CHAPTER VII

THE ROLE OF c IN SPACE-TIME TRANSFORMATIONS : RELATIVITY IN A SUBSTRATE*

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7.1. INTRODUCTION

The present chapter also deals with the so-called Reichenbach-Grünbaum thesis (Reichenbach 1958; Grünbaum 1963) of conventionality of distant simultaneity of Special Relativity (SR). We have already noted that the Conventionality of Simultaneity (C-S) thesis observes that in Einstein's formulation of SR, the synchronization procedure for spatially distant clocks in an inertial frame contains an element of convention . The conventional ingredient of SR which logically cannot have any empirical content, gives rise to results that are often erroneously construed as the new philosophical imports of special relativity theory. For example, the issue of relativity of distant simultaneity which according to the C-S thesis can be proved to be empirically empty (Winnie 1970; Cavalleri & Bernasconi 1989; Ghosal et al 1991b; Wegener 1992) is still popularly regarded as one of the most fundamental imports of SR. As is well known, the source of this conventionality lies in the fact that in SR distant clocks in a *given* inertial frame are synchronized by light signals, the one-way speed (Ghosal et al 1991a & 1991b) of which has to be known beforehand for the purpose. To know the one-way speed of light on the other hand one requires to have presynchronized clocks and the whole process of synchronization ends up in a logical circularity which forces us to introduce a degree of arbitrariness in assigning the value for the one-way velocity of light. However it may be borne in mind that there is no arbitrariness regarding the round-trip speed of a signal, since the round-trip speed can be measured with one clock only and therefore it is independent of conventionality of distant clock synchrony (Winnie 1970; Ghosal et al 1991a & 1991b).

There has now been a substantial amount of clarification of the C-S thesis due to a number of authors. Possibility of using synchronization convention other than that adopted by Einstein to describe the relativistic world has also been much discussed. Papers by Winnie (1970) and more recently by Mansouri and Sexl (1977) and successive development by Podlaha (1978) and by Sjödin (1979) in particular are some of the recent important expositions in this regard.

Unfortunately still, apart from some casual mention of the problem of clock synchrony in SR, text books of relativity hardly devotes a paragraph on the conventionality thesis or the possibility of using non-Einsteinian synchrony to describe the relativistic world.

The fact that the C-S thesis has not yet gained wide spread attention among Physicists may be attributed to the fact that there is a tendency to regard the C-S thesis as an antithesis of SR and anything that seems contrary to the standard formulation of relativity is viewed with skepticism. Indeed in our opinion the C-S thesis complements SR and the understanding of the former helps clear out confusions that sometimes occur in SR. As we have pointed out, the claim that the relativity of distant simultaneity is a new non-classical philosophical import is just one example of various such confusions. In a recent paper (Ghosal et al 1991a) we have discussed another common misconception that under the small velocity approximation, SR goes over to the Galilean relativity. Indeed we have noted that the small velocity approximation cannot alter simultaneity convention.

Misconstruction of the C-S thesis itself is also not uncommon. For example in a recent article (Cavalleri & Bernasconi 1989) it has been erroneously suggested that light speed invariance in special relativity is a trivial matter and as if, by virtue of the C-S thesis, even Galilean Physics can be reformulated so that light speed remains invariant! In another paper Rodrigues (1985), in connection with the enquiry whether Lorentz invariant clocks can exist without violating the principle of relativity, incorrectly remarked that the possibility of having absolute synchrony is an antithesis of the relativity principle! In some of our recent papers (Ghosal et al 1991b ; Ghosal & Chakraborty 1992) we addressed ourselves to the task of clarifying these issues.

Sometimes in connection with the C-S thesis, the debatable issue of ether (as a *hypothetical* substrate providing a preferred inertial frame) often crops up (Sjödin 1979; Mansouri & Sexl 1977; Cavalleri & Bernasconi 1989). But question have been raised whether considerations of synchronization alone can distinguish an ether frame or not (Spinelli 1983; Cavalleri & Spinelli 1983; Stone 1991). As it stands now, as if the existence of a real physical ether as a preferred frame would have placed the C-S thesis on a stronger footing. In fact efforts are still on to give a physical support to this preferred frame of ether (Spinelli 1983; Cavalleri & Spinelli 1983). (We shall later see that for the understanding of the C-S thesis at least, one can bypass the debate concerning the existence of ether by introducing at the out- set a real physical substrate (water for example) through which different inertial frames may be considered to be in relative motion).

Given this perspective of confusion, misconstruction and polemics regarding the C-S thesis or SR for that matter, we are led to conclude that everything of SR is still not well understood. We therefore feel that it is necessary to provide some additional clarifications in this regard. It is to this task the present paper addresses itself.

However, before we proceed let us first make the following observations: In the standard formulation of SR (and in particular in the derivation of Lorentz transformations) light has two roles to play. In the one hand it offers the peculiar property viz the constancy of its Two-Way-Speed (TWS), which any transformation will have to honour. On the other hand in SR light also acts as a synchronizing agent since, according to SR distant clocks in any *given* inertial frame are synchronized with it following a particular convention. The convention is known as the standard synchronization according to which the One-Way-Speed (OWS) of light is assumed to be independent of direction along a given line.

It is obvious that while the first role (TWS-role) has its basis on the empirically verifiable property of light, the second one is purely prescriptive in origin.

In SR these two roles of light overlap in a sense that they work together to render 'c', the speed of light in vacuum to appear in a particular way in Lorentz Transformations (LT). Therefore by looking at LT it becomes difficult to separate the real relativistic effects from the apparent ones where the latter have its origin in the prescriptive inputs alone.

In the standard formulation of SR it is the non-separatebility of these two roles of light which is largly responsible for a lot of confusion that still prevails. In order to gain a clearer perspective it will be benificial in our opinion to somehow delink the synchronizing role of the optical signal from its TWS role so that one can visualize the relative effects of each of these roles separately in the space-time transformations.

Keeping this in mind we propose in the present text to choose a different signal (whose TWS is not equal to c) for synchronization at the out-set. This new signal will be assumed to be a wave (Podlaha 1975,1976) so that its TWS with respect to different inertial frames will have definite values. The TWS of a wave can always be measured in principle by the reflection technique which requires only one clock and therefore it is independent of the distant clock synchrony. Such a signal whose round-trip speed is different from c, though not available in an empty space, can always be thought to exist within a medium. One may consider a fluid substrate and may single out any particular wave mode characteristic to the medium as a synchronizing agent. This can be the acoustic mode or it can even be the optical mode with speed c′ different from c. For the following analysis however we shall use the term "acoustic signal" to mean any signal (chosen for clock synchrony) whose speed is not equal to c. We may then enquire what the nature of the transformation equations will be if the synchronization is performed by the "acoustic signal".

For definiteness let us imagine that some intelligent Dolphins equipped with standard rods and clocks under water are trying to "discover" relativity. For the time being let us suppose that the Dolphins are unable to communicate with the outside world so that only the "acoustic signal" with speed c' is available for the distant clock synchrony. One may then ask— what will be the nature of the transformation laws? Will they contain c, the vacuum speed of light? If so how? Or, will the Dolphins land up with a c'-relativity theory* analogous to the Einstein (c-relativity) theory with the speed c' replacing c?

The purpose of the present article is to obtain heuristically the transformation equations between inertial frames in the medium from the Dolphins' perspective and thereby all the questions posed above will be answered. As we go along some important and subtle aspects of the role of optical synchrony in SR will automatically be revealed.

We shall be particularly interested in understanding the role of Einstein's standard synchrony in the structure of LT. This will be best understood if the procedure for the standard synchrony adopted by Einstein is mimicked by taking the "acoustic signal" as the synchronizing agent. We shall then be able to see clearly, among other things that the well known γ-factors of SR, partly has its origin in the synchronization convention alone.

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* c'-relativity was considered by Rosen (1952) who developed the so-called c'-Lorentz transformations (c' replacing c in ordinary LT) between two inertial frames in relative motion within a medium.

7.2. TRANSFORMATION EQUATIONS (TE) IN A SUBSTRATE :

(a) Pseudo-Standard Synchrony

In order to obtain the transformation equations let us suppose that the Dolphins follow the procedure similar to the so-called light sphere derivation of LT commonly discussed in standard text books. The essential content of this derivation may be outlined as follows:

Two inertial frames S_i and S_k are travelling with a relative speed v_{ik} along the positive direction of their common x-axis and it is supposed that a flash of light be emitted from their common origin at $t_i = t_k = 0$ Then it is asked which linear transformations of the form

$$\begin{aligned} x_k &= \alpha_{ik}\, x_i + \delta_{ik}\, t_i \\ y_k &= y_i \qquad\qquad (7.1) \\ \text{and} \quad t_k &= \xi_{ik}\, x_i + \beta_{ik}\, t_i \end{aligned}$$

preserves the form of the spherical wave front equations:

$$x_k^2 + y_k^2 + z_k^2 = c^2\, t_k^2 \qquad (7.2).$$

In other words the task here is to find out the unknown coefficients α_{ik} ,δ_{ik} etc. for which the equation (7.2) goes over to

$$x_i^2 + y_i^2 + z_i^2 = c^2\, t_i^2 \qquad \text{after the transformations.}$$

Note that while the last requirement assumes the second relativity postulate (that the TWS of light is independent of the reference frame (Ghosal et al 1991b)), the very choice of the spherical wave front equation implicitly assumes Einstein's prescription of standard synchrony according to which the OWS of light is independent of "direction". This is more evident if one takes the one dimensional equivalent of (7.2):

$$x_k^2 = c^2 t_k^2 \tag{7.3}$$

or,
$$x_k = \pm ct_k \tag{7.4}$$

One can now see that the equation (7.4) clearly states that in a *given frame* the one-way speed of light in the forward (+ve x) and as well as in the reverse (-ve x) directions is the same and is equal to the two-way average speed c. This is precisely known as the standard synchronization convention.

Now suppose, instead of light, an acoustic wave is generated at t=0 at the common origin of the frames S_i , S_k etc. and ask what the (one-dimensional) wavefront equation will be. Now, except for the frame S_0 which is at rest relative to the medium the speeds of sound in the positive and in the negative x-directions will not be equal if the clocks were somehow synchronized beforehand by light signal in vacuum. However, according to the C-S thesis the Dolphins are free to adopt the synchrony which assumes the equality of these speeds in all the frames. This synchrony will be termed as pseudo-standard synchrony as opposed to Einstein's standard synchrony which assumes the equality of the one-way *vacuum light speed* in all directions.

According to pseudo-standard synchrony along the x-axis the one-dimensional wavefront equation will be

$$x_k^2 = a_{kx}^2 t_k^2 \tag{7.5}$$

where a_{kx} is the two-way speed of the Acoustic Signal (AS) along the x-axis. Note that in general the two-way speeds will not be the same along other directions. For example, along the y-axis we shall have to write

$$y_k^2 = a_{ky}^2 \; t_k^2 \tag{7.6}$$

where a_{ky} , the two-way speed of AS in the y-direction may differ in value with its x-direction counter part.

In SR one has the Constancy of Velocity of Light (CVL) postulate. However one cannot assume such a thing to be valid for an acoustic signal. Therefore, in absence of a similar postulate the values of a_{kx} & a_{ky} will differ from that of a_{ix} & a_{iy} where the subscripts k & i refer to different inertial frames S_k & S_i which are in relative motion.

(b) <u>One-Dimensional Sound-Sphere Derivation Of TE</u>

Since S_k's spatial origin moves with the velocity v_{ik} with respect to S_i, the transformations (7.1) may now be written as

$$\begin{aligned} x_k &= \alpha_{ik} \; (x_i - v_{ik} \; t_i) \\ y_k &= y_i \\ t_k &= \xi_{ik} \; x_i + \beta_{ik} \; t_i \end{aligned} \tag{7.7}$$

where we have replaced, as usual, δ_{ik} by $-\alpha_{ik} \; v_{ik}$.

In the present case in order to find out the coefficients of the above TE, the Dolphins are not able to use a 3-sphere wavefront equation since the sound wave in a general frame will not be spherical. However by virtue of the chosen synchrony they are able to take the one-sphere equation of the form (7.5) and subject the equation (7.7) to the condition

$$x_k^2 - a_{kx}^2\ t_k^2 = \lambda_{ik}^2\ (x_i^2 - a_{ix}^2\ t_i^2) \tag{7.8}$$

where the scale factor λ_{ik} is independent of the space and time coordinates. Using (7.7) in (7.8) one obtains the transformation coefficients as

$$\alpha_{ik} = \lambda_{ik}\ \gamma_{ik} \tag{7.9}$$

$$\beta_{ik} = \alpha_{ik}\ /\ \rho_{ik} \tag{7.10}$$

$$\xi_{ik} = -\ (\alpha_{ik}/\rho_{ik})\ \ (v_{ik}/a_{ix}^2) \tag{7.11}$$

with

$$\gamma_{ik} = (1-v_{ik}^2/a_{ix}^2)^{-1/2} \tag{7.12}$$

and

$$\rho_{ik} = a_{kx}/a_{ix} \tag{7.13}$$

The transformation equations can thus be written as

$S_i \rightarrow S_k$:

$$x_k = \lambda_{ik}\ \gamma_{ik}\ \ (x_i\ -\ v_{ik}\ t_i) \tag{7.14a}$$

$$t_k = (\lambda_{ik}/\rho_{ik})\ \gamma_{ik}\ \ (t_i\ -\ (v_{ik}/a_{ix}^2)x_i) \tag{7.14b}$$

where λ_{ik} is yet to be determined. Even at this stage one can make some important observations:

(A) The transformation equations contain the two-way speeds of the synchronizing signal in the frames S_i & S_k.

(B) Factors γ_{ik} , akin to the γ factor in the TE of special relativity [$\gamma = (1-v^2/c^2)^{-1/2}$] appear with c replaced by the acoustic speed.

(C) Simultaneity is relative (which follows from 7.14b).

Uptill now no special property (like CVL in SR) of the signal speed has been assumed. It therefore follows that the observations A,B & C are the consequences of the assumed distant clock synchrony.

(D) Further by inverting the above equations (7.14a & 7.14b) one may verify that

$$v_{ki} = -\rho_{ik}\, v_{ik} \tag{7.15}$$

which illustrates that under the assumed synchrony relative speeds are not reciprocal since $\rho_{ik} \neq 1$ in general.

From (7.15) it also follows [by virtue of (7.12)] that

$$\gamma_{ik} = \gamma_{ki} \tag{7.16}$$

and

$$\lambda_{ik}\, \lambda_{ki} = 1 \tag{7.17}$$

Now suppose there is another frame S_l moving with respect to S_i & S_k with uniform speeds v_{il} & v_{kl} respectively. One can immediately write down by suitably changing subscripts in equations (7.14a) & (7.14b) the following transformations:

$S_k \rightarrow S_l$:

$$x_l = \lambda_{kl}\, \gamma_{kl}\, (x_k - v_{kl}\, t_k) \tag{7.18a}$$

$$t_l = (\lambda_{kl}/\rho_{kl})\, \gamma_{kl}\, (t_k - (v_{kl}/a_{kx}^2)\, x_k) \tag{7.18b}$$

$S_k \rightarrow S_i$:

$$x_i = \lambda_{ki}\, \gamma_{ki}\, (x_k - v_{ki}\, t_k) \tag{7.19a}$$

$$t_i = (\lambda_{ki}/\rho_{ki})\, \gamma_{ki}\, (t_k - (v_{ki}/a_{kx}^2)\, x_k) \tag{7.19b}$$

$S_i \rightarrow S_l$:

$$x_l = \lambda_{il}\, \gamma_{il}\, (x_i - v_{il}\, t_i) \tag{7.20a}$$

$$t_l = (\lambda_{il}/\rho_{il})\, \gamma_{il}\, (t_i - (v_{il}/a_{ix}^2)\, x_i) \tag{7.20b}$$

The transformations between S_l & S_k ($S_k \rightarrow S_l$) can also be obtained by inserting equations (7.19) in equations (7.20). For example the spatial part of this transformation (between S_l & S_k via S_i) will assume the form

$$x_l = \frac{\lambda_{il}\,\lambda_{ik}^{-1}\,(1-(v_{ik}v_{il}/a_{ix}^2))}{(1-v_{il}^2/a_{ix}^2)^{1/2}\,(1-v_{ik}^2/a_{ix}^2)^{1/2}}\left[x_k+\frac{a_{kx}}{a_{ix}}\,\frac{(v_{ik}-v_{il})}{(1-v_{ik}\,v_{il}/a_{ix}^2)}\,t_k\right] \tag{7.21}$$

where we have explicitly used together with (7.15) & (7.16) the expressions for γ_{ik} & ρ_{ik} for different subscripts from equations (7.12) & (7.13).

Comparing (7.21) with (7.18a) one can easily find the velocity transformation formula

$$v_{kl} = \frac{a_{kx}}{a_{ix}}\,\frac{v_{il}-v_{ik}}{1-(v_{ik}\,v_{il}/a_{ix}^2)} \tag{7.22}.$$

Also we find an important relation

$$\lambda_{kl} = \frac{\lambda_{il}}{\lambda_{ik}} \tag{7.23}$$

which will be useful later. Note that in order to obtain (7.23) we used the algebraic identity

$$(1-v_{ik}v_{il}/a_{ix}^2)\,\gamma_{ik}\,\gamma_{il} = \gamma_{kl} \tag{7.24}.$$

Note that we could not say, as yet anything about λ_{ik} except some relations (viz (7.17) & (7.23)) which they satisfy. Indeed, we cannot proceed any further through one dimensional analysis since the factor λ_{ik} remains to be determined. However, later we shall see that λ_{ik} will contain two-way speeds of the signal along other directions as well and to understand all the subtleties & the full implications of the C-S thesis viz-a-viz SR and related theories one should in our opinion work with two spatial dimensions (one may leave out the z-direction because of symmetry) even though some notable earlier works (Grünbaum 1963; Winnie 1970) on the C-S thesis were based on 1-dimensional analysis.

7.3. PREFERRED FRAME AND THE TWO-WAY VELOCITY TRANSFORMATIONS

In order to find out λ_{ik} it will be essential first to introduce the concept of a preferred frame. In a general frame S_k the TWS of AS will not be isotropic, however it will be the same in all directions with respect to a frame S_0, stationary with respect to the medium. Therefore one may call the reference frame S_0 to be a preferred one. It therefore follows that in S_0 , if this isotropic signal speed is assumed to be a_0 , one is able to write along any given ray direction (x,y)

$$a_x^2 + a_y^2 = a_0^2 \tag{7.25}$$

where a_x and a_y are the x & y components of the velocity of the wavefront along the direction. We shall use this relation in due course.

The velocity transformations that follow from (7.7) may be written as

$$u_{kx} = \frac{\alpha_{ik}\,(u_{ix} - v_{ik})}{\xi_{ik}\,u_{ix} + \beta_{ik}} \tag{7.26}$$

and

$$u_{ky} = \frac{u_{iy}}{\xi_{ik}\, u_{ix} + \beta_{ik}} \tag{7.27}$$

where, u_{ix} and u_{iy} are the components of velocities of a particle (or a signal) as observed from S_i , whereas u_{kx} and u_{ky} represent the same as observed from S_k.

Let us now calculate the transformation for the two-way speeds of a signal in the longitudinal (x) as well as in the transverse (y) direction. To be specific consider the propagation of the acoustic signal as viewed from S_0 and S_k. Let us recall the velocity transformations (7.26) & (7.27). Between S_0 & S_k , the index 'i' is to be replaced by '0' in these equations. Now suppose in order to measure the TWS (longitudinal) of sound from S_k , a sound pulse is sent away from the origin of S_k along the x-direction (a_y=0) and finally received back at the same point of S_k. The relative speeds u_{kx} for the forward and as well as for the reverse journey may be obtained from (7.26) by replacing u_{ix} by $\pm a_x = \pm a_0$ (since a_y=0 in (7.25) implies $a_x = a_0$). The harmonic mean of these values give the value for the TWS (longitudinal) for sound as

$$a_{kx} = \frac{\alpha_{0k} a_0 \; (1 - v_{0k}^2/a_0^2)}{(\beta_{0k} + \xi_{0k}\; v_{0k})} \tag{7.28}$$

In order to measure the TWS (transverse) from S_k , one sends the sound signal along the y-axis of S_k and receives back the reflected signal. With respect to S_0 however, these "ray" directions for the forward and the return journey of the signal will not be along the y-axis. Now since u_{kx}, the x-component of the one-way sound speed in this case is zero, from (7.26) it is evident that

$$u_{0x} = a_x = v_{0k} \tag{7.29}$$

Inserting (7.29) in (7.25) one obtains a_y as

$$a_y = \pm\, a_0\ (1-v_{0k}^2/a_0^2)^{1/2} \tag{7.30}$$

Inserting these values for a_y $(=u_{0y})$ in (7.27) (and taking the harmonic mean as usual) one obtains

$$a_{ky} = \frac{a_0\ (1-v_{0k}^2/a_0^2)^{1/2}}{(\beta_{0k} + \xi_{0k}\ v_{0k})} \tag{7.31}$$

Observe that in obtaining the equations (7.28) and (7.31) the explicity expressions for α_{0k}, β_{0k}, ξ_{0k} which contain a_0 have not yet been used. One may therefore easily verify that the general two-way velocity transformation laws for any "other" signal whose *isotropic* speed (two-way) in S_0 is a_0', which may numerically differ from a_0 may be given by

TWS (longitudinal) $$a_{kx}' = \frac{\alpha_{0k}\ a_0'\ (1-v_{0k}^2/a_0'^2)}{(\beta_{0k} + \xi_{0k}\ v_{0k})} \tag{7.32}$$

and

TWS (transverse) $$a_{ky}' = \frac{a_0'\ (1-v_{0k}^2/a_0'^2)^{1/2}}{(\beta_{0k} + \xi_{0k}\ v_{0k})} \tag{7.33}$$

where a'_{kx} and a'_{ky} are the TWS of the signal as measured from S_k in the longitudinal and in the transverse directions respectively. However it is clear that in order to arrive at these relations one assumes that with respect to S_0 under the chosen synchrony, the OWS of the "other" signal is isotropic and hence is equal to its TWS. In other words it has been tacitly assumed that in S_0 the pseudo-standard synchrony with AS and that with the "other" signal are equivalent.

7.4 THE COMPLETE TE

Having obtained the transformation laws for the two-way speeds we are now in a position to obtain the undetermined parameter λ_{ik} as a function of the two-way speeds of the synchronizing signal. Using the relations (7.9 – 7.13) the equation (7.31) , after simplification reads

$$a_{ky} = \frac{a_{kx}}{\lambda_{0k}}$$

which gives

$$\lambda_{0k} = \frac{a_{kx}}{a_{ky}} \qquad (7.34).$$

Combining (7.34) with (7.23) yields

$$\lambda_{ik} = \frac{\lambda_{0k}}{\lambda_{0i}} = \frac{a_{kx}}{a_{ky}} \frac{a_{iy}}{a_{ix}} \qquad (7.35).$$

With this value for λ_{ik} when put in equations (7.14a) & (7.14b) give us the complete transformation equations which can be written explicitly as

$S_i \rightarrow S_k$:

$$x_k = \frac{a_{kx}}{a_{ix}} \frac{a_{iy}}{a_{ky}} (1-v_{ik}^2/a_{ix}^2)^{-1/2} [x_i - v_{ik} t_i] \tag{7.36a}$$

$$t_k = \frac{a_{iy}}{a_{ky}} (1-v_{ik}^2/a_{ix}^2)^{-1/2} [t_i - (v_{ik}/a_{ix}^2)x_i] \tag{7.36b}$$

With respect to the preferred frame S_0 (where $a_{0x}=a_{0y}=a_0$) these equations take simpler forms:

$S_0 \rightarrow S_k$:

$$x_k = \frac{a_{kx}}{a_{ky}} (1-v_{0k}^2/a_0^2)^{-1/2} [x_0 - v_{0k} t_0] \tag{7.37a}$$

$$t_k = \frac{a_0}{a_{ky}} (1-v_{0k}^2/a_0^2)^{-1/2} [t_0 - (v_{0k}/a_0^2)x_0] \tag{7.37b}$$

The above equations (7.36a - 7.37b) represent the space-time transformations as perceived by the Dolphins. In a lighter vein one may call them Dolphin Transformations (DT). One may observe that except for few factors the DT look very similar to LT. It is obvious that the γ-factors (γ_{ik}, γ_{0k} etc.) and the spatial part appearing in the time transformations (equations 7.36b or 7.37b) have their origin in choosing the standard synchrony with AS.

In the present form of the DT it is evident that one is able to use the space-time relations between two frames provided one knows the TWS of AS in these two frames. Note that if instead of AS one chooses light signal (in vacuum) for synchrony one is able, by virtue of CVL postulate in SR, to put in equations (7.36a) & (7.36b)

$$a_{ix} = a_{iy} = a_{kx} = a_{ky} = c$$

and one immediately obtains the familiar Lorentz transformations. However, in absence of any communication with the outside world, *apparently* 'c' does not play any role in DT. This may appear surprising since we (and the Dolphins too) live in the relativistic world where we know 'c' plays a fundamental role!

Indeed the Dolphins will discover the physical constant 'c' to appear in their transformations through a_{kx} and a_{ky} (equations 7.37a & 7.37b). In order to make the DT usable, the Dolphins will have to measure the TWS of AS in S_k as a function of velocity v_{0k} and one can anticipate that they will eventually find that

$$a_{kx} = a_{kx}\,(v_{0k}\,,\,c) \tag{7.38}$$

and

$$a_{ky} = a_{ky}\,(v_{0k}\,,\,c) \tag{7.39}$$

where 'c' would appear in these functions not as the speed of light but as some physical constant.

To understand this consider the synchronizing signal to be the optical signal in the *medium*. The Dolphins may conduct experiments similar to Fizeau's experiment (velocity of light determination in a moving liquid) to obtain a_{kx} & a_{ky} as function of the liquid speed and we know how they should contain the *vacuum* speed of light c through the refractive index terms (Kacser 1967). In order to make this remark we have implicitly assumed that relativity is intrinsically true and one or the other formulations of SR, just because of the possibility of different synchronizations, would not predict different results for measurements which do not require distant clock synchrony. Remember that one such synchrony independent quantity is the TWS of a signal and in Fizeau's experiment one essentially measures the TWS only.

7.5. RELATIVISTIC AND GALILEAN WORLDS :

To understand the whole thing clearly we consider two possible "worlds" – the relativistic world and the Galilean world. The relativistic world is defined to be a world where the space-time admits an *invariant TWS*, on the other hand the world will be called Galilean if TWS of any signal will obey the transformation law that one would have obtained by using Galilean velocity addition theorem.

Now let us suppose that the Dolphins are sometimes able to communicate with the outside world and discover that their world admits an invariant speed 'c'. Recall the formulae for the two-way velocity transformations (7.32) & (7.33) and put $a'_{kx} = a'_{ky} = a'_{O} = c$. Now using the equations (7.9–7.12) one may easily demonstrate that

$$\rho_{Ok} = \frac{(1-v_{Ok}^2/a_O^2)}{(1-v_{Ok}^2/c^2)} \tag{7.40}$$

and

$$\lambda_{Ok} = \frac{(1-v_{Ok}^2/a_O^2)^{1/2}}{(1-v_{Ok}^2/c^2)^{1/2}} \tag{7.41}$$

or by equations (7.13) and (7.34)

$$a_{kx} = a_O \frac{(1-v_{Ok}^2/a_O^2)}{(1-v_{Ok}^2/c^2)} \tag{7.42}$$

and

$$a_{ky} = a_0 \frac{(1-v_{0k}^2/a_0^2)^{1/2}}{(1-v_{0k}^2/c^2)^{1/2}} \qquad (7.43).$$

Indeed (7.42) & (7.43) are in accordance with our previous stipulation [vide equations (7.38) & (7.39)] made earlier in the previous section.

One may also verify that one would have obtained the same expressions (7.42) & (7.43) for the TWS of a signal using LT. This is not surprising since the TWS only depends on the world type and it cannot depend on distant synchrony.

Now inserting (7.42) & (7.43) in (7.37a) and (7.37b) gives the Dolphins' Transformations for the relativistic world.

$S_0 \rightarrow S_k$:

$$x_k = \frac{x_0 - v_{0k}\, t_0}{(1-v_{0k}^2/c^2)^{1/2}} \qquad (7.44a)$$

and

$$t_k = \frac{(1-v_{0k}^2/c^2)^{1/2}}{(1-v_{0k}^2/a_0^2)} [t_0 - (v_{0k}/a_0^2) x_0] \qquad (7.44b).$$

Or inverting one obtains

$S_k \rightarrow S_o$:

$$x_0 = \frac{(1-v_{0k}^2/c^2)^{1/2}}{(1-v_{0k}^2/a_0^2)} x_k + \frac{v_{0k}}{(1-v_{0k}^2/c^2)^{1/2}} t_k \qquad (7.45a)$$

and

$$t_0 = \frac{v_{0k}}{a_0^2} \frac{(1-v_{0k}^2/c^2)^{1/2}}{(1-v_{0k}^2/a_0^2)} x_k + \frac{1}{(1-v_{0k}^2/c^2)^{1/2}} t_k \qquad (7.45b).$$

From the above transformation equations (7.44a – 7.45b) it is clear that the time dilatation and length contraction factors as observed from S_k for standard clocks and rods stationary with respect to the medium are given by

$$\text{Length Contraction Factor (LCF)} = \frac{(1-v_{0k}^2/a_0^2)}{(1-v_{0k}^2/c^2)^{1/2}} \tag{7.46}$$

$$\text{Time Dilatation Factor (TDF)} = \frac{(1-v_{0k}^2/c^2)^{1/2}}{(1-v_{0k}^2/a_0^2)} \tag{7.47}$$

With respect to S_0 however LCF & TDF are the same as predicted by SR. This is not surprising as we have pointed out earlier (section 7.3) that in S_0 Einstein synchrony and the pseudo-standard synchrony tally.

In a similar way one can write down the general expressions for LCF and TDF for moving rods and clocks with respect to any arbitrary frame. However, to understand the role of pseudo-standard synchrony qualitatively at least it is enough to continue the discussions by confining ourselves to the relations (7.46) & (7.47). From these equations we can see how in the Dolphins' relativistic world the light signal & the synchronizing signal play together to give us the kinematics effects. Note that as $a_0 \to c$ i.e. instead of pseudo -standard synchrony if one chooses Einstein (standard) synchrony one gets back the usual relativistic results for LCF & TDF.

Since the birth of SR there is a considerable disagreement among Physicists as to whether the time dilatation and length contraction are real or whether they are solely due to the process of synchronization (Sjödin 1979). We now observe that the factor

$(1-v_{0k}^2/c^2)$ which appear in (7.46) & (7.47) are due to real effects whereas the factor $(1-v_{0k}^2/a_0^2)$ arises due to the special way chosen for the signal synchrony. Indeed one can even demonstrate that the "apparent" LCF & TDF can be obtained in the Galilean world because of signal synchrony. This can be seen as follows.

From the Galilean velocity addition formulae one can obtain the TWS of any signal with respect to any frame S_k moving with respect to the medium. We may write (Ghosal et al 1991b) for example the TWS of the AS, as

$$a_{kx} = a_0 \ (1-v_{0k}^2/a_0^2) \tag{7.48}$$

and

$$a_{ky} = a_0 \ (1-v_{0k}^2/a_0^2)^{1/2} \tag{7.49}.$$

Inserting these expressions for a_{kx} & a_{ky} in the DT (and in particular in 7.37a & 7.37b) we obtain

$$x_k = x_0 - v_{0k}\, t_0 \tag{7.50a}$$

and

$$t_k = (1-v_{0k}^2/a_0^2)^{-1} \ [t_0-(v_{0k}/a_0^2)x_0] \tag{7.50b}.$$

Whereas the inverse transformations may be written as

$$x_0 = \frac{1}{(1-v_{0k}^2/a_0^2)} \ [x_k + (1-v_{0k}^2/a_0^2)v_{0k}\, t_k] \tag{7.51a}$$

and

$$t_0 = t_k + \frac{v_{ok}}{a_0^2 \ (1-v_{0k}^2/a_0^2)} \, x_k \tag{7.51b}.$$

The above equations (7.50a - 7.51b) represent DT for the Galilean world.

It is thus evident that there is no length contraction or time dilatation effects as observed from S_0. However, for rods & clocks stationary with respect to the medium, the LCF & TDF as observed from a moving frame S_k are respectively given by

$$LCF = (1-v_{0k}^2/a_0^2) \qquad (7.52)$$

and

$$TDF = (1-v_{0k}^2/a_0^2)^{-1} \qquad (7.53).$$

Clearly in the present case the length contraction and the time dilatation effects are contrived and they arise because of the synchronization alone.

Note that the transformations, (7.50a) & (7.50b) [when a_0 is replaced by c] are much discussed (Zahar 1977; Sjödin 1979; Ghosal et al 1991a) and they are known as Zahar Transformations (ZT). ZT thus represent Galilean world with Einstein (standard) synchrony (pseudo standard synchrony becomes standard synchrony when a_0 is replaced by c).

One observes that in (7.50a) & (7.50b) if a_0 is taken to be infinitely large, ZT becomes Galilean Transformations (GT).*

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*

However remember that instead of putting $a_0 \to \infty$ in these equations, if v^2/c^2 terms are neglected in ZT we do not obtain GT, instead one obtains the so-called Approximate Lorentz Transformations (Ghosal et al 1991a).

On the other hand if one assumes $a_0 \to \infty$ in equations (7.44a–7.45b) which represent DT in the relativistic world one obtains the so-called Tangherlini transformations (Sjödin 1979; Ghosal et al 1991a).

$S_0 \to S_k$:

$$x_k = \frac{x_0 - v_{0k}\, t_0}{(1-v_{0k}^2/c^2)^{1/2}} \tag{7.54a}$$

$$t_k = (1-v_{0k}^2/c^2)^{1/2}\, t_0 \tag{7.54b}$$

and

$S_k \to S_0$:

$$x_0 = (1-v_{0k}^2/c^2)^{1/2}\, [x_k + \frac{v_{0k}}{(1-v_{0k}^2/c^2)}\, t_k \tag{7.55a}$$

$$t_0 = (1-v_{0k}^2/c^2)^{-1/2}\, t_k \tag{7.55b}.$$

The above transformations (7.54a–7.55b) represent relativistic world with absolute synchrony. According to this synchrony the simultaneity is absolute. On the contrary Zahar transformations represent Galilean world manifesting relativity of simultaneity.

7.6. EPILOGUE

Before we end this chapter, let us take stock of what we have achieved so far. We have essentially clarified the role of conventionality in special relativity. In order to do so we have tried to heuristically derive SR in a fluid medium (Dolphin's World!) where clock synchronization can be done by acoustic signal, optical signal or any other wave mode available in the medium. Remember that once a signal has been picked up for synchrony we call it acoustic signal just to keep us reminding that speed of the synchronizing signal is different from that of light in empty space. This has been done in order to understand by contrast, specifically the role of the optical signal as a synchronizing agent, in the standard formulation of SR.

We have argued in secs.7.4 & 7.5 that for the appearance of γ-factor in SR, the Einstein's choice of synchrony is partly responsible. Indeed by deliberately opting for non-luminal synchrony, but at the same time preserving Einstein's procedure for it we have been able to see visually that the factor $(1-v^2/a^2_0)$ are present in the transformation equations. Obviously if a_0 happens to be equal to c this factor becomes γ^{-2} which when combined with the γ-factors (whose origin is independent of convention) already existing in the TE for the relativistic world gives the well known form for the TE viz LT.

Note the condition $a_0 \rightarrow c$ does not necessarily mean that the Dolphins will have to synchronize clocks with a signal whose speed is c. In fact one need not have to step out of the fluid world. The assumption $a_0=c$ practically means that the to and fro speeds (a_+ & a_- say) of the AS in a "moving" frame are *unequal*. This is because they are now subjected to the usual relativistic one-way velocity transformations. In other words the condition $a_0 \rightarrow c$, that was necessary to arrive at LT from equations (7.44a) & (7.44b), may be replaced by the assumption of relativistically correct forms for a_+ and a_- as function of the speed (with respect to S_0) of the moving frame.*

*However, it is evident from sec.7.2b that the sound-sphere derivation will not be applicable in the present situation, but this is unimportant for the present discussion.

We may therefore conclude that even if it is given that the Dolphins are not able to communicate with the outside world it is possible to synchronize clocks with any signal available in the medium but by exercising the option to take (by virtue of the c-s thesis) any functional form of a_+ and a_-, the Dolphins may, by trial, eventually strike upon the standard synchrony.

So far as the TWS of the AS is concerned, if relativity is correct, by empirical observations one would be able to verify in principle that $a_{kx} = a_{kx}(v_{0k}, c)$ and $a_{ky} = a_{ky}(v_{0k}, c)$ with their correct forms given by the equations (7.42) & (7.43). We have seen that when these are inserted in the DT we obtain equations (7.44a-7.45b) which again goes over to LT as a special case.

The above observations clarify the role of 'c' in the space-time transformations. We see how eventually the Dolphins may arrive at the Lorentz transformations without even uttering the word "light" since as we have remarked in sec.7.4, c enters in eqns. (7.44a) & (7.44b) (and hence in LT) just as a physical constant.*

Here we may once again support the Sjödin interpretation which holds that the relativistic transformation equations really describe the behaviour of standard rods and clocks in motion where in the present case the term "motion" now refers to the motion relative to the frame at rest with the *medium*.

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*This remark apparently has a striking similarity with that usually given in connection with the one-postulate-derivations of LT (Lee & Kalotas 1975; Srivastava 1981), however, it may be noted that these derivations do not take into account the C-S thesis.

The arguments used throught this chapter may also be extended in the case of a non-homogeneous medium where light rays are not straight lines in general. One may obtain transformations analogous to DT (valid for a homogeneous substrate). The calculations may be a bit more involved but will be straight forward nevertheless. Also as in the previous situation, here standard SR will show up as a special case.

The whole developement will find its direct applicability in the context of flat-space-time theories of gravity where velocity of light as we have seen is not equal to c. This is so as because the empirical content of SR is independent of synchrony and at the same time it resides in the empirical behaviour of "standard rods & clocks" in motion. These arguments may be seen to be indirectly substantiated by a recent claim that LT are not valid if one uses non-standard instruments viz. the so-called "unbonded instruments" (vide Sherwin 1992 + references therein). Note that in the first part of the present volume we considered standard rods and clocks and we were therefore justified in assuming the correctness of SR globally even in presence of gravity.

Another importance of the DT for the relativistic world is that we understand that the c′-relativity is not a correct proposition. What is c′-relativity? Rosen (1952) considered a homogeneous isotropic medium in which some phenomena take place with which is associated a limiting speed c′. It was then claimed that within the medium a set of space-time transformations will hold which are Lorentzlike i.e Lorentz transformations with c replaced by c′. It was called c′- relativity. Even though it can have some limited applicability as claimed by Rosen, we hold that

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