APPENDIX

Appendix I

The Idea of Set Theory and other Related Matters

Mathematics and logic depend on each other. It is usually considered that logic ends and mathematics begins. Mathematics has been connected with science, logic with Greek. Both have developed in modern times, logic has become more mathematical and mathematics has become more logical. Modern mathematical work is obviously on the borderline of logic, so much of modern logic is symbolic and formal, that the very close relationship of logic and mathematics has become obvious to every instructed student.

Mathematics is the science of "quantity". "Quantity" is a vague word, but for the sake of argument we may replace it by the word "number". Probably only a person with some mathematical knowledge would think of beginning with 0 instead of with 1, but we will presume this degree of knowledge; we will take as our starting point the series :

0,1,2,3, ... n, n+1, ... (pace Peano)

Which is known as the "series of natural numbers". Very few people are prepared with a definition of what is meant by "number", or "0" or "1". It is not very difficult to see that starting from 0, any other of the natural numbers can be reached by repeated additions of 1,

Peano showed that the entire theory of the natural numbers could be derived from **three primitive** ideas and five primitive propositions in addition to those of pure logic.

The three primitive ideas of Peano's arithmetic are :

0, number, successor.

The five primitive propositions which Peano assumes are

(1) 0 is a number

(2) The successor of any number is a number

(3) No two numbers have the same successor

(4) 0 is not the successor of any number'

(5) Any property which belongs to 0, and also to the successor of every number which has the property, to belong to all numbers.

The most elementary properties of numbers are concerned with one-one relations, and similarity between classes. Addition is concerned with the construction of mutually exclusive classes respectively similar to a set of classes which are not known to be mutually exclusive. Multiplication is merged in the theory of "selections" i.e., of a certain kind of one-many relations. The ordinal properties of the various kinds of number-series, and the elements of the theory of continuity of functions and the limits of functions, can be generalised. So as to involve no longer any essential reference to numbers.

In any given branch of science or mathematics, one of the most powerful methods for eliminating conceptual vagueness is to isolate a small number of concepts basic to the subject at hand and then to define the other concepts of the discipline in terms of the basic set. Set theory, or the general theory of classes as it is sometimes called, is the basic discipline of mathematics, for with a few rare exceptions the entities which are studied and analyzed in mathematics may be regarded as certain particular sets or classes of objects. Any part of mathematics may be called a special branch of Set theory.

The aximoatization of a theory within set theory is an important initial step in making its structure both exact and explicit. Logic is the theory of valid arguments or the theory of deductive inference. The theory of definition together with the theory of sets provides an exact foundation for the axiomatic method, the study of which is informally considered part of logic by most mathematicians.

Every logician has recognized that logic is concerned with form but only recently that it has been recognized that there is a science of pure logic which is concerned with nothing but form. Logic is regarded as the art of thinking. Consequently, much stress has been laid upon the fallacies incident to language and upon the causes of erroneous beliefs. The port Royal Logicians regarded themselves as carrying on the tradition of Aristotle but there is good ground for believing that Aristotle himself was the first logician to be primarily interested in form. He saw clearly that propositions have form and that it is their form that is essential to deduction. Again in their theory of form, both Aristotle and subsequent logicians came very near to the theory of the logical variable. We have already laid stress upon the importance of form and have seen that the syllogism is a form of implication, deductive solely in virtue of its form.

If we believe that the premises are true, we shall accept the conclusion as true. But the pure logician is not interested in their truth or falsity; he is concerned only with the implication, that is, the form. The form of every syllogism which consists of a premises relating two classes and a premise relating an individual to a class of which it is a member. In the pure propositional form of implication the material constituents are replaced by variables, the form is expressed by logical constants. The historical development of a science reflects the mental development of man. Science begins by establishing a connection between one particular fact and another, and only gradually disentangles those properties upon which the connection depends. It is not otherwise with logic. Only as the result of a long process of development has it been possible to realize that all deduction depends upon the formal. The different kinds of logical constants represent the different types of deduction as the propositional form determines what material constants can be fitted into the form.

All deduction depends upon the logical properties of relations. The concept of relation is of fundamental importance. Any object of which we can think of possesses characteristics that enables us to distinguish it from other objects. These characteristics are of two kinds; qualities and relations. By a relation we mean a characteristic that belongs to A considered with reference to some other object B. The great importance of relations deductive theory is due to the fact that they possess certain formal, i.e., purely logical, properties that lie at the basis of all inference. It is worth stating that one of Russell's achievements is to have presented in PM a theory relations in fullness. Though we are not directly concerned with relations, yet we propose to give a brief statement of relations below. We consider only those properties of relations that are important for inference.

(1) Symmetrical relation, Asymmetrical relation and non-symmetrical relation.

Transitiveness :

(2) Transitive relation, intransitive relation and non-transitive relation.

The properties of symmetry and transitiveness and their opposites are independent.

Relations that are both transitive and symmetrical have the formal properties of equality. There is a third important property that belongs to such relations known as reflexiveness.

A relation is reflexive when it holds between a term and itself. The relation of identity is reflexive - Russell has pointed out that the only relation that can be said to be reflexive without limitation is the relation of identity. The properties of reflexiveness, symmetry, transitiveness are the formal properties that belong to the relations of identity and equality. We shall accordingly distinguish four kinds of relations, defined as follows :

(1) Many - many relation

(2) Many - one relation

(3) One - many relation

(4) One - one relation

We have now to consider the combination of two relations. Such a mode of combination is called relative multiplication and the relation thus obtained is called the relative product of R and S. Russell symbolizes the relative product of R and S by R | S. R and S are called the factors of their relative product. The converse of a relative product is obtained by reversing the order of the factors and then substituting their converses.

$$(R \mid S) = \breve{S}/\breve{R}$$

That (R | S) = S/R.

The relative product of R and R is the square of R. That is $R \mid R = R^2$.

For example, the relative product of father and father is parental grand father. The converse of the square of father is son's son. The square of ancestor is ancestor.

We may now say something about the idea of system, as the theory of members and sets are often spoken as a system.

A system consists of elements standing in certain relations. For example, the solar system is a system consisting of certain elements viz. the sun, the planets and their satellites, standing in certain relations. In any given system the fact that an element stands in a given relation can be expressed by a proposition. A deductive system is a special kind of system in which the elements are propositions and the relations between the elements are logical relations. Self evidence seems to combine two elements, obviousness and logical priority, though it is not a notion useful for logic. It might, however, be supposed that we could define axioms in terms of logical priority. But logical priority is not absolute. The notion of logical priority is somewhat obscure.

A primitive concept and a primitive proposition are, then, primitive only in relation to a given system. The selection of these primitive notions and primitive propositions determines a given deductive system. This may be made clearer by considering the logical relation of physics and mathematics. There is a sense in which physics presupposes mathematics, since physics cannot be developed without reference to mathematics, where as mathematics can be developed without reference to physics. But it would be a mistake to argue from this that mathematics is in an unqualified sense logically prior to, or necessarily presupposed by physics. The propositions of a deductive system are established as true only by means of inductive verification. Such verification is never complete, it could not amount to demonstration. The complete generality of a deductive system is due to the fact that the primitive propositions do not determine a unique set of objects. When such deductive system can be constructed it becomes possible to develop a part of several abstract sciences at the same time.

A calculus is an instrument for reasoning. Its purpose is to economize thought by providing a mechanical method of obtaining results, which can then be interpreted in a manner analogous to the way in which a mathematical equation can be interpreted. As a matter of historical fact recent work on the logical foundations of mathematics has grown out of the attempt to develop a logical calculus. In order to obtain a calculus a well-defined symbolism must be used. In this way an almost mechanical process of calculation takes the place of reasoning.

Thus the development of a calculus prepared the way for the analysis of deductive systems. We shall consider the calculus only in so far as it throws light upon the process of the generalization of logic. The most fundamental relation between propositions is implication, the most fundamental relation between classes is inclusion. The connection of two elements by a relation does not yield a result which is of the same kind as the elements related. For example, the relation between classes yields a proposition, not a class. A deductive system is generated by logical relations, not by operations. The calculus was first worked out for classes and was then interpreted so as to apply to propositions.

We have two special relations, the UNIVERSAL Relations $\vee$ and the NULL Relation $\wedge$,

- the first relation holds between any two individuals and the second relation between none. Calculus of relations are of different types :

Different relations on calculus of relations :

1. Relation of inclusion : $R \subset S$
2. Relation of Diversity : $X < Y$

   $X \neq Y$
3. Relation of Identity : $R = S$
4. Relation of Union : RUS

   $|X\ (R \cup S) Y|$
5. Relation of Intersection : $R \cap S$

   $X\ (R \cap S) Y$
6. Relation of Negation $\sim$

   or

   Complement /

7. Relative product or Composition of R and S : R | S

8. Relative sum of two relations $R \cup S$

9. Converse relation $\overset{\cup}{R}$

We may distinguish relations of different orders.

1. First order relations are those relation which hold between individuals

2. Second order relations - are those relation which hold between classes or relations, of the first order

3. Mixed relation - are those relation whose predecessor are, for instance, classes of the fist order and its successors classes of the second order.

The domain of a relation is the first components of the ordered pairs of the relation.

The range of a relation is the second components of the ordered pairs of the relations.

Again we can say the class of all predecessors with respect to the relation - is known as domain and the class of all successors - is known as Counter domain

or

Converse domain

There is a familiar relation not by means of which from a given proposition p we obtain another proposition not ~ p. The proposition not ~ p will be called the contradictory of p.

There modes of combining two, or more propositions are familiar, viz. the mode of combining by means of the logical relations and, or, 'implies'. Thus, given any two propositions there is a third proposition which consists in their simultaneous assertion, i.e., the assertion of both together. This proposition is derived from combining p,q by means of and, which yields p and q.

These mode of combinations are called functions of the propositions so combined, and the derivation of a proposition by means of 'not' is called a function of the original proposition. These four functions have appropriate names and symbols :

Logical Relation	Functions	Symbolized by
(1) not	negation or contradiction	$\sim p$
(2) or	addition or disjunction	$p \vee q$
(3) and	multiplication or conjunction	$p . q$
(4) implies	implication	$p \supset q$

It is not necessary to take all the modes of combination as undefined modes. Given negation, we can define disjunction if we assume conjunction; conversely, we can define conjunction if we assume disjunction; finally we can define implication if we assume conjunction or if we assume disjunction.

There is a paradox in sentential or propositional calculus as well we may state that in passing.

As Russell has admitted, 'whenever p is false, "not p or q" is true, but is useless for inference, which requires that p should be true. Whenever q is already known to be true, "not-p or q" is of course also known to be true, but is again useless for inference, since q is already known, and therefore does not need to be inferred'. He draws a distinction between the validity of the inference and what he calls the 'practical feasibility of the inference'. False propositions imply other propositions, but they cannot be made the basis of valid inferences in which the conclusion is asserted to be true. There is, then no reason to regard the consequences of the definition of implication as·paradoxical. They would be paradoxical only if they were taken to be paradoxes of inference. But in that case they would not strictly speaking be paradoxical; they would be false.

Appendix II

Some definition of concepts related with the Thesis

The set theory was developed by the German mathematician GEORGE CANTOR (1845 - 1918).

Arithmetic involves the study of sets of numbers, Algebra involves the study of variables. Geometry involves the study of the set of points on a straight line or curved line.

Cantor is regarded as the founder of the theory of sets and it was he who introduced it to the mathematical world.

1. **Class :** Any (kind of)cellection of entities made on the basis of shared character of any sort is known as class. Many other words are used synonymously with sets :

Example : (i) Set of numbers

(ii) Bunch of vegetables

(iii) A class of student

That means any group or set of living or non-living things which is not completely specified is known as a class.

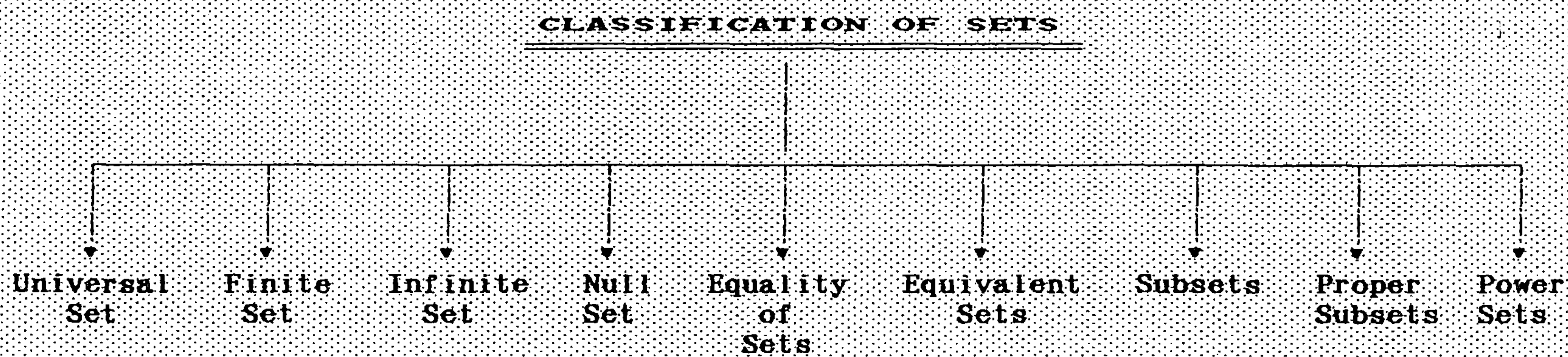
2. **Set :** In mathematics, we use the word 'set' to denote any "well - defined" collection of objects, things or symbols.

By "well - defined" we mean that it must be possible to tell beyond doubt whether or not a given object belongs to the collection that we are considering.

Example : (i) Set of rational numbers.

(ii) Bunch of green vegetables.

(iii) A class of M.A. students who had mathematics combinations in 1990.


CLASSIFICATION OF SETS
Universal Set
Finite Set
Infinite Set
Null Set
Equality of Sets
Equivalent Sets
Subsets
Proper Subsets
Power Sets

Set Operations

Union of two sets	Intersection of two sets	Difference of two sets	Complement of a set
$A \cup B$	$A \cap B$	$A - B$	A'

3. **Cardinal Number :** Cardinal number of afinite set 'A' is the number of elements in the set 'A'. It is denoted by n(A).

All infinite sets were considered to be equivalent. The following definition, which has revolutionized the entire theory of sets, is attributed to the German mathematician George Cantor (1845-1918).

Let A be any set and let denote the family of sets which are equivalent to A. Then α is called a cardinal number and is denoted by

$$\alpha = \#(A)$$

The symbol alpha null (ς) used to denote cardinally of denumberable sets. i.e., = # (N)

4. **Denumerable Set :** If a set is equivalent to natural numbers (N), then D is called denumberable and is said to have cardinality α.

i.e., $D \simeq N$ (have a cardinality α)

5. **Non-denumerable Set :** A set is called non-denumerable if it is infinite and is not equivalent to N, the set of natural numbers.

i.e., $D \not\simeq N$