Chapter 1

INTRODUCTION

1.1. A brief introductory survey

Scalar fields have long been conjectured to give rise to long-range gravitational fields [1]. In a phenomenological approach, scalar fields are often introduced as a source of gravitation in the field equations of the theory (usually in the right hand side of Einstein equations of general relativity), but there are stronger theoretical motivations for inclusion of fundamental scalar fields in gravitational theories. For instance, scalar partner to the graviton generically arises in almost every modern theoretical attempt to unify gravity with the remaining interactions [2-4]. Another independent motivation for scalar fields is furnished by inflationary models of cosmology in the framework of scalar-tensor (ST) theories of gravitation, in which the overall interaction is mediated by one or several long-range scalar fields in addition to the usual tensor fields present in Einstein's theory. This mechanism has been found to provide a technical but natural way of terminating inflationary eras of the early universe by the nucleation of bubbles of the true vacuum [5].

ST theories of gravitation do satisfy all the weak-field solar system experiments or binary pulsar tests and also the cosmological observations to date. Moreover, in contrast to most of the other available alternative theories of gravity, these theories in the Jordan frame respect most of the well-known symmetries: conservation laws, constancy of non-gravitational constants, local Lorentz invariance and they also satisfy the weak equivalence principle (universality of free fall of laboratory size objects). Thus ST theories are regarded as a viable alternative to Einstein's general relativity (GR) and they provide an important theoretical framework against which results of various gravitational experiments, old and new, can be compared [6]. It is known that several new experiments are either currently in operation or going to take place in the near future (e.g. Gravity Probe B [7], LATOR [8], APOLLO [9], LISA [10] etc.)

ST theories have been widely studied in recent years also because of their great relevance in cosmology, especially in the early epochs. As mentioned already, these theories are very successful in describing the so-called inflationary era of the universe. The evolution of scalar field slows down the expansion rate of the universe during inflation, and allows nucleation of bubbles to end the inflationary era thus overcoming the fine-tuning problem of "new" inflationary model of the universe [5]. The present acceleration of the universe, as revealed from the observational data of the type 1a supernova [11], can also be accommodated within the scalar field coupled gravity theories [12]. There have also been suggestions that the dark matter problem might be resolved within the framework of a suitable scalar-tensor cosmological model in which some components of matter field couple with a different strength to gravitation than does ordinary matter [13]. Also ST theories provide a rich arena for investigations into wormhole physics [14-15]. The above possibilities are by no means exhaustive but represent only some of the useful applications of ST theories.

A generic aspect of the theories involving scalar fields is that one faces an inherent question as to how one can select a physical conformal frame out of two obvious alternatives, viz., the Jordan frame (JF) and the Einstein frame (EF), as they are conveniently called. The former is a conformal frame in which the scalar field is coupled non-minimally and plays the role of a spin-0 component of gravity. In contrast, in the Einstein frame, scalar fields appear as a source of gravitation in the field equations of the theory (usually the source term is an abnormally coupled scalar matter field in the

Einstein equations of general relativity). Theories in these two frames are related to each other by means of conformal transformations of the metric part and a re-definition of the scalar part [16]. However, the question whether the two formulations in the two conformal frames are really equivalent or not has been the issue of continuing lively debates [17].

The advantage of the Jordan frame is that the laws of evolution of the matter (i.e., non-gravitational) fields take the same form as in GR. This frame has the disadvantage that the propagation mode of metric tensor and scalar fields are mixed together i.e., gravitational waves contain both helicity-2 and helicity-0 excitations [4,18] and as a result the variables of the frame are inconvenient for the formulation a Cauchy problem [18]. Despite such limitations, the Jordan frame is regarded as the physical frame because here matter couples universally to the metric tensor (normal coupling). The test particle rest mass is constant and consequently the particle trajectories follow the usual geodesic equations in the metric field. On the other hand, in the Einstein frame, it is mathematically more convenient to analyze the field equations though this is not always the case. For instance, in this frame, rest mass of a particle is *not* constant but depends on the scalar field and consequently the weak principle of equivalence for ordinary matter is not satisfied. However, the helicities are not mixed in this frame; the propagation is described only by spin-2 gravitons.

1.2. Scalar-tensor theories in the Jordan frame

Scalar-tensor (ST) theories have an extensive history. The first ones to appear were those of Jordan [19], Fierz [20] and Brans-Dicke [21]. These are the simplest theories in the sense that they consist of just one massless scalar field and its coupling strength to matter is constant. Later, Bergmann [22], Nordtvedt [23] and Wagoner [24] generalized the theory. In their version the scalar field has a dynamic coupling to matter and/or an arbitrary self- interaction. More recently, ST theories are further generalized to the case of multiple scalar fields [18].

In the ST theories, the scalar field (φ) plays the role of the (local) gravitational coupling with $G \sim \langle\varphi\rangle^{-1}$ and consequently the gravitational "constant" is not in fact a constant but is determined by the total matter in the universe (Mach's principle) through an auxiliary scalar field equation. The scalar field couples to both matter and space-time geometry, the strength of the coupling is represented by an arbitrary dimensionless function $\omega(\varphi)$.

The general form of the action describing a massless scalar-tensor theory of gravity is [23] (We shall use units G=c=1 unless otherwise specified):

$$S[g_{\mu\nu},\varphi]=\frac{1}{16\pi}\int\sqrt{-g}\,d^4x\left(\varphi R-\frac{\omega(\varphi)}{\varphi}g^{\mu\nu}\varphi_{,\mu}\varphi_{,\nu}+16\pi S_m\right), \tag{1}$$

where R is the Ricci scalar constructed from the metric $g_{\mu\nu}$, and S_m is the Lagrangian density of ordinary matter which could include electromagnetic field, nuclear field etc. The principle of equivalence is guaranteed by requiring that the matter field Lagrangian can depend only on the metric $g_{\mu\nu}$ but not on φ. For the Brans-Dicke theory, $\omega(\varphi)=$ constant.

A variation of (1) with respect to $g^{\mu\nu}$ and φ yields the following field equations

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R=\frac{8\pi}{\varphi}T_{\mu\nu}+\frac{\omega}{\phi^2}\left[\varphi_{,\mu}\varphi_{,\nu}-\frac{1}{2}g_{\mu\nu}\varphi_{,\sigma}\varphi^{,\sigma}\right]+\frac{1}{\varphi}\left[\varphi_{;\mu;\nu}-g_{\mu\nu}\Box\varphi\right] \tag{2}$$

$$\Box\varphi=\frac{8\pi}{2\omega(\varphi)+3}T-\frac{\omega'}{2\omega(\varphi)+3}g^{\mu\nu}\varphi_{,\mu}\varphi_{,\nu} \tag{3}$$

with the energy momentum conservation equation

$$T^{\mu\nu}_{;\nu} = 0 \tag{4}$$

where $\Box \equiv g_{\mu\nu}\nabla_\mu\nabla_\nu$, $T = T^\mu_\mu$ is the trace of the matter energy momentum tensor and $\omega' \equiv \dfrac{d\omega}{d\varphi}$. Although the coupling function $\omega(\varphi)$ in general ST theories can be any arbitrary function of scalar field φ but the numerical value of $\omega(\varphi)$ is constrained by the classical tests of general relativity as well as by cosmological considerations such as the Big Bang Nucleosynthesis [25]. Solar system tests put limits on the Post-Newtonian parameters γ,β [5] which, in the general ST theories, translate into limits on $\omega(\varphi)$ and $\omega'(\varphi)$. The first post-Newtonian (PPN) parameters are given by [23]

$$\beta = 1 + \frac{\omega'}{(2\omega(\varphi)+4)(2\omega(\varphi)+3)^2} \tag{5}$$

and

$$\gamma = 1 - \frac{1}{\omega(\varphi)+2} \tag{6}$$

The Eddington parameter γ, whose value in GR is unity, is the most fundamental PPN parameter. Observational limits on this parameter are $|\gamma\text{-}1| \approx 0.002$ from radio timing delays [26], $|\gamma\text{-}1| = 0.0003$ from light deflection using VLBI observations of quasars [27], $|\gamma\text{-}1| = 0.02$ from lunar laser ranging [28]. Analysis of planetary ranging data recently yielded $\gamma = 0.0015 \pm 0.0021$ [29]. A more precise value of γ has been recently obtained from solar conjunction experiment with the Cassini spacecraft which gives $|\gamma - 1| = (2.1 \pm 2.3) \times 10^{-5}$ [30]. Consequently, the lower limit on the coupling function is $\omega(\varphi) < |5 \times 10^4|$.

On the other hand the planetary ranging data yields $|\beta-1|=-0.0010 \pm 0.0012$ [29]. Hence there is only a rather weak limit on ω' at present.

As evident from above, solar system experiments place the coupling parameter ω in ST theories to a large value, thus making these theories practically indistinguishable from GR in local situations. This may cast doubt on the existence of scalar fields in the gravitational sector but as shown by Nordtvedt and Damour [31], most of the ST theories cosmologically evolve toward a state with no scalar admixture to gravity during matter-dominated era. This means that, during cosmological evolution, ω approaches to a very large value (the so called "attractor mechanism"). However, even for large ω, ST theories can produce interesting departures from GR at the strong field scenario as well as at the cosmological level. It is clear from the field equations that under Nordtvedt conditions, i.e., when $\omega(\varphi) \to\infty$ and $\omega'/\omega^3\to 0$, the ST theories tends to general relativity unless the energy-momentum tensor is traceless [32].

1.3. Two particular ST theories

So far, there is no unique way to choose the functional form of $\omega(\varphi)$. As a result, there exists a plethora of ST gravity theories characterized by the different functional form of $\omega(\varphi)$ which include the celebrated Brans-Dicke theory [21], Barker's constant-G theory [33], Bekenstein's variable rest mass theory [34], or Schmidt-Greiner-Heinz-Muller's theory [35]. Besides, for $\omega=-1$, ST theories turn out to be the low energy effective four-dimensional superstring theory. Below we discuss two of these theories in some details, the Brans-Dicke theory and the low energy effective four- dimensional superstring theory.

1.4. The Brans-Dicke theory

Historically, most of the interest has been focused upon the first and the simplest ST theory, presented by Brans and Dicke (BD) [21], in which the coupling function is a constant i.e. $\omega'=0$. Since Birkhoff's theorem does not hold in the presence of a scalar field, several classes of static solutions of the BD theory are possible even in spherically symmetric vacuum situation. Brans himself provided [36] four forms of static spherically symmetric vacuum solution of the BD theory. However, later it has been found that only two of them, class I and class IV solutions, are really independent [37, 38]. These solutions are in "isotropic" coordinates, (t,ρ,θ,φ) are given by (We shall use the signature (-,+,+,+):

Class I solutions:

$$ds^2=-\left[\frac{1-\frac{B}{2\rho}}{1+\frac{B}{2\rho}}\right]^{\frac{2}{\lambda}}dt^2+\left(1+\frac{B}{2\rho}\right)^4\left[\frac{1-\frac{B}{2\rho}}{1+\frac{B}{2\rho}}\right]^{\frac{2(\lambda-C-1)}{\lambda}}\left[d\rho^2+\rho^2d\theta^2+\rho^2\sin^2\theta d\varphi^2\right], \qquad (7)$$

$$\varphi=\varphi_0\left[\frac{1-\frac{B}{2\rho}}{1+\frac{B}{2\rho}}\right]^{\frac{C}{\lambda}}, \qquad (8)$$

where λ, C, φ_o, B are constants, and the first two relate to ϖ as the constraint

$$\lambda^2\equiv(C+1)^2-C\left(1-\frac{\omega C}{2}\right). \qquad (9)$$

The class IV solutions are:

$$ds^2=-e^{-2/(B\rho)}dt^2+e^{2(C+1)/(B\rho)}\left[d\rho^2+\rho^2d\theta^2+\rho^2\sin^2\theta d\varphi^2\right], \qquad (10)$$

$$\varphi=\varphi_0 e^{C/(B\rho)}, \tag{11}$$

with the constraint condition

$$C=\frac{-1\pm\sqrt{-2\omega-3}}{\omega+2}. \tag{12}$$

Class I solution received more attention as it is the only one which is permitted for all values of ω. This class of solution in general gives rise to naked singularity [39] though for some particular choices of the solution parameters, it represents a black hole different from the Schwarzschild one [40]. The class I solution may be written in the EF. Identifying $M=m\delta$ where $\delta=\frac{C+2}{2\lambda}, B=m$, the solution may be compared with the Robertson expansion of any centrally symmetric field give, in isotropic coordinates, by

$$ds^2=-\left(1-2\alpha\frac{M}{\rho}+2\beta\frac{M}{\rho}+...\right)dt^2+\left(1+2\gamma\frac{M}{\rho}+...\right)[d\rho^2+\rho^2 d\theta^2+\rho^2\sin^2\theta d\phi^2] \tag{13}$$

One immediately finds the values

$$\alpha=\beta=\gamma=1, \tag{14}$$

which are just the Schwarzschild values. In the JF, these are

$$\alpha=\beta=1, \gamma=\frac{\omega+1}{\omega+2}. \tag{15}$$

The class IV is valid only for ω <-3/2 and is a nonsingular solution. There is a non-positive contribution of matter to effective gravitational constant and thus there is a violation of some of the energy conditions. In fact, the solution has the topology of a wormhole [14]. Nevertheless, in the Einstein frame, one obtains exactly the Schwarzschild values as in Eq.(14). In some respects, Class IV solution, which gives rise to what is called cold black holes [41], exhibits better behavior than class I solution. For example, the tidal forces do not diverge on the horizon for this spacetime unlike the class I metric [42].

The cosmological scenario in BD theory is the following. Here the solution is analogous to Einstein-de Sitter model of GR and is given by [21]

$$a = a_o \left(\frac{t}{t_o} \right)^{\frac{2\omega+2}{3\omega+4}}, \tag{16}$$

and

$$\varphi = \varphi_o \left(\frac{t}{t_o} \right)^{\frac{2}{3\omega+4}}. \tag{17}$$

As $\omega \to \infty$, the above solution tends to the Einstein-de Sitter model. But if ω is not too large, the BD cosmology could have interesting departures from GR when t is small. As mentioned before, inflationary scenario in the BD cosmology is particularly interesting.

1.5. Low energy effective four-dimensional Superstring theory

Superstring theory is regarded as the most promising candidate for a unified theory of the fundamental interactions, including gravity. However, there exist five anomaly free, supersymmetric perturbative string theories known as the type I, type IIA, type IIB, SO(32) heterotic and $E_8 \times E_8$ heterotic theories. There is now evidence that these theories are related by a set of dualities and may in fact represent different manifestations of a more fundamental quantum theory, often termed as M-theory.

A definite prediction of superstring theory is the existence of a scalar field. This is referred to as the dilaton and it couples directly to matter. A typical scheme in superstring theory is to compactify from ten dimensions (in the framework of supersymmetry, the quantization of the string is only consistent if spacetime is ten dimensional) onto an isotropic six torus to obtain the effective four-dimensional action given by [43]

$$S = -\int d^4x\sqrt{-g}e^{-\Phi}\left(-R + \frac{1}{12}H_{\mu\nu\rho}H^{\mu\nu\rho} - g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi + \frac{1}{8}F_{\mu\nu}F^{\mu\nu}\right), \tag{18}$$

where $g_{\mu\nu}$ is the metric that arises naturally in the σ-model, R is the Ricci scalar, $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength of the Maxwell field A_μ, Φ is the dilaton field, and,

$$H_{\mu\nu\rho} \equiv \partial_\mu B_{\nu\rho} + \partial_\nu B_{\rho\mu} + \partial_\rho B_{\mu\nu} - [\Omega_3(A)]_{\mu\nu\rho} \tag{19}$$

where $B_{\mu\nu}$ is the antisymmetric tensor gauge field and

$$[\Omega_3(A)]_{\mu\nu\rho} \equiv \frac{1}{4}(A_\mu F_{\nu\rho} + A_\nu F_{\rho\mu} + A_\rho F_{\mu\nu}) \tag{20}$$

is the gauge Chern-Simons term.

As long as curvature is small compared to the Planck scale, all vacuum solutions of GR are approximate solutions of effective four-dimensional string theory except in the region near the singularity. However, the situation is quite different in the presence of electromagnetic field since the dilaton field has a linear coupling to F^2. Consequently charged black hole in string theory differs significantly from that in GR. The static charged black hole solution of the effective string theory is given by the Gibbons-Maeda-Garfinkle-Horowitz-Strominger (GMGHS) solution [44]

$$d\tau^2 = -\left(1 - \frac{2M}{r}\right)dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r\left(r - \frac{Q^2 e^{-2\phi_0}}{M}\right)d\Omega \tag{21}$$

$$e^{-2\phi} = e^{-2\phi_0}\left(1 - \frac{Q^2 e^{-2\phi_0}}{Mr}\right) \tag{22}$$

$$F = Q \sin\theta d\theta \wedge d\varphi, \tag{23}$$

where ϕ_o is the asymptotic constant value of the dilaton, Q is the electromagnetic charge, M is the mass, $d\Omega$ is the metric on a unit sphere. The solution has been studied for possible string signature in gravitational lensing phenomena [45]. The wormhole physics with the above spacetime is also addressed in [46].

1.6. Broad outline of the aim and plan of the thesis

The present investigation is aimed at studying local situations in the framework of ST theories and look for the scalar field induced effects. The target here is mainly twofold:

(a) In this part, certain aspects of a few static spherically symmetric solutions of different ST theories have been investigated as well as a new class of solutions for the generalized theories have been advanced. According to the scalar "no-hair" theorem [47] the Schwarzschild solution is the only static spherically symmetric black hole solution of the ST gravity theories. The proof relies on the assumption that the conditions of the Hawking-Ellis singularity theorems hold which include the weak energy condition. However, it is well known now that many physical matter fields violate the weak energy condition (squeezed matter field or the matter arising out of the quantum Casimir effect, for instance). As a result existence of several black hole solutions in the ST theories has been claimed - the black hole nature of the solutions is judged by critically examining the non-energetic characteristics of black holes.

(b) In this part, the effect of the scalar field in physical observations in a Sagnac type experiment has been studied. This is motivated by the fact that the future or present onboard experiments would be able to measure the PPN parameters with unprecedented accuracy thus creating the possibility of measuring small deviations from the predictions of GR. For instance the Gravity Probe-B (GP-B) mission will be able to measure the geodetic precision effect to 2×10^{-5} and to measure frame-dragging effect to an accuracy 3×10^{-3} [7]. As a result, the experiment will permit direct measurement of γ with accuracy of 5×10^{-5}. LATOR mission is expected to measure γ to the unprecedented levels of accuracy of 10^{-8} [8].

Sagnac-like rotational scenario (Sagnac effect can be simply described as a shift of the interferometer fringes appearing in a suitable interferometer placed on a rotating platform [48]) has been selected due to the fact that the conventional *static* cases so far have not succeeded to isolate any scalar field induced effects in a real observation despite huge efforts [45, 49]. Hence effects of scalar field in rotational scenario have been explored. For this purpose, rotating solutions of two representative ST theories, the BD theory and the low energy effective field theory describing heterotic string theory, are considered.

The present thesis is planned as follows. In the next Chapter, the behavior of a few spherically symmetric acclaimed black hole solutions of different ST theories as well as some other non-Einsteinian theories are investigated in respect of tidal forces in the geodesic frame. In Chapter 3, a new class of solutions is obtained in the generalized ST theories using the Bekenstein black hole of conformal scalar field theory [50] as the seed solution. In Chapter 4, the effect of the scalar field on the corrections to the Sagnac effect, geodetic and Lense-Thirring precession are calculated using a Kerr-like solution of the BD theory. The same has also been estimated for the low energy effective string theory rotating black holes in 4-dimensions (Kerr-Sen metric) [43] in the Chapter 5. Finally the results are briefly summarized in Chapter 6. References are given at the end of each chapter for easy reading. All references and equation numbers are specific to the concerned chapter.

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