INTRODUCTION

The present thesis is directed towards a statement and examination of the three paradoxes of Set Theory, namely, Cantor's Paradox, Russell's Paradox and Burali-Forti Paradox. The thesis will concentrate Specially on Russell's Philosophy of Mathematics.

Since the publication of Kurt Göedel's theorem concerning incompleteness of arithmetic in 1931 much of the lure of Axiomatic Systems has diminished. Göedel proved the existence of formally undecidable propositions in any formal system of arithmetic. His second theorem, derived as corollary from the first, states that the consistency of a formal system of arithmetic cannot be proved by means formalizable within that system.

This result was damazing to the prospects of completing Hilbert's programme for the foundations of mathematics. Under the spell of enthusiasm of those enamoured by axiomatic method it had come to be believed that every field of inquiry, if it generates knowledge worth the name, must be formalizable axiomatically. And it is well-known the construction can be axiomatized. Further, it had come to be admitted that axiomatic systems are to be identified in terms of the **consistency, completeness** and **independence** of the axioms. What Göedel brought to notice is the fact that there exists at least one formal system, that of arithmetic, which connot be axiomatized. In course of time axiomatic systems were proposed for logic, social sciences, and even for physics.

But If mathematics includes arithmetic, then meta-mathematics, or the axiomatic system for mathematics should remain essentially incomplete. It should be noticed also that axiomatic system for logic differes from axiomatic system for arithmetic. Logic can express contingent truths, whereas arithmetic can only express thruths or their negations. Again, axiomatic system for Physics should include observation statements. Arithmetic has nothing to do with statements such as these.

To say that there cannot be a complete axiom system for all arithmetic means that we can always find some truth of arithmetic which is not a theorem of that system. It also has been proved that there can be no decision procedure for axiom systems for arithmetic. That is, we can never construct a computer which will solve all of the problems of arithmetic in a purely mechanical way.

Not much attention has however been paid to connect or link up incomplete nature of the formal system for arithmetic with the paradoxes of set theory. There are sets of ordinal and cardinal members, and it is in connection with these that the paradoxes arise. The proposed thesis will pay special attention to that aspect of the linkage.

The thesis is to be divided into chapters as the schema stated under.

I. The Concept and Types of Paradox

Mathematics has been used to clarify the so-called logical paradoxes, some of which date back to the time of the Greeks. For example, the paradox of Zeno's arrow.

A Paradox is a situation arising, when from a number of premises all generally accepted as true, a conclusion is reached by valid deductive argument that is either an outright contradiction or conflicts with other generally held or accepted beliefs. Such a result is both perplexing and disturbing because it is not clear which of one's well entrenched beliefs should be rejected, while it is plain that in the interests of consistency some modification must be made [A set of statement is inconsistent if it entails a contradiction or has contradicting consequences, and is consistent otherwise]. Paradoxes crop up in many theoretical disciplines relativity has its clock paradox and mathematics the Skolem paradox. The proof provided by Lowenheim in 1915 that any finite set Γ of sentences which has a model has a denumberable model, which result Skolem generalized in 1920 to the case where Γ is a denumerably infinite set of sentences. This gives rise to the Skolem paradox, because it is possible to formalize the theory of real numbers in a system with denumerably many axioms. Within this system it is possible to prove that the set of real numbers is non-denumerably infinite. Yet application of Skolem's result to the system means that if the system is consistents (has a model), it has a denumerable model (one in which there are denumerably many real numbers). There is however, a whole family of paradoxes, known as the self-referential paradoxes, some of which have played a crucial role in the historical development of the foundations of mathematics. One of the best and oldest of these is the "liar paradox" of Epimenides of Crete. Russell's paradox is a logical paradox that has serious repurcussions in the theory of classes. Consider the class of all classes that do not belong to themselves. Does it belong to itself? Answering either 'Yes' or 'No' results in a contradictions. From this Russell drew the conclusion that no such class exists, but it is not easily to justify this conclusion.

II. Basic Concepts : They are Class, Set, power set, ordinal and cardinal numbers, Real numbers and their notations

III. Statement of the Paradoxes

(A) **Cantor's Paradox** : George Cantor discovered a paradox in set theory. It is generated by the question of whether the set of all sets is (a) equal to or (b) greater than its own power set. If (a), a contradiction arises in that it can be proved that the power set of any given set is greater than the set itself. If (b), a contradiction arises since the power set of any set is itself a set of sets and therefore a subset of the set of all sets, and it can be proved that the subset of any given set is not greater than the given set.

(B) **Rusell's Paradox** : It is an important paradox in set theory. Some sets (classes or collection) are members of themselves and some are not. For instance, the set of horses is not a member of itself since it is a set and not a horse, where as the set of non-horses is a member of itself. Is the set of all sets which are not members of themselves, a member of itself ? If it is then it is, then it is not. If it is not, then it is. The paradox was discovered by Russell in 1901, and it had a profound influence on both the development of set theory and on our understanding of what sets are. Increasingly sets were conceived of as being determined by their members rather than determined by specifying conditions.

(C) **Burali-Forti Paradox** : The paradox of the greatest ordinal. Consider the set of all ordinals; it can be 0. 0 can be well-ordered, by the ' less than' relation, and there is a proof that all well ordered sets have an ordinal. So the ordinal of a set of consecutive ordinals (starting from the lowest) will be greater than every ordinal in the set. Thus 0 has an ordinal, ω, greater than all the ordinals in 0. But ω is itself an ordinal, and therefore is a member of 0. So ω both is, and is not a member of 0. In most versions of set theory, paradox is avoided by insisting that there is no set of all ordinals.

IV. **Examination of the Paradoxes** : Russell's theory of types deals with the problem of self reference. He became interested in his attempted definition of numbers in terms of classes. He encountered paradoxes generated by the notion of a class of classes, which includes itself as a member. Not all classes are members of themselves. But is the class of classes which are not members of themselves a member of itself or not? Whichever answer we choose, we are led into contradictions.

Russell's solution to the paradox was to say that self-referring statements are without meaning, and in particular, to speak of "all statements" is meaningless. Instead, we must speak of sets of statement that form a genuine totality. A statement referring to other statements must, Russell says, be of a different type from a higher order than the statement it is about. So we must say that the class of all first order classes which are not members of themselves is a second order class, and hence it will be "obvious nonsense" to say of a class either that it is or that it isn't a member of itself. Thus the paradox disappears.

Similarly Cantor's paradox and Burali-Forti paradox also may be made to disappear, and attempts would be made in that direction. An examination of the theory of types will also be undertaken.