Chapter 5

STRING THEORY CORRECTIONS TO THE SAGNAC EFFECT

5.1. Introduction

Low energy effective field theory describing heterotic string theory has now become an indispensable part of the frontiers of theoretical physics [1]. An interesting result is that black hole solutions exist also in the string theory and that they exhibit qualitatively different properties than those of Einstein's General Relativity [2]. A rotating black hole solution that reduces to the Kerr solution for a constant dilaton field, has been constructed and analyzed by Horowitz *et al* [3].

A more general classical exact solution has been found by Sen [4]. The action underlying the theory has a U(1) gauge symmetry and contains antisymmetric tensor gauge field. Also, 6 of the 10 dimensions are compactified to a suitable manifold, but the resulting massless fields are not included in the action. The absence of these fields enhances the possibility of the black hole nature of the solution; otherwise, naked singularities could arise. Kerr-Sen solution describes a rotating black hole carrying finite amount of charge and angular momentum and it differs from the Horowitz-Horne black hole even in the limit of small angular momentum. This difference arises due to the coupling of the antisymmetric tensor gauge field to the Chern-Simons term in the action considered by Sen [4]. These, and other developments taken together, indicate that there have been tremendous theoretical advances in the understanding and utility of the string theory. However, relatively much less is discussed in the literature as to how a black hole in the string theory could possibly affect physical observations in practice. It has been shown by Gegenberg [5] that *static* spherically symmetric solutions do not lead to string effects in the PPN approximation of the solar system scenario but that, he conjectured, *rotating* solutions might lead to observable string effects, however tiny.

The present chapter aims to undertake an investigation precisely in this direction. For this purpose, we consider the Kerr-Sen metric and examine how the black hole parameters appear in the correction terms in a Sagnac-type experiment. We shall find *exact* expressions for the time delay by following the procedure of Tartaglia [6] which we had also adopted in our recent investigation of the Brans-Dicke correction factors for different types of orbits [7]. An estimate of the possible terrestrial dilatonic charge is also attempted.

We have chosen the Sagnac effect because of its simplicity and its easy adaptability to rotating sources. The effect stems from the basic physical fact that the round trip time of light around a closed contour, when the source is fixed on a turntable, depends on the angular velocity, say Ω, of the turntable. Using Special Theory of Relativity, and assuming $\Omega r \ll c$, one obtains the proper time difference $\delta\tau_s$ when the two beams meet again at the starting point as [6]:

$$\delta\tau_s \cong \frac{4\Omega}{c^2} S, \tag{1}$$

where c is the vacuum speed of light, S $(=\pi r^2)$ is the projected area of the contour perpendicular to the axis of rotation. It is a real physical effect in the sense that it does not involve any arbitrary synchronization convention that is required between two distant clocks [8]. The effect is also universal as it manifests not only for light rays but also for all kinds of waves including matter waves [9]. The formula (1) has been tested to a good accuracy and the remarkable degree of precision attained lately by the advent of ring laser interferometry raises the hope that measurements of higher order corrections to this effect might be possible in near future [6,10].

The string theory effective action in 4 dimensions, considered by Sen [4], is

$$S = -\int d^4x\sqrt{-g}e^{-\Phi}\left(-R + \frac{1}{12}H_{\mu\nu\rho}H^{\mu\nu\rho} - g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi + \frac{1}{8}F_{\mu\nu}F^{\mu\nu}\right) \tag{2}$$

where $g_{\mu\nu}$ is the metric that arises naturally in the σ-model, R is the Ricci scalar, $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength of the Maxwell field A_μ, Φ is the dilaton field, and,

$$H_{\mu\nu\rho} \equiv \partial_\mu B_{\nu\rho} + \partial_\nu B_{\rho\mu} + \partial_\rho B_{\mu\nu} - [\Omega_3(A)]_{\mu\nu\rho} \tag{3}$$

where $B_{\mu\nu}$ is the antisymmetric tensor gauge field and

$$[\Omega_3(A)]_{\mu\nu\rho} \equiv \frac{1}{4}(A_\mu F_{\nu\rho} + A_\nu F_{\rho\mu} + A_\rho F_{\mu\nu}) \tag{4}$$

is the gauge Chern-Simons term. The Einstein frame metric is obtained from the relation

$$g_{\mu\nu}^{(E)} = e^{-\Phi}g_{\mu\nu}.$$

For our purposes, we recast the Einstein frame Sen metric into a form that closely resembles the familiar Kerr solution in Boyer-Lindquist coordinates. The result is:

$$d\tau^2 = \left(1 - \frac{2M\rho}{\Sigma}\right)dt^2 - \Sigma\left(\frac{d\rho^2}{\Delta} + d\theta^2\right)$$

$$-\left[\rho(\rho+\xi) + a^2 + \frac{2M\rho a^2\sin^2\theta}{\Sigma}\right]\sin^2\theta d\varphi^2 + \frac{4M\rho a\sin^2\theta}{\Sigma}dtd\varphi, \tag{5}$$

$$\Phi = -\ln\left[\frac{\Sigma}{\rho^2 + a^2\cos^2\theta}\right], A_\varphi = -\frac{2\sqrt{2}a\rho Q\sin^2\theta}{\Sigma}, A_t = \frac{2\sqrt{2}\rho Q}{\Sigma},$$

$$B_{t\varphi} = \frac{a\rho Q^2\sin^2\theta}{M\Sigma},$$

$$\Sigma = \rho(\rho+\xi) + a^2\cos^2\theta, \Delta = \rho(\rho+\xi) + a^2 - 2M\rho, \xi = \frac{Q^2}{M}, \tag{6}$$

The metric describes a black hole with mass M, dilatonic charge Q, angular momentum aM, and magnetic dipole moment aQ. For $Q=0$, the metric reduces to the Kerr solution of GR and for $a=0$, it reduces to the Gibbons-Maeda-Garfinkle-Horowitz-Strominger black hole solution [2] with the redefinition: $\rho \to r-\xi$. Using the metric (5), we proceed to calculate the proper Sagnac delay for three types of source/receiver orbits: Equatorial, polar and geodesic circular orbits.

5.2. Equatorial orbit

Suppose that the source/receiver of two oppositely directed light beams is moving around an uncollapsed normal gravitating body, along a circumference at a radius $\rho = R = const.$, on the equatorial plane $\theta = \pi/2$. Suitably placed mirrors send back to their origin both beams after a circular trip about the central rotating body. Let us further assume that the source/receiver is moving with uniform orbital angular speed ω_0 with respect to distant stars such that the rotation angle is

$$\varphi_0 = \omega_0 t. \tag{7}$$

Under these circumstances, the metric (5) reduces to

$$d\tau^2 = \left[\left(1-\frac{2M}{X}\right)+\frac{4Ma\omega_0}{X}-\left\{X(X-\xi)+a^2+\frac{2Ma^2}{X}\right\}\omega_0^2\right]dt^2 \tag{8}$$

where $X \equiv R+\xi$. The trajectory of a light ray is given by $d\tau^2=0$ which gives

$$\left(1-\frac{2M}{X}\right)+\frac{4Ma\omega}{X}-\left\{X(X-\xi)+a^2+\frac{2Ma^2}{X}\right\}\omega^2 = 0 \tag{9}$$

where ω is the angular speed of photons. The two roots $\Omega_\pm$ of the quadratic Eq.(9) provide the rotation angles for the light rays:

$$\varphi_{\pm} = \Omega_{\pm} t = \frac{\Omega_{\pm}}{\omega_0} \varphi_0. \tag{10}$$

The first intersection of the world lines of the two light rays with the world line of the orbiting observer after emission at time $t = 0$ occurs when

$$\varphi_{+} = \varphi_0 + 2\pi, \varphi_{-} = \varphi_0 - 2\pi, or, \frac{\Omega_{\pm}}{\omega_0} \varphi_0 = \varphi_0 \pm 2\pi, \tag{11}$$

where + and – refer to co-rotating and counter-rotating beams. Solving for φ_0, one finds from the equations (11) for the two $\pm$ beams

$$\varphi_{0\pm} = \pm \frac{2\pi\omega_0}{\Omega_{\pm} - \omega_0}. \tag{12}$$

The proper time as measured the orbiting observer is found from Eq.(8) by using $dt = \frac{d\varphi_0}{\omega_0}$, and integrating between φ_{0+} and φ_{0-}. The final result, which is the Sagnac delay, is given by

$$\delta\tau_E = \left(\frac{4\pi}{X}\right) \times \left(\frac{\omega_0 (X^3 + a^2 X + 2Ma^2 - X^2 \xi) - 2Ma}{\left[\left(1 - \frac{2M}{X}\right) + \frac{4Ma\omega_0}{X} - \left\{ X(X - \xi) + a^2 + \frac{2Ma^2}{X} \right\} \omega_0^2 \right]^{1/2}} \right) \tag{13}$$

This is an exact expression for the delay and it is zero if the angular speed ω_0 of the orbiting observer is such that the numerator in Eq.(13) is zero. On the other hand, if the observer keeps fixed positions with respect to distant stars so that ω_0 =0, then the Sagnac delay becomes

$$\delta\tau_0 = -\frac{8\pi a M}{X\left(1 - \frac{2M}{X}\right)^{1/2}}. \tag{14}$$

The effect of the dilatonic charge Q is evident in Eqs.(13) and (14) through the appearance of the factor ξ and the values of $\delta\tau_E, \delta\tau_0$ are different from the Kerr case if $\xi \neq 0$.

Assuming $\varepsilon = \frac{M}{R} << 1$ and $\beta = \omega_0 R << 1$, the correction terms to the basic Sagnac delay $\delta\tau_S (\equiv 4\pi\beta R)$ to first order in ε and β are obtained from Eq.(13):

$$\delta\tau_E \cong \delta\tau_S - 8\pi a\varepsilon + 4\pi\beta R(3\eta - 2\varsigma), \tag{15}$$

where

$$\eta = \frac{M}{R+\xi}, \varsigma = \frac{2M-\xi}{2R}. \tag{16}$$

The second term in Eq.(15) is a contribution purely due to the angular momentum J while the last term displays the contribution from $\xi \; (\equiv \frac{Q^2}{M})$.

5.3. Polar orbit

We shall now investigate the effect when the light rays move along a circular trajectory passing over the poles. In this case, too, we may take $\rho = R$=constant and φ=constant. Assuming uniform motion again, we take $\theta = \omega_0 t$. Then, we have, using $d\rho = 0, d\varphi = 0, d\theta = \omega_0 dt$ and $d\tau^2 = 0$, from the metric (5):

$$\frac{d\theta}{dt} = \pm \frac{\left(R^2 - 2MR + a^2\cos^2\theta\right)^{1/2}}{R(R+\xi) + a^2\cos^2\theta}. \tag{17}$$

Assuming that $a^2/R^2 << 1$, $t = 0$ when $\theta = 0$, we have, on integration,

$$t \cong \frac{R+\xi}{\left(1-\frac{2M}{R}\right)^{1/2}}\theta + \frac{a^2}{4R}.\frac{\left(1-\frac{4M+\xi}{R}\right)}{\left(1-\frac{2M}{R}\right)^{3/2}}(\cos\theta\sin\theta+\theta). \tag{18}$$

During this time t, the rotating observer describes an angle θ_0 (say) while light travels an angle $2\pi \pm \theta_0$ (once again, + for co-rotating beam and - for the counter-rotating beam) so that, one obtains, after some manipulations [7],

$$\frac{\theta_0}{\omega_0} = (p+q)(2\pi \pm \theta_0) \pm \frac{q}{2}\sin 2\theta_0 \tag{19}$$

where

$$p \equiv \frac{R+\xi}{\left(1-\frac{2M}{R}\right)^{1/2}}, q \equiv \frac{a^2}{4R}.\frac{\left(1-\frac{4M+\xi}{R}\right)}{\left(1-\frac{2M}{R}\right)^{3/2}}. \tag{20}$$

Assuming a low speed observer and that the angle $2\theta_0$ be so small as to justify $\sin 2\theta_0 \cong 2\theta_0$, we get, on solving for θ_0, from Eq.(19):

$$\theta_{0\pm} = 2\pi\frac{p+q}{\frac{1}{\omega_0} \mp (p+q) \mp q}. \tag{21}$$

Finally, the difference between two round trip "coordinate" times (recalling the approximations already used) comes to

$$t_+ - t_- = \frac{\theta_{0+} - \theta_{0-}}{\omega_0}$$

$$\cong \pi\omega_0 (R+\xi)^2 \left\{4+8\varepsilon+\frac{3a^2}{R(R+\xi)}\right\}. \tag{22}$$

where we have retained terms of the order of ω_0 and ε only.

It is now necessary to express the above time difference in terms of the proper time τ of the rotating observer. This is done using in the metric (5), $\theta = \omega_0 t, d\theta = \omega_0 dt$:

$$\tau = \int \left[\frac{\Sigma - 2MR}{\Sigma} - \omega_0^2 \Sigma\right]^{\frac{1}{2}} dt, \Sigma \equiv R(R+\xi) + a^2 \cos^2(\omega_0 t). \tag{23}$$

Under the assumption of small $(\omega_0 t)$ such that $\cos(\omega_0 t) \cong 1$, $\sin(\omega_0 t) \cong 0$ and $\frac{a^2}{R(R+\xi)} << 1$, we get,

$$\tau \cong \left(1 - \frac{2M}{R+\xi} - \omega_0{}^2 R(R+\xi)\right)^{\frac{1}{2}} t. \tag{24}$$

Therefore, the time delay in the polar case, denoted by $\delta\tau_P$, is given by

$$\delta\tau_P \cong \left(1 - \frac{2M}{R(R+\xi)} - \omega_0{}^2 R(R+\xi)\right)^{\frac{1}{2}} (t_+ - t_-). \tag{25}$$

Using the result in Eq.(23), we get

$$\delta\tau_P \cong \pi\omega_0 (R+\xi)^2 \left[4 + \frac{3a^2}{(R+\xi)} + 4M\left\{\frac{R+2\xi}{R(R+\xi)}\right\}\right].$$

$$\cong 4\pi\omega_0 R^2 + \pi\omega_0 (3a^2 + 4MR) + \pi\omega_0 \left(8R\xi + 4\xi^2 + \frac{3a^2\xi}{R} + 12M\xi + \frac{8M\xi^2}{R}\right). \tag{26}$$

The first term is the basic Sagnac delay $\delta\tau_s$, the second term is the Kerr contribution and the third term is the string correction.

The absolute difference between equatorial and polar Sagnac delay is:

$$\left|\delta\tau_E - \delta\tau_P\right| \cong 8\pi a\varepsilon + 3\pi\omega_0 a^2 + 4\pi\omega_0\xi(R+6M). \tag{27}$$

5.4. Geodesic orbit

Consider the geodesic motion of the source/receiver having a 4-velocity $u^\mu\left(\equiv \dfrac{dx^\mu}{d\tau}\right)$ so that the equations are

$$\frac{\partial u^\mu}{\partial x^\nu}u^\nu + \Gamma^\mu_{\nu\alpha}u^\nu u^\alpha = 0 \tag{28}$$

where $\Gamma^\mu{}_{\nu\alpha}$ are the Christoffel symbols formed from the metric (5). The problem can be simplified by choosing $\theta = \pi/2$, so that $u^\theta = 0$. The geodesic equations this case, do allow such a solution [11]. Then, for a circular geo orbit with a constant radius $\rho = R$, the condition is $u^\rho = 0$. Then the radial equation turns out to be:

$$\Gamma^\rho_{tt}(u^t)^2 + \Gamma^\rho_{\varphi\varphi}(u^\varphi)^2 + 2\Gamma^\rho_{t\varphi}u^t u^\varphi = 0. \tag{29}$$

Defining the angular speed of rotation of the source/receiver as $\omega = u^\varphi / u^t$, we get, after a simplification, the two roots $\omega_\pm$ from Eq.(29):

$$\omega_\pm = \frac{1}{\dfrac{\partial g_{\varphi\varphi}}{\partial\rho}}\left[-\frac{\partial g_{t\varphi}}{\partial\rho} \pm \sqrt{\left(\frac{\partial g_{t\varphi}}{\partial\rho}\right)^2 - \frac{\partial g_{tt}}{\partial\rho}\cdot\frac{\partial g_{\varphi\varphi}}{\partial\rho}}\right] \tag{30}$$

Using the metric tensor from (5), we get

$$\omega_{\pm} = \left[\frac{1}{2Ma^2 - (R+\xi)^2(2R+\xi)}\right] \times \left[2Ma \pm \{2M(R+\xi)^2(2R+\xi)\}^{1/2}\right]. \tag{31}$$

We can put this $\omega_{\pm}$ in place of ω_0 in Eq. (13) to get the exact expression for the Sagnac delay. The approximation follows from Eq. (15):

$$\delta\tau_G \cong -\frac{8\pi a M}{R} + 4\pi\omega_{\pm} R^2 (1 + 3\eta - 2\varsigma). \tag{32}$$

The leading terms are

$$\begin{aligned} \delta\tau_G &\cong -\frac{8\pi a M}{R} \mp 4\pi\sqrt{MR} \pm \frac{5\pi Q^2}{\sqrt{RM}} \\ &= \mp\delta\tau_s - \frac{8\pi a M}{R} - \frac{4\pi a M^2}{R^2} \pm 5\pi\xi\sqrt{\frac{M}{R}} \end{aligned} \tag{33}$$

where $\delta\tau_s \left(\equiv 4\pi\sqrt{MR}\right)$ is the basic Sagnac delay in the polar case [6]. In the above approximation, we have neglected terms of the order $\left(\frac{\xi}{R}\right)^2, \frac{a^2}{R^3}$ etc. because of their smallness.

5.5. An estimate of the dilatonic charge

In the above, we provided exact formulations of the Sagnac delay for three types of orbits. These lead to correction terms embodied in Eqs.(15), (26) and (33), which reveal, especially, the role of the dilaton parameter ξ. The terms involving only a and M are the Kerr corrections. It might be of some interest to try to have an idea of the possible estimate of the dilatonic charge Q.

One possibility is the following: The existence of a horizon requires that $M^2 > J + \frac{1}{2}Q^2$. Thus, if $J \geq 0$, one has $Q < M$, that is, the mass of the black hole itself provides the upper limit. But this need not be the case when one considers orbits not around a black hole but around an ordinary uncollapsed object like the Earth. Therefore, the other possibility is to look for values of Q in Earth bound experiments.

Consider, for instance, a polar orbit around the Earth, say at a radius $R = 7\times10^6 m$. We also restore G and c in the expressions that follow and recall the relevant Earth values, designated by the subscript $\oplus$:

$$R_\oplus = 6.37\times10^6 m, \Omega_\oplus = 7.27\times10^{-5} m/s, GM_\oplus/c^2 = 4.4\times10^{-3} m, a_\oplus = 9.81\times10^8 m^2/s,$$

$$c = 3\times10^8 m/s. \qquad (34)$$

For the angular velocity $\omega_0 = \frac{c}{R}\sqrt{\frac{GM_\oplus}{Rc^2}}$, the main Sagnac term becomes

$$\delta\tau_S = \frac{4\pi\omega_0 R^2}{c^2} \approx 7.35\times10^{-6} s,$$

while the Kerr contribution, designated by $\delta\tau_K$, is

$$\delta\tau_K = \frac{\pi\omega_0}{c^2}\left(3a^2 + 4MGR\right)$$

The first term is $\approx 1.39\times10^{-18} s$, while the second term is $\approx 4.84\times10^{-15} s$, respectively. The dilatonic contribution, designated by $\delta\tau_\xi$, is

$$\delta\tau_\xi = \frac{\pi\omega_0}{c^2}\left(8R + \frac{12MG}{c^2} + \frac{3a^2}{Rc^2}\right)\xi + \frac{\pi\omega_0}{c^2}\left(4 + \frac{8MG}{Rc^2}\right)\xi^2. \qquad (35)$$

Note that ξ has the dimension of length, and so has Q. For the above expansion to be meaningful, we have assumed $\xi < 1m$.

For the values cited in (34), the leading term in Eq.(35) is

$$\delta\tau_{\xi} = \frac{8\pi\omega_0 R}{c^2}\xi \approx 4.75\times 10^{-10} Q^2 s/m^2.$$

Since $\xi = 2.27\times 10^2 Q^2 m^{-1}$, and that $\xi < 1m$, we can, in general, take $Q^2 = 10^{-\beta} m^2, \beta \geq 2$ so that $\delta\tau_{\xi} \approx 4.75\times 10^{-(10+\beta)} Q^2 s/m^2$. This implies that $Q = 10^{26-\frac{\beta}{2}} esu.$ Obviously, the value of β depends on the accuracy of observation of $\delta\tau_P$. Suppose, we are able to measure $\delta\tau_P$ up to an accuracy of $10^{-14} s$, which might be technologically possible in the near future. Then, $\beta = 4$ and it leads to $Q = 10^{24} esu.$ This is really a huge value for the terrestrial dilatonic charge, in fact exceeding the electronic charge of the Earth's magnetosphere $(2\times 10^{22} esu)$ by 200 times! A large value of $\beta >> 4$ would certainly reduce the dilatonic charge, but then the corrections to $\delta\tau_P$ would be extremely small and would go far beyond the Kerr corrections.

5.6. Some remarks

Correction to the Sagnac delay beyond the basic value $\delta\tau_s$ could also come from the non-Einsteinian theories such as the Brans-Dicke or other nonminimally coupled scalar tensor theories [7] but, unfortunately, these theories produce only naked singularities instead of black holes. This situations leaves us only with a few physically viable options in the form of either Kerr-Newman or dilation gravity. The best Kerr correction to the earth bound experiments is of the order of ~ 10^{-15} s and beyond, as we saw above. Therefore, a good candidate for the interpretation of corrections within the intermediate range (between 10^{-6} and 10^{-15} s) could be the terrestrial dilatonic charge which, however, seems rather huge in electrostatic units. On the other hand, we still do not know of a basic unit of the dilatonic charge given by an independent theory. Consequently, it is really not known what the actual magnitude would be in that basic unit.

It has come to our attention that in a recent paper, Bini, Jantzen and Mashhoon [12] have elaborately studied, from the relative observer point of view, the gravitomagnetic clock effect and the Sagnac effect for circularly rotating orbits in a general class of axisymmetric spacetimes. Although not specifically mentioned therein, the Kerr-Sen spacetime is automatically a member of this class due to its symmetries and much of the calculations in the present Letter follow directly from Ref.[12]. For instance, consider the metric of the circular orbit cylinder in the threading and slicing notation:

$$d\tau^2{}_{(t,\varphi)} = N^2 dt^2 - g_{\varphi\varphi}(d\varphi + N^{\varphi} dt)^2 \tag{37}$$

with

$$N^{\varphi} = -2Ma/(X^3 + a^2 X + 2Ma^2 - X^2 \xi).$$

Eq.(8) turns out to be exactly the same as Eq.(3.5)of Ref.[12]. Also, note that Eq.(12) gives the coordinate time difference as

$$\Delta t \equiv t_+ - t_- = \frac{\varphi_{0+} - \varphi_{0-}}{\omega_0}$$

$$= -2\pi[\frac{2\omega_0 - (\Omega_+ + \Omega_-)}{\omega_0 - \Omega_+)(\omega_0 - \Omega_-)} \tag{38}$$

which is precisely the same as Eq.(6.2) of Ref.[12] if the following identifications $\zeta \equiv \omega_0$, $\zeta_{\pm} \equiv \Omega_{\pm}$ are made in Eq.(38).

References:

[1]. B. Green, J.M. Schwarz and E. Witten, *Superstring Theory* (Cambridge University Press, Cambridge, 1987).

[2]. G.W. Gibbons, Nucl. Phys. **B 207**, 337 (1982); G.W. Gibbons and K. Maeda, Nucl. Phys. **B 298**, 741 (1988); D. Garfinkle, G. T. Horowitz and A. Strominger, Phys. Rev. **D 43**, 3140 (1991); G.T. Horowitz and A. Strominger, Nucl.Phys., **B 360**, 197 (1991); A. Shapere, S. Trivedi and F. Wilczek, Mod. Phys. Lett. **A 6**, 2677 (1991). See also: K.K. Nandi, T.B. Nayak, A. Bhadra and P.M. Alsing, Int. J. Mod. Phys. **D 10**, 529 (2001).

[3]. J. Horne, G.T. Horowitz and A. Steif, Phys. Rev. Lett. **68**, 568 (1992); J. Horne, G.T. Horowitz, preprint UCSBTH-92-11.

[4].A. Sen, Phys. Rev. Lett. **69**, 1006 (1992).

[5]. J.D. Gegenberg, Gen. Relat. Grav. **21**, 155 (1989).

[6]. A. Tartaglia, Phys. Rev. **D 58**, 064009 (1998).

[7]. K.K. Nandi, P.M. Alsing, J.C. Evans and T.B. Nayak, Phys. Rev. **D 63**, 084027 (2001).

[8]. J.M. Cohen and B. Mashhoon, Phys. Lett. **A 181**, 353 (1993); J.M. Cohen and H.E. Moses, Phys. Rev. Lett. **39**, 1641 (1977); J.M. Cohen, C. Ftaclas and H.E. Moses, Phys. Rev. **D 19**, 1273 (1979): J.M. Cohen, A. Rosenblum and Y. Clifton, Phys. Lett. **A 131**, 163 (1988).

[9]. J.E. Zimmermann and J.E. Mercereau, Phys. Rev. Lett. **14**, 887 (1965); A. Werner, J.L. Staudenmmn and R. Colella, Phys. Rev. Lett. **42**, 1103 (1979); F. Riehle, Th. Kisters, A. Witte, J. Helmcke and Ch. Borde, Phys. Rev. Lett. **67**, 177 (1991).

[10]. G.E. Stedman, Rep. Prog. Phys. **60**, 615 (1997).

[11]. L.D. Landau and E.M. Lifshitz, *The Classical Theory of Fields* (Pergamon Press, Oxford, Reprinted in 1987) p.329.