

CHAPTER VIII

MINIMISATION OF COST OF RUNNING A VEHICLE PER DAY : THE LINEAR PROGRAMMING MODEL WITH SENSITIVITY ANALYSIS OF THE COST AND REQUIREMENT VECTORS

Various manufacturers, firms, industrial producers, and many business and economic situations are often concerned with problems of planning activities. They deal with situations where a number of resources, such as manpower, machines, land, capital, etc. are available, and are to be combined to yield products. But in each case, there are limited resources at their disposal and within the limited resources there may exist many feasible allocations. Out of all available courses of action (i.e. permissible allocations of resources), it is desired to select one or ones which will give the maximum profit or minimum cost of production. Such problems are referred to as the optimization problems. Recently, many new and important optimization problems have emerged in the field

of economics and have received much attention. Classical optimization techniques have been found to be of little assistance in solving these problems. Therefore, new methods had to be developed and one of the very important of them is Linear Programming problems and the most widely used and best known method for solving them is simplex method.

8.1 Origin and Background :

The origin and background of linear programming has been in the field of econometrics and research into economic relationships. Its roots go back to 1874 and the works of the mathematical economists Leon Walras. Linear Programming as it is known to-day, is descendant from the input output method of analysis developed by the economist Wassily W. Leontief in the 1920s. The present day development stems primarily from the work of George B. Dantzig, who is credited with developing the simplex method¹.

8.2 Definition and Framework of L.P :

Linear Programming in mathematical terms is concerned with maximizing or minimizing a linear objective function of variables constrained by linear inequalities. It

provides a method for the determination of some best course or courses of action. In this way it is analogous to the producers of marginal analysis in economics. It is one of the most important optimization techniques developed in the field of operations Research. Its theoretical developments have been initiated and supported by a countless variety of practical applications in economics and Industrial management. In management terms, the definition can be stated as below :

"Linear Programming (L.P) is a technique for specifying how to use limited resources or capacities of a business to obtain a particular objective, such as least cost, highest margin, or least time, when these resources have alternate uses. It is a technique that systematizes for certain conditions the process of selecting the most desirable course of action from a number of available courses of action, thereby giving management information for making a more effective decision about the resources under its control"².

Linear Programming problems have the following framework :

1. There is some objective to be attained such as maximum profit, minimum cost, or minimum elapsed time, of the system being studied.

2. There are a large number of variables to be handled simultaneously.

3. There are many interactions between the variables.

4. Most linear programming problems are also characterised by the presence of objectives that conflict with the principal objective of the problem.

8.3 Application of Linear Programming :

Linear Programming developed within the last twenty five years has attracted much attention. It is applied to a wide variety of problems. We shall show how linear programming problems can arise in practice to different situations.

1. Determining most profitable mix to be obtained from existing facilities.
2. Determining which parts to make and which to buy to obtain maximum profit margin.
3. Scheduling orders to machine centres at least cost consistent with delivery promises and schedules.
4. Establishing best location of warehouses to minimise transportation costs.

5. Selecting equipment and evaluating methods improvements that maximize profit margin.
6. Planning profits on a fiscal year basis to maximize net return on an investment in plant, facilities, equipment, cash on hand, and inventory.
7. Supplying a fluctuating sales demand at least inventory cost considering a fixed level of production and stabilised employment.
8. Allocating production releases among several plants so as to maximize profit margin, considering manufacturing and distribution costs.
9. Determining equitable sales and incentive compensation.
10. Determining the feed mix that satisfies nutritional requirements and minimises the cost of raising livestock.
11. Programming a Chemical distilling type operation, including the processing of purchased material, to obtain the highest manufacturing margin with limits of sales demand.
12. Planning the most profitable combinations of sales requirements and plant capacity that obtain a fair share of the market"³.

8.4 Advantages to Management From Using L.P :

The benefits derived by management from using L.P. are as follows :

1. More time to manage, by providing him with information quicker and permitting more delegation.
2. Less pressure to get things done, because he can do more accurate planning and avoid pitfalls that create crisis situations.
3. Better qualified personnel, because the techniques provide insight into management problems and decision making that formerly would have required years of experience and practice to obtain.

Each and all of these factors will add to the ability of the manager to manage better, which, in turn, will reflect profitability for the firm.

8.5 Problems in Introducing and Applying Linear Programme :

Experience so far indicates that to get most out of Linear Programme requires the solution of two major groups of problems.

1. The direct problem of providing the personnel, the know how, and the data for setting up and solving problems.

2. To attain the most benefits from the use of Linear Programme, it must be assimilated into the organisation as a working tool and as an integral part of management thinking⁴.

8.6 Future of Linear Programme :

Experience so far indicates that Linear programming will move forward as a part of the increased use of scientific and mathematical tools rather than as a particular technique. Following predications can be made of Linear Programme :

1. The increase of OR will increase the use of L.P.
2. The use of L.P for solving operating problems will increase.
3. Educational Institutions will expand the fields of L.P.
4. Electronic Computer will expand the use of L.P.⁵.

8.7 Linear Programming Model with Sensitivity Analysis :

Our main aim is to minimise the cost of running a vehicle per day. The cost of running a vehicle per day

should be less than the earning from a vehicle per day. There are so many variables. But we are considering only the important variables for our purpose :- (1) number of staff per vehicle per day (Y_1), (2) quantity of fuel (litres) required per vehicle per day (Y_2), and (3) number of tyres and tubes (fraction) required for a vehicle per day (Y_3).

Let C_1 be the pay per staff per day :

C_2 be the price of 1 (one) litre of fuel;

C_3 be the price of 1 (one) tyre/tube.

Then our objective function will be,

$$Z = C_1 Y_1 + C_2 Y_2 + C_3 Y_3$$

Now our problem is to

$$\text{Min. } Z = C_1 Y_1 + C_2 Y_2 + C_3 Y_3$$

subject to some constraints.

The values of costs (C_1, C_2, C_3) will be obtained from the previous records of the corresponding transport corporation.

Now about the constraints :

1. $Y_1 \geq b_1$ = Minimum number of staff per vehicle per day; (we have taken this as 5.54 because Haryana SRTC takes 5.54 employees per vehicle per day which is least among all

the SRTU's reported.

Source : Performance Statistic - 1986-87 and
1987-88 ... (1)

2. $Y_2 \gg b_2$ = Minimum quantity of fuel required per vehicle per day; (we have taken this as 71.3198 litres per vehicle per day because Gujrat SRTC continued to report the best performance of 4.99 kms. per litre.

Source : Performance statistics of STU's 1986-87 and 1987-88.

Note : Quantity consumed per vehicle per day in Gujrat SRTC in 1987-88 = Total cost of fuel in 1987-88.

Rate per litre x No. of days in a year x No. of vehicles in 1987-88

$$= \frac{\text{Rs. } 6132.93 \text{ lakhs}}{\text{Rs. } 3.56 \times 365 \times 6437} = 71.3198 \text{ litres.}$$

3. $Y_3 \gg b_3$ = minimum number (fraction) of tyres and tubes required per vehicle per day (we have taken this as 0.0106648 because Uttar Pradesh SRTC reports the lowest cost i.e. 19.3 paise per km. and average cumulative kms. performance per tyre is

95458 km which is the best among all the STU's reported.

Source : Performance Statistics of STU's - 1986-87 and 1987-88 ... (III)

Note : Number of tyres and tubes in Uttar Pradesh State Road Transport Corporation per vehicle per day in 1987-88.

$$= \frac{\text{Total cost of tyres and Tubes in 1987-88}}{\text{Cost of one tyre and tube x No. of days in a year x Vehicle in 1987-88.}}$$

$$= \frac{\text{Rs. } 1017.90 \text{ lakhs}}{\text{Rs. } 4150 \times 365 \times 6301} = 0.0106648$$

4. Sometimes it happens that

$$C_1 Y_1 \gg C_2 Y_2 + C_3 Y_3$$

(i.e. cost of personnel \gg cost of fuel + cost of tyres/tubes)

$$\text{or } C_1 Y_1 - C_2 Y_2 - C_3 Y_3 \gg 0 = b_4 \text{ (say) ... (IV)}$$

5. Sometimes it happens that

$$C_2 Y_2 \gg C_3 Y_3$$

(i.e. fuel cost \gg cost of tyres and tubes)

$$\text{or } C_2 Y_2 - C_3 Y_3 \gg 0 = b_5 \text{ (say) ... (V)}$$

6. We should set our programme in such a way that the cost of running a vehicle per day will be lesser than the earnings per vehicle per day.

(the earning per vehicle per day varies for different transport corporations)

$$\text{i.e. } C_1 Y_1 + C_2 Y_2 + C_3 Y_3 \leq b_6 = \text{earning per vehicle per day ... (VI)}$$

$$Y_1, Y_2, Y_3 \geq 0$$

Among the above six (6) constraints, constraints number 1,2,3 respectively are considered for all combinations, but from the constraints 4,5,6; we have considered all possible combinations. We have calculated the optimum (minimum) value of the objective function Z (in overall cost) as well as the corresponding Y_i 's.

We have also done the sensitivity analysis of the changes in (i) cost vector, (ii) requirement vector for each combination.

(i) Changes in cost vector

Let ΔC_K be the amount of which is added to the K th component C_K of cost vector, so that the new value of the K th cost component becomes $C_K^A = C_K + \Delta C_K$

The range of values of ΔC_K is found so that the new basic feasible solution shall remain optimum.

(ii) Changes in requirement vector

Let Δb_K be added to K th component b_K of requirement vector, so that the new value of the K th requirement

component becomes $b_K^* = b_K + \Delta b_K$.

The range of the values of Δb_K is found so that the new basic feasible solution shall remain optimum. Now we shall consider the different road transport corporations.

A. FOR NBSTC (for the year 1987-88) :

$$C_1 = \text{Rs. } 40.40$$

$$C_2 = \text{Rs. } 3.66 \text{ (approx).}$$

$$C_3 = \text{Rs. } 4150 \text{ (approx).}$$

The average earnings per vehicle per day for NBSTC (for the year 1987-88) is Rs. 656.31.

Case - I : Considering constraints - 1, 2 and 3

Here $b = (b_1, b_2, b_3)$

Min cost = Rs. 528.67

Where $Y_1 = 5.54$

$Y_2 = 71.3198$

$Y_3 = 0.01066$

Sensitivity analysis of cost vector.

$$\Delta C_1 \leq 40.375$$

$$\Delta C_2 \leq 3.6875$$

$$\Delta C_3 \leq 4150$$

Sensitivity analysis of requirement vector.

$$\begin{aligned} \Delta b_1 &\leq 5.54 \\ \Delta b_2 &\leq 71.3198 \\ \Delta b_3 &\leq 0.0107 \end{aligned}$$

Case No. 2 : Considering constraints 1,2,3,4.

$$\begin{aligned} \text{Min Cost} &= 610.69 \\ \text{Where } Y_1 &= 7.56 \\ Y_2 &= 71.3198 \\ Y_3 &= 0.01066 \end{aligned}$$

Sensitivity analysis of cost vector.

$$\begin{aligned} \Delta C_1 &\leq 42.8709 \\ \Delta C_2 &\leq 7.3568 \\ \Delta C_3 &\leq 8190 \end{aligned}$$

Sensitivity Analysis of Requirement vector.

$$\begin{aligned} -2.0167 &\leq \Delta b_1 \\ \Delta b_2 &\leq 22.2594 \\ \Delta b_3 &\leq 0.0107 \\ \Delta b_4 &\leq 81.3185 \end{aligned}$$

Case No.3 : Considering Constraints 1,2,3,5

$$\begin{aligned} \text{Min Cost} &= \text{Rs. } 529.67 \\ \text{Where } Y_1 &= 5.54 \\ Y_2 &= 71.3198 \\ Y_3 &= 0.01066 \end{aligned}$$

Sensitivity Analysis of Cost Vector.

$$\Delta C_1 \leq 40.3750$$

$$\Delta C_2 \leq 3.7365$$

$$\Delta C_3 \leq 3906.3694$$

Sensitivity Analysis of Requirement Vector.

$$\Delta b_1 \leq 5.54$$

$$\Delta b_2 \leq 59.2272$$

$$-.0522 \leq \Delta b_3 \leq 0.0107$$

$$-216.7716 \leq \Delta b_4$$

Case No.4 : Considering Constraints 1,2,3,6

$$\text{Min Cost} = \text{Rs. } 528.67$$

$$\text{Where } Y_1 = 5.54$$

$$Y_2 = 71.3198$$

$$Y_3 = 0.01066$$

Sensitivity Analysis of Cost Vectors.

$$\Delta C_1 \leq 40.3750$$

$$\Delta C_2 \leq 3.6875$$

$$\Delta C_3 \leq 4150$$

Sensitivity Analysis of Requirement Vectors.

$$-3.1485 \leq \Delta b_1 \leq 5.54$$

$$-34.7553 \leq \Delta b_2 \leq 71.3198$$

$$-0.0306 \leq \Delta b_3 \leq 0.0107$$

$$-127.2046 \leq \Delta b_4$$

Case No.5 : Considering Constraints 1,2,3,4,5

$$\begin{aligned} \text{Min Cost} &= 610.69 \\ \text{Where } Y_1 &= 7.56 \\ Y_2 &= 71.3198 \\ Y_3 &= 0.01066 \end{aligned}$$

Sensitivity Analysis of Cost Vectors.

$$\begin{aligned} \Delta C_1 &\leq 42.2217 \\ \Delta C_2 &\leq 7.3549 \\ \Delta C_3 &\leq 7911.9159 \end{aligned}$$

Sensitivity Analysis of Requirement Vectors.

$$\begin{aligned} -2.0167 &\leq \Delta b_1 \\ \Delta b_2 &\leq 22.25 \\ \Delta b_3 &\leq 0.0107 \\ \Delta b_4 &\leq 81.3185 \\ -216.7716 &\leq \Delta b_5 \end{aligned}$$

Case No.6 : Considering Constraints 1,2,3,4,6

$$\begin{aligned} \text{Min Cost} &= \text{Rs. } 610.69 \\ \text{Where } Y_1 &= 7.55 \\ Y_2 &= 71.3198 \\ Y_3 &= 0.01066 \end{aligned}$$

Sensitivity Analysis of Cost Vectors.

$$\begin{aligned} \Delta C_1 &\leq 42.87 \\ \Delta C_2 &\leq 7.3568 \\ \Delta C_3 &\leq 8190 \end{aligned}$$

Sensitivity Analysis of Requirement Vectors.

$$\begin{aligned} -2.0167 &\leq \Delta b_1 \\ -6.2474 &\leq \Delta b_2 \leq 22.2593 \\ -0.0055 &\leq \Delta b_3 \leq 0.0107 \\ -45.7312 &\leq \Delta b_4 \leq 81.3185 \\ -45.7312 &\leq \Delta b_5 \end{aligned}$$

Case No.7 : Considering Constraints 1,2,3,5,6

$$\text{Min Cost} = \text{Rs. } 529.58$$

$$\text{Where } Y_1 = 5.54$$

$$Y_2 = 71.3198$$

$$Y_3 = 0.01066$$

Sensitivity Analysis of Cost Vector :

$$\begin{aligned} \Delta C_1 &\leq 40.3880 \\ \Delta C_2 &\leq 3.7286 \\ \Delta C_3 &\leq 3906.8364 \end{aligned}$$

Sensitivity Analysis of Requirement Vector.

$$\begin{aligned} - 3.1486 &\leq \Delta b_1 \leq 5.54 \\ - 34.7553 &\leq \Delta b_2 \leq 59.2227 \\ - 0.0306 &\leq \Delta b_3 \leq 0.0107 \end{aligned}$$

$$-216.7716 \leq \Delta b_4$$

$$-127.2046 \leq \Delta b_5$$

Case No.8 : Considering Constraints 1,2,3,4,5,6

$$\text{Min cost} = 610.69$$

$$\text{Where } Y_1 = 7.55$$

$$Y_2 = 71.3198$$

$$Y_3 = 0.01066$$

Sensitivity Analysis of Cost Vector.

$$\Delta c_1 \leq 42.8427$$

$$\Delta c_2 \leq 7.3342$$

$$\Delta c_3 \leq 0.0107$$

Sensitivity Analysis of Requirement Vectors.

$$-2.0167 \leq \Delta b_1$$

$$-6.2474 \leq \Delta b_2 \leq 22.2593$$

$$-0.0055 \leq \Delta b_3 \leq 0.0107$$

$$-45.7312 \leq \Delta b_4 \leq 81.3185$$

$$-216.7716 \leq \Delta b_5$$

$$-45.7312 \leq \Delta b_6$$

Note : The actual cost per vehicle per day is Rs. 1138 in

1987-88.

For Rajasthan SRTC :

Taking the same principles and conditions applied for NBSTC, we get the following :

Our problem is,

$$\text{Min } Z = C_1 Y_1 + C_2 Y_2 + C_3 Y_3$$

Subject to some constraints,

$$\text{Here, } C_1 = \text{Rs. } 39.40 \text{ (in 87-88)}$$

$$C_2 = \text{Rs. } 3.66 \text{ (approx) (in 87-88)}$$

$$C_3 = \text{Rs. } 4,150 \text{ (in 87-88)}$$

About Constraints,

$$(1) \quad Y_1 \geq 5.54 \quad (b_1) \quad \dots \quad (1)$$

$$(2) \quad Y_2 \geq 71.3198 \quad (b_2) \quad \dots \quad (2)$$

$$(3) \quad Y_3 \geq .0106648 \quad (b_3) \quad \dots \quad (3)$$

$$(4) \quad C_1 Y_1 - C_2 Y_2 - C_3 Y_3 \geq 0 = (b_4) \quad \dots \quad (4)$$

$$(5) \quad C_2 Y_2 - C_3 Y_3 \geq 0 = (b_5) \quad \dots \quad (5)$$

$$(6) \quad C_1 Y_1 + C_2 Y_2 + C_3 Y_3 \leq 1319 = (b_6) \dots \quad (6)$$

Among six (6) constraints, number 1,2,3 respectively are considered for all combinations, but from the constraints 4,5,6 we have considered all possible combinations.

Case No.1 : Considering constraints 1,2 and 3

$$\text{Min Cost} = \text{Rs. } 523.13$$

$$\text{Where } Y_1 = 5.54$$

$$\begin{aligned}
 Y_2 &= 71.3198 \\
 Y_3 &= .01066
 \end{aligned}$$

Sensitivity Analysis of Cost Vectors.

$$\begin{aligned}
 \Delta C_1 &\leq 39.3750 \\
 \Delta C_2 &\leq 3.6875 \\
 \Delta C_3 &\leq 4150
 \end{aligned}$$

B : Sensitivity Analysis of Requirement Vectors.

$$\begin{aligned}
 \Delta b_1 &\leq 5.54 \\
 \Delta b_2 &\leq 71.3198 \\
 \Delta b_3 &\leq 0.0107
 \end{aligned}$$

Case No.2 : Considering Constraints 1,2,3 and 4

$$\begin{aligned}
 \text{Min Cost} &= \text{Rs. } 610.22 \\
 \text{Where } Y_1 &= 7.74 \\
 Y_2 &= 71.3198 \\
 Y_3 &= 0.01066
 \end{aligned}$$

Sensitivity Analysis of Cost Vectors.

$$\begin{aligned}
 \Delta C_1 &\leq 36.3937 \\
 \Delta C_2 &\leq 7.1210 \\
 \Delta C_3 &\leq 7795.6875
 \end{aligned}$$

B : Sensitivity Analysis of Requirement Vectors.

$$\begin{aligned}
 -2.2085 &\leq \Delta b_1 \\
 \Delta b_2 &\leq 23.7728 \\
 \Delta b_3 &\leq 0.0107 \\
 \Delta b_4 &\leq 86.9488
 \end{aligned}$$

Case No.3 : Considering constraints 1,2,3 and 5

$$\begin{aligned}
 \text{Min Cost} &= \text{Rs. } 524.12 \\
 \text{Where, } Y_1 &= 5.54 \\
 Y_2 &= 71.3198 \\
 Y_3 &= 0.01066
 \end{aligned}$$

Sensitivity Analysis of Cost Vectors.

$$\begin{aligned}
 \Delta c_1 &\leq 39.3750 \\
 \Delta c_2 &\leq 3.7365 \\
 \Delta c_3 &\leq 3906.3694
 \end{aligned}$$

Sensitivity Analysis of Requirement Vectors.

$$\begin{aligned}
 \Delta b_1 &\leq 5.54 \\
 \Delta b_2 &\leq 59.2272 \\
 -.0522 \leq \Delta b_3 &\leq 0.0107 \\
 -216.7716 \leq \Delta b_4 &
 \end{aligned}$$

Case No.4 : Considering Constraints 1,2,3,4,5

$$\begin{aligned}
 \text{Min Cost} &= 610.22 \\
 \text{Where, } Y_1 &= 7.74
 \end{aligned}$$

$$\begin{aligned}
 Y_2 &= 71.3198 \\
 Y_3 &= 0.01066
 \end{aligned}$$

Sensitivity Analysis of Cost Vectors.

$$\begin{aligned}
 \Delta C_1 &\leq 37.0590 \\
 \Delta C_2 &\leq 7.1259 \\
 \Delta C_3 &\leq 7472.9268
 \end{aligned}$$

B : Sensitivity Analysis of Requirement Vectors.

$$\begin{aligned}
 -2.2085 &\leq \Delta b_1 \\
 \Delta b_2 &\leq 23.7728 \\
 -0.0522 &\leq \Delta b_3 \leq 0.0107 \\
 \Delta b_4 &\leq 86.9488 \\
 -216.7716 &\leq \Delta b_5
 \end{aligned}$$

Case No.5 : Considering Constraints 1,2,3 and 6

$$\begin{aligned}
 \text{Min Cost} &= 523.13 \\
 \text{Where } Y_1 &= 5.54 \\
 Y_2 &= 71.3198 \\
 Y_3 &= .01066
 \end{aligned}$$

Sensitivity Analysis of cost vectors.

$$\begin{aligned}
 \Delta C_1 &\leq 39.3750 \\
 \Delta C_2 &\leq 3.6875 \\
 \Delta C_3 &\leq 4150
 \end{aligned}$$

B : Sensitivity Analysis of Requirement Vectors.

$$\begin{aligned}
 - 20.1886 &\leq \Delta b_1 \leq 5.54 \\
 -217.3318 &\leq \Delta b_2 \leq 71.3198 \\
 - 0.1916 &\leq \Delta b_3 \leq .0107 \\
 -795.4346 &\leq \Delta b_4
 \end{aligned}$$

Case No.6 : Considering Constraints 1,2,3,4,6

$$\begin{aligned}
 \text{Min Cost} &= \text{Rs. } 610.22 \\
 \text{Where } Y_1 &= 7.74 \\
 Y_2 &= 71.3198 \\
 Y_3 &= 0.01066
 \end{aligned}$$

Sensitivity Analysis of Cost Vectors.

$$\begin{aligned}
 \Delta C_1 &\leq 36.3937 \\
 \Delta C_2 &\leq 7.1210 \\
 \Delta C_3 &\leq 7795.6875
 \end{aligned}$$

B : Sensitivity Analysis of Requirement Vectors.

$$\begin{aligned}
 - 2.2085 &\leq \Delta b_1 \\
 - 36.7788 &\leq \Delta b_2 \leq 23.7728 \\
 - 0.0853 &\leq \Delta b_3 \leq 0.0107 \\
 -708.4213 &\leq \Delta b_4 \leq 86.9488 \\
 -708.4213 &\leq \Delta b_5
 \end{aligned}$$

Case No.7 : Considering Constraints 1,2,3,5,6

$$\begin{aligned}
 \text{Min Cost} &= \text{Rs. } 524.12 \\
 \text{Where } Y_1 &= 5.54 \\
 Y_2 &= 71.3198 \\
 Y_3 &= 0.01066
 \end{aligned}$$

Sensitivity Analysis of Cost Vectors

$$\begin{aligned} \Delta C_1 &\leq 39.3750 \\ \Delta C_2 &\leq 3.7365 \\ \Delta C_3 &\leq 3906.3694 \end{aligned}$$

B : Sensitivity Analysis of Requirement Vectors.

$$\begin{aligned} -20.1886 &\leq \Delta b_1 \leq 5.54 \\ -217.3318 &\leq \Delta b_2 \leq 59.2272 \\ -0.0522 &\leq \Delta b_3 \leq .0107 \\ -216.7716 &\leq \Delta b_4 \\ -795.4346 &\leq \Delta b_5 \end{aligned}$$

Case No.8 : Considering Constraints 1,2,3,4,5,6

$$\begin{aligned} \text{Min Cost} &= \text{Rs. } 610.22 \\ \text{where } Y_1 &= 7.74 \\ Y_2 &= 71.3198 \\ Y_3 &= 0.01066 \end{aligned}$$

Sensitivity Analysis of Cost Vectors.

$$\begin{aligned} \Delta C_1 &\leq 37.0590 \\ \Delta C_2 &\leq 7.1259 \\ \Delta C_3 &\leq 7472.9258 \end{aligned}$$

B : Sensitivity Analysis of Requirement Vectors.

$$\begin{array}{rcll}
 - 2.2085 & \leq & \triangle & b_1 \\
 - 36.7788 & \leq & \triangle & b_2 \leq 23.7728 \\
 - 0.0522 & \leq & \triangle & b_3 \leq 0.0107 \\
 -708.4213 & \leq & \triangle & b_4 \leq 86.9488 \\
 -216.7716 & \leq & \triangle & b_5 \\
 -708.4213 & \leq & \triangle & b_6
 \end{array}$$

For Pepsu SRTC :

Taking the same principles and conditions applied for NBSTC we get the following :-

Our problem is,

$$\text{Min } Z = C_1 Y_1 + C_2 Y_2 + C_3 Y_3$$

Subject to some Constraints,

$$\begin{array}{rcl}
 \text{Here } C_1 & = & \text{Rs. } 61.61 \text{ (in 1987-88)} \\
 C_2 & = & \text{Rs. } 3.66 \text{ (in 1987-88)} \\
 C_3 & = & 4,150 \text{ (in 1987-88)}
 \end{array}$$

About Constraints,

$$\begin{array}{rcl}
 (1) & Y_1 & \geq 5.54 (b_1) \quad \dots (1) \\
 (2) & Y_2 & \geq 71.3198 (b_2) \quad \dots (2) \\
 (3) & Y_3 & \geq .0106648 (b_3) \quad \dots (3) \\
 (4) & C_1 Y_1 - C_2 Y_2 - C_3 Y_3 & \geq 0 = (b_4) \quad \dots (4)
 \end{array}$$

$$(5) \quad C_2 Y_2 - C_3 Y_3 \geq 0 = (b_5) \quad \dots (5)$$

$$(6) \quad C_1 Y_1 + C_2 Y_2 + C_3 Y_3 \leq 799 = (b_6) \quad \dots (6)$$

Among six (6) Constraints, number 1,2,3 respectively are considered for all combinations, but from the Constraints A, 5,6, we have considered all possible combinations.

B.

Case No.1 : Considering Constraints 1,2,3

$$\text{Min Cost} = \text{Rs. } 646.74$$

$$\text{Where } Y_1 = 5.54$$

$$Y_2 = 71.3198$$

$$Y_3 = 0.01066$$

Sensitivity Analysis of Cost Vectors.

$$\Delta C_1 \leq 61.6875$$

$$\Delta C_2 \leq 3.6875$$

$$\Delta C_3 \leq 4150$$

B : Sensitivity Analysis of Requirement Vectors.

$$\Delta b_1 \leq 5.54$$

$$\Delta b_2 \leq 71.3198$$

$$\Delta b_3 \leq 0.0107$$

Case No.2 : Considering Constraints 1,2,3,4

$$\text{Min Cost} = \text{Rs. } 646.74$$

$$\text{Where } Y_1 = 5.54$$

$$\begin{aligned}
 Y_2 &= 71.3198 \\
 Y_3 &= .01066
 \end{aligned}$$

Sensitivity Analysis of Cost Vectors.

$$\begin{aligned}
 \Delta C_1 &\leq 60.5536 \\
 \Delta C_2 &\leq 3.6682 \\
 \Delta C_3 &\leq 4017.5457
 \end{aligned}$$

B : Sensitivity Analysis of Requirement Vectors.

$$\begin{aligned}
 \Delta b_1 &\leq 0.58 \\
 -9.9199 \leq \Delta b_2 &\leq 71.3198 \\
 -0.0087 \leq \Delta b_3 &\leq 0.0107 \\
 -36.3070 \leq \Delta b_4 &
 \end{aligned}$$

Case No.3 : Considering Constraints 1,2,3,5

$$\begin{aligned}
 \text{Min Cost} &= 647.74 \\
 \text{Where } Y_1 &= 5.54 \\
 Y_2 &= 71.3198 \\
 Y_3 &= 0.01066
 \end{aligned}$$

Sensitivity Analysis of Cost Vectors.

$$\begin{aligned}
 \Delta C_1 &\leq 61.6375 \\
 \Delta C_2 &\leq 3.7365 \\
 \Delta C_3 &\leq 3906.3694
 \end{aligned}$$

B : Sensitivity Analysis of Requirement Vectors.

$$\begin{aligned} \Delta b_1 &\leq 5.54 \\ \Delta b_2 &\leq 59.22 \\ -0.0522 \leq \Delta b_3 &\leq 0.0107 \\ -216.7716 \leq \Delta b_4 & \end{aligned}$$

Case No.4 : Considering Constraints 1,2,3,4,5

$$\begin{aligned} \text{Min Cost} &= 656.66 \\ \text{Where } Y_1 &= 5.54 \\ Y_2 &= 71.3198 \\ Y_3 &= 0.01066 \end{aligned}$$

Sensitivity Analysis of Cost Vectors..

$$\begin{aligned} \Delta c_1 &\leq 60.2660 \\ \Delta c_2 &\leq 3.9589 \\ \Delta c_3 &\leq 3696.0938 \end{aligned}$$

B : Sensitivity Analysis of Requirement Vectors :

$$\begin{aligned} \Delta b_1 &\leq 0.58 \\ -9.9199 \leq \Delta b_2 &\leq 59.2271 \\ -0.0037 \leq \Delta b_3 &\leq 0.0107 \\ -36.3070 \leq \Delta b_4 & \\ -216.7715 \leq \Delta b_5 & \end{aligned}$$

Case No.5 : Considering Constraints 1,2,3,6

$$\text{Min Cost} = 646.74585$$

$$\text{Where } Y_1 = 5.54$$

$$Y_2 = 71.3198$$

$$Y_3 = 0.01056$$

Sensitivity Analysis of Cost Vectors.

$$\Delta C_1 \leq 61.6875$$

$$\Delta C_2 \leq 3.6875$$

$$\Delta C_3 \leq 4150$$

B : Sensitivity Analysis of Requirement Vectors.

$$-2.4669 \leq \Delta b_1 \leq 5.54$$

$$-41.5612 \leq \Delta b_2 \leq 71.3198$$

$$-0.0356 \leq \Delta b_3 \leq 0.0107$$

$$-152.1142 \leq \Delta b_4$$

Case No.6 : Considering Constraints 1,2,3,4,6

$$\text{Min Cost} = \text{Rs. } 646.74$$

$$\text{Where } Y_1 = 5.54$$

$$Y_2 = 71.3198$$

$$Y_3 = 0.01056$$

Sensitivity Analysis of Cost Vectors.

$$\begin{aligned} \Delta c_1 &\leq 61.66 \\ \Delta c_2 &\leq 3.6682 \\ \Delta c_3 &\leq 4017.5457 \end{aligned}$$

B : Sensitivity Analysis of Requirement Vectors.

$$\begin{aligned} - 2.4569 &\leq \Delta b_1 \leq 0.58 \\ - 0.0087 &\leq \Delta b_2 \leq 0.0107 \\ - 9.9199 &\leq \Delta b_3 \leq 71.3198 \\ - 36.3070 &\leq \Delta b_4 \\ -152.1142 &\leq \Delta b_5 \end{aligned}$$

Case No.7 : Considering Constraints 1,2,3,5,6

$$\begin{aligned} \text{Min Cost} &= \text{Rs. } 647.72 \\ \text{Where } Y_1 &= 5.54 \\ Y_2 &= 71.3198 \\ Y_3 &= 0.01066 \end{aligned}$$

Sensitivity Analysis of Cost Vectors.

$$\begin{aligned} \Delta c_1 &\leq 61.7094 \\ \Delta c_2 &\leq 3.7389 \\ \Delta c_3 &\leq 3906.3354 \end{aligned}$$

B : Sensitivity Analysis of Requirement Vectors.

- $2.4569 \leq \Delta b_1 \leq 5.54$
- $41.5612 \leq \Delta b_2 \leq 59.2272$
- $0.0522 \leq \Delta b_3 \leq .0107$
- $-215.7716 \leq \Delta b_4$
- $-152.1142 \leq \Delta b_5$

Case No.8 : Considering Constraints 1,2,3,4,5,6

$$\text{Min Cost} = \text{Rs. } 658.57$$

$$\text{Where } Y_1 = 5.54$$

$$Y_2 = 71.31980$$

$$Y_3 = 0.01066$$

Sensitivity Analysis of Cost Vectors.

$$\Delta C_1 \leq 60.5956$$

$$\Delta C_2 \leq 3.9736$$

$$\Delta C_3 \leq 3528.5098$$

B : Sensitivity Analysis of Requirement Vectors.

- $2.4669 \leq \Delta b_1 \leq 0.58$
- $9.9199 \leq \Delta b_2 \leq 59.2271$
- $0.0087 \leq \Delta b_3 \leq 0.0107$
- $-36.3070 \leq \Delta b_4$
- $-215.7715 \leq \Delta b_5$
- $-152.1142 \leq \Delta b_6$

For Gujrat SRTC :

Taking the same principles and conditions applied for NBSTC, we get the following :

Our problem is,

$$\text{Min } Z = C_1 Y_1 + C_2 Y_2 + C_3 Y_3$$

Subject to some Constraints

Here, C_1	=	Rs. 66.29	(in 1987-38)
C_2	=	Rs. 3.56	"
C_3	=	Rs. 4,150	"

About Constraints :

- | | | | | | |
|-----|------------------------------------|---------|---------|-----|-----|
| (1) | $Y_1 \geq$ | 5.54 | (b_1) | ... | (1) |
| (2) | $Y_2 \geq$ | 71.3198 | (b_2) | ... | (2) |
| (3) | $Y_3 \geq$ | 0106648 | (b_3) | ... | (3) |
| (4) | $C_1 Y_1 - C_2 Y_2 - C_3 Y_3 \geq$ | 0 | (b_4) | ... | (4) |
| (5) | $C_2 Y_2 - C_3 Y_3 \geq$ | 0 | (b_5) | ... | (5) |
| (6) | $C_1 Y_1 + C_2 Y_2 + C_3 Y_3 \leq$ | 1355 | (b_6) | ... | (6) |

Among six (6) constraints, number 1,2,3 respectively are considered for all combinations, but from the constraints 4,5,6 we have considered all possible combinations.

Case No.1 : Considering constraints 1,2,3

$$\begin{aligned} \text{Min Cost} &= \text{Rs. } 672.36 \\ \text{Where } Y_1 &= 5.54 \\ Y_2 &= 71.3198 \\ Y_3 &= 0.01066 \end{aligned}$$

Sensitivity Analysis of Cost Vectors :

$$\begin{aligned} \Delta C_1 &\leq 66.3125 \\ \Delta C_2 &\leq 3.6875 \\ \Delta C_3 &\leq 4150 \end{aligned}$$

/of

Sensitivity Analysis/Requirements Vectors :

$$\begin{aligned} \Delta b_1 &\leq 5.54 \\ \Delta b_2 &\leq 71.3198 \\ \Delta b_3 &\leq 0.0107 \end{aligned}$$

Case No.2 : Considering Constraints 1,2,3 and 4

$$\begin{aligned} \text{Min cost} &= 672.36 \\ \text{Where } Y_1 &= 5.54 \\ Y_2 &= 71.3198 \\ Y_3 &= .01066 \end{aligned}$$

Sensitivity Analysis of Cost Vectors.

$$\begin{aligned} \Delta C_1 &\leq 64.2550 \\ \Delta C_2 &\leq 3.7773 \\ \Delta C_3 &\leq 4070.4006 \end{aligned}$$

B : Sensitivity Analysis of Requirement Vector .

$$\begin{aligned} \Delta b_1 &\leq .9346 \\ -16.928 \leq \Delta b_2 &\leq 71.3198 \\ -0.0149 \leq \Delta b_3 &\leq 0.0107 \\ -61.9572 \leq \Delta b_4 & \end{aligned}$$

Case No.3 : Considering Constraints 1,2,3,5

$$\begin{aligned} \text{Min Cost} &= \text{Rs. } 673.36 \\ \text{Where, } Y_1 &= 5.54 \\ Y_2 &= 71.3198 \\ Y_3 &= 0.01066 \end{aligned}$$

Sensitivity Analysis of Cost Vectors.

$$\begin{aligned} \Delta C_1 &\leq 66.3125 \\ \Delta C_2 &\leq 3.7365 \\ \Delta C_3 &\leq 3906.3694 \end{aligned}$$

B : About Requirements of Cost Vector.

$$\begin{aligned} \Delta b_1 &\leq 5.54 \\ \Delta b_2 &\leq 59.2272 \\ -0.0522 \leq \Delta b_3 &\leq 0.0107 \\ -216.7716 \leq \Delta b_4 & \end{aligned}$$

Case No.4 : Considering Constraints 1,2,3,4,5

$$\begin{aligned} \text{Min Cost} &= \text{Rs. } 689.97 \\ \text{Where } Y_1 &= 5.54 \end{aligned}$$

$$\begin{aligned}
 Y_2 &= 71.3198 \\
 Y_3 &= 0.01066
 \end{aligned}$$

Sensitivity Analysis of Cost Vector.

$$\begin{aligned}
 \Delta C_1 &\leq 64.2173 \\
 \Delta C_2 &\leq 4.1132 \\
 \Delta C_3 &\leq 3760.9377
 \end{aligned}$$

B : Sensitivity Analysis of Requirement Vector

$$\begin{aligned}
 \Delta b_1 &\leq 0.9346 \\
 -16.9281 \leq \Delta b_2 &\leq 59.2272 \\
 -0.0149 \leq \Delta b_3 &\leq 0.0107 \\
 -61.9572 \leq \Delta b_4 & \\
 -216.7716 \leq \Delta b_5 &
 \end{aligned}$$

Case No.5 : Considering Constraints 1,2,3,6

$$\text{Min Cost} = \text{Rs. } 672.36$$

$$\text{Where, } Y_1 = 5.54$$

$$Y_2 = 71.3198$$

$$Y_3 = 0.01066$$

Sensitivity Analysis of cost vector.

$$\begin{aligned}
 \Delta C_1 &\leq 66.3125 \\
 \Delta C_2 &\leq 3.6875 \\
 \Delta C_3 &\leq 4150.0000
 \end{aligned}$$

B : Sensitivity Analysis of Requirement Vector.

$$\begin{aligned}
 - 10.2951 &\leq \Delta b_1 \leq 5.54 \\
 -186.4655 &\leq \Delta b_2 \leq 71.3198 \\
 - 0.1544 &\leq \Delta b_3 \leq 0.0107 \\
 -582.4540 &\leq \Delta b_4
 \end{aligned}$$

Case No.5 : Considering Constraints 1,2,3,4,5

$$\text{Min Cost} = \text{Rs. } 672.36.$$

$$\text{Where, } Y_1 = 5.54$$

$$Y_2 = 71.3198$$

$$Y_3 = 0.01066$$

Sensitivity Analysis of Cost Vectors.

$$\Delta C_1 \leq 66.29$$

$$\Delta C_2 \leq 3.7773$$

$$\Delta C_3 \leq 4150$$

B : Sensitivity Analysis of Requirement Vector.

$$- 10.2951 \leq \Delta b_1 \leq 0.9345$$

$$- 16.9281 \leq \Delta b_2 \leq 71.3198$$

$$- 0.0149 \leq \Delta b_3 \leq 0.0107$$

$$- 61.9572 \leq \Delta b_4$$

$$-582.4641 \leq \Delta b_5$$

Case No.7 : Considering Constraints 1,2,3,5,6

$$\begin{aligned} \text{Min Cost} &= \text{Rs. } 673.36 \\ \text{Where, } Y_1 &= 5.54 \\ Y_2 &= 71.3198 \\ Y_3 &= 0.01066 \end{aligned}$$

Sensitivity Analysis of Cost Vector.

$$\begin{aligned} \Delta C_1 &\leq 66.3125 \\ \Delta C_2 &\leq 3.7365 \\ \Delta C_3 &\leq 3906.3694 \end{aligned}$$

B: Sensitivity Analysis of Requirement Vector.

$$\begin{aligned} -10.295 &\leq \Delta b_1 \leq 5.54 \\ -186.4656 &\leq \Delta b_2 \leq 59.2272 \\ -0.0522 &\leq \Delta b_3 \leq 0.0107 \\ -216.7716 &\leq \Delta b_4 \\ -682.4641 &\leq \Delta b_5 \end{aligned}$$

Case No.8 : Considering Constraints 1,2,3,4,5,6

$$\begin{aligned} \text{Min Cost} &= \text{Rs. } 689.97 \\ \text{Where, } Y_1 &= 5.54 \\ Y_2 &= 71.3198 \\ Y_3 &= 0.01066 \end{aligned}$$

Sensitivity Analysis of Cost Vector

$$\begin{aligned} \Delta C_1 &\leq 64.2173 \\ \Delta C_2 &\leq 4.1132 \\ \Delta C_3 &\leq 3760.9377 \end{aligned}$$

B : Sensitivity Analysis of Requirement Vector.

$$- 10.2951 \leq \Delta b_1 \leq 0.9346$$

$$- 16.9281 \leq \Delta b_2 \leq 59.2271$$

$$- 0.0149 \leq \Delta b_3 \leq 0.0107$$

$$- 61.9572 \leq \Delta b_4$$

$$-216.7716 \leq \Delta b_5$$

$$-682.4641 \leq \Delta b_6$$

For Andhra SRTC :

Taking the same principles and conditions applied for NBSTC we get the following :

Our problem is,

$$\text{Min } Z = C_1 Y_1 + C_2 Y_2 + C_3 Y_3$$

Subject to some constraints,

$$\text{Here } C_1 = \text{Rs. } 53.56 \text{ (in 1987-88)}$$

$$C_2 = \text{Rs. } 3.56 \quad "$$

$$C_3 = \text{Rs. } 4150 \quad "$$

About Constraints,

$$(1) \quad Y_1 \geq 5.54 \quad (b_1) \quad \dots (1)$$

$$(2) \quad Y_2 \geq 71.3198 \quad (b_2) \quad \dots (2)$$

$$(3) \quad Y_3 \geq 0.0106648 \quad (b_3) \quad \dots (3)$$

$$(4) \quad C_1 Y_1 - C_2 Y_2 - C_3 Y_3 \geq 0 \quad (b_4) \quad \dots (4)$$

$$(5) \quad C_2 Y_2 - C_3 Y_3 \geq 0 = (b_5) \quad \dots (5)$$

$$(6) \quad C_1 Y_1 + C_2 Y_2 + C_3 Y_3 \leq 1399 = (b_6) \quad \dots (5)$$

Among six (6) constraints number 1,2,3 respectively are considered for all combinations, but from the constraints 4,5,6 we have considered all possible combinations.

Case No.1 : Considering constraints 1,2,3

$$\text{Min Cost} = \text{Rs. } 602.42$$

$$\text{Where, } Y_1 = 5.54$$

$$Y_2 = 71.3198$$

$$Y_3 = 0.0106648$$

Sensitivity Analysis of Cost Vectors.

$$\triangle C_1 \leq 53.6875$$

$$\triangle C_2 \leq 3.6875$$

$$\triangle C_3 \leq 4150$$

B : Sensitivity Analysis of Requirement Vector.

$$\triangle b_1 \leq 5.54$$

$$\triangle b_2 \leq 71.3198$$

$$\triangle b_3 \leq 0.0107$$

Case No.2 : Considering Constraints 1,2,3,4

$$\text{Min Cost} = \text{Rs. } 610.05$$

$$\text{Where, } Y_1 = 5.58$$

$$Y_2 = 71.3198$$

$$Y_3 = 0.01066$$

A: Sensitivity Analysis of Cost Vector.

$$\triangle C_1 \leq 50.9623$$

$$\triangle C_2 \leq 7.0801$$

$$\triangle C_3 \leq 8098.5361$$

B : Sensitivity Analysis of Requirement Vector.

$$\triangle b_1 \leq 2.1891$$

$$\triangle b_2 \leq .0019$$

$$\triangle b_3 \leq .0268$$

$$\triangle b_4 \leq .1493$$

Case No.3 : Considering Constraints 1,2,3,5

$$\text{Min Cost} = \text{Rs. } 603.42$$

$$\text{Where, } Y_1 = 5.54$$

$$Y_2 = 71.3198$$

$$Y_3 = 0.01066$$

Sensitivity Analysis of Cost Vectors.

$$\triangle C_1 \leq 53.5875$$

$$\triangle C_2 \leq 3.7365$$

$$\triangle C_3 \leq 3906.3694$$

B : Sensitivity Analysis of Requirement Vector.

$$\triangle b_1 \leq 5.54$$

$$\triangle b_2 \leq 59.2272$$

$$-0.0522 \leq \triangle b_3 \leq 0.0107$$

$$-216.7716 \leq \triangle b_4$$

Case No.4 : Considering Constraints 1,2,3,4,5

$$\begin{aligned} \text{Min Cost} &= \text{Rs. } 615.80 \\ \text{Where, } Y_1 &= 5.68 \\ Y_2 &= 71.3198 \\ Y_3 &= 0.01066 \end{aligned}$$

Sensitivity Analysis of Cost Vector :

$$\begin{aligned} \Delta C_1 &\leq 50.0483 \\ \Delta C_2 &\leq 7.4450 \\ \Delta C_3 &\leq 7988.7002 \end{aligned}$$

B : Sensitivity Analysis Requirement Vector.

$$\begin{aligned} - .1493 &\leq \Delta b_1 \\ \Delta b_2 &\leq 2.1891 \\ - 0.0522 &\leq \Delta b_3 \leq .0019 \\ \Delta b_4 &\leq 8.0268 \\ -216.7716 &\leq \Delta b_5 \end{aligned}$$

Case No.5 : Considering Constraints 1,2,3,6

$$\begin{aligned} \text{Min Cost} &= \text{Rs } 602.4255 \\ \text{Where, } Y_1 &= 5.54 \\ Y_2 &= 71.3198 \\ Y_3 &= 0.01066 \end{aligned}$$

Sensitivity Analysis of Cost Vector.

$$\begin{aligned} \Delta C_1 &\leq 53.6875 \\ \Delta C_2 &\leq 3.6875 \\ \Delta C_3 &\leq 4150 \end{aligned}$$

B : Sensitivity Analysis of Requirement Vector.

$$\begin{aligned} -14.8422 &\leq \Delta b_1 \leq 5.54 \\ -217.5049 &\leq \Delta b_2 \leq 71.3198 \\ -0.1919 &\leq \Delta b_3 \leq 0.0107 \\ -796.4342 &\leq \Delta b_4 \end{aligned}$$

Case No.6 : Considering Constraints 1,2,3,4,6

$$\begin{aligned} \text{Min Cost} &= \text{Rs. } 610.05 \\ \text{Where, } Y_1 &= 5.68 \\ Y_2 &= 71.3198 \\ Y_3 &= 0.01066 \end{aligned}$$

Sensitivity Analysis of Cost Vector.

$$\begin{aligned} \Delta C_1 &\leq 50.9623 \\ \Delta C_2 &\leq 7.0801 \\ \Delta C_3 &\leq 8098.5361 \end{aligned}$$

B : Sensitivity Analysis of Requirement Vector.

$$\begin{aligned} -0.1493 &\leq \Delta b_1 \\ -107.7078 &\leq \Delta b_2 \leq 2.1891 \\ -0.0949 &\leq \Delta b_3 \leq 0.0019 \end{aligned}$$

$$-788.4213 \leq \Delta b_4 \leq 8.0268$$

$$-788.4213 \leq \Delta b_5$$

Case No.7 : Considering Constraints 1,2,3,5,6.

$$\text{Min Cost} = \text{Rs. } 603.42$$

$$\text{Where, } Y_1 = 5.54$$

$$Y_2 = 71.3198$$

$$Y_3 = 0.01066$$

Sensitivity Analysis of Cost Vector.

$$\Delta c_1 \leq 53.6865$$

$$\Delta c_2 \leq 3.7365$$

$$\Delta c_3 \leq 3906.3694$$

B : Sensitivity Analysis of Requirement Vector.

$$-14.8422 \leq \Delta b_1 \leq 5.54$$

$$-217.605 \leq \Delta b_2 \leq 59.2272$$

$$-0.0522 \leq \Delta b_3 \leq 0.0107$$

$$-216.7716 \leq \Delta b_4$$

$$-796.4343 \leq \Delta b_5$$

Case No.8 ; Considering Constraints 1,2,3,4,5,6

$$\text{Min Cost} = \text{Rs. } 615.80$$

$$\text{Where, } Y_1 = 5.68$$

$$Y_2 = 71.3198$$

$$Y_3 = 0.01066$$

Sensitivity Analysis of Cost Vector.

ΔC_1	\leq	50.0483
ΔC_2	\leq	7.4450
ΔC_3	\leq	7988.7002

B : Sensitivity Analysis of Requirement Vector.

- 0.1493	\leq	Δb_1	
-107.7078	\leq	Δb_2	\leq 2.1891
- 0.0522	\leq	Δb_3	\leq 0.0019
-788.4213	\leq	Δb_4	\leq 8.0268
-216.7716	\leq	Δb_5	
-788.4213	\leq	Δb_6	

For Kerala SRTC :

Taking the same principles and conditions applied for NBSTC we get the following :

Our problems is,

Here,	C_1	=	Rs. 54.80 (in 1987-88)
	C_2	=	Rs. 3.56 "
	C_3	=	Rs. 4,150 "

About constraints,

$$(1) Y_1 \geq 5.54 (b_1) \quad \dots (1)$$

$$(2) Y_2 \geq 71.3198 (b_2) \quad \dots (2)$$

$$(3) \quad Y_3 \geq .0106648 \quad (b_3) \quad \dots (3)$$

$$(4) \quad C_1 Y_1 - C_2 Y_2 - C_3 Y_3 \geq 0 \quad (b_4) \quad \dots (4)$$

$$(5) \quad C_2 Y_2 - C_3 Y_3 \geq 0 \quad (b_5) \quad \dots (5)$$

$$(6) \quad C_1 Y_1 + C_2 Y_2 + C_3 Y_3 \leq 1344 \quad (b_6) \quad \dots (6)$$

Among six (6) constraints, number 1,2,3 respectively are considered for all combinations, but from the constraints 4,5,6 we have considered all possible combinations.

Case No.1 : Considering constraints 1,2,3

Min Cost	=	Rs. 608.65
Where, Y_1	=	5.54
Y_2	=	71.3198
Y_3	=	0.01066

Sensitivity Analysis of Cost Vectors.

$\Delta C_1 \leq$	-	54.8125
$\Delta C_2 \leq$		3.6875
$\Delta C_3 \leq$		4150

B:Sensitivity Analysis of Requirement Vector.

$\Delta b_1 \leq$		5.54
$\Delta b_2 \leq$		71.3198
$\Delta b_3 \leq$		0.0107

Case No.2 : Considering Constraints 1,2,3,4.

$$\begin{aligned} \text{Min Cost} &= \text{Rs. } 610.50 \\ \text{Where, } Y_1 &= 5.57 \\ Y_2 &= 71.3199 \\ Y_3 &= 0.01066 \end{aligned}$$

Sensitivity Analysis of cost Vector.

$$\begin{aligned} \triangle C_1 &\leq 53.68 \\ \triangle C_2 &\leq 7.3186 \\ \triangle C_3 &\leq 8347.3252 \end{aligned}$$

B : Sensitivity Analysis of Requirement Vector.

$$\begin{aligned} \triangle b_1 &\leq .0310 \\ \triangle b_2 &\leq .4640 \\ \triangle b_3 &\leq .0004 \\ \triangle b_4 &\leq 1.7032 \end{aligned}$$

Case No.3 : Considering Constraints 1,2,3,5

$$\begin{aligned} \text{Min Cost} &= \text{Rs. } 609.65 \\ \text{Where, } Y_1 &= 5.54 \\ Y_2 &= 71.3198 \\ Y_3 &= 0.01066 \end{aligned}$$

A : Sensitivity Analysis of Cost Vector.

$$\Delta c_1 \leq 54.8125$$

$$\Delta c_2 \leq 3.7365$$

$$\Delta c_3 \leq 3906.3694$$

B : Sensitivity Analysis of Requirement Vector.

$$\Delta b_1 \leq 5.54$$

$$\Delta b_2 \leq 59.2272$$

$$-0.0522 \leq \Delta b_3 \leq 0.0107$$

$$-216.7716 \leq \Delta b_4$$

Case No.4 : Considering Constraints 1,2,3,4,5

$$\text{Min Cost} = \text{Rs. } 624.42$$

$$\text{Where, } Y_1 = 5.57$$

$$Y_2 = 71.3198$$

$$Y_3 = 0.01066$$

Sensitivity Analysis of Cost Vector.

$$\Delta c_1 \leq 52.6648$$

$$\Delta c_2 \leq 7.5881$$

$$\Delta c_3 \leq 7991.0850$$

B : Sensitivity Analysis of Requirement Vector.

$$-0.0310 \leq \Delta b_1$$

$$\Delta b_2 \leq 0.4640$$

$$\begin{aligned}
 & - .0522 \leq \Delta b_3 \leq .0004 \\
 \Delta b_4 & \leq 1.7032 \\
 & -216.7716 \leq \Delta b_5
 \end{aligned}$$

Case No.5 : Considering Constraints 1,2,3,6

$$\begin{aligned}
 \text{Min Cost.} & = \text{Rs. } 608.65 \\
 \text{Where, } Y_1 & = 5.54 \\
 Y_2 & = 71.3198 \\
 Y_3 & = 0.01066
 \end{aligned}$$

Sensitivity Analysis of Cost Vector.

$$\begin{aligned}
 \Delta c_1 & \leq 54.8125 \\
 \Delta c_2 & \leq 3.6875 \\
 \Delta c_3 & \leq 4150
 \end{aligned}$$

B : Sensitivity Analysis of Requirement Vector.

$$\begin{aligned}
 & - 13.4145 \leq \Delta b_1 \leq 5.54 \\
 & -200.8520 \leq \Delta b_2 \leq 71.3198 \\
 & - 0.1771 \leq \Delta b_3 \leq .0107 \\
 & -735.1186 \leq \Delta b_4
 \end{aligned}$$

Case No.6 : Considering constraints, 1,2,3,4,6

$$\begin{aligned}
 \text{Min Cost} & = \text{Rs. } 610.50 \\
 \text{Where, } Y_1 & = 5.57 \\
 Y_2 & = 71.3198 \\
 Y_3 & = 0.01066
 \end{aligned}$$

Sensitivity Analysis of Cost Vector.

$$\begin{aligned} \Delta C_1 &\leq 53.6813 \\ \Delta C_2 &\leq 7.3186 \\ \Delta C_3 &\leq 8347.3252 \end{aligned}$$

B : Sensitivity Analysis of Requirement Vector.

$$\begin{aligned} - .0310 &\leq \Delta b_1 \\ -100.1941 &\leq \Delta b_2 \leq .4640 \\ - 0.0883 &\leq \Delta b_3 \leq 0.0004 \\ -733.4212 &\leq \Delta b_4 \leq 1.7032 \\ -733.4212 &\leq \Delta b_5 \end{aligned}$$

Case No.7 : Considering Constraints 1,2,3,5,6

$$\begin{aligned} \text{Min Cost} &= \text{Rs. } 609.65 \\ \text{Where, } Y_1 &= 5.54 \\ Y_2 &= 71.3198 \\ Y_3 &= 0.01066 \end{aligned}$$

Sensitivity Analysis of Cost Vector.

$$\begin{aligned} \Delta C_1 &\leq 54.8125 \\ \Delta C_2 &\leq 3.7365 \\ \Delta C_3 &\leq 4150 \end{aligned}$$

B : Sensitivity Analysis of Requirement Vector.

$$\begin{aligned} - 13.4145 &\leq \Delta b_1 \leq 5.54 \\ -200.8521 &\leq \Delta b_2 \leq 59.2272 \end{aligned}$$

$$- .0522 \leq \Delta b_3 \leq 0.0107$$

$$-216.7716 \leq \Delta b_4$$

$$-735.1187 \leq \Delta b_5$$

Case No.8 : Considering constraints 1,2,3,4,5,6

$$\text{Min Cost} = \text{Rs. } 624.42$$

$$\text{Where, } Y_1 = 5.57$$

$$Y_2 = 71.3198$$

$$Y_3 = 0.01066$$

Sensitivity Analysis of Cost Vectors.

$$\Delta C_1 \leq 52.6648$$

$$\Delta C_2 \leq 7.5881$$

$$\Delta C_3 \leq 7991.0850$$

B : Sensitivity Analysis of Requirement Vector.

$$- 0.0310 \leq \Delta b_1$$

$$-100.1941 \leq \Delta b_2 \leq .4640$$

$$- 0.0522 \leq \Delta b_3 \leq .0004$$

$$-733.4213 \leq \Delta b_4 \leq 1.7032$$

$$-216.7716 \leq \Delta b_5$$

$$-733.4213 \leq \Delta b_6$$

Results at a Glance

Constraints	NBSTC	Pepsa SRTC	Rajasthan SRTC	Gujrat SRTC	Andhra SRTC	Kerala SRTC
1. 1,2,3	O.C. ^{R₁} =528.67 Y ₁ = 5.54 Y ₂ =71.3198 Y ₃ = 0.01066	O.C. ^{R₁} =546.74 Y ₁ = 5.54 Y ₂ =71.3198 Y ₃ = 0.01066	O.C. ^{R₁} =523.13 Y ₁ = 5.54 Y ₂ =71.3198 Y ₃ = 0.01066	O.C. ^{R₁} =672.36 Y ₁ = 5.54 Y ₂ =71.3198 Y ₃ = 0.01066	O.C. ^{R₁} =602.42 Y ₁ = 5.54 Y ₂ =71.3198 Y ₃ = 0.01066	O.C. ^{R₁} =608.65 Y ₁ = 5.54 Y ₂ =71.3198 Y ₃ = 0.01066
2. 1,2,3,4	O.C. ^{R₁} =610.69 Y ₁ = 7.56 Y ₂ =71.3198 Y ₃ = 0.01066	O.C. ^{R₁} =546.74 Y ₁ = 5.54 Y ₂ =71.3198 Y ₃ = 0.01066	O.C. ^{R₁} =610.22 Y ₁ = 7.74 Y ₂ =71.3198 Y ₃ = 0.01066	O.C. ^{R₁} =672.36 Y ₁ = 5.54 Y ₂ =71.3198 Y ₃ = 0.01066	O.C. ^{R₁} =610.05 Y ₁ = 5.68 Y ₂ =71.3198 Y ₃ = 0.01066	O.C. ^{R₁} =610.50 Y ₁ = 5.57 Y ₂ =71.3198 Y ₃ = 0.01066
3. 1,2,3,5	O.C. ^{R₁} =529.67 Y ₁ = 5.54 Y ₂ =71.3198 Y ₃ = 0.01066	O.C. ^{R₁} =547.74 Y ₁ = 5.54 Y ₂ =71.3198 Y ₃ = 0.01066	O.C. ^{R₁} =524.12 Y ₁ = 5.54 Y ₂ =71.3198 Y ₃ = 0.01066	O.C. ^{R₁} =673.36 Y ₁ = 5.54 Y ₂ =71.3198 Y ₃ = 0.01066	O.C. ^{R₁} =603.42 Y ₁ = 5.54 Y ₂ =71.3198 Y ₃ = 0.01066	O.C. ^{R₁} =609.65 Y ₁ = 5.54 Y ₂ =71.3198 Y ₃ = 0.01066
4. 1,2,3,4,5	O.C. ^{R₁} =610.69 Y ₁ = 7.56 Y ₂ =71.3198 Y ₃ = 0.01066	O.C. ^{R₁} =656.66 Y ₁ = 5.55 Y ₂ =71.3198 Y ₃ = 0.01066	O.C. ^{R₁} =610.22 Y ₁ = 7.74 Y ₂ =71.3198 Y ₃ = 0.01066	O.C. ^{R₁} =689.97 Y ₁ = 5.54 Y ₂ =71.3198 Y ₃ = 0.01066	O.C. ^{R₁} =615.80 Y ₁ = 5.68 Y ₂ =71.3198 Y ₃ = 0.01066	O.C. ^{R₁} =624.42 Y ₁ = 5.58 Y ₂ =71.3198 Y ₃ = 0.01066

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5. 1,2,3,6	O.C.=528.67 Y ₁ = 5.54 Y ₂ =71.3198 Y ₃ = 0.01066	O.C.=646.74 Y ₁ = 5.54 Y ₂ =71.3198 Y ₃ = 0.01066	O.C.=523.13 Y ₁ = 5.54 Y ₂ =71.3198 Y ₃ = 0.01066	O.C.=672.36 Y ₁ = 5.54 Y ₂ =71.3198 Y ₃ = 0.01066	O.C.=602.42 Y ₁ = 5.54 Y ₂ =71.3198 Y ₃ = 0.01066	O.C.=608.65 Y ₁ = 5.54 Y ₂ =71.3198 Y ₃ = 0.01066
6. 1,2,3,4,6	O.C.=610.69 Y ₁ = 7.56 Y ₂ =71.3198 Y ₃ = 0.01066	O.C.=647.7 Y ₁ = 5.54 Y ₂ =71.3198 Y ₃ = 0.01066	O.C.=610.22 Y ₁ = 7.74 Y ₂ =71.3198 Y ₃ =0.01066	O.C.=672.36 Y ₁ = 5.54 Y ₂ =71.3198 Y ₃ =0.01066	O.C.=610.85 Y ₁ = 5.68 Y ₂ =71.3198 Y ₃ = 0.01066	O.C.=610.50 Y ₁ = 5.57 Y ₂ =71.3198 Y ₃ = 0.01066
7. 1,2,3,5,6	O.C.=529.67 Y ₁ = 5.54 Y ₂ =71.3198 Y ₃ = 0.01066	O.C.=647.74 Y ₁ = 5.54 Y ₂ =71.3198 Y ₃ = 0.01066	O.C.=524.12 Y ₁ = 5.54 Y ₂ =71.3198 Y ₃ = 0.01066	O.C.=673.36 Y ₁ = 5.54 Y ₂ =71.3198 Y ₃ = 0.01066	O.C.=603.42 Y ₁ = 5.54 Y ₂ =71.3198 Y ₃ = 0.01066	O.C.=609.65 Y ₁ = 5.54 Y ₂ =71.3198 Y ₃ = 0.01066
8. 1,2,3,4,5,6	O.C.=610.69 Y ₁ = 7.56 Y ₂ =71.3198 Y ₃ = 0.01066	O.C.=658.57 Y ₁ = 5.54 Y ₂ =71.3198 Y ₃ = 0.01066	O.C.=610.22 Y ₁ = 7.74 Y ₂ =71.3198 Y ₃ = 0.01066	O.C.=689.97 Y ₁ = 5.54 Y ₂ =71.3198 Y ₃ = 0.01066	O.C.=615.80 Y ₁ = 5.68 Y ₂ =71.3198 Y ₃ = 0.01066	O.C.=624.42 Y ₁ = 5.57 Y ₂ =71.3198 Y ₃ = 0.01066
9. Actual cost	1138	1220	1308	1489	1349	1483
10. Actual Earning	656.31	799	1319	1355	1399	1344

The following conclusions are drawn from the above table and the analysis made in detail earlier.

a) For NBSTC :

- 1) If we add the constraint $C_1 Y_1 \gg C_2 Y_2 + C_3 Y_3$, then the total cost changes rapidly.
- 2) Out of all the cases, the minimum of optimum cost is Rs. 528.57 (Case No.1 and Case No.5), the maximum of optimum cost is Rs. 610.59 (Case No.2,4,6 and 8).
- 3) The values of Y_2 and Y_3 are same in all cases and are same as the specified minimum values.
- 4) In Case No. 1,7,5 and in Case No. 2,3,4,6,8 the costs are different slightly but their corresponding Y_1, Y_2, Y_3 respectively are recorded same which is happening due to fraction.
- 5) The difference between the Actual cost per bus per day and the lowest hypothetical cost per bus per day is (Rs. 1138 - Rs. 528.57) = Rs. 609.33 and the difference between the Actual cost per bus per day and highest hypothetical cost per bus per day = Rs. 527.31.
- 6) The difference between the earning per bus per day and the lowest hypothetical cost per bus per

day is Rs. 127.64 and the earning per bus per day and highest hypothetical cost per bus per day is Rs. 45.62.

- 7) The actual cost per bus per day is almost double than the hypothetical cost per bus per day.
- 8) The difference between the actual cost per bus per day and the actual earning per bus per day is Rs. (-) 481.69.

For Pepsa SRTC :

- 1) The effects of the individual constraints are more or less the same.
- 2) Out of all the cases, the minimum of optimum cost is Rs. 646.74 (Case No. 1,2,5,6) the maximum of optimum cost is Rs. 658.57 (Case No.8).
- 3) The values of Y_1 , Y_2 and Y_3 are same in all cases and are same as specified number.
- 4) Here, in all the cases the costs are different slightly but their corresponding Y_1 , Y_2 and Y_3 respectively are recorded same which is happening due to fraction.
- 5) The difference between the actual cost per bus per day and the lowest hypothetical cost per bus per

day is Rs. 573.26 and the difference between the actual cost per bus per day and the highest hypothetical cost per bus per day is Rs. 561.43. So the Actual Cost per bus per day is more than double than the hypothetical cost per bus per day.

- 6) The difference between the earning per bus per day and the lowest hypothetical cost per bus per day is Rs. 152.26 and the difference between the earning per bus per day and the highest hypothetical cost per bus per day is Rs. 140.43.
- 7) The difference between the actual cost per bus per day and the actual earnings per bus per day is (-) Rs. 421.

For Rajasthan SRTC :

- 1) If we add the constraint $C_1 Y_1 \gg C_2 Y_2 + C_3 Y_3$, then the total cost changes rapidly.
- 2) Out of all the cases, the minimum of optimum cost Rs. 523.13 (Case No.1 and 5) the maximum of optimum cost is Rs. 610.22 (Case No. 2,4,6 and 8).
- 3) The values of Y_2 and Y_3 are same in all cases and the same as specified number.

- 4) In Case No. 1,3,5 and in Case No. 2,4,6,8 the costs are different slightly but their corresponding Y_1 , Y_2 , Y_3 respectively are recorded same which is happening due to fraction.
- 5) The difference between the actual cost per bus per day and the lowest hypothetical cost per bus per day is Rs. 1308 - Rs. 523.13 = Rs. 784.87 and the actual cost per bus per day and the highest hypothetical cost per bus per day is Rs. 697.70. The actual cost per bus per day is more than double than the hypothetical cost per bus per day.
- 6) The difference between the earning per bus per day and the lowest hypothetical cost per bus per day is Rs. 795.87 and the difference between the earning per bus per day and highest hypothetical cost per bus per day is Rs. 708.78.
- 7) The difference between the actual cost per bus per day and the actual earning per bus per day is Rs.11.

For Gujrat SRTC:

- 1) The effects of the individual constraints are more or less the same.

- 2) Out of all the cases, the minimum of optimum cost is Rs. 672.36 (Case No. 1,2,5,6), the maximum of optimum cost is Rs. 689.97 (Case No. 8,4).
- 3) The values of Y_1 , Y_2 and Y_3 are same in all cases and are same as the specified minimum values.
- 4) In Case No. 1,2,3,5,6,7 and in Case No. 4,8 the costs are different slightly but their corresponding Y_1 , Y_2 and Y_3 respectively are recorded same which is happening due to fraction.
- 5) The difference between the actual cost per bus per day and the lowest hypothetical cost per bus per day is Rs. (1489 - 672.36) or Rs. 816.64 and the difference between the actual cost per bus per day and the highest hypothetical cost per bus per day is Rs. (1489 - 689.97) or Rs. 799.03. The actual cost per bus per day is more than double than the hypothetical cost per bus per day.
- 6) The difference between the actual cost per bus per day and the actual earning per bus per day is (-) Rs. 134.
- 7) The difference between the actual earning per bus per day and the lowest hypothetical cost per bus per day is Rs. 682.64 and the difference between the actual earning and the highest hypothetical

cost per vehicle per day is Rs. 665.03.

For Andhra SRTC :

- 1) The effects of the individual constraints are more or less the same
- 2) Out of all the cases, the minimum of optimum cost is Rs. 602.42 (Case No.1,5) and the maximum of optimum cost is Rs. 615.80 (Case No. 4,8).
- 3) The values of Y_2 and Y_3 are same in all the cases and are same as the specified number
- 4) In Case No. 1,3,5,7, in Case No. 2,6 and in Case No. 4,8, the costs are different slightly but their corresponding Y_1 , Y_2 , Y_3 respectively are recorded same which is happening due to fraction.
- 5) The difference between the actual cost per vehicle per day and the lowest hypothetical cost per vehicle per day is Rs. 740.50 and the difference between the actual cost per vehicle per day and the highest hypothetical cost per vehicle per day is Rs. 727.20. The actual cost per vehicle per day is more than double than the hypothetical cost per vehicle per day.

- 6) The difference between the actual earning per vehicle per day and the lowest hypothetical cost per vehicle per day is Rs. 796.58 and the difference between the actual earning per vehicle per day and the highest hypothetical cost per vehicle per day is Rs. 783.20.
- 7) The difference between the actual cost per vehicle per day and the actual earnings per vehicle per day is Rs. 56.

For Kerala SRTC :

- 1) The effects of the individual constraints are more or less the same.
- 2) Out of all the cases, the minimum of optimum cost is Rs. 608.65 (Case No. 1,5), the maximum of optimum cost is Rs. 624.42 (Case No. 4,8).
- 3) The values of Y_2 and Y_3 are same in all cases and are same as the specified minimum values.
- 4) In Case No. 1,3,5,7, in Case no. 2,6 and in Case No. 4,8, the costs are different slightly but their corresponding Y_1 , Y_2 , Y_3 respectively are recorded same which is happening due to fraction.

- 5) The difference between the actual cost per vehicle per day and the lowest hypothetical cost per vehicle per day is Rs. 874.35 and the difference between the actual cost per vehicle per day and the highest hypothetical cost per vehicle per day is Rs. 853.58.
- 6) The difference between the actual earning per vehicle per day and the lowest hypothetical cost per vehicle per day is 735.35 and the difference between the actual earning per vehicle per day and the highest hypothetical cost per vehicle per day is Rs. 719.58.
- 7) The difference between the actual cost per bus per day and the actual earning per vehicle per day is Rs. (-) 139.

Among six SRTCs, the hypothetical cost (lowest) calculated for NBSTC is ranked as second. But the difference between the actual earning per vehicle per day and the hypothetical cost per vehicle per day of NBSTC is the lowest among all SRTCs i.e. Rs. 127.64 (lowest taking hypothetical cost per vehicle per day) and Rs. 45.62 (Taking highest hypothetical cost per vehicle per day). The highest is for Rajasthan SRTC i.e. Rs. 796.58 and Rs. 783.20.

The difference between the actual cost per vehicle per day and the actual earnings per vehicle per day of NBSTC

is the highest i.e. (-) Rs. 481.69 among all the SRTCs taken for consideration. The difference between the actual cost per vehicle per day and hypothetical cost per vehicle per day of NBSTC is also ranked as second i.e. Rs. 609. So keeping the cost per vehicle per day in a considerable position in NBSTC, the reasons for low profitability are low fleet utilisation, low vehicle utilisation, low occupation ratio, unwise route planning, leakage in Revenue etc. The earning per vehicle per day is the lowest in NBSTC among all the SRTCs taken for comparison.

NOTES AND REFERENCES

1. Robert O Ferguson and Louren F. Sargent, Linear Programming : Fundamentals and Applications, McGraw Hill Book Company, Inc, New York, Toronto London, 1958, pp.6-7.
2. Ibid., p.3.
3. Ibid., pp.300-307.
4. Ibid., p.302.
5. Ibid., pp.308-309.