Concluding remarks

Multiparticle production at $E_{\rm lab} = 200A$ GeV/c is predominantly a soft hadronic process that pertains to very late stages of a high-energy AB collision. By studying the debris of particles produced in the final state particles, most of which are hadrons, it is extremely difficult to draw definite conclusions about an early and/or intermediate stage. One only hopes that the characteristic features of the early stages can withstand the impact of the collision and retain their foot prints even in the later stages. The method of analysis should therefore be sensitive enough so that it can disentangle the characteristic features of a signal from all kinds of background noise. One can develop theories and/or simulation codes based on some presumptive dynamical processes, come up with predictions and try to match the experimental observations, and then fine tune the theory and/or code. It is claimed that a strongly interacting QGP matter has already been created in very high-energy AB experiments ($\sqrt{s}_{NN} \sim$ TeV) held at the RHIC/LHC. The QGP state so created, is almost baryon free and has a very high temperature, a state perhaps similar to that filled up the entire universe a few microsceonds after its birth. Such a claim however does not make the experiments at lower collision energies redundant. It is quite possible that even at much lower energies (typical SPS, FAIR or NICA energies, or the lower end of RHIC energy) there will be large baryon stopping. As a result an extended QCD matter at high baryon density and moderate temperature will be created. A similar QGP state perhaps can be found within the cores of very compact stars. AB collisions should therefore be examined from different perspectives using a wide variety of colliding systems over a vastly different collision energies.

An investigation on the local structures in the phase space distribution of singly charged particles produced in high-energy AB collisions has been made. The data used in this investigation are collected from a nuclear emulsion experiment (EMU-08) held at the Super-proton Synchrotron at CERN, Geneva. Several statistical methods are employed to analyze the multiparticle emission data in ^{16}O-Ag/Br interaction at $p_{\rm lab} = 200A$ GeV/c. A comparison with the ^{32}S-Ag/Br results at the same incident momentum per nucleon is made. The experiments are compared with a set of simulated data generated by the Ultra-relativistic Quantum Molecular Dynamics (UrQMD). A charge reassignment algorithm that mimics the Bose-Einstein Correlation (BEC), is implemented to the UrQMD output as an after burner. The experimental results are also compared with the simulation after the UrQMD output is modified by the charge reassignment algorithm. The UrQMD model(s) set a good reference baseline to the experiment(s). In this thesis we review some aspects of the high-energy AB collision, i.e. its importance in the context of QGP formation, describe gross characteristics of the experimental data samples used, explain some general features of the UrQMD model, outline the charge reassignment scheme, describe the statistical techniques adopted in our analysis, and discuss the results obtained thereof. We now summarize the major conclusions of this investigation and conclude the thesis.

We observe that the gross features of the ^{16}O-Ag/Br experiment like, the η-distribution, the φ-distribution, the distribution of particle density, $N_{ev}^{-1}\,(dn_s/d\eta)$-values of the shower tracks, are more or less well reproduced by the UrQMD model. Corresponding distributions obtained for the ^{32}S-Ag/Br interaction are similar in nature, but are different from the ^{16}O-Ag/Br results only quantitatively. UrQMD reproduces the ^{32}S-Ag/Br results too. There is however a little mismatch between the φ-distributions obtained from the experiments and that from the corresponding simulations. The differences observed in the $0 \le \varphi \le \pi/2$ and $3\pi/2 \le \varphi \le 2\pi$ regions, may be attributed to an inefficiency in measurements, a smaller statistics used in the experiments and/or to the collective flow of hadronic matter. The η-distributions of charged hadrons can be approximated by single Gaussian curves. Corresponding UrQMD generated distributions also have similar characteristics. In both experiments the initial energy density values estimated from the central pseudorapidity densities, are very close to the LQCD predicted threshold of QGP formation, which is $\sim$ a few GeV/fm^3. The single Gaussian shape of the η-distributions, as expected at such collision energies, is compatible to a significant baryon stopping in the central region. Large values of η-density in the central particle producing regions are seen in the η-distributions of shower tracks in individual events. One of the main objectives of this investigation is to characterize these high particle densities.

The η-distributions of charged hadrons belonging to individual events, fluctuate at random between sharp spikes and deep valleys apparently devoid of any definite pattern. For a large event sample, these fluctuations are however averaged out resulting in smooth Gaussian shaped distributions. With decreasing size of phase space intervals down to the experimental resolution, such fluctuations increase in magnitude. Particles in large numbers accumulate in narrow phase space intervals for three different reasons, due to statistical uncertainties, due to kinematic conservation rules and due to some underlying dynamics. A part of the dynamics may be known whereas a part of it may still be unkown. We use the scaled factorial moments (SFM) and characterize the fluctuating distributions in terms of a finite set of regularly behaving parameters. For the experimental distributions, the SFMs follow a power law and increase in magnitude with decreasing phase space resolution size. This is observed in both η and φ-spaces. In high-energy physics the phenomenon is known as intermittency. On the other hand, for the UrQMD generated samples we do not see any significant change in the SFM values. Inclusion of BEC in the UrQMD output only marginally recovers the power law type of scaling. The main sources of short range particle correlations in UrQMD are resonance decays and mini-jet fragmentation. Inclusion of BEC perhaps enhances the correlation to a small extent, which is not there in the UrQMD. The intermittency strengths are however much larger in the experiments than in the simulations. There is a definite indication that some degree of short range correlations present in the experiments cannot be accounted for by the simulation. We speculate that a kind of hadronic/partonic cascade mechanism or some other unknown dynamics is responsible for the differences. Some other observations regarding the intermittency are listed below.

1. Intermittency in the φ-space is stronger than that in the η-space. The observation may partially be attributed to a stronger influence of kinematic conservation in the transverse (azimuthal) plane.

2. At the same incident momentum per nucleon, intermittency in the ^{16}O-Ag/Br interaction is stronger than that in the ^{32}S-Ag/Br interaction. The observation is attributed to a larger number of particle producing sources in a larger colliding system.

3. Intermittency in the (η, φ)-plane is stronger than that either in the η or φ-space. In $1d$ intermittency is self-similar, whereas in $2d$ it is self-affine. Self-similarity in $2d$ is retrieved when the phase space is partitioned unequally along the η and φ-directions by using the roughness (Hurst) parameter.

The intermittency technique also allows us to characterize the bin-to-bin correlations. A two fold SFM or the factorial correlator (FC) is used for this purpose. In the η-space, as the bin-to bin distance (the correlation length) is decreased, the FCs follow a power-law type of scaling. The FCs however are independent of the phase space interval size. Our results on the FCs are consistent with the predictions of a simple intermittency model (α-model). The experimental results could not be reproduced either by the UrQMD or by the UrQMD+BEC model. Once again presence of short range correlations in the data is established that the models can not account for.

Presence of genuine higher order correlations are examined by using the factorial cumulant moments or the normalized cumulant moments, where contributions coming from lower order correlations are eliminated. According to a QCD-based parton shower cascade model, the normalized cumulant moments should undergo an oscillation with the order number. This prediction has been verified in our experiments in small sized η-intervals ($\Delta\eta \lesssim 0.25$) in the central region. With increasing width of the η-interval considered, the oscillatory behavior gradually disappears, and the experiment starts to coincide with the respective simulation. However, as long as the η-interval remains small sized, our experimental results on the oscillatory moments are not exactly reproduced by the UrQMD model(s).

We have obtained the distributions of single event SFMs for a particular partition number in the η-space. We observe that while most of the SFM-values are restricted within a small range, events with quite high SFM-values are also not very rare. It is speculated that this kind of event space fluctuations of the SFMs is chaotic in nature. Such fluctuations can be characterized in terms of the erraticity moments. We find that the erraticity moments in both experiments and simulations increase with increasing partition number following once again a power-law type of scaling. A chaoticity parameter, similar to the entropy index used in the *pp* interaction, was obtained. We find that the event space fluctuations are more chaotic in the ^{32}S-Ag/Br than in the ^{16}O-Ag/Br interaction. Fluctuations in the simulated event samples are significantly less chaotic than that in the experiment.

We have used several statistical methods to characterize the particle density fluctuations in the η-space in terms of a set of (multi)fractal parameters. The results of ^{16}O-Ag/Br interaction are sometimes compared with the ^{32}S-Ag/Br results. We also use the detrended methods and visibility graph(s) for our multifractal analysis. Our observations on multifractality are summarized below.

1. With diminishing phase space resolution size the G_q-moments increase in magnitude following a scale invariant power-law. The UrQMD and UrQMD+BEC simulated moments show similar trends. This is in contrast to our intermittency results for the same sets of data, where the simulations fail to reproduce the experiments. The SFMs take care of the statistical noise which the multifractal moments do not. Perhaps it is this reason that makes all the differences in our observations between intermittency and multifractality.

2. The multifractal spectrum, consistent in all respects with its expected behavior, has slightly smaller width in the UrQMD than in the corresponding experiment. Width of the UrQMD+BEC generated spectrum lies in between. The multifractal spectrum may be considered as a sensitive tool that can distinguish the dynamics of experiment from the dynamics of simulation.

3. Takagi's multifractal moments also exhibit a power-law scaling with decreasing phase space resolution size for the experiments and simulations. In the ^{16}O-Ag/Br interaction the fractal dimensions obtained from the experiment are marginally different from the UrQMD simulated values. The D_q results are consistent with a thermodynamic interpretation of a monofractal to multifractal phase transition.

4. Multifractal analysis has been performed by using the MFDFA and MFDMA techniques too. The results show that for both interactions the η-distributions are long-range correlated and multifractal in nature. The multifractality observed in the experiments goes beyond the UrQMD+BEC prediction. The MFDFA method too is not quite capable of filtering out the statistical noise from the signal. Unlike the first order MFDFA method, the MFDMA analysis produces a complete and stable singularity spectrum, which indicates the presence of a small number of events with large fluctuations. Coarse fluctuations are attributed mostly to statistical reasons.

5. Using the visibility graph and sandbox algorithm, event-wise η-distributions of shower tracks are analyzed. Degree distributions of visibility graphs obtained for each experiment and the respective UrQMD simulation are similar in nature. The scale freeness of the degree distribution indicates presence of long-range correlated signals in our multiparticle emission data. The sandbox algorithm applied to the visibility graphs successfully reproduces the multifractal properties. We observe that the graph theoretical approach has the potential to differentiate between the dynamical and statistical components.

We have looked for unusual structures in the azimuthal space. The S-parameters used for this purpose, behave consistently with ring and/or jet-like structures in the experiments. Small but significant departures from an independent emission model are seen in the experiments. In the framework of a Cerenkov gluon emission model we conclude that, in some events only a few gluons are emitted, whereas in some others their numbers are large. It would be worthwhile to find out the nuclear refractive index and use the same to constrain the nuclear equation of state. In our wavelet analysis once again we find statistically significant differences between the experiment and corresponding UrQMD+BEC simulation. All the parameters studied in connection with the g_2 and g_4-wavelets indicate that large and ordered fluctuations are present in both ^{16}O-Ag/Br and ^{32}S-Ag/Br data. The differences are attributed to short range correlations.

The forward-backward multiplicity correlations and e-by-e fluctuations in various correlation measures are investigated. The roughness distribution of particles are examined through a χ^2 test. Large concentration of particles within small η-intervals are observed, that cannot be reproduced by the simulations used. On some occasions the particle densities are too large to be merely due to statistical and/or kinematic reasons.

Fluctuations in the forward-backward asymmetry parameter show dominance of short-range correlations in the η-space. Small sized clusters are responsible for the short-range correlations observed. The UrQMD simulation, even after inclusion of the BEC effect, can not reproduce the experimental results. At large η, re-scattering among hadrons significantly dilutes the fluctuations. Our observation on the correlation strength parameter is more or less consistent with that of the fluctuations in the FB asymmetry parameter. The UrQMD and UrQMD+BEC simulations are close to the ^{16}O-Ag/Br experiment, but they over predict the ^{32}S-Ag/Br results.

Presence of particle correlations is reaffirmed through the ω-measure. Non-Poissonian multiplicity distribution within small η-intervals is established through this test. The experimental results however are very close to the simulations. The Φ_η-measure of dynamical fluctuations indicate that both ^{16}O-Ag/Br and ^{32}S-Ag/Br collision events can perhaps be approximated as incoherent superpositions of many particle producing sources. Hadronic re-scattering dilutes the simulated values to a noticeable extent.