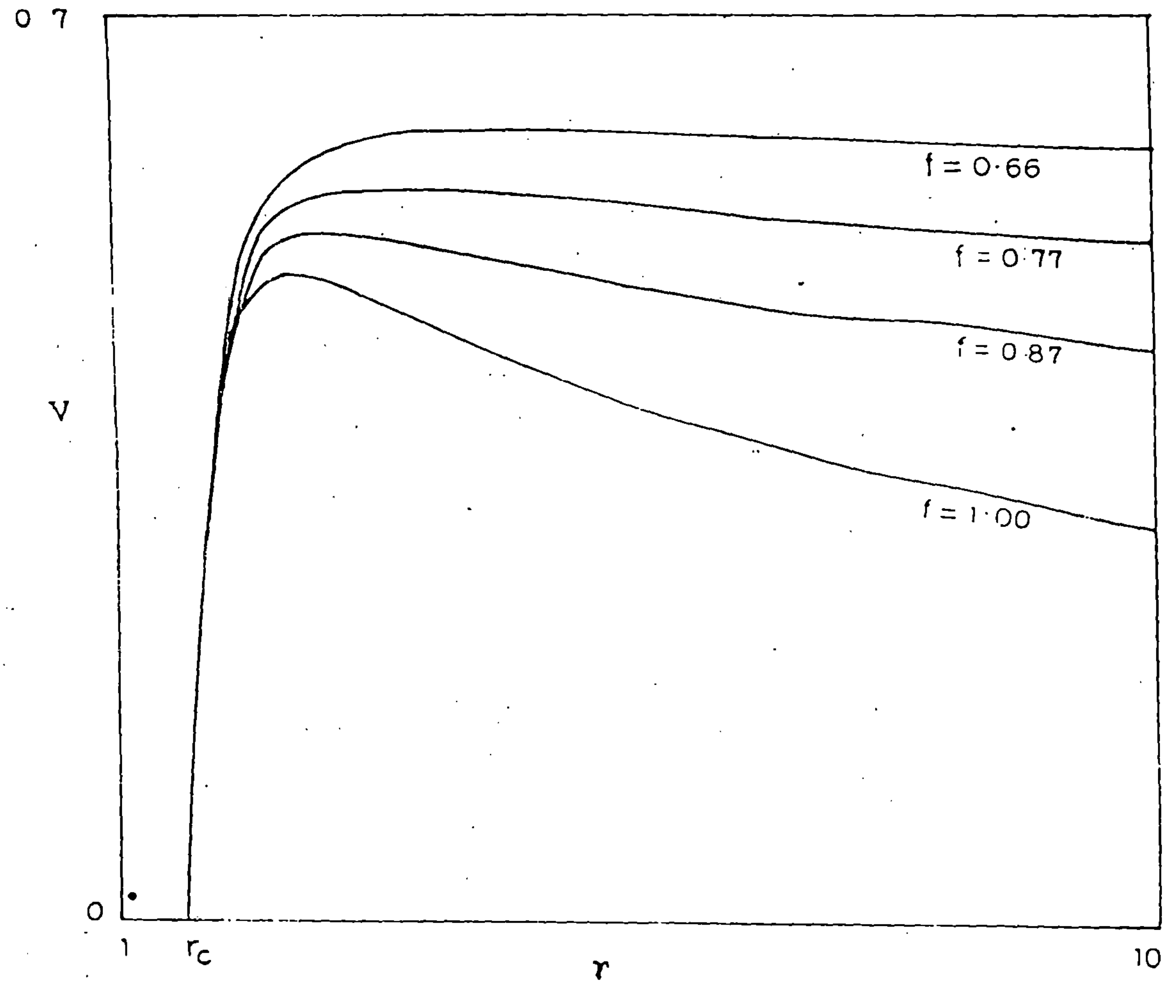

Fig I.

r–v curves for different particle energies (Solution I).

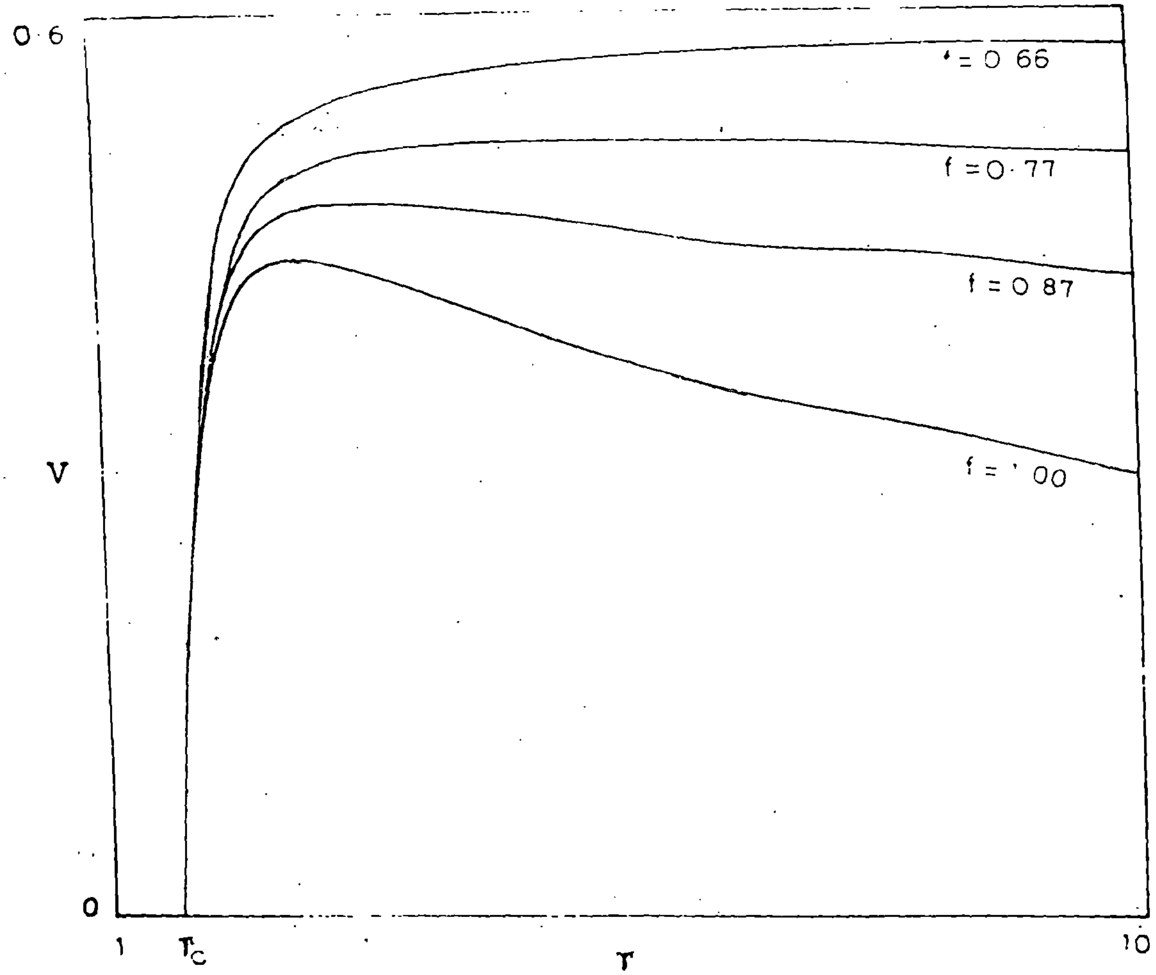

Fig II

r–v curves for different particle energies (Solution II).

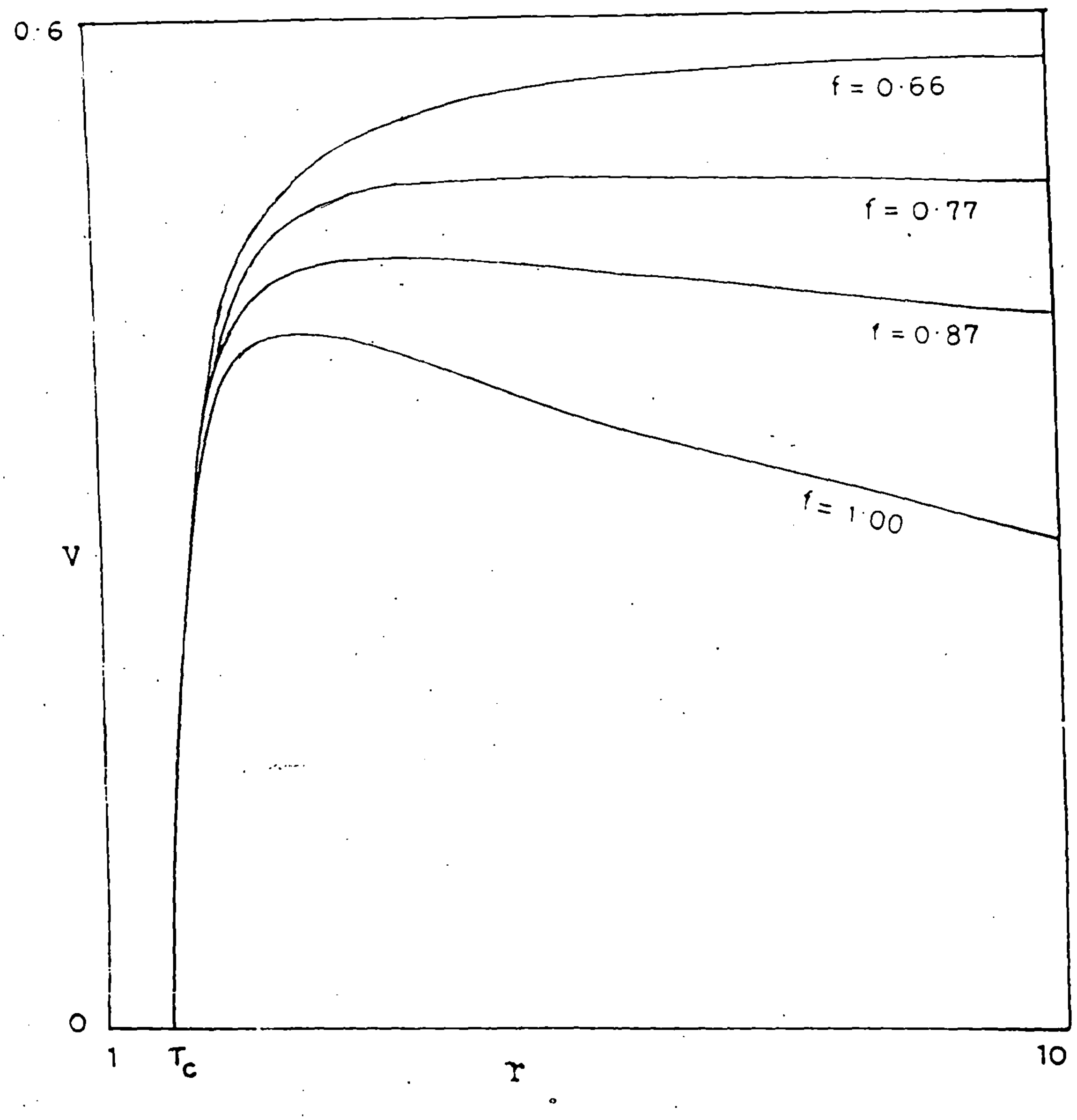

Fig III.

r-v curves for different particle energies (Solution III).

towards the source. However, as the velocity increases and the particle penetrates deep into the field, the - non-Newtonian features of gravity begin to show up. From some point along the radial trajectory, depending on the initial energy, the particle starts getting retarded and finally stops at r_c. It is therefore observed that the radial fall in general is characterised by a "terminal velocity" in the velocity field. Another non-Newtonian character of the present flat-space-time gravity concerns the question of existence of circular orbits. Calculations revealed that circular orbits cannot exist below a limiting radius which is approximately equal to $2.23r_s$.

3.6 EPILOGUE

In chapter II, in connection with discussions on gravitational red-shift we have seen that the concept of global inertial frame is not inconsistent with gravity even though Schield (1960) argued on the contrary. Having been taken care of the arguments against the existence of global inertial frames in a gravitational field, the problem seems to be over and one can happily declare that gravitation and special relativity can (logically at least) go together. But that is not to be. The objection may come from other directions. In a gravitational field light assumes a curved path. Hence one may ask how one is going to synchronize distant clocks in a given inertial frame according to Einstein's prescription (Winnie 1970) which implicitly assumes the rectilinear propagation of light.

Again there is another problem. According to SRNG, even locally the speed of light is not equal to c inside a gravitational field. To understand this quantitatively let us consider for example the radial trajectory of a photon, which can directly be obtained from (2.45) by substituting dt/ds by $(a_{00}\varepsilon')^{-1}$ where the constant of motion $\varepsilon' = 1/\varepsilon$ is defined through the relation (2.21). One thus obtains the following integral of motion for a photon.

$$\varepsilon' = \frac{[1+(a_{11}/a_{00})v^2]^{1/2}}{(-a_{00})^{1/2}} \qquad (3.24).$$

Inserting explicitly the expressions for a_{00} & a_{11} one writes

$$\varepsilon' = \frac{[1-\frac{1-\ln(1-2GM/r)}{1+\ln(1-2GM/r)}\ v^2]^{1/2}}{[\ 1 + \ln(1-2GM/r)\ ]^{1/2}} \qquad (3.25).$$

The condition that as $r\to\infty$ (where gravitation is absent), $v\to1$ (i.e the speed of light equals to c) gives $\varepsilon'=0$. Therefore, from (3.25) one obtains the radial velocity of light at any point r as

$$v^2 = \frac{1+\ln(1-2GM/r)}{1-\ln(1-2GM/r)} \qquad (3.26).$$

The relation clearly illustrates that the speed of light is a function of r.

Now is it consistent with special relativity? The answer is apparently no, since it is known that the special relativity rests on the well-known Constancy of Velocity of Light (CVL) assumption!

These two problems to our knowledge are never addressed in the literature. The answer to these problems cann't be fully given till we come to the end of the present volume. However, at this point it will be enough to say that the first problem i.e the problem of synchrony arising out of non-rectilinear propagation of light can be easily dealt with by virtue of the Conventionality of distant Simultaneity thesis of Reichenbach-Grunbaum, which will form the focal theme of the next part of this volume. It will be noted that Einstein's choice of distant clock synchrony is nothing but a convention.

It will be seen that the problem arising out of the non-constancy of the optical speed can also be taken care of. The question of CVL and its role in special relativity will be dealt thoroughly in chapter VII. It will be observed that one can always arrive at the standard form of special relativity even if one starts with the description of space-time in a medium (Ghosal et al 1992) where the optical speed is not equal to c, the speed of light in empty space.

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PART - II

CHAPTER - IV

PASSAGE FROM EINSTEINIAN TO GALILEAN RELATIVITY AND CLOCK SYNCHRONY

4.1 INTRODUCTION

In most of the text books of Special Relativity (SR) there seems to be a prevailing belief that special relativity goes over to Galilean Relativity (GR) for relative speeds that are very small compared to the speed of light in vacuum. This belief is *typically* expressed by Rindler (1979) in the form that as comparison of Lorentz Transformations (LT) with Galilean Transformations (GT) shows, the GT approximates well to the LT when v that is relative velocity is small. Bergmann (1969) observed that for small values of v/c, the Lorentz transformation equations are approximated by the Galilean transformations. Similar belief may be found to be expressed by other authors as well (Kacser 1967; Sokolovsky 1962). But this belief leads to an interesting fallacy and it is shown that the resolution of this fallacy lies in the proper understanding of the role of clock synchronization convention adopted by Einstein. In this chapter it has been shown that a misconception could easily arise that would stem from overlooking the role of conventionality ingredients of special theory of relativity . We have shown first that the small velocity approximation cannot alter the convention of distant simultaneity. In course of discussion the approximated Lorentz transformations are critically compared, under the same approximation, with two other space-time transformations, one of which represents an Einstein world with Galilean synchrony whereas the other describes a Galilean world with Einsteinian synchrony.

4.2 CLOCK SYNCHRONY AND ROUTE TO GALILEAN RELATIVITY

The assumption, that under small velocity approximation, SR goes over into GR is not strictly correct and the aim of the present article is to demonstrate this statement. And we feel that the most straightforward approach is to start from a fallacy posed below, which the students of relativity may find interesting.

Consider two events $E_1:(x_1,t_1)$ and $E_2:(x_2,t_2)$ in an inertial frame S. Represented in a Minkowski diagram, the invariant interval between these two events is

$$\Delta s^2 = (\Delta x_1)^2 + (\Delta x_2)^2 + (\Delta x_3)^2 - c^2(\Delta t)^2$$

$$= (\Delta \bar{x}_1)^2 + (\Delta \bar{x}_2)^2 + (\Delta \bar{x}_3)^2 - c^2(\Delta \bar{t})^2 \qquad (4.1)$$

where $\Delta x_i = x_{i2} - x_{i1}$, $\Delta t = t_2 - t_1$ and bars represent the corresponding quantities in another reference frame $\bar{S}$ moving relative to S with the uniform non-zero speed v. If v^2/c^2 is neglected and if it were true that LT goes over into GT for $v^2/c^2 \to 0$, then one would usually expect the time to be absolute i.e it should hold that $\Delta \bar{t} = \Delta t$. It follows then from the equation (4.1) that

$$(\Delta x_1)^2 + (\Delta x_2)^2 + (\Delta x_3)^2 = (\Delta \bar{x}_1)^2 + (\Delta \bar{x}_2)^2 + (\Delta \bar{x}_3)^2.$$

This appears to be all very fine since it looks as if we are merely going from Minkowski metric to Euclidean metric. But this is only an illusion and students often make such a mistake. We will see that this leads to a contradiction since, according to GT

$$\bar{x} = x - vt, \qquad \bar{y} = y, \qquad \bar{z} = z, \qquad \bar{t} = t \qquad (4.2).$$

So that

$$\Delta \bar{x} = \Delta x - v\Delta t, \quad \Delta \bar{y} = \Delta y, \quad \Delta \bar{z} = \Delta z, \quad \Delta \bar{t} = \Delta t$$

and clearly, for any two non-simultaneous ($\Delta t \neq 0$) events, $(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2$ is not an invariant under equation (4.2). The above fallacious situation can not be resolved unless one rejects the notion that alone the neglect of v^2/c^2 in LT leads to Galilean Relativity. Indeed, if v^2/c^2 is neglected in the Lorentz factor, the LT reduces to the Approximate Lorentz Transformation [ALT] (Landau and Lifshitz 1975).

$$\bar{x} = x - vt, \qquad \bar{t} = t - (vx/c^2). \tag{4.3}$$

Thus, for any pair of events

$$\Delta\bar{x} = \Delta x - v\Delta t, \qquad \Delta\bar{t} = \Delta t - (v/c^2)\Delta x \tag{4.4}$$

Notice here that for any chosen spatial separation Δx between two events, we can take v sufficiently small, so that Δt becomes very large compared to $(v/c^2)\Delta x$ and hence the latter may be neglected implying $\Delta\bar{t}=\Delta t$. On the other hand, the approximation $v^2/c^2 \ll 1$ is certainly not dependent on the space time separation of two arbitrary and independent events. In fact, for any preassigned small value of v, one is free to consider a pair of sufficiently distant events so that one cannot ignore the $(v/c^2)\Delta x$ term in (4.4). Therefore absolute nature of distant simultaneity ($\Delta\bar{t}=\Delta t$) can never be retrieved. That is, simultaneity is still relative. This means, that distant events which are simultaneous in a given inertial frame of reference are not simultaneous events in any other inertial frame in constant (non-zero) motion with respect to the first. This is not surprising since we should realize that the relative character of distant simultaneity is the result of a synchronization convention (Reichenbach 1957; Grünbaum 1963; Winnie 1970; Mansouri and Sexl 1977; Sjödin 1979,1980,1982; Podlaha 1980; Ghosal and Mukhopadhyay 1984).

Recent analyses of the special theory initiated by Reichenbach (1924,1957) and carried on by Grünbaum (1963) have brought attention to the status of simultaneity *within* an inertial frame of reference by virtue of their claim that the relation of simultaneity *within* each inertial reference frame contains an ineradicable element of convention which reveals itself in our ability to select (within certain limits) the value to be assigned to the one-way speed of light in that inertial frame. This thesis, which shall here be called the thesis of the *Conventionality of distant Simultaneity* (the C-S thesis), focuses directly upon the conventions attaching to distant *simultaneity* within the special theory. A convention once chosen *a priori*, is unlikely to change into a different one, merely due to an approximative assumption on the relative velocity alone.

Let us recall that the standard Einstein synchronization procedure requires that the spatially distant clocks to be so adjusted that in any *given* inertial frame the to & fro speeds of light appear to be the same and equal to the round-trip speed of light (Reichenbach 1957; Grünbaum 1963; Winnie 1970; Mansouri and Sexl 1977; Sjödin 1979,1980,1982; Podlaha 1980; Ghosal and Mukhopadhyay 1984). In this context it is now worthwhile to examine,in some detail the nature of ALT [equation (4.3) for *all* v.

The velocity addition laws can be obtained from (4.3) as

$$\bar{W}_x = (W_x - v)/(1 - vW_x/c^2), \qquad \bar{W}_y = W_y/(1 - vW_x/c^2), \qquad \bar{W}_z = W_z/(1 - vW_x/c^2)$$

As expected, W_y and W_z do not transform as in SR. Now, if a light pulse is sent back and forth along the x-direction alone, that is,

$$W_x = \pm c \quad \text{and} \quad W_y = W_z = 0$$

then the to & fro speed of light in $\bar{S}$, parallel to the direction of motion, is given by

$$\bar{C}_{\parallel} = \pm c \tag{4.5}$$

If, on the other hand, a light pulse is sent back and forth in S in such a direction that the signals travel back and forth only in the y-direction in $\bar{S}$, then

$$\bar{W}_x = \bar{W}_z = 0.$$

Now using the fact that $W_x^2+W_y^2= C^2$ in S, one obtains the speed of light in $\bar{S}$, perpendicular to the direction of motion, the value

$$C_{\perp} = \pm \frac{c}{(1-v^2/c^2)^{1/2}} \tag{4.6}$$

These results, i.e. equations (4.5) and (4.6), certainly do not agree with the corresponding classical results unless v = 0 strictly (NB, the classical result $C_{\parallel}= c(1 \pm \frac{v}{c})$ differs from equation (4.5) in the first order of v/c !). Furthermore, from equations (4.5) and (4.6) we see that the to and fro speeds are individually equal both in the longitudinal direction and in the transverse direction. In fact, it can be shown that the same conclusion holds also for any arbitrary direction in $\bar{S}$. This is precisely the *standard synchronization* convention. We call a synchronization *standard synchronization* in a given direction if it renders the one-way velocity of light in that direction equal to the one-way velocity in the opposite direction. Thus Einsteinian synchrony inherent in LT is preserved (even under the approximation $v^2/c^2 \ll 1$). This is exactly in accordance with our earlier assertion.

However, one may still suspect whether the transformation (4.3) represents a Galilean world in essence, save the synchronization convention. In order to decide this, one must compare synchrony independent quantities obtained from (4.3) with those obtained from the usual Galilean transformations. One such quantity is the round trip speed of any signal. In fact, two sets of transformations may appear structurally very different depending on the choice of synchrony, but when synchrony independent quantities are compared one might discover that they are essentially the same. In that case we say that these two transformations represent the same kinematical "World". From the Galilean transformation, it follows that the two-way average speed of light in the direction parallel and perpendicular to the direction of relative motion are given, respectively, by

$$\overset{\leftrightarrow}{C}_{\parallel} = c(1 - v^2/c^2) \tag{4.7}$$

and

$$\overset{\leftrightarrow}{C}_{\perp} = c\ (1 - v^2/c^2)^{1/2} \tag{4.8}$$

Whereas we see from (4.5) and (4.6) that they are given by

$$\overleftrightarrow{C}_{\parallel} = c \qquad (4.9), \qquad \overleftrightarrow{C}_{\perp} = c/(1-v^2/c^2)^{1/2} \qquad (4.10)^{*}.$$

Thus, eqn. (4.3) for all v in general, does not represent a Galilean World (GW). Of course one may choose $v^2/c^2 \ll 1$ again in equations (4.7), (4.8) and (4.10), and it becomes clear that (4.3) represents GW approximately. But then there is a subtle point that must be carefully noted. The resulting GW is not a GW *in totality* but it is limited by the very approximation. To exemplify this point, consider the Tangherlini Transformation (TT), which represents an Einstein World (EW) with absolute (Galilean) synchrony (Tangherlini 1961):

$$\bar{x} = (x-vt)/(1-\beta^2)^{1/2}, \text{ with } \beta = v/c, \quad \bar{t} = t(1-\beta^2)^{1/2} \qquad (4.11).$$

Note here that if $v^2/c^2 \ll 1$, the resulting transformations represent a GT in totality. This is expected because we mentioned before that any set of transformations depends structurally on the choice of synchrony. Since here we consider Galilean synchrony it is natural that under the condition $\beta^2 \ll 1$ it gives GT in totality. Obviously, this fact is absent in (4.3). Hence it proves again that a convention once chosen does not change into a different one due to an approximate assumption on the relative velocity alone.

Thus we have demonstrated that the LT does not lead under

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*Note that the eqns. (4.9) & (4.10) refer to two-way speeds whereas $C_{\parallel}$ & $C_{\perp}$ of equations (4.5) & (4.6) refer to one-way speeds. The expressions do not differ however because of standard synchrony.

the small velocity approximation to Galilean (absolute) synchrony. As a result, the Galilean transformation law for *one – way* velocities could not be obtainted unless v = 0 strictly. However, eqn. (4.3) represents a GW only for small velocities but not for the entire velocity range, in contrast to the Tangherlini case just mentioned above.

Finally, one may raise the question whether it is at all possible to construct a transformation which represents a GW in totality having Einstein (standard) synchrony. Indeed, one may verify that the transformation (ZST) due to Zahar and Sjödin (Sjödin 1979; Ghosal and Mukhopadhyay 1984; Zahar 1977)*, satisfies the above characteristics which are just complementary to those of the Tangherlini Transformations.

$$\bar{x} = x - vt \qquad \text{and} \qquad \bar{t} = \frac{t-(vx/c^2)}{(1-v^2/c^2)} \tag{4.12}$$

It is evident that the (ZST) transformation reduces to ALT from (4.3) if the v^2/c^2 term is neglected. Note that here again the Poincare-Einstein synchrony is preserved.

Thus we see that LT under the small velocity approximation does not go over to GT but instead, it becomes, as it should be equivalent to ZST from (4.12) under the same approximation. In contrast, TT from (4.11) directly goes over to GT. Therefore, in order to fully comprehend the passage of SR to GR one should examine LT vis-a-vis ZST and GT vis-a-vis TT in the context of the small speed approximation.

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*Tangherlini transformations and Zahar transformations have been discussed in some detail in the last chapter of this volume also.

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Passage from Einsteinian to Galilean Relativity and Clock Synchrony

S. K. Ghosal *, K. K. Nandi **, and Papia Chakraborty *

University of North Bengal, Darjeeling (W. B.), India

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There is a general belief that under small velocity approximation, Special Relativity goes over into Galilean Relativity. Should this be interpreted exclusively in terms of the kinematical symmetry transformations (Lorentz vs. Galilei) a misconception could easily arise that would stem from overlooking the role of conventionality ingredients of Special Relativity Theory. It is observed that the small velocity approximation cannot alter the convention of distant simultaneity. In order to exemplify this point further, the Lorentz transformations are critically compared, under the same approximation, with two other space time transformations, one of which represents an Einstein world with Galilean synchrony whereas the other describes a Galilean world with Einsteinian synchrony.

There seems to be a prevailing belief that Special Relativity (SR) goes over to Galilean Relativity (GR) for relative speeds that are very small compared to the speed of light in vacuum [1–4]. The belief is *typically* expressed in the form that the Lorentz Transformation (LT) goes over to the Galilean Transformation (GT) when β^2 terms, where $\beta = v/c$, are neglected in LT [1, 2]. This assumption, however, is not strictly correct. The aim of the present paper is to demonstrate this statement. We feel that the most straightforward approach is to start from an interesting fallacy posed below.

Consider two events E_1: (x_1, t_1) and E_2: (x_2, t_2) in an inertial frame S. Represented in a Minkowski diagram, the invariant interval between these two events is

$$\begin{aligned} \Delta s^2 &= (\Delta x_1)^2 + (\Delta x_2)^2 + (\Delta x_3)^2 - c^2 (\Delta t)^2 \\ &= (\Delta \bar{x}_1)^2 + (\Delta \bar{x}_2)^2 + (\Delta \bar{x}_3)^2 - c^2 (\Delta \bar{t})^2, \end{aligned} \tag{1}$$

where $\Delta x_i = x_{i2} - x_{i1}$, $\Delta t = t_2 - t_1$ and bars represent the corresponding quantities in another reference frame $\bar{S}$ moving relative to S with the uniform non-zero speed v. If β^2 is neglected and if it were true that LT goes over into GT for $\beta^2 \to 0$, then it should hold that $\Delta \bar{t} = \Delta t$. It follows then from (1) that

$$(\Delta x_1)^2 + (\Delta x_2)^2 + (\Delta x_3)^2 = (\Delta \bar{x}_1)^2 + (\Delta \bar{x}_2)^2 + (\Delta \bar{x}_3)^2.$$

This leads to a contradiction since, according to GT

$$\Delta \bar{x} = \Delta x - v \Delta t, \quad \Delta \bar{y} = \Delta y, \quad \Delta \bar{t} = \Delta t,$$

and clearly, for any two non-simultaneous ($\Delta t \neq 0$) events, $(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2$ is not an invariant. The above argument can not be resolved unless one rejects the notation that alone the neglect of β^2 in LT leads to Galilean Relativity. Indeed, if β^2 is neglected in the Lorentz factor, the LT reduces to the Approximate Lorentz Transformation (ALT) [5].

$$\bar{x} = x - v t, \quad \bar{t} = t - (v x / c^2). \tag{2}$$

Thus, for any pair or events

$$\Delta \bar{x} = \Delta x - v \Delta \bar{t}, \quad \Delta \bar{t} = \Delta t - (v/c^2) \Delta x. \tag{3}$$

Notice here that for any chosen spatial separation Δx between two events, we can take v sufficiently small, so that Δt becomes very large compared to $(v/c^2) \Delta x$ and hence the latter may be neglected implying $\Delta \bar{t} = \Delta t$. On the other hand, the approximation $v^2/c^2 \ll 1$ is certainly independent of the space time separation of two arbitrary and independent events. In fact, for any preassigned small value of v one is free to consider a pair of sufficiently distant events so that one cannot ignore the $(v/c^2) \Delta x$ term in (3). Therefore absolute nature of distant simultaneity ($\Delta \bar{t} = \Delta t$) can never be retrieved. That is, simultaneity is still relative. This is not surprising since we should realize that the relative character of distant simultaneity is the result of a synchronization convention [6–13]. A convention once chosen a priori is unlikely to change into a different

* Department of Physics.
** Department of Mathematics.

Reprint requests to Dr. S. K. Ghosal, Department of Physics, University of North Bengal, Darjeeling (W. B.) 734430, Indien.

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one merely due to an approximative assumption on the relative velocity alone.

Let us recall that the standard Einstein synchronization procedure requires spatially distant clocks to be so adjusted that in any given inertial frame the to and fro speeds of light appear to be the same and equal to the round trip speed of light [6–12]. In this context it is now worthwhile to examine, in some detail the nature of ALT(2) for *all v*.

The velocity addition laws can be obtained from (2) as

$$\overline{W}_x=(W_x-v)/[1-(vW_x/c^2)],$$
$$\overline{W}_y=W_y/[1-(vW_x/c^2)].$$

As expected, W_y does not transform as in SR. Now, if a light pulse is sent back and forth along the x-direction alone, the to and fro speed of light in $\overline{S}$, parallel to the direction of motion, is given by

$$C_{\parallel}=c\,. \tag{4}$$

If, on the other hand, a light pulse is sent back and forth in S in such a direction that the signals travel back and forth only in the y-direction in $\overline{S}$, one obtains, using the fact that $W_x^2+W_y^2=C^2$ in S, for the speed of light in $\overline{S}$, perpendicular to the direction of motion, the value

$$C_{\perp}=\frac{c}{(1-\beta^2)^{1/2}}\,. \tag{5}$$

These results, i.e. (4) and (5), certainly do not agree with the corresponding classical results unless $v=0$ strictly (NB, the classical result $C_{\parallel}=c(1\pm\beta)$ differs from (4) in the first order of β!). Furthermore, from (4) and (5) we see that the to and fro speeds are individually equal both in the longitudinal direction and in the transverse direction. In fact, it can be shown that the same conclusion holds also for any arbitrary direction in $\overline{S}$. This is precisely the standard synchronization convention. Thus Einsteinian synchrony inherent in LT is preserved (even under the approximation $\beta^2\ll 1$). This is exactly in accordance with our earlier assertion.

However, one may still suspect whether the transformation (2) represents a Galilean world in essence, save the synchronization convention. In order to decide this, one must compare synchrony independent quantities obtained from (2) with those obtained from the usual Galilean transformations. One such quantity is the round trip speed of any signal. In fact, two sets of transformations may appear structurally very different depending on the choice of synchrony, but when synchrony independent quantities are compared one might discover that they are essentially the same. In that case we say that these two transformations represent the same kinematical "World". From the Galilean transformation, it follows that the two-way average speed of light in the direction parallel and perpendicular to the direction of relative motion are given, respectively, by

$$\overset{\leftrightarrows}{C}_{\parallel}=c(1-\beta^2) \tag{6}$$

and

$$\overset{\leftrightarrows}{C}_{\perp}=c(1-\beta^2)^{1/2}, \tag{7}$$

whereas we see from (4) and (5) that they are given by

$$\overset{\leftrightarrows}{C}_{\parallel}=c\,, \tag{8}$$

$$\overset{\leftrightarrows}{C}_{\perp}=c/(1-\beta^2)^{1/2}. \tag{9}$$

Thus, (2) for all v in general, does not represent a Galilean World (GW). Of course one may choose $\beta^2\ll 1$ again in (6), (7), and (9), and it becomes clear that (2) represents a GW approximately. But then there is a subtle point that must be carefully noted. The resulting GW is not a GW *in totality* but it is limited by the very approximation. To exemplify this point, consider the Tangherlini Transformation (TT), which represents an Einstein World (EW) with absolute (Galilean) synchrony [14]:

$$\bar{x}=(x-vt)/(1-\beta^2)^{1/2},\quad \bar{t}=t(1-\beta^2)^{1/2}. \tag{10}$$

Note here that if $\beta^2\ll 1$, the resulting transformations represent a GT in totality. Obviously, this fact is absent in (2).

Thus we have demonstrated that the LT does not lead under the small velocity approximation to Galilean (absolute) synchrony. As a result, the Galilean transformation law for *one way* velocities could not be obtained unless $v=0$ strictly. However, (2) represents a GW only for small velocities but not for the entire velocity range, in contrast to the Tangherlini case just mentioned above.

Finally, one may raise the question whether it is at all possible to construct a transformation which represents a GW in totality having Einstein synchrony. Indeed, one may verify that the transformation (ZST)

$$\bar{x}=x-vt,\quad \bar{t}=\frac{t-(vx/c^2)}{1-\beta^2}, \tag{11}$$

due to Zahar and Sjödin [10, 12, 15], satisfies the above characteristics which are just complementary to those of the Tangherlini Transformation. It is evident that the ZST transformation reduces to ALT from (2) if the β^2 term is neglected. Note that here again the Poincaré-Einstein synchrony is preserved.

Thus we see that LT under the small velocity approximation does not go over to GT but instead, it becomes, as it should be equivalent to ZST from (11) under the same approximation. In contrast, TT from (10) directly goes over to GT. Therefore, in order to fully comprehend the passage of SR to GR one should examine LT vis-a-vis ZST and GT vis-a-vis TT in the context of the small speed approximation.

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CHAPTER - V

CONVENTIONALITY OF DISTANT SIMULTANEITY AND LIGHT SPEED INVARIANCE

5.1 INTRODUCTION

In section 1.3 of chapter I we have introduced the *Conventionality of Simultaneity* thesis (C-S thesis) proposed by Reichenbach (1958) and Grünbaum (1963). It has been remarked that various misconceptions and prejudices that still prevail in Special Relativity (SR) arise mainly out of two reasons : (1) Misconstruing of the subtlities of the C-S thesis and (2) Overlooking of the C-S thesis. In chapter IV we have dicussed one common misconception which arises out of (2). Here we give an example of (1) i.e. we show here that confusions may occur which stem from misconstruing of the Reichenbach-Grünbaum thesis of *Conventionality of distant Simultaneity.*

Recently Cavalleri and Bernasconi (CB) (1989) have claimed that the two fundamental properties of Special Relativity (SR) viz the Constancy of Velocity of Light (CVL) and the relativity of Distant Simultaneity (DS) are not peculiar to Relativistic Physics (RP) alone and it is possible to formulate the prerelativistic or Galilean Physics (GP) which can demonstrate both CVL and relativity of DS. Conversely it has also been argued that the Relativistic Physics (RP) can be formulated in such a way that CVL be no longer valid and distant simultaneity be absolute. While we have nothing against the claims by CB regarding the question of relativity of DS in GP and RP, we hold that the remarks made about CVL in their works (1989) are rather unfortunate. The arguments of CB seem to have been based on the Reichenbach-Grünbaum (RG) thesis (1958,1963) of the conventionality of distant simultaneity in SR. According to this thesis it is held that in Einstein's formulation of SR, the synchronization procedure for spatially distant clocks in any inertial frame contains an element of convention which is obviously devoid of any empirical content. This observation inevitably leads to the possibility of other synchronyzation conventions different from that adopted by Einstein. This fact reveals itself in our ability to choose any value (with certain restrictions) to be assigned to the one-way speed of light in a given inertial frame. The CS thesis has later been upheld and clarified by several authors [Winnie 1970; Tangherlini 1961; Mansouri and Sexl 1977; Podlaha 1978] and has been further developed and consolidated in a beautiful paper by Sjödin (1979).

Now since the question of simultaneity of any two spatially separated events in a *given* inertial frame depends on the chosen synchronization convention the issue of relativity of DS, which is often considered as one of the most fundamental imports of SR, has little significance. Thus for example, one is able to synchronize clocks and formulate relativistic physics in such a way that the simultaneity remains absolute. In the same token it is possible to adopt a synchronization procedure in GP so that DS becomes relative. CB in their paper (1989), among other things, seem to have rediscovered this fact which is indeed correct. However the C-S thesis has been misconstrued by CB when they remark that the CVL in SR is "trivial".

To prove the triviality of CVL in SR, the authors (1989), in one hand, formulated a transformation between inertial systems in GP and hence claimed that for the chosen synchrony the signal speed remains invariant even in GP. On the other hand, CB have taken the Tangherlini transformations [Tangherlini 1961; Sjödin 1979; Ghosal et al 1991a] representing absolute synchrony in particular to show that relativistic physics also can demonstrate the non-invariance of light speed.

We shall show below that these claims as they should be are not quite correct.

5.2 GALILEAN PHYSICS AND SIGNAL SYNCHRONY

CB consider transformations of coordinates between two frames S and S′ where S is stationary with respect to a quiet sea representing the etherial fluid and S′ is moving with respect to the former with a non-zero relative speed v. The clocks of both the frames are synchronized with a submarine (signal) which has a constant speed c along any direction with respect to the sea water. The clocks in S′ frame are synchronized in such a way that the signal speed appears to be c in the direction away from the origin of S′. This is always permitted according to C-S thesis when applied to GP.

The transformation equations are

$$\mathbf{r}' = \mathbf{r} - \mathbf{v}t \qquad (5.1)$$

and

$$t = t' + \lambda r' \qquad (5.2)$$

where

$$\lambda = |c-v|^{-1} - c^{-1} \qquad (5.3).$$

For the discussions that will follow we shall assume the relative velocity v to be along the common x-axis of the S and S′. One may thus write the equations (5.1) and (5.2) explicitly in terms of the coordinates x and y as

$$x' = x - vt \tag{5.4}$$

$$y' = y \tag{5.5}$$

$$t = t' + \lambda(x^2+y^2)^{1/2} \tag{5.6}$$.

Since the last equation is non-linear, the general velocity transformation is expected to be somewhat complicated. However, the transformations of velocities for the longitudinal and transverse motion of particles passing through the origin at t=0 can be obtained with less difficulty from equations (5.4),(5.5) and (5.6) as

$$\omega_L' = \frac{(\omega_L - v)}{1 - \varepsilon_x\lambda_L(\omega_L - v)} \tag{5.7}$$

$$\omega_T' = \frac{\omega_T}{1 - \varepsilon_y\lambda_T\omega_T} \tag{5.8}$$

where ω_L and ω_T are the velocities in the longitudinal and transverse directions as measured from S and the corresponding quantities with prime denote the same as measured from S'. Note that the values of λ (λ_L and λ_T) are different for the above two cases. The values of ε_x (or ε_y) are ± 1 for x' (or y') $\gtrless$ 0. The origin of these terms in (5.7) and (5.8) will be evident soon.

In the present case (when v is parallel to x-axis) λ can be written also as

$$\lambda = \left|c\frac{\mathbf{r}}{r} - \mathbf{v}\right|^{-1} - c^{-1} = [(c\cos\theta - v)^2 + c^2\sin^2\theta]^{-1/2} - c^{-1} \tag{5.9}$$

where θ is given by $\tan\theta = y/x$.

Thus λ, being function of θ alone, remains constant for the rectilinear motion of a particle which passes through the origin of S and S' at t=0. In order to derive equations (5.7) and (5.8) this constancy of λ has been assumed. Note that for the longitudinal and transverse motion, equation (5.6) takes the following two forms, respectively:

$$t = t' + \lambda_L|x'| = t' + \varepsilon_x\lambda_L x \qquad (5.10a)$$

$$t = t' + \lambda_T|y'| = t' + \varepsilon_y\lambda_T y \qquad (5.10b).$$

The above simple forms are guaranteed by the fact that the particle passes through the origin at t=0. The velocity transformation equations (5.7) and (5.8) then follow rather easily as usual from the transformation equations (5.4),(5.5) and (5.10).

For the longitudinal direction, for $x'>0$, λ can be calculated from (5.9) by putting $\theta=0$:

$$\lambda_L = \frac{v}{(c-v)c} \qquad (5.11)$$

for $x'<0$, $\theta=180^0$, λ_L is given by

$$\lambda_L = -\frac{v}{(c+v)c} \qquad (5.12).$$

However, unlike λ_L, λ_T does not depend on the spatial domain in S'. λ_T can be calculated directly from (5.3), by assuming

$$c = vi + uj \qquad (5.13)$$

(i and j are unit vectors along x and y respectively) where u is given by the relation

$$c^2 = u^2 + v^2 \qquad (5.14).$$

For the synchronization of clocks on the y′ axis the light (submarine) must travel along the y′ axis. To ensure that, it is necessary that the x component of c should be taken as v [equation (5.13)], i.e the speed of S′ with respect to S. This gives

$$\lambda_T = \frac{1}{c}\,[\{1 / (1 - \beta^2)^{1/2}\} - 1\,], \quad \beta = v/c \qquad (5.15).$$

In table I we put these values of λ_L and λ_T in the 4th column against different spatial domains (2nd column) in S′.

TABLE I

Signal Direction	Domain in S′	Velocity Transformation Formula	λ	Submarine speed (outward) as measured from S′ (magnitude only)	Submarine speed (inward) as measured from S′ (magnitude only)	Round-trip average speed
Longitudinal	$x' > 0$	$\omega_L' = \frac{\omega_L - v}{1 - \lambda_L(\omega_L - v)}$	$\lambda_L = \frac{v}{c(c-v)}$	c	$\frac{c(1-\beta^2)}{(1+\beta^2)}$	$c(1-\beta^2)$
	$x' < 0$	$\omega_L' = \frac{\omega_L - v}{1 + \lambda_L(\omega_L - v)}$	$\lambda_L = \frac{-v}{c(c+v)}$	c	$\frac{c(1-\beta^2)}{(1+\beta^2)}$	$c(1-\beta^2)$
Transverse	$y' > 0$	$\omega_T' = \frac{\omega_T}{1 - \lambda_T \omega_T}$	$\lambda_T = \frac{1}{c}[(1-\beta^2)^{-1/2} - 1]$	c	$\frac{c(1-\beta^2)^{1/2}}{2 - (1-\beta^2)^{1/2}}$	$c(1-\beta^2)^{1/2}$
	$y' < 0$	$\omega_T' = \frac{\omega_T}{1 + \lambda_T \omega_T}$	$\lambda_T = \frac{1}{c}[(1-\beta^2)^{-1/2} - 1]$	c	$\frac{c(1-\beta^2)^{1/2}}{2 - (1-\beta^2)^{1/2}}$	$c(1-\beta^2)^{1/2}$

In order to understand the implications of CB's synchronization in GP it is necessary to calculate the signal (submarine) speed as observed from S′. Note that by assumption the submarine travels along any direction, with a constant speed c as observed from S. For the longitudinal direction the calculation is straightforward. If the signal travels along x-axis *away from the origin*, in order to obtain ω_L' we put

$$\omega_L = \pm c \text{ (for } x' \gtrless 0) \tag{5.16}$$

in (5.7). On the other hand, if the submarine travels along y′ axis *away from the origin* we shall have to put using (5.14)

$$\omega_T = u = \pm c(1-\beta^2)^{1/2} \text{ (for } y' \gtrless 0) \tag{5.17}$$

in (5.8) in order to compute ω_T'. The obtained ω_L' and ω_T', the velocities of the submarine for the longitudinal and transverse directions as observed from S′ and as shown in the fifth column of the table I, are found to be c. This is not surprising since this trivial result is the outcome of the assumed synchronization convention in S′. However, one may now enquire what happens if instead of going away from the origin the submarine comes towards the origin. To calculate ω_L' and ω_T', i.e the longitudinal and transverse speeds as observed from S′, for the case when the submarine comes towards the origin of S′ we shall have to put

$$\omega_L = \mp c \text{ (for } x' \gtrless 0) \tag{5.18}$$

and

$$\omega_T = \mp c(1-\beta^2)^{1/2} \text{ (for } y' \gtrless 0) \tag{5.19}$$

in (5.7) and (5.8) respectively. The results [signal speed (inward) in S′] are summarised in the sixth column and it can be seen that they are by no means equal to c. This clearly contradicts the claim by CB that the transformations (5.1) and (5.2) which represent GP keep the light speed invariant. The speed of light coming towards the origin of S′ is not only different from c, but also its values depend on the direction (cf. ω_L' and ω_T' in column 6).

Indeed this is precisely what is expected of any transformation representing GP, since, one may calculate the round- trip average speed of the submarine (by taking row-wise harmonic average of columns (5) and (6)) and verify that they (represented in the last column : round -trip average speed) represent the to and fro average signal speeds that follow from Galilean transformations (Ghosal et al 1991a). Had the to and fro speeds of the signal been equal to c in all directions, the transformations (5.1) and (5.2) would not have represented GP. Note that the C-S thesis gives us option to choose any value to be assigned to the one-way speed of the synchronizing signal but there is no scope for any manipulation with regard to the two-way average speeds. This is because the two-way average speed of a signal can be measured with a single clock in a *given* inertial frame without any ambiguity and hence it should be convention independent.

5.3 SECOND RELATIVITY POSTULATE AND ITS EMPIRICAL CONTENT

The similar arguments can be given against the incorrect claim made by CB in their article (1989) that "SR can be formulated in such a way that c invariance be no longer valid". The implication of the above claim is very serious indeed, since it-questions the correctness of the second relativity postulate of Einstein which forms the very basis for the foundation of SR. In the light of C-S thesis however the CVL postulate of Einstein loses its conventional interpretation. According to the second postulate of SR light speed is independent of the inertial frame chosen and its value is c in all directions. Now the term "speed", if it means the one-way speed, has no empirical significance since according to C-S thesis it could be chosen arbitrary. However, if the term "speed" is interpreted as two-way average speed, there seems to be no problem because as we have already discussed in section (5.2) there is no conventionality ingredient in the measurement of round- trip speeds. The validity of the second relativity postulate, when interpreted in this fashion, clearly distinguishes RP from other non-relativistic transformations.

To advance their erroneous claim CB have chosen the Tangherlini transformations representing RP and shown that in the longitudinal direction light speed away from the origin in S' is given by

$$\omega_L' \text{ (outward)} = c/(1+\beta) \tag{5.20}$$

and argue that CVL is not maintained! Clearly according to CB, CVL refers to one-way speed of light and according to C-S thesis this observation [equation (5.20)], though correct, is trivial. The last equation refers to the one-way speed of light moving away from the origin as observed from S'. If the light travels towards the origin, the transformed speed with respect to S' can be obtained by putting $u_x = -c$ in the equation preceding the equation (11) of the article by CB (1989) (there is a printimg mistake in this equation) and this is given by

$$\omega_L' \text{ (inward)} = c/(1-\beta) \qquad (5.21).$$

Hence one can verify that the longitudinal round-trip speed obtained by taking the harmonic mean of ω_L'(outward) and ω_L'(inward) is clearly c. One can also check that the conclusion regarding the value of the round-trip speed of light holds for any direction in general. This fact reaffirms CVL (if interpreted correctly) in RP and invalidates CB's contrary claim in this regard.

5.4 AN INTERESTING FEATURE OF THE CB TRANSFORMATIONS

In Galilean Physics (GP) we assume that the distance between two points —— they may be particles ——— at a given time is quite independent of any particular frame of reference; that is we assume that we can construct rigid measuring rods whose length is independent of their state of motion. But in Special Relativity, every rigid body appears to be largest when at rest relatively to the observer. When it is not at rest, it appears contracted in the direction of its relative motion by the factor $(1-v^2/c^2)^{1/2}$, while its dimensions perpendicular to the direction of motion are unaffected.

Let us now investigate what happens if we consider the CB transformations i.e

$$x' = x - vt \tag{5.4}$$

and
$$t' = t - \lambda_\theta x \tag{5.6}$$

where λ_θ is θ dependent (vide eqn. (5.9)).

At a given time the equation (5.4) gives, for any spatial separation Δx of S (i.e for a rod of length Δx parallel to the x-axis)

$$\Delta x' = \Delta x \tag{5.22}$$.

This means that there is no length contraction if the moving rigid rod is measured from S.

Now the inverse transformations of (5.4) and (5.6) are given by

$$x = x'(1+v\lambda_\theta) + vt' \tag{5.4a}$$

and
$$t = t' + \lambda_\theta x' \tag{5.6a}$$

From these equations it is evident that a rod fixed with respect to S, when measured from the point of view of S′, will appear to have contracted. The amount of contraction however cannot be given in terms of a fixed proportionality factor (i.e a factor dependent on the relative velocity of S & S′ alone) because of the term λ_θ, which depends on θ. This is obvious since the end points of a rod parallel to x-axis will have different values of θ depending on the rod length. However if the rod is placed on the x-axis (i.e $y'=0$), one obtains

$$\Delta x = \Delta x'(1+v)\lambda_L \tag{5.23}$$

where λ_L is given by (5.11).

This equation (5.23) shows that there is a length contraction for the inverse transformations. This implies that for CB transformations, there is length contraction which is one-way! Similar thing happens for Zahar transformations also. However this issue will be discussed in some detail in the last chapter.

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CHAPTER VI

ON THE DEFINITION AND EXISTENCE OF LORENTZ INVARIANT CLOCKS*

*S.K.Ghosal and Papia Chakraborty., Proc. the 3rd Conference on "Physical Interpretations Of Relativity Theory", BRITISH SOCIETY FOR THE PHILOSOPHY OF SCIENCE, Imperial College, London, September 1992.

6.1 INTRODUCTION

This chapter deals with the indepth study of the question of the existence of Lorentz - Invariant - Clocks (LIC) in a relativistic world. This is prompted by some recent claims and counter claims (Schlegel 1973,1975,1977; Rodrigues 1985) regarding LIC. Here we have reexamined the definition of a Lorentz - Invariant (LI) clock in the light of the Conventionality of distant Simultaneity thesis (C-S thesis). The main purpose of the present text is to first present a synchrony independent definition of a LI clock and then to prove generally that the existence of an invariant clock is indeed incompatible with Special Relativity (SR). We hope that the present exposition will also help readers grasp and appreciate once more the importance of the C-S thesis in understanding the foundational questions of relativity.

In an article Schlegel (1973,1975,1977) claimed that it is possible to construct theoretically a Lorentz Invariant (LI) clock whose rate does not depend on its state of motion. The author further stated that the Principle of Relativity (PR) does not come in the way in conceiving such a clock. To advance his thesis, Schlegel theoretically built his gedanken clock (ball and track clock) and analysed its functioning to show that the clock rate was indeed Lorentz invariant.

Rodrigues (1985) correctly replies to Schlegel's paper by pointing out explicitly the fault in Schlegel's analysis. However, we find it rather difficult to understand his "general proof" that the theory of relativity forbids the existence of a LI clock. Rodrigues's "general proof" is unsatisfactory on two counts. First, his definition of LI clock seems to be ambiguous. For, as we shall see later that the authors' definition itself presupposes a preferred inertial frame and therefore any effort to prove further that PR forbids the existence of a LI clock becomes meaningless. In our opinion a careful definition of a LI clock is no less crucial than the issue of its existence. In section 6.2 we shall elaborate on what is wrong with Rodrigues' definition and then in section 6.3 we shall seek a more profound definition of an invariant clock.

The second drawback of the paper concerns the author's indifference to the Conventionality of distant Simultaneity thesis (C-S thesis) of special relativity (Reichenbach 1958; Grünbaum 1963; Winnie 1970; Sjödin 1979; Ghosal et al 1991a, 1991b; Mansouri & Sexl 1977). The c-s thesis observes that Einstein's formulation of SR rests on a special synchronization convention and since a convention is devoid of any empirical content, one is free to choose any value (with certain restrictions) to be assigned to the One-Way Speed (OWS) of light in a given inertial frame.

However we take this occasion to comment here once again that the above observation by no means imply that the Relativistic Physics (RP) is empirically empty and on the contrary, as we have observed in a recent paper (Ghosal et al 1991b), we believe that the validity of Einstein's second relativity postulate, when reinterpreted in the light of the C-S thesis, distinguishes RP from other non-relativistic transformations.

Now Rodrigues argues in his article that if LI clocks (as defined by the author) exist it will be possible to synchronize standard clocks in any inertial frame S with the help of a master LI clock and the resulting synchrony will not be equivalent to the standard Einstein synchrony or the so-called slow transport synchrony. This would immediately imply that the velocity of light will not remain isotropic in S. The author then argues that the principle of relativity as if forbids such a situation to arise! Obviously this argument is contrary to the spirit of the C-S thesis. It is evident that when Rodrigues speaks about the velocity of light he refers to the OWS alone and as its value, according to the C-S thesis, can be assigned arbitrarily, any conclusion based on the OWS of light definitely falls through.

Now our aim is to first present a synchrony independent definition of a LI clock and then to prove generally that the existence of such an invariant clock is indeed incompatible with special relativity. The other sections of this text will discuss the problem in details.

6.2 LI CLOCK : A DEFINITIONAL PROBLEM

In trying to refute Schlegel's "claim" that the existence of a Lorentz invariant clock is consistent with special relativity Rodrigues defines his LI clock in the following way :

(A) It is a clock such that when in motion relative to an inertial frame S_O, does not lag behind relative to a series of clocks synchronized according to Einstein in S_O.

Rodrigues' analysis is based on the above definition; therefore before one proceeds it is necessary first to examine its content carefully. Note that the statement (A) refers to a particular inertial frame S_0 ; the set of standard clocks belonging to S_0 are used as reference clocks with which the LI clock (non-standard clock) rate is to be compared. Now the question is whether the inertial frame S_0 is a preferred one or not. Obviously it should be a tacit stipulation that S_0 be *any* inertial frame, otherwise the whole exercise of refuting Schlegel's "claim" becomes meaningless. Unfortunately we shall see in a moment that the frame S_0 is a preferred one by definition. But before giving the formal proof to this effect let us first point out another feature of the definition (A), the cognizance of which itself will provide us some preparedness for the journey to the next section. In the definition (A) the rate of a LI clock in motion is compared with that of standard clocks at rest in S_0. It is evident therefore that any such comparison automatically calls for a particular procedure for distant clock synchrony in S_0. Since the experimental clock moves from one point to another in S_0 the rate of the single LI clock is actually compared with the difference of readings of the two spatially separated clocks in S_0 which are to be synchronized beforehand. Obviously the result of such a comparison depends on the adopted synchronization convention for the separated clocks in S_0. The above fact thus makes the definition (A) convention dependent. This is another undesirable feature of the definition.

Now consider an inertial frame S in motion relative to S_0. The proper period $\Delta\tau$ of a standard clock C_s comoving with S is given by

$$\Delta\tau = (1-u^2/c^2)^{1/2}\Delta t \tag{6.1}$$

where Δt is the period of C_s as observed from S_0 and u is the relative speed between the reference frames in question. Let us suppose that an Intelligent Observer (IO) sitting on S is able to change the time rate of C_s with the help of a rate adjuster. If, by using the adjuster the rate of C_s is continuously changed in some arbitrary way depending on the state of motion of S with respect to S_0, C_s can no longer be considered to be a standard one. If in particular the adjusted time rate $\Delta\tau'$ is given by

$$\Delta\tau' = (1-u^2/c^2)^{1/2}\Delta\tau \tag{6.2}$$

equations (6.1) and (6.2) yield

$$\Delta\tau' = \Delta t \quad . \tag{6.3}$$

The above relation will continue to hold as long as the observer "knows" the state of motion of S relative to S_0 and adjusts the clock rate according to equation (6.2). Equation (6.3) now implies that the clock C_s behaves like a LI clock according to Rodrigues' definition.

If we now consider another inertial frame S_1 moving with respect to S_0 with a relative speed v along their common x-axis, we obtain for the proper period of a standard clock in S

$$\Delta\tau = (1-w^2/c^2)^{1/2}\Delta t_1 \tag{6.4}$$

where Δt_1 is the period of the same clock as measured from S_1 and w is given by the well known velocity transformation formula

$$w = \frac{v - u}{1 - uv/c^2} \quad (6.5).$$

Inserting (6.2) in (6.4) we obtain

$$\Delta\tau' = [(1-w^2/c^2)/(1-u^2/c^2)]^{1/2}\,\Delta t_1 \quad (6.6)$$

which implies that the clock $C_{\equiv}$, even though invariant from the point of view of S_0, now differs in its rate with respect to the series of clocks synchronized a' la Einstein in S_1 unless v=0 i.e when S_1 and S_0 coincide. This means, in other words that as if the frame S_0 has some special significance!

The above fact thus makes the definition (A) uninteresting if not totally meaningless, since now the existence of the so-called LI clock of Rodrigues, the definition of which itself gives a preferential status to S_0 can no longer be cited as an evidence for or against PR.

6.3 INVARIANT CLOCKS AND TRANSFORMATION LAWS

From the last section it is clear that a logically consistent definition of an invariant clock should not refer to any particular reference frame or to any particular synchronization convention. Keeping this in mind let us proceed to present our definition (Definition B) : A clock is such that when initially compared and adjusted (rate and phase) with an inertial standard clock at any given point in space does not differ its phase with the standard clock when they meet after the former performs a round-trip. Note that the above definition makes no reference to any particular inertial frame and the fact that the comparison of clocks are performed between the same pair of clocks (at the same space point) does away with any need for synchronization of distant clocks.

We shall now first look for the transformation laws between inertial frames if standard clocks are replaced by the invariant ones (according to definition B). In order to do so we shall take the so-called conventionalists' view point of special relativity and follow Sjödin's approach in particular (Sjödin 1979). According to Sjödin, relativistic physics can be described consistently in terms of real length contr ction and time dilatation of moving rods and standard clocks with respect to a given inertial frame S_0 and it is held following the C-S thesis (vide sec.6.1) that a plethora of transformation equations between inertial frames with different synchronization parameters may describe the same physical reality. Sjödin derived from some simple considerations the general form of the transformation equations between two inertial frames S_0 and S' (when the latter moves with a velocity u with respect to S_0 along their common x-axis) as

$$\begin{aligned} x' &= \phi_u^{-1}\ (x-ut) \\ y' &= y \\ t' &= Ax + (\Omega_u - uA)t \end{aligned} \tag{6.7}$$

where the values for the length contraction factor ϕ and time dilatation factor Ω depend on the world that the above equations represent and A is the synchronization parameter which according to C-S thesis' can be assigned arbitrarily.

For the relativistic world (Sjödin 1979; Ghosal et al 1991a, 1991b)

$$\phi_u = \gamma_u^{-1} = \Omega_u \tag{6.8}$$

where as usual $\gamma_u = (1-u^2/c^2)^{-1/2}$.

For the Galilean world (Sjödin 1979) on the other hand

$$\phi_u = \Omega_u = 1 \tag{6.9}.$$

It may appear at a first sight that since equations (6.7) are not formally symmetric between S_0 and S', S_0 might have been given a preferred status; but that is not so. For example for a relativistic world ($\phi = \Omega = \gamma^{-1}$) one may choose the standard synchrony so that (6.7) would represent Lorentz Transformations (LT) in which case S_0 is just *any* inertial frame.

One may now also write down the transformations if standard clocks (but not the rods) are replaced by non-standard ones. In this case the value of $\phi_u = \gamma_u^{-1}$ is retained but Ω_u remains arbitrary in (6.7) :

$$\begin{aligned} x' &= \gamma_u (x-ut) \\ y' &= y \\ t' &= Ax + (\Omega_u - uA)t \end{aligned} \tag{6.10}.$$

We may now define a "Semi Relativistic World" (SRW) when the non-standard clocks of (6.10) are the invariant clocks in particular.

One may easily find out the value of Ω_u in this case by explicitly using definition (B) given at the begining of this section for invariant clocks. However for the present analysis, in order to obtain a better insight into the present problem we first intuitively postulate the transformations for the SRW and hence prove afterwards that the clocks of the said world are indeed invariant (according to definition B).

The proposed transformations are

$$\begin{aligned} x' &= \gamma_u \,(x-ut) \\ y' &= y \\ t' &= Ax + (1-uA)t \end{aligned} \tag{6.11}$$

Note that we have taken the value of ϕ_u from (6.8) and that of Ω_u from (6.9) and inserted these values in (6.10).

6.4 <u>Standard Synchrony in SRW.</u>- Velocity transformation equations that follow from (6.11) are given by

$$v_x' = \frac{\gamma_u (v_x - u)}{1+A(v_x - u)} \tag{6.12}$$

$$v_y' = \frac{v_y}{1+A(v_x - u)} \tag{6.13}$$

where v_x and v_y are the components of velocity of a particle (for instance) as observed from S_0 and the corresponding quantities with prime denote the same as observed from S'.

Now, since our definition of the invariant clock is independent of clock synchrony one is free to adopt any convention for distant synchrony in SRW without any loss of generality. We shall use for the following analysis the so-called Standard Synchrony (SS) (Winnie 1970; Sjödin 1979; Ghosal et al 1991a; Mansouri and Sexl 1977), according to which one way speed of light is independent of direction in any inertial frame. We can explicitly use the above definition of SS to obtain the value of A as follows. Consider a light signal travels in the x-direction. Now if one puts, for the "one way speeds" of light in S_0, $u=\pm c$ in (6.12), according to the SS the corresponding speeds of light v_x' (in S′) will also be equal in magnitude i.e

$$\frac{\gamma_u (c - u)}{1 + A(c - u)} = \frac{\gamma_u (c + u)}{1 - A(c + u)} \tag{6.14}$$

which gives

$$A = -(u/c^2)\gamma_u^{\,2} \tag{6.15}$$

Inserting this value of the parameter for synchrony in (6.11) we obtain the transformations with standard synchrony in SRW :

$$x' = \frac{x - ut}{(1-u^2/c^2)^{1/2}}$$
$$y' = y \tag{6.16}$$
$$t' = \frac{t - ux/c^2}{1 - u^2/c^2}$$

Now we shall proceed to prove that the clocks of SRW as represented by the transformations (6.16) are indeed invariant (according to definition B). From (6.16) we can write down the transformations between *any* two frames S′ and S″, for example, where the later moves with a relative velocity v with respect to the first frame S_0. They are obtained as

$$x'' = \gamma_v[\gamma_u(1-uv/c^2)x'-(v-u)t'] \tag{6.17}$$

and $$t'' = \gamma_v^2[(1-uv/c^2)t'-(\gamma_u/c^2)(v-u)x']$$

and also inverting the above equations we obtain

$$x' = \gamma_u[\gamma_v(1-uv/c^2)x''+(v-u)t''] \tag{6.18a}$$

and $$t' = \gamma_u^2[(1-uv/c^2)t''+(\gamma_v/c^2)(v-u)x''] \tag{6.18b}.$$

Suppose now a clock C_1 of SRW is placed at the origin ($x''=0$) of S″. From eqn. (6.18b) the observed period $\Delta t'$ of C_1 from S′ may be obtained in terms of its "proper" period (i.e the period as observed from the clock's own rest frame) as

$$\Delta t' = \gamma_u^2\ (1-uv/c^2)\ \Delta\tau'' \tag{6.19}.$$

Note that in order to arrive at (6.19) we have put $\Delta x''=0$ & $\Delta t''=\Delta\tau''$ in (6.18b).

Observe that the time dilatation factor in (6.19) differs from unity in general. If for the moment we anticipate that the clocks under question are invariant, the time dilatation effect (eqn. 6.19) appears to be a bit surprising. However, there is little to be surprised since here the invariance of the clock period is clearly masked by the process of distant clock synchrony in S'. This is the precise reason why it is necessary to define invariant clock in a synchrony independent way.

Now suppose C_1 completes its journey in the following way. It first starts from the origin of S' at $t=0$, travels with a uniform speed v with respect to S_0 a distance x' of S' and after a brief period of deceleration g, returns to the origin of S', with a velocity $-v$ with respect to S_0. Now if there is another clock C at the origin of S', one is able to measure the time required for the round-trip journey of C_1 by C and as well as by C_1. Note that the time measurements in this case are independent of the process of distant clock synchrony in the concerned inertial frames. We rewrite (6.19) with a change of notations

$$\Delta t'_f = \gamma_u^2 \; (1-uv/c^2)\Delta\tau''_f \tag{6.20}$$

and

$$\Delta t'_r = \gamma_u^2 \; (1+uv/c^2)\Delta\tau''_r \tag{6.21}$$

where the indices f and r represent the respective quantities for the forward and return journey of C_1 respectively. Note that the last equation is obtained by replacing v by $-v$ for the return journey in (6.19).

For the round-trip journey of C_1, C time and C_1 time are given by

$$\Delta\tau'' = \Delta\tau''_f + \Delta\tau''_r \tag{6.22}$$

and $$\Delta t' = \Delta t'_f + \Delta t'_r \tag{6.23}$$

respectively.

In the above calculations we have considered onl; uniform motion for C_1 both for the forward and for the return jouney and omitted any effect of its retardation at the end of the forward journey, because the retardation effect in time dilatation (in this case apparent) calculations can be made arbitrarily small as the retardation $g\to\infty$ at constant v (vide clock paradox calculations due to Moler (1972), for example).

To prove that C_1 represents an invariant clock we shall have to prove $\Delta\tau'' = \Delta t'$ i.e

$$\Delta t'_f + \Delta t'_r = \Delta\tau''_f + \Delta\tau''_r \tag{6.24}$$

or, in other words, using (6.20) and (6.21)

$$\Delta t'_f + \Delta t'_r = \gamma_u^{-2}[\Delta t'_f/(1-uv/c^2) + \Delta t'_r/(1+uv/c^2)] \tag{6.25}$$.

The above identity may easily be verified by noting that

$$\omega_f \Delta t'_f = \omega_r \Delta t'_r = x' \quad \text{or,} \quad \Delta t'_r = (\omega_f/\omega_r)\Delta t'_f \tag{6.26}$$

where ω_f and ω_r are the speeds of C_1 with respect to S' for the forward and the return journies respectively which, from (6.17) are given by

$$\omega_f = (v-u)/\gamma_u(1-uv/c^2) \tag{6.27}$$

and $$\omega_r = (v+u)/\gamma_u(1+uv/c^2) \tag{6.28}$$

By virtue of (6.26), the $\Delta t'_f$ cancels from both sides of (6.25) giving

$$1 + (\omega_f/\omega_r) = \gamma_u^{-2}[1/(1-uv/c^2) + (\omega_f/\omega_r)(1+uv/c^2)^{-1}] \tag{6.29}$$

If one now makes use of (6.27) and (6.28), the verification of the above identity becomes a trivial exercise.

Observe that the identity (6.24) or (6.25) thus established is not generally valid since similar calculations with Lorentz transformations for example, unlike the case in hand, would display real time dilatation effect (Moller 1972).

6.5 <u>SRW and its incompatibility with PR.</u>- In section 6.1 we observed that Rodrigues in his "general proof" did not take into account the C-S thesis properly and on the contrary he equated the possible existence of a LI clock with the possibility of a particular non-standard synchronization giving rise to anisotropy of OWS of light in a general frame. We shall however work with the "Two-Way-Speed" (TWS) of light the definition of which does not call for distant synchrony (Ghosal et al 1991a & 1991b). The following steps will provide the proof that the invariant clocks are really incompatible with PR.

Consider that the light speed in S_0 is c and is the same in all directions. Recall the velocity transformation formula (6.12)

$$v'_x = \frac{\gamma_u(v_x - u)}{1+A(v_x - u)} \tag{6.12}$$

and assume in particular that in S_0, $v_x=-c$ for the return journey (obtained by reflection) of a light signal in the x-direction of S_0. The corresponding speeds of light as observed for S′ will be given (using 6.12) by

$$c'_f = \frac{\gamma_u(c - u)}{1 + A(c-u)} \qquad (6.30)$$

for the forward journey of light and for the return journey

$$c'_r = \frac{\gamma_u(c + u)}{1 - A(c+u)} \qquad (6.31).$$

The harmonic mean of last two speeds give the TWS of light as observed from S′ which is given by

$$c' = \frac{1}{2}\left(\frac{1}{c'_f} + \frac{1}{c'_r}\right)^{-1} = c(1 - v^2/c^2)^{1/2} \qquad (6.32).$$

Note that the synchronization parameter A has no role in the last expression. This is expected since TWS requires a single clock for its measurement and therefore its value ought to be independent of synchrony. Similarly (Ghosal et al 1991b) one may obtain the value of TWS in the y-direction or any direction for that matter and verify that the observed round-trip speed is independent of direction. But it is evident that while in a general frame S′ the isotropy of light speed (TWS) is maintained, its magnitude differs and therefore in violation of PR one can tell one frame from the other just by performing experiments to determine TWS of light in different inertial frames.

However in tune with Rodrigues' final remark (1985) we also hold that the above proof cannot forbid invariant clocks to exist in nature but for that, only PR has to be discarded once and for all.

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CHAPTER VII

THE ROLE OF c IN SPACE-TIME TRANSFORMATIONS : RELATIVITY IN A SUBSTRATE*

*——— Papia Chakraborty et al., Presented by S.K.Ghosal at the 3rd Conference on "Physical Interpretations Of Relativity Theory", BRITISH SOCIETY FOR THE PHILOSOPHY OF SCIENCE, Imperial College, London, September 1992.

7.1. INTRODUCTION

The present chapter also deals with the so-called Reichenbach-Grünbaum thesis (Reichenbach 1958; Grünbaum 1963) of conventionality of distant simultaneity of Special Relativity (SR). We have already noted that the Conventionality of Simultaneity (C-S) thesis observes that in Einstein's formulation of SR, the synchronization procedure for spatially distant clocks in an inertial frame contains an element of convention . The conventional ingredient of SR which logically cannot have any empirical content, gives rise to results that are often erroneously construed as the new philosophical imports of special relativity theory. For example, the issue of relativity of distant simultaneity which according to the C-S thesis can be proved to be empirically empty (Winnie 1970; Cavalleri & Bernasconi 1989; Ghosal et al 1991b; Wegener 1992) is still popularly regarded as one of the most fundamental imports of SR. As is well known, the source of this conventionality lies in the fact that in SR distant clocks in a *given* inertial frame are synchronized by light signals, the one-way speed (Ghosal et al 1991a & 1991b) of which has to be known beforehand for the purpose. To know the one-way speed of light on the other hand one requires to have presynchronized clocks and the whole process of synchronization ends up in a logical circularity which forces us to introduce a degree of arbitrariness in assigning the value for the one-way velocity of light. However it may be borne in mind that there is no arbitrariness regarding the round-trip speed of a signal, since the round-trip speed can be measured with one clock only and therefore it is independent of conventionality of distant clock synchrony (Winnie 1970; Ghosal et al 1991a & 1991b).

There has now been a substantial amount of clarification of the C-S thesis due to a number of authors. Possibility of using synchronization convention other than that adopted by Einstein to describe the relativistic world has also been much discussed. Papers by Winnie (1970) and more recently by Mansouri and Sexl (1977) and successive development by Podlaha (1978) and by Sjödin (1979) in particular are some of the recent important expositions in this regard.

Unfortunately still, apart from some casual mention of the problem of clock synchrony in SR, text books of relativity hardly devotes a paragraph on the conventionality thesis or the possibility of using non-Einsteinian synchrony to describe the relativistic world.

The fact that the C-S thesis has not yet gained wide spread attention among Physicists may be attributed to the fact that there is a tendency to regard the C-S thesis as an antithesis of SR and anything that seems contrary to the standard formulation of relativity is viewed with skepticism. Indeed in our opinion the C-S thesis complements SR and the understanding of the former helps clear out confusions that sometimes occur in SR. As we have pointed out, the claim that the relativity of distant simultaneity is a new non-classical philosophical import is just one example of various such confusions. In a recent paper (Ghosal et al 1991a) we have discussed another common misconception that under the small velocity approximation, SR goes over to the Galilean relativity. Indeed we have noted that the small velocity approximation cannot alter simultaneity convention.

Misconstruction of the C-S thesis itself is also not uncommon. For example in a recent article (Cavalleri & Bernasconi 1989) it has been erroneously suggested that light speed invariance in special relativity is a trivial matter and as if, by virtue of the C-S thesis, even Galilean Physics can be reformulated so that light speed remains invariant! In another paper Rodrigues (1985), in connection with the enquiry whether Lorentz invariant clocks can exist without violating the principle of relativity, incorrectly remarked that the possibility of having absolute synchrony is an antithesis of the relativity principle! In some of our recent papers (Ghosal et al 1991b ; Ghosal & Chakraborty 1992) we addressed ourselves to the task of clarifying these issues.

Sometimes in connection with the C-S thesis, the debatable issue of ether (as a *hypothetical* substrate providing a preferred inertial frame) often crops up (Sjödin 1979; Mansouri & Sexl 1977; Cavalleri & Bernasconi 1989). But question have been raised whether considerations of synchronization alone can distinguish an ether frame or not (Spinelli 1983; Cavalleri & Spinelli 1983; Stone 1991). As it stands now, as if the existence of a real physical ether as a preferred frame would have placed the C-S thesis on a stronger footing. In fact efforts are still on to give a physical support to this preferred frame of ether (Spinelli 1983; Cavalleri & Spinelli 1983). (We shall later see that for the understanding of the C-S thesis at least, one can bypass the debate concerning the existence of ether by introducing at the out- set a real physical substrate (water for example) through which different inertial frames may be considered to be in relative motion).

Given this perspective of confusion, miscoostruction and polemics regarding the C-S thesis or SR for that matter, we are led to conclude that everything of SR is still not well understood. We therefore feel that it is necessary to provide some additional clarifications in this regard. It is to this task the present paper addresses itself.

However, before we proceed let us first make the following observations: In the standard formulation of SR (and in particular in the derivation of Lorentz transformations) light has two roles to play. In the one hand it offers the peculiar property viz the constancy of its Two-Way-Speed (TWS), which any transformation will have to honour. On the other hand in SR light also acts as a synchronizing agent since, according to SR distant clocks in any *given* inertial frame are synchronized with it following a particular convention. The convention is known as the standard synchronization according to which the One-Way-Speed (OWS) of light is assumed to be independent of direction along a given line.

It is obvious that while the first role (TWS-role) has its basis on the empirically verifiable property of light, the second one is purely prescriptive in origin.

In SR these two roles of light overlap in a sense that they work together to render 'c', the speed of light in vacuum to appear in a particular way in Lorentz Transformations (LT). Therefore by looking at LT it becomes difficult to separate the real relativistic effects from the apparent ones where the latter have its origin in the prescriptive inputs alone.

In the standard formulation of SR it is the non-separatebility of these two roles of light which is largly responsible for a lot of confusion that still prevails. In order to gain a clearer perspective it will be benificial in our opinion to somehow delink the synchronizing role of the optical signal from its TWS role so that one can visualize the relative effects of each of these roles separately in the space-time transformations.

Keeping this in mind we propose in the present text to choose a different signal (whose TWS is not equal to c) for synchronization at the out-set. This new signal will be assumed to be a wave (Podlaha 1975,1976) so that its TWS with respect to different inertial frames will have definite values. The TWS of a wave can always be measured in principle by the reflection technique which requires only one clock and therefore it is independent of the distant clock synchrony. Such a signal whose round-trip speed is different from c, though not available in an empty space, can always be thought to exist within a medium. One may consider a fluid substrate and may single out any particular wave mode characteristic to the medium as a synchronizing agent. This can be the acoustic mode or it can even be the optical mode with speed c′ different from c. For the following analysis however we shall use the term "acoustic signal" to mean any signal (chosen for clock synchrony) whose speed is not equal to c. We may then enquire what the nature of the transformation equations will be if the synchronization is performed by the "acoustic signal".

For definiteness let us imagine that some intelligent Dolphins equipped with standard rods and clocks under water are trying to "discover" relativity. For the time being let us suppose that the Dolphins are unable to communicate with the outside world so that only the "acoustic signal" with speed c' is available for the distant clock synchrony. One may then ask— what will be the nature of the transformation laws? Will they contain c, the vacuum speed of light? If so how? Or, will the Dolphins land up with a c'-relativity theory* analogous to the Einstein (c-relativity) theory with the speed c' replacing c?

The purpose of the present article is to obtain heuristically the transformation equations between inertial frames in the medium from the Dolphins' perspective and thereby all the questions posed above will be answered. As we go along some important and subtle aspects of the role of optical synchrony in SR will automatically be revealed.

We shall be particularly interested in understanding the role of Einstein's standard synchrony in the structure of LT. This will be best understood if the procedure for the standard synchrony adopted by Einstein is mimicked by taking the "acoustic signal" as the synchronizing agent. We shall then be able to see clearly, among other things that the well known γ-factors of SR, partly has its origin in the synchronization convention alone.

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* c'-relativity was considered by Rosen (1952) who developed the so-called c'-Lorentz transformations (c' replacing c in ordinary LT) between two inertial frames in relative motion within a medium.

7.2. TRANSFORMATION EQUATIONS (TE) IN A SUBSTRATE :

(a) Pseudo-Standard Synchrony

In order to obtain the transformation equations let us suppose that the Dolphins follow the procedure similar to the so-called light sphere derivation of LT commonly discussed in standard text books. The essential content of this derivation may be outlined as follows:

Two inertial frames S_i and S_k are travelling with a relative speed v_{ik} along the positive direction of their common x-axis and it is supposed that a flash of light be emitted from their common origin at $t_i = t_k = 0$ Then it is asked which linear transformations of the form

$$x_k = \alpha_{ik}\, x_i + \delta_{ik}\, t_i$$
$$y_k = y_i \qquad (7.1)$$

and

$$t_k = \xi_{ik}\, x_i + \beta_{ik}\, t_i$$

preserves the form of the spherical wave front equations:

$$x_k^2 + y_k^2 + z_k^2 = c^2\, t_k^2 \qquad (7.2).$$

In other words the task here is to find out the unknown coefficients α_{ik} ,δ_{ik} etc. for which the equation (7.2) goes over to

$$x_i^2 + y_i^2 + z_i^2 = c^2\, t_i^2$$ after the transformations.

Note that while the last requirement assumes the second relativity postulate (that the TWS of light is independent of the reference frame (Ghosal et al 1991b)), the very choice of the spherical wave front equation implicitly assumes Einstein's prescription of standard synchrony according to which the OWS of light is independent of "direction". This is more evident if one takes the one dimensional equivalent of (7.2):

$$x_k^2 = c^2 \, t_k^2 \tag{7.3}$$

or,

$$x_k = \pm\, ct_k \tag{7.4}$$

One can now see that the equation (7.4) clearly states that in a *given frame* the one-way speed of light in the forward (+ve x) and as well as in the reverse (−ve x) directions is the same and is equal to the two-way average speed c. This is precisely known as the standard synchronization convention.

Now suppose, instead of light, an acoustic wave is generated at t=0 at the common origin of the frames S_i , S_k etc. and ask what the (one-dimensional) wavefront equation will be. Now, except for the frame S_0 which is at rest relative to the medium the speeds of sound in the positive and in the negative x-directions will not be equal if the clocks were somehow synchronized beforehand by light signal in vacuum. However, according to the C-S thesis the Dolphins are free to adopt the synchrony which assumes the equality of these speeds in all the frames. This synchrony will be termed as pseudo-standard synchrony as opposed to Einstein's standard synchrony which assumes the equality of the one-way *vacuum light speed* in all directions.

According to pseudo-standard synchrony along the x-axis the one-dimensional wavefront equation will be

$$x_k^2 = a_{kx}^2 \, t_k^2 \tag{7.5}$$

where a_{kx} is the two-way speed of the Acoustic Signal (AS) along the x-axis. Note that in general the two-way speeds will not be the same along other directions. For example, along the y-axis we shall have to write

$$y_k^2 = a_{ky}^2 \; t_k^2 \tag{7.6}$$

where a_{ky} , the two-way speed of AS in the y-direction may differ in value with its x-direction counter part.

In SR one has the Constancy of Velocity of Light (CVL) postulate. However one cannot assume such a thing to be valid for an acoustic signal. Therefore, in absence of a similar postulate the values of a_{kx} & a_{ky} will differ from that of a_{ix} & a_{iy} where the subscripts k & i refer to different inertial frames S_k & S_i which are in relative motion.

(b) <u>One-Dimensional Sound-Sphere Derivation Of TE</u>

Since S_k's spatial origin moves with the velocity v_{ik} with respect to S_i, the transformations (7.1) may now be written as

$$\begin{aligned} x_k &= \alpha_{ik} \; (x_i - v_{ik} \; t_i) \\ y_k &= y_i \\ t_k &= \xi_{ik} \; x_i + \beta_{ik} \; t_i \end{aligned} \tag{7.7}$$

where we have replaced, as usual, δ_{ik} by $-\alpha_{ik} \; v_{ik}$.

In the present case in order to find out the coefficients of the above TE, the Dolphins are not able to use a 3-sphere wavefront equation since the sound wave in a general frame will not be spherical. However by virtue of the chosen synchrony they are able to take the one-sphere equation of the form (7.5) and subject the equation (7.7) to the condition

$$x_k^2 - a_{kx}^2 \; t_k^2 = \lambda_{ik}^2 \; (x_i^2 - a_{ix}^2 \; t_i^2) \tag{7.8}$$

where the scale factor λ_{ik} is independent of the space and time coordinates. Using (7.7) in (7.8) one obtains the transformation coefficients as

$$\alpha_{ik} = \lambda_{ik} \, \gamma_{ik} \tag{7.9}$$

$$\beta_{ik} = \alpha_{ik} \, / \, \rho_{ik} \tag{7.10}$$

$$\xi_{ik} = - \, (\alpha_{ik}/\rho_{ik}) \; (v_{ik}/a_{ix}^2) \tag{7.11}$$

with
$$\gamma_{ik} = (1-v_{ik}^2/a_{ix}^2)^{-1/2} \tag{7.12}$$

and
$$\rho_{ik} = a_{kx}/a_{ix} \tag{7.13}$$

The transformation equations can thus be written as

$S_i \rightarrow S_k$:

$$x_k = \lambda_{ik} \, \gamma_{ik} \; (x_i - v_{ik} \, t_i) \tag{7.14a}$$

$$t_k = (\lambda_{ik}/\rho_{ik}) \; \gamma_{ik} \; (t_i - (v_{ik}/a_{ix}^2) x_i) \tag{7.14b}$$

where λ_{ik} is yet to be determined. Even at this stage one can make some important observations:

(A) The transformation equations contain the two-way speeds of the synchronizing signal in the frames S_i & S_k.

(B) Factors γ_{ik} , akin to the γ factor in the TE of special relativity $[\gamma = (1-v^2/c^2)^{-1/2}]$ appear with c replaced by the acoustic speed.

(C) Simultaneity is relative (which follows from 7.14b).

Uptill now no special property (like CVL in SR) of the signal speed has been assumed. It therefore follows that the observations A, B & C are the consequences of the assumed distant clock synchrony.

(D) Further by inverting the above equations (7.14a & 7.14b) one may verify that

$$v_{ki} = -\rho_{ik} v_{ik} \tag{7.15}$$

which illustrates that under the assumed synchrony relative speeds are not reciprocal since $\rho_{ik} \neq 1$ in general.

From (7.15) it also follows [by virtue of (7.12)] that

$$\gamma_{ik} = \gamma_{ki} \tag{7.16}$$

and

$$\lambda_{ik} \lambda_{ki} = 1 \tag{7.17}$$

Now suppose there is another frame S_l moving with respect to S_i & S_k with uniform speeds v_{il} & v_{kl} respectively. One can immediately write down by suitably changing subscripts in equations (7.14a) & (7.14b) the following transformations:

$S_k \rightarrow S_l$:

$$x_l = \lambda_{kl} \gamma_{kl} (x_k - v_{kl} t_k) \tag{7.18a}$$

$$t_l = (\lambda_{kl}/\rho_{kl}) \gamma_{kl} (t_k - (v_{kl}/a_{kx}^2) x_k) \tag{7.18b}$$

$S_k \rightarrow S_i$:

$$x_i = \lambda_{ki} \gamma_{ki} (x_k - v_{ki} t_k) \tag{7.19a}$$

$$t_i = (\lambda_{ki}/\rho_{ki}) \gamma_{ki} (t_k - (v_{ki}/a_{kx}^2) x_k) \tag{7.19b}$$

$S_i \rightarrow S_l$:

$$x_l = \lambda_{il} \gamma_{il} (x_i - v_{il} t_i) \tag{7.20a}$$

$$t_l = (\lambda_{il}/\rho_{il}) \gamma_{il} (t_i - (v_{il}/a_{ix}^2) x_i) \tag{7.20b}$$

The transformations between S_l & S_k ($S_k \rightarrow S_l$) can also be obtained by inserting equations (7.19) in equations (7.20). For example the spatial part of this transformation (between S_l & S_k via S_i) will assume the form

$$x_l = \frac{\lambda_{il}\,\lambda_{ik}^{-1}\,(1-(v_{ik}v_{il}/a_{ix}^2))}{(1-v_{il}^2/a_{ix}^2)^{1/2}\,(1-v_{ik}^2/a_{ix}^2)^{1/2}}\,[x_k + \frac{a_{kx}}{a_{ix}}\,\frac{(v_{ik}-v_{il})}{(1-v_{ik}\,v_{il}/a_{ix}^2)}\,t_k] \tag{7.21}$$

where we have explicitly used together with (7.15) & (7.16) the expressions for γ_{ik} & ρ_{ik} for different subscripts from equations (7.12) & (7.13).

Comparing (7.21) with (7.18a) one can easily find the velocity transformation formula

$$v_{kl} = \frac{a_{kx}}{a_{ix}}\,\frac{v_{il}-v_{ik}}{1-(v_{ik}\,v_{il}/a_{ix}^2)} \tag{7.22}.$$

Also we find an important relation

$$\lambda_{kl} = \frac{\lambda_{il}}{\lambda_{ik}} \tag{7.23}$$

which will be useful later. Note that in order to obtain (7.23) we used the algebraic identity

$$(1 - v_{ik}v_{il}/a_{ix}^2)\,\gamma_{ik}\,\gamma_{il} = \gamma_{kl} \tag{7.24}.$$

Note that we could not say, as yet anything about λ_{ik} except some relations (viz (7.17) & (7.23)) which they satisfy. Indeed, we cannot proceed any further through one dimensional analysis since the factor λ_{ik} remains to be determined. However, later we shall see that λ_{ik} will contain two-way speeds of the signal along other directions as well and to understand all the subtleties & the full implications of the C-S thesis viz-a-viz SR and related theories one should in our opinion work with two spatial dimensions (one may leave out the z-direction because of symmetry) even though some notable earlier works (Grünbaum 1963; Winnie 1970) on the C-S thesis were based on 1-dimensional analysis.

7.3. PREFERRED FRAME AND THE TWO-WAY VELOCITY TRANSFORMATIONS

In order to find out λ_{ik} it will be essential first to introduce the concept of a preferred frame. In a general frame S_k the TWS of AS will not be isotropic, however it will be the same in all directions with respect to a frame S_0, stationary with respect to the medium. Therefore one may call the reference frame S_0 to be a preferred one. It therefore follows that in S_0 , if this isotropic signal speed is assumed to be a_0 , one is able to write along any given ray direction (x,y)

$$a_x^2 + a_y^2 = a_0^2 \tag{7.25}$$

where a_x and a_y are the x & y components of the velocity of the wavefront along the direction. We shall use this relation in due course.

The velocity transformations that follow from (7.7) may be written as

$$u_{kx} = \frac{\alpha_{ik}\,(u_{ix} - v_{ik})}{\xi_{ik}\,u_{ix} + \beta_{ik}} \tag{7.26}$$

and

$$u_{ky} = \frac{u_{iy}}{\xi_{ik}\, u_{ix} + \beta_{ik}} \tag{7.27}$$

where, u_{ix} and u_{iy} are the components of velocities of a particle (or a signal) as observed from S_i , whereas u_{kx} and u_{ky} represent the same as observed from S_k.

Let us now calculate the transformation for the two-way speeds of a signal in the longitudinal (x) as well as in the transverse (y) direction. To be specific consider the propagation of the acoustic signal as viewed from S_0 and S_k. Let us recall the velocity transformations (7.26) & (7.27). Between S_0 & S_k , the index 'i' is to be replaced by '0' in these equations. Now suppose in order to measure the TWS (longitudinal) of sound from S_k , a sound pulse is sent away from the origin of S_k along the x-direction (a_y=0) and finally received back at the same point of S_k. The relative speeds u_{kx} for the forward and as well as for the reverse journey may be obtained from (7.26) by replacing u_{ix} by $\pm a_x = \pm a_0$ (since a_y=0 in (7.25) implies $a_x = a_0$). The harmonic mean of these values give the value for the TWS (longitudinal) for sound as

$$a_{kx} = \frac{\alpha_{0k} a_0 \;(1 - v_{0k}^2 / a_0^2)}{(\beta_{0k} + \xi_{0k}\; v_{0k})} \tag{7.28}$$

In order to measure the TWS (transverse) from S_k , one sends the sound signal along the y-axis of S_k and receives back the reflected signal. With respect to S_0 however, these "ray" directions for the forward and the return journey of the signal will not be along the y-axis. Now since u_{kx}, the x-component of the one-way sound speed in this case is zero, from (7.26) it is evident that

$$u_{0x} = a_x = v_{0k} \tag{7.29}$$

Inserting (7.29) in (7.25) one obtains a_y as

$$a_y = \pm\, a_0\, (1 - v_{0k}^2 / a_0^2)^{1/2} \tag{7.30}$$

Inserting these values for a_y ($=u_{0y}$) in (7.27) (and taking the harmonic mean as usual) one obtains

$$a_{ky} = \frac{a_0\, (1 - v_{0k}^2 / a_0^2)^{1/2}}{(\beta_{0k} + \xi_{0k}\, v_{0k})} \tag{7.31}$$

Observe that in obtaining the equations (7.28) and (7.31) the explicity expressions for α_{0k}, β_{0k}, ξ_{0k} which contain a_0 have not yet been used. One may therefore easily verify that the general two-way velocity transformation laws for any "other" signal whose *isotropic* speed (two-way) in S_0 is a_0', which may numerically differ from a_0 may be given by

TWS (longitudinal) $$a_{kx}' = \frac{\alpha_{0k}\, a_0'\, (1 - v_{0k}^2 / a_0'^2)}{(\beta_{0k} + \xi_{0k}\, v_{0k})} \tag{7.32}$$

and

TWS (transverse) $$a_{ky}' = \frac{a_0'\, (1 - v_{0k}^2 / a_0'^2)^{1/2}}{(\beta_{0k} + \xi_{0k}\, v_{0k})} \tag{7.33}$$

where a'_{kx} and a'_{ky} are the TWS of the signal as measured from S_k in the longitudinal and in the transverse directions respectively. However it is clear that in order to arrive at these relations one assumes that with respect to S_0 under the chosen synchrony, the OWS of the "other" signal is isotropic and hence is equal to its TWS. In other words it has been tacitly assumed that in S_0 the pseudo-standard synchrony with AS and that with the "other" signal are equivalent.

7.4 THE COMPLETE TE

Having obtained the transformation laws for the two-way speeds we are now in a position to obtain the undetermined parameter λ_{ik} as a function of the two-way speeds of the synchronizing signal. Using the relations (7.9 – 7.13) the equation (7.31) , after simplification reads

$$a_{ky} = \frac{a_{kx}}{\lambda_{0k}}$$

which gives

$$\lambda_{0k} = \frac{a_{kx}}{a_{ky}} \tag{7.34}$$

Combining (7.34) with (7.23) yields

$$\lambda_{ik} = \frac{\lambda_{0k}}{\lambda_{0i}} = \frac{a_{kx}}{a_{ky}} \frac{a_{iy}}{a_{ix}} \tag{7.35}$$

With this value for λ_{ik} when put in equations (7.14a) & (7.14b) give us the complete transformation equations which can be written explicitly as

$S_i \to S_k$:

$$x_k = \frac{a_{kx}}{a_{ix}} \frac{a_{iy}}{a_{ky}} (1-v_{ik}^2/a_{ix}^2)^{-1/2} [x_i - v_{ik} t_i] \tag{7.36a}$$

$$t_k = \frac{a_{iy}}{a_{ky}} (1-v_{ik}^2/a_{ix}^2)^{-1/2} [t_i - (v_{ik}/a_{ix}^2) x_i] \tag{7.36b}$$

With respect to the preferred frame S_0 (where $a_{0x}=a_{0y}=a_0$) these equations take simpler forms:

$S_0 \to S_k$:

$$x_k = \frac{a_{kx}}{a_{ky}} (1-v_{0k}^2/a_0^2)^{-1/2} [x_0 - v_{0k} t_0] \tag{7.37a}$$

$$t_k = \frac{a_0}{a_{ky}} (1-v_{0k}^2/a_0^2)^{-1/2} [t_0 - (v_{0k}/a_0^2) x_0] \tag{7.37b}$$

The above equations (7.36a – 7.37b) represent the space-time transformations as perceived by the Dolphins. In a lighter vein one may call them Dolphin Transformations (DT). One may observe that except for few factors the DT look very similar to LT. It is obvious that the γ-factors (γ_{ik}, γ_{0k} etc.) and the spatial part appearing in the time transformations (equations 7.36b or 7.37b) have their origin in choosing the standard synchrony with AS.

In the present form of the DT it is evident that one is able to use the space-time relations between two frames provided one knows the TWS of AS in these two frames. Note that if instead of AS one chooses light signal (in vacuum) for synchrony one is able, by virtue of CVL postulate in SR, to put in equations (7.36a) & (7.36b)

$$a_{ix} = a_{iy} = a_{kx} = a_{ky} = c$$

and one immediately obtains the familiar Lorentz transformations. However, in absence of any communication with the outside world, *apparently* 'c' does not play any role in DT. This may appear surprising since we (and the Dolphins too) live in the relativistic world where we know 'c' plays a fundamental role!

Indeed the Dolphins will discover the physical constant 'c' to appear in their transformations through a_{kx} and a_{ky} (equations 7.37a & 7.37b). In order to make the DT usable, the Dolphins will have to measure the TWS of AS in S_k as a function of velocity v_{Ok} and one can anticipate that they will eventually find that

$$a_{kx} = a_{kx} (v_{Ok} , c) \tag{7.38}$$

and

$$a_{ky} = a_{ky} (v_{Ok} , c) \tag{7.39}$$

where 'c' would appear in these functions not as the speed of light but as some physical constant.

To understand this consider the synchronizing signal to be the optical signal in the *medium*. The Dolphins may conduct experiments similar to Fizeau's experiment (velocity of light determination in a moving liquid) to obtain a_{kx} & a_{ky} as function of the liquid speed and we know how they should contain the *vacuum* speed of light c through the refractive index terms (Kacser 1967). In order to make this remark we have implicitly assumed that relativity is intrinsically true and one or the other formulations of SR, just because of the possibility of different synchronizations, would not predict different results for measurements which do not require distant clock synchrony. Remember that one such synchrony independent quantity is the TWS of a signal and in Fizeau's experiment one essentially measures the TWS only.

7.5. RELATIVISTIC AND GALILEAN WORLDS :

To understand the whole thing clearly we consider two possible "worlds" – the relativistic world and the Galilean world. The relativistic world is defined to be a world where the space-time admits an *invariant TWS*, on the other hand the world will be called Galilean if TWS of any signal will obey the transformation law that one would have obtained by using Galilean velocity addition theorem.

Now let us suppose that the Dolphins are sometimes able to communicate with the outside world and discover that their world admits an invariant speed 'c'. Recall the formulae for the two-way velocity transformations (7.32) & (7.33) and put $a'_{kx} = a'_{ky} = a'_0 = c$. Now using the equations (7.9–7.12) one may easily demonstrate that

$$\rho_{0k} = \frac{(1-v_{0k}^2/a_0^2)}{(1-v_{0k}^2/c^2)} \tag{7.40}$$

and

$$\lambda_{0k} = \frac{(1-v_{0k}^2/a_0^2)^{1/2}}{(1-v_{0k}^2/c^2)^{1/2}} \tag{7.41}$$

or by equations (7.13) and (7.34)

$$a_{kx} = a_0 \frac{(1-v_{0k}^2/a_0^2)}{(1-v_{0k}^2/c^2)} \tag{7.42}$$

and

$$a_{ky} = a_0 \frac{(1-v_{0k}^2/a_0^2)^{1/2}}{(1-v_{0k}^2/c^2)^{1/2}} \qquad (7.43).$$

Indeed (7.42) & (7.43) are in accordance with our previous stipulation [vide equations (7.38) & (7.39)] made earlier in the previous section.

One may also verify that one would have obtained the same expressions (7.42) & (7.43) for the TWS of a signal using LT. This is not surprising since the TWS only depends on the world type and it cannot depend on distant synchrony.

Now inserting (7.42) & (7.43) in (7.37a) and (7.37b) gives the Dolphins' Transformations for the relativistic world.

$S_0 \rightarrow S_k$:

$$x_k = \frac{x_0 - v_{0k}\, t_0}{(1-v_{0k}^2/c^2)^{1/2}} \qquad (7.44a)$$

and

$$t_k = \frac{(1-v_{0k}^2/c^2)^{1/2}}{(1-v_{0k}^2/a_0^2)} [t_0 - (v_{0k}/a_0^2)x_0] \qquad (7.44b).$$

Or inverting one obtains

$S_k \rightarrow S_o$:

$$x_0 = \frac{(1-v_{0k}^2/c^2)^{1/2}}{(1-v_{0k}^2/a_0^2)}\, x_k + \frac{v_{0k}}{(1-v_{0k}^2/c^2)^{1/2}}\, t_k \qquad (7.45a)$$

and

$$t_0 = \frac{v_{0k}}{a_0^2} \frac{(1-v_{0k}^2/c^2)^{1/2}}{(1-v_{0k}^2/a_0^2)}\, x_k + \frac{1}{(1-v_{0k}^2/c^2)^{1/2}}\, t_k$$

(7.45b).

From the above transformation equations (7.44a - 7.45b) it is clear that the time dilatation and length contraction factors as observed from S_k for standard clocks and rods stationary with respect to the medium are given by

$$\text{Length Contraction Factor (LCF)} = \frac{(1-v_{0k}^2/a_0^2)}{(1-v_{0k}^2/c^2)^{1/2}} \qquad (7.46).$$

$$\text{Time Dilatation Factor (TDF)} = \frac{(1-v_{0k}^2/c^2)^{1/2}}{(1-v_{0k}^2/a_0^2)} \qquad (7.47).$$

With respect to S_0 however LCF & TDF are the same as predicted by SR. This is not surprising as we have pointed out earlier (section 7.3) that in S_0 Einstein synchrony and the pseudo-standard synchrony tally.

In a similar way one can write down the general expressions for LCF and TDF for moving rods and clocks with respect to any arbitrary frame. However, to understand the role of pseudo-standard synchrony qualitatively at least it is enough to continue the discussions by confining ourselves to the relations (7.46) & (7.47). From these equations we can see how in the Dolphins' relativistic world the light signal & the synchronizing signal play together to give us the kinematics effects. Note that as $a_0 \to c$ i.e. instead of pseudo -standard synchrony if one chooses Einstein (standard) synchrony one gets back the usual relativistic results for LCF & TDF.

Since the birth of SR there is a considerable disagreement among Physicists as to whether the time dilatation and length contraction are real or whether they are solely due to the process of synchronization (Sjödin 1979). We now observe that the factor

$(1-v_{0k}^2/c^2)$ which appear in (7.46) & (7.47) are due to real effects whereas the factor $(1-v_{0k}^2/a_0^2)$ arises due to the special way chosen for the signal synchrony. Indeed one can even demonstrate that the "apparent" LCF & TDF can be obtained in the Galilean world because of signal synchrony. This can be seen as follows.

From the Galilean velocity addition formulae one can obtain the TWS of any signal with respect to any frame S_k moving with respect to the medium. We may write (Ghosal et al 1991b) for example the TWS of the AS, as

$$a_{kx} = a_0 \; (1-v_{0k}^2/a_0^2) \tag{7.48}$$

and

$$a_{ky} = a_0 \; (1-v_{0k}^2/a_0^2)^{1/2} \tag{7.49}$$

.

Inserting these expressions for a_{kx} & a_{ky} in the DT (and in particular in 7.37a & 7.37b) we obtain

$$x_k = x_0 - v_{0k}\, t_0 \tag{7.50a}$$

and

$$t_k = (1-v_{0k}^2/a_0^2)^{-1} \; [t_0-(v_{0k}/a_0^2)x_0] \tag{7.50b}$$

.

Whereas the inverse transformations may be written as

$$x_0 = \frac{1}{(1-v_{0k}^2/a_0^2)} \;\; [x_k + (1-v_{0k}^2/a_0^2)v_{0k}\, t_k] \tag{7.51a}$$

and

$$t_0 = t_k + \frac{v_{ok}}{a_0^2 \;(1-v_{0k}^2/a_0^2)}\, x_k \tag{7.51b}$$

.

The above equations (7.50a – 7.51b) represent DT for the Galilean world.

It is thus evident that there is no length contraction or time dilatation effects as observed from S_0. However, for rods & clocks stationary with respect to the medium, the LCF & TDF as observed from a moving frame S_k are respectively given by

$$\text{LCF} = (1-v_{0k}^2/a_0^2) \tag{7.52}$$

and

$$\text{TDF} = (1-v_{0k}^2/a_0^2)^{-1} \tag{7.53}.$$

Clearly in the present case the length contraction and the time dilatation effects are contrived and they arise because of the synchronization alone.

Note that the transformations, (7.50a) & (7.50b) [when a_0 is replaced by c] are much discussed (Zahar 1977; Sjödin 1979; Ghosal et al 1991a) and they are known as Zahar Transformations (ZT). ZT thus represent Galilean world with Einstein (standard) synchrony (pseudo standard synchrony becomes standard synchrony when a_0 is replaced by c).

One observes that in (7.50a) & (7.50b) if a_0 is taken to be infinitely large, ZT becomes Galilean Transformations (GT).*

---

* However remember that instead of putting $a_0 \to \infty$ in these equations, if v^2/c^2 terms are neglected in ZT we do not obtain GT, instead one obtains the so-called Approximate Lorentz Transformations (Ghosal et al 1991a).

On the other hand if one assumes $a_0 \to \infty$ in equations (7.44a–7.45b) which represent DT in the relativistic world one obtains the so-called Tangherlini transformations (Sjödin 1979; Ghosal et al 1991a).

$S_0 \to S_k$:

$$x_k = \frac{x_0 - v_{0k}\, t_0}{(1-v_{0k}^2/c^2)^{1/2}} \tag{7.54a}$$

$$t_k = (1-v_{0k}^2/c^2)^{1/2}\, t_0 \tag{7.54b}$$

and

$S_k \to S_0$:

$$x_0 = (1-v_{0k}^2/c^2)^{1/2}\, [x_k + \frac{v_{0k}}{(1-v_{0k}^2/c^2)}\, t_k \tag{7.55a}$$

$$t_0 = (1-v_{0k}^2/c^2)^{-1/2}\, t_k \tag{7.55b}.$$

The above transformations (7.54a–7.55b) represent relativistic world with absolute synchrony. According to this synchrony the simultaneity is absolute. On the contrary Zahar transformations represent Galilean world manifesting relativity of simultaneity.

7.6. EPILOGUE

Before we end this chapter, let us take stock of what we have achieved so far. We have essentially clarified the role of conventionality in special relativity. In order to do so we have tried to heuristically derive SR in a fluid medium (Dolphin's World!) where clock synchronization can be done by acoustic signal, optical signal or any other wave mode available in the medium. Remember that once a signal has been picked up for synchrony we call it acoustic signal just to keep us reminding that speed of the synchronizing signal is different from that of light in empty space. This has been done in order to understand by contrast, specifically the role of the optical signal as a synchronizing agent, in the standard formulation of SR.

We have argued in secs.7.4 & 7.5 that for the appearance of γ-factor in SR, the Einstein's choice of synchrony is partly responsible. Indeed by deliberately opting for non-luminal synchrony, but at the same time preserving Einstein's procedure for it we have been able to see visually that the factor $(1-v^2/a_0^2)$ are present in the transformation equations. Obviously if a_0 happens to be equal to c this factor becomes γ^{-2} which when combined with the γ-factors (whose origin is independent of convention) already existing in the TE for the relativistic world gives the well known form for the TE viz LT.

Note the condition $a_0 \rightarrow c$ does not necessarily mean that the Dolphins will have to synchronize clocks with a signal whose speed is c. In fact one need not have to step out of the fluid world. The assumption $a_0=c$ practically means that the to and fro speeds (a_+ & a_- say) of the AS in a "moving" frame are *unequal*. This is because they are now subjected to the usual relativistic one-way velocity transformations. In other words the condition $a_0 \rightarrow c$, that was necessary to arrive at LT from equations (7.44a) & (7.44b), may be replaced by the assumption of relativistically correct forms for a_+ and a_- as function of the speed (with respect to S_0) of the moving frame.*

*However, it is evident from sec.7.2b that the sound-sphere derivation will not be applicable in the present situation, but this is unimportant for the present discussion.

We may therefore conclude that even if it is given that the Dolphins are not able to communicate with the outside world it is possible to synchronize clocks with any signal available in the medium but by exercising the option to take (by virtue of the c-s thesis) any functional form of a_+ and a_-, the Dolphins may, by trial, eventually strike upon the standard synchrony.

So far as the TWS of the AS is concerned, if relativity is correct, by empirical observations one would be able to verify in principle that $a_{kx} = a_{kx}(v_{Ok}, c)$ and $a_{ky} = a_{ky}(v_{Ok}, c)$ with their correct forms given by the equations (7.42) & (7.43). We have seen that when these are inserted in the DT we obtain equations (7.44a–7.45b) which again goes over to LT as a special case.

The above observations clarify the role of 'c' in the space-time transformations. We see how eventually the Dolphins may arrive at the Lorentz transformations without even uttering the word "light" since as we have remarked in sec.7.4, c enters in eqns. (7.44a) & (7.44b) (and hence in LT) just as a physical constant.*

Here we may once again support the Sjödin interpretation which holds that the relativistic transformation equations really describe the behaviour of standard rods and clocks in motion where in the present case the term "motion" now refers to the motion relative to the frame at rest with the *medium*.

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*This remark apparently has a striking similarity with that usually given in connection with the one-postulate-derivations of LT (Lee & Kalotas 1975; Srivastava 1981), however, it may be noted that these derivations do not take into account the C-S thesis.

The arguments used throught this chapter may also be extended in the case of a non-homogeneous medium where light rays are not straight lines in general. One may obtain transformations analogous to DT (valid for a homogeneous substrate). The calculations may be a bit more involved but will be straight forward nevertheless. Also as in the previous situation, here standard SR will show up as a special case.

The whole developement will find its direct applicability in the context of flat-space-time theories of gravity where velocity of light as we have seen is not equal to c. This is so as because the empirical content of SR is independent of synchrony and at the same time it resides in the empirical behaviour of "standard rods & clocks" in motion. These arguments may be seen to be indirectly substantiated by a recent claim that LT are not valid if one uses non-standard instruments viz. the so-called "unbonded instruments" (vide Sherwin 1992 + references therein). Note that in the first part of the present volume we considered standard rods and clocks and we were therefore justified in assuming the correctness of SR globally even in presence of gravity.

Another importance of the DT for the relativistic world is that we understand that the c′-relativity is not a correct proposition. What is c′-relativity? Rosen (1952) considered a homogeneous isotropic medium in which some phenomena take place with which is associated a limiting speed c′. It was then claimed that within the medium a set of space-time transformations will hold which are Lorentzlike i.e Lorentz transformations with c replaced by c′. It was called c′- relativity. Even though it can have some limited applicability as claimed by Rosen, we hold that

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