Preface

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The purpose of this work is to apply the Laplace transform method in combination with Wiener-Hopf technique, to the exact solutions of equations of Radiative transfer . The method has received much less attention even though it has been succesfully applied in the past to solve transport problems by Placzek and Seidel (1947) and Marshak (1947) , Shu-Su Huang (1955) , Dasgupta(1956, 1957, 1958, 1965) and Boffi (1970) . Laplace transform method is undoubtedly an interesting alternative to the other methods described in the following introductory chapter. The domain of the mathematical analysis is the complex plane . The Wiener-Hopf technique is one of the techniques which are based on the properties of the function of the complex variable , and is unique in its aesthetic beauty . The success of Wiener-Hopf technique as applied here to the exact solution of linear singular integral equations depends on the appropriate decomposition of complex functions into two parts with two domains of regularity such that the two parts are equal on the non-null intersection of the two domains of regularity and satisfy certain analytic properties at the origin and at infinity . The final solution is then served on a plate by a modified form of Liouville's theorem [Titchmarsh, Theory of functions (Oxford, 1932) , art 2.83]. Attention is diverted to the elegance of the method in that it at no time depends on transformations of the complicated integral part of the basic equation but only on the analytic properties of the functionsrepresented by the integral . At the same time it may be pointed out that the success of this method rests on the presence of the unknown function outside the sign of integration . This work is concerned with the exact solution of the Radiative transfer problems in a homogeneous semi-infinite plane-parallel atmosphere scattering either isotropically or anisotropically . The previous investigations of the problems, have mostly (excepting a few employing the method of eigen-function expansion) obtained only the emergent intensity of radiation from the bounding face of the atmosphere and did not attempt to obtain the intensity at any level inside the atmosphere . We have however obtained here the emergent intensity from the bounding face of the atmosphere and also the intensity at any level inside the atmosphere in a closed and exact form . The transfer equations are solved under different phase functions peculiar to the problems considered . The integrodifferential equations are converted into linear singular integral equations by Laplace transformation and approprate boundary conditions. These singular linear integral equations are then exactly solved by Wiener-Hopf technique for emergent intensities which are obtained in terms of Chandrasekhar's H-functions(Chandrasekhar 1950), (Busbridge 1960), Dasgupta (1974, 1977)) . The constants appearing in the solution are determined by appealing to the analytic properties of the H- functions . Finallyn the intensity at any depth is derived by inversion with the help of Plemelj's formula(Muskelishvilli 1946 chap-II art-17 The intensity at any depth is expressed in a more tractable form by using the closed explicit form of H-functions given by Dasgupta (1974) .

The results thus obtained are exact analogue of the results obtained by different authors applying the Eigen-functionexpansion approach of Case (1960) and the two sided Laplace transform approach of Boffi (1970) .

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The present work has been completed under the supervision of Dr. Santi Ranjan Dasgupta, Department of Mathematics, University of North Bengal . It is a pleasure to be able to acknowledge my enormous debt to him . Throughout my attempt to understand the Mathematical and physical problems of Radiative Transfer , he has been a source of constructive suggestions, new ideas and friendly encouragement. I express my grateful thanks to the North Bengal University Library and the Department of Mathematics for the facilities I received from time to time .

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7 th September,1977.