Chapter-3

A Mechanism for Simultaneous Generation of Magnetic Fields in a Laser Produced Plasma : Inclusion of Ion Motion

3.1 Introduction

In Chapter-2, we have discussed a mechanism of simultaneous generation of magnetic fields both in axial and in lateral directions in an electron plasma. In this chapter, we shall extend the previous work of Chapter-2 by including the ion contributions. The incident laser field is a transverse mode of vibration. The field generation mechanism here relies on the linear conversion of at least a part of this laser energy into the energy of the induced magnetic field which is longitudinal in nature. The mechanism involves conversion of ordered kinetic energy of the charged particles in the presence of the electromagnetic field into the energy of magnetic fields in both poloidal (axial) and toroidal (lateral) directions. The entire process is observed in the underdense region of plasma. Mathematical techniques remain same as that in Chapter-2 to evaluate spontaneous and simultaneous generation of the axial (poloidal) and lateral (toroidal) magnetic fields for a hot, nondissipative , two-component plasma involving both electrons and ions.

3.2 Formulation of the problem

(A) Basic assumptions : The motions of both electrons and ions are important. The electrons and ions are assumed to be equal and uniform in the plasma of our interest. So, the temperature gradients for electron and ion plasma do not exist. Thus the thermoelectric effect [Stamper etal., (1971), Stamper and Ripin (1975)] can be ignored even for two component

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plasmas. The collision frequency ν is assumed to be much lower than the laser frequency ω ($\nu / \omega << 1$) so that the plasma is essentially nondissipative, even

though both the species of electron and ion are present. The width of the resonance layer is defined as $\Delta x \simeq (\nu/\omega)L$, where L= density scale length. If Δx is small, we can neglect the phenomena occurring at the resonance layer [Thompson etal., (1975), Mora and Pellat (1981), Woo and DeGroot (1978)] . Thermal velocities of electrons and ions are small compared with the phase velocity of laser field. The internal energy of electrons and ions consists of the kinetic energies and their Coulomb interaction energy. In the hot plasma, the ratio of average ion energy and ionization energy is much greater than unity. Kinetic theory establishes relationship between temperature and average kinetic energy. All the particles do not possess the same velocity. Electrons being smaller in size and mass than the ions, have much greater thermal velocities than the ions. The effect of plasma inhomogeneity may be neglected as the Debye length is less than the density scale length.

We have assumed long density scale length and uniform temperature of the plasma so that energy of the laser is absorbed mainly in the underdense region of plasma. Powerful electromagnetic waves change the form of amplitude which are called self-focussing and self-trapping of the waves. The threshold power of laser for initiating self-focussing is larger for smaller pulse length. Below this power limit, self-action effects are insignificant [Max (1973), (1976)]. When an intense laser beam is incident on a plasma, the following observations would be made: (1) the laser is absorbed, (2) coherent scattering occurs and (3) molecular vibration is excited. These changes lead to decay instabilities, known as Stimulated Raman Scattering (SRS) and Stimulated Brillouin Scattering (SBS). These instabilities are also being neglected in our case [Shen (1976), Kruer (1987)]. We have considered only Kerr type nonlinearity [Aleksic etal., (1992), Newell and Moloney (1992)]. A perturbation scheme [Bellman (1964), Ames (1965)], which has been used in Chapter-2, is also to be used here to find out the induced magnetic field resulting from the interaction of intense laser field with a two component, one temperature and nondissipative plasma.

(B) <u>Basic equations</u> : Under the above assumptions, we start with the equations of continuity for electrons and ions as,

$$n_e + \nabla.(n_e \dot{\mathbf{r}}_e) = 0 \qquad (3.2.1)$$

$$n_i + \nabla.(n_i \dot{\mathbf{r}}_i) = 0 \qquad (3.2.2)$$

The equations of momentum for electrons and ions are,

$$\ddot{\mathbf{r}}_e + (\dot{\mathbf{r}}_e.\nabla)\dot{\mathbf{r}}_e + \frac{e}{m_e}\mathbf{E} + \frac{e}{m_e c}(\dot{\mathbf{r}}_e \times \mathbf{H}) + \frac{\nabla P_e}{m_e n_e} = 0 \qquad (3.2.3)$$

$$\ddot{\mathbf{r}}_i + (\dot{\mathbf{r}}_i.\nabla)\dot{\mathbf{r}}_i - \frac{e}{m_i}\mathbf{E} - \frac{e}{m_i c}(\dot{\mathbf{r}}_i \times \mathbf{H}) + \frac{\nabla P_i}{m_i n_i} = 0 \qquad (3.2.4)$$

and the four Maxwell's equations are

$$\nabla \times \mathbf{E} = -(\frac{\dot{\mathbf{H}}}{c}) \qquad (3.2.5)$$

$$\nabla \times \mathbf{H} = (\frac{\dot{\mathbf{E}}}{c}) + \frac{4\pi e}{c}(n_i \dot{\mathbf{r}}_i - n_e \dot{\mathbf{r}}_e) \qquad (3.2.6)$$

$$\nabla.\mathbf{E} = 4\pi e(n_i - n_e) \qquad (3.2.7)$$

$$\nabla.\mathbf{H} = 0 \qquad (3.2.8)$$

where n_e and n_i are the densities of the electrons and the ions respectively, $\dot{\mathbf{r}}_e$ and $\dot{\mathbf{r}}_i$ are the respective velocities of electrons the ions, m_e and m_i are the mass of electron and the ion respectively, P_e and P_i are electron and ion pressures respectively and all other symbols have their usual meanings.

The interaction process is so fast that it can be assumed to be isothermal. Then equation of state for electrons is

$$P_e = n_e k_B T_e \qquad (3.2.9)$$

and that for ions is

$$P_i = n_i k_B T_i \tag{3.2.10}$$

where k_B is Boltzmann constant and T_e and T_i are temperatures of electrons and ions respectively.

The perturbation technique, similar to Chapter-2, is used to find out the solutions of these basic equations (3.2.1) to (3.2.10) in different orders for studying the nonlinear phenomena in a laser produced two-component, one temperature and nondissipative plasma.

3.3 First order equations and solutions

The first order equations of continuity for electrons and ions are as

$$\dot{n}_{e1} + n_0(\nabla.\dot{\mathbf{r}}_{e1}) = 0 \tag{3.3.1}$$

$$\dot{n}_{i1} + n_0(\nabla.\dot{\mathbf{r}}_{i1}) = 0 \tag{3.3.2}$$

and that of momentum equations for electrons and ions are as

$$\ddot{\mathbf{r}}_{e1} + \frac{e}{m_e}\mathbf{E}_1 + \frac{\nabla \mathbf{p}_{e1}}{m_e n_0} = 0=0 \tag{3.3.3}$$

$$\ddot{\mathbf{r}}_{i1} + \frac{e}{m_i}\mathbf{E}_1 + \frac{\nabla \mathbf{p}_{i1}}{m_i n_0} = 0=0 \tag{3.3.4}$$

and, so, the first order Maxwell equations are written as

$$\nabla\times\mathbf{E}_1 = -\left(\frac{\dot{\mathbf{H}}_1}{c}\right) \tag{3.3.5}$$

$$\nabla\times\mathbf{H}_1 = \left(\frac{\dot{\mathbf{E}}_1}{c}\right) + \frac{4\pi e n_0}{c}(\dot{\mathbf{r}}_{i1} - \dot{\mathbf{r}}_{e1}) \tag{3.3.6}$$

$$\nabla.\mathbf{E}_1 = 4\pi e(n_{i1} - n_{e1}) \tag{3.3.7}$$

$$\nabla.\mathbf{H}_1 = 0 \tag{3.3.8}$$

Also, the first order forms of the equations (3.2.9) and (3.2.10) are written as

$$P_{e1} = k_B T_0 n_{e1} + k_B n_0 T_{e1} \approx k_B T_0 n_{e1} \tag{3.3.9}$$

and $$P_{i1} = k_B T_0 n_{i1} + k_B n_0 T_{i1} \approx k_B T_0 n_{i1} \tag{3.3.10}$$

where T_0 is the unperturbed plasma temperature and would be treated constant, whereas, the perturbed electron temperature T_{e1} and ion temperature T_{i1} are neglected because our study is confined to isothermal processes. It is evident from the relations (3.3.9) and (3.3.10) that the temperature in the tangential direction, but not in the direction of propagation of laser. This would help us to avoid thermoelectric effect [Stamper etal., (1971)] in the magnetic field evolutions.

Multiplying by the vector operator ∇ on both sides of (3.3.9) and (3.3.10), we have

$$\nabla P_{e1} = k_B T_0 \nabla n_{e1} \tag{3.3.11}$$

and $$\nabla P_{i1} = k_B T_0 \nabla n_{i1} \tag{3.3.12}$$

Using the relations (3.3.11) and (3.3.12) in the equations (3.3.3) and (3.3.4), respectively, we have

$$\ddot{\mathbf{r}}_{e1} + \frac{e}{m_e}\mathbf{E}_1 + v_{the}^2 \frac{\nabla n_{e1}}{2n_0} = 0 \tag{3.3.13}$$

$$\ddot{\mathbf{r}}_{i1} + \frac{e}{m_i}\mathbf{E}_1 + v_{thi}^2 \frac{\nabla n_{i1}}{2n_0} = 0 \tag{3.3.14}$$

Let the first order electric field be assumed as in (2.4.1), namely,

$$\mathbf{E}_1 = (Mc\omega/2e)[\hat{\mathbf{x}}\alpha_\parallel e^{i\theta_\parallel} + (\hat{\mathbf{y}}\alpha_\perp - i\hat{\mathbf{z}}\beta_\perp)e^{i\theta_\perp}] + \text{c.c.} \tag{3.3.15}$$

where $\alpha_\parallel(=a_\parallel e/Mc\omega)$, $\alpha_\perp(=ea_\perp/Mc\omega)$, $\beta_\perp(=eb_\perp/Mc\omega)$. $\alpha_\parallel$ is the dimensionless amplitude of the longitudinal electric field, $\alpha_\perp$ and $\beta_\perp$ are that of transverse electric fields, $\theta_\perp = k_\perp x - \omega t$ and $\theta_\parallel = k_\parallel x - \omega t$ are the phases of transverse and longitudinal field respectively. $k_\perp$ and $k_\parallel$ are the respective transverse and the longitudinal wave numbers, ω is the frequency of the wave. $M(=\frac{m_e m_i}{m_e + m_i})$ is the mean mass, $\hat{x}, \hat{y}$ and $\hat{z}$ are the unit vectors along the three mutually perpendicular directions, the term c.c. represents the complex conjugate. In eqn. (3.3.15), the last two terms in the third bracket arise directly from the laser fields while the first component arises from the converted mode for a thermal plasma showed by Kull [(1981), (1983)]. It may be noted that the initial field as stated in equation (3.3.15) is the same to that of equation (2.4.1) for electric pulse. Therefore, the physical justifications of the equation (2.4.1) led down in Chapter-2 would be the same as that of the equation (3.3.15). Hence, the reason for using (3.3.15) will not be mentioned in this Chapter again.

When light is incident on a spatially homogeneous plasma, electrostatic waves are generated whenever the light has a component of electric field **E** along the density gradient $\nabla n_{e,i}$. When a p-polarized light (with **E** parallel to the plane of incidence) is incident obliquely on density gradient, then the component of the light electric field parallel to $\nabla n_{e,i}$ of plasma can drive plasma waves. Near $n_{e,i} = n = n_c$, the critical density , **E** becomes very large and will resonantly excite these waves. Hence, an energy transfer mechanism occurs from light into the waves and through the damping of waves into the electron and ion temperatures. This process is known as resonance absorption. But we have ignored this absorption because we are finding fields in underdense region of plasma. Further it may be pointed out that the mode conversion is possible even in the underdense region for thermal plasma and the width of the conversion layer plays an important role in such conversion. So, the amplitude of the electromagnetic mode of laser light will be modified in a thermal plasma [Kull (1981, 1983)] and, hence, the linear form of the electric field ($\mathbf{E}_1$) be chosen as (3.3.15). Moreover, our analysis is confined only in the region where the effect of resonance layer and the Landau damping effects are not important as mentioned in Chapter-2.

Solving (3.3.7) together with (3.3.15) we have the linearized magnetic field as

$$\mathbf{H}_1 = (Mc\omega/2e)[(i\hat{y}\beta_\perp + \hat{z}\alpha_\perp)n_\perp e^{i\theta_\perp} + \text{c.c.}] \qquad (3.3.16)$$

where $n_\perp = (k_\perp c/\omega)$, and the term c.c. represents the complex conjugate.

Solving (3.3.3), (3.3.4), (3.3.8) together with (3.3.15) and (3.3.16) we have the linearized density for electron as

$$n_{i1} = I\,(n_0/2)(G_e \alpha_{\parallel} e^{i\theta_{\parallel}} + c.c.) \qquad (3.3.17)$$

where , $G_e = [2(X-1) + V_e^2 n_{\parallel}^2]/[n_{\parallel}(V_i^2 - V_e^2)]$.

and the ion density as

$$n_{e1} = i\,(n_0/2)(G_i \alpha_{\parallel} e^{i\theta_{\parallel}} + c.c.) \qquad (3.3.18)$$

where , $G_i = [2(X-1) + V_i^2 n_{\parallel}^2]/[n_{\parallel}(V_i^2 - V_e^2)]$.

Solving (3.3.3) or (3.3.13) we have the linearized electron velocity as

$$\dot{\mathbf{r}}_{e1} = i(M_e c/2)[D_{e1}\hat{x}\alpha_{\parallel} e^{i\theta_{\parallel}} + D_{e2}(\hat{y}\alpha_{\perp} - i\hat{z}\beta_{\perp})e^{i\theta_{\perp}}] + c.c. \qquad (3.2.25)$$

where $\qquad D_{e1} = 1 - V_i^2 G_e n_{\parallel}/2M_i,\ D_{e2} = (n_{\perp}^2 - 1)/X,\ M_e = M/m_e$

and also, solving (3.3.4) or (3.2.14) we have the linearized ion velocity as

$$\dot{\mathbf{r}}_{i1} = i(M_i c/2)[D_{i1}\hat{x}\alpha_{\parallel} e^{i\theta_{\parallel}} + D_{i2}(\hat{y}\alpha_{\perp} - i\hat{z}\beta_{\perp})e^{i\theta_{\perp}}] + c.c. \qquad (3.2.26)$$

where $\qquad D_{i1} = 1 - V_e^2 G_i n_{\parallel}/2M_e,\ D_{i2} = D_{e2},\ M_i = M/m_i$

3.4 Linear dispersion relations and preliminary analysis

Using the above set of linearized equations and their solutions, we have the linearized dispersion relation for transverse waves as

$$n_{\perp}^2 - 1 + X_p = 0 \qquad (3.4.1)$$

where $n_{\perp} = (k_{\perp} c/\omega)$, $X_p = X_e + X_i$, $X_e = \omega_{pe}^2/\omega^2$,$X_i = \omega_{pi}^2/\omega^2$, $\omega_{pe}^2 = 4\pi\, n_0 e^2/m_e$,
$\omega_{pi}^2 = 4\pi\, n_0 e^2/m_i$,$M_e = M/m_e$,$M_i = M/m_i$.

Also the dispersion relation for longitudinal wave is

$$(1-\frac{V_i^2 n_\|^2}{2})(1-\frac{V_e^2 n_\|^2}{2}) = X_i(1-\frac{V_e^2 n_\|^2}{2})+X_e(1-\frac{V_i^2 n_\|^2}{2}).$$

$$\text{or,}\quad (\frac{V_i^2 n_\|^2}{2}-1)(\frac{V_e^2 n_\|^2}{2}-1) + X_i(\frac{V_e^2 n_\|^2}{2}-1)+X_e(\frac{V_i^2 n_\|^2}{2}-1) = 0. \qquad (3.4.2)$$

where $n_\| = k_\| c/\omega$, $V_e^2 = v_{the}^2/c^2$, $V_i^2 = v_{thi}^2/c^2$, $v_{the}^2 = 2k_B T_0/m_e$, $v_{thi}^2 = 2k_B T_0/m_i$.

From the two dispersion relations (3.4.1) and (3.4.2) we see that the two relations are not coupled. So, we conclude that there occurs no exchange of energy between the transverse and longitudinal mode of wave. The first order solution is a linear solution. So, no exchange of energy is possible in linear case between transverse and longitudinal modes. The two vibration modes are independent of wave amplitudes. The transverse vibration is independent of the thermal velocities i.e. temperature has no direct effect on thermal velocities. Temperature of plasma affects the longitudinal vibration directly.

3.5 Second order field equations and solutions

Applying the perturbation technique, given in Chapter-2, the second order equations are written for the continuity equations of electron and ion as

$$\dot{n}_{e2} + n_0(\nabla.\dot{\mathbf{r}}_{e2}) + \nabla.(n_{e1}\dot{\mathbf{r}}_{e1}) = 0 \qquad (3.5.1)$$

$$\dot{n}_{i2} + n_0(\nabla.\dot{\mathbf{r}}_{i2}) + \nabla.(n_{i1}\dot{\mathbf{r}}_{i1}) = 0 \qquad (3.5.2)$$

and for the momentum equations of electron and ion as

$$\ddot{\mathbf{r}}_{e2} + \frac{e}{m_e}\mathbf{E}_2 + \frac{v_{the}^2}{2n_0}\nabla n_{e2} = -(\dot{\mathbf{r}}_{e1}.\nabla)\dot{\mathbf{r}}_{e1} - \frac{e}{m_e c}(\dot{\mathbf{r}}_{e1}\times\mathbf{H}_1) + \frac{v_{the}^2}{2n_0^2}n_{e1}(\nabla n_{e1}) \qquad (3.5.3)$$

$$\ddot{\mathbf{r}}_{i2} - \frac{e}{m_e}\mathbf{E}_2 + \frac{v_{thi}^2}{2n_0}\nabla n_{i2} = -(\dot{\mathbf{r}}_{i1}.\nabla)\dot{\mathbf{r}}_{i1} + \frac{e}{m_i c}(\dot{\mathbf{r}}_{i1}\times\mathbf{H}_1) + \frac{v_{thi}^2}{2n_0^2}n_{i1}(\nabla n_{i1}) \qquad (3.5.4)$$

After operating ∇ on both sides of the second order equations of state for electrons and ions, deduced from (3.2.9) and (3.2.10), we have

$$\nabla \frac{p_{e2}}{n_0} = k_B T_0 \frac{\nabla n_{e2}}{n_0} + k_B T_0 \frac{n_{e1} \nabla n_{e1}}{n_0^2} \tag{3.5.5}$$

and

$$\nabla \frac{p_{i2}}{n_0} = k_B T_0 \frac{\nabla n_{i2}}{n_0} + k_B T_0 \frac{n_{i1} \nabla n_{i1}}{n_0^2} \tag{3.5.6}$$

The second order Maxwell equations are as below

$$\nabla \times \mathbf{E}_2 = -(\frac{\dot{\mathbf{H}}_2}{c}) \tag{3.5.7}$$

$$\nabla \times \mathbf{H}_2 = (\frac{\dot{\mathbf{E}}_2}{c}) + \frac{4\pi e\, n_0}{c}(\dot{\mathbf{r}}_{i2} - \dot{\mathbf{r}}_{e2}) + \frac{4\pi e}{c}(n_{i1}\dot{\mathbf{r}}_{i1} - n_{e1}\dot{\mathbf{r}}_{e1}) \tag{3.5.8}$$

$$\nabla . \mathbf{E}_2 = 4\pi e(n_{i2} - n_{e2}) \tag{3.5.9}$$

$$\nabla . \mathbf{H}_2 = 0 \tag{3.5.10}$$

Solving the above set of second order equations (3.5.1) to (3.5.10), by using the first order solutions given in the equations (3.3.16) to (3.3.26), we have the solutions of the second order variables as below:

The nonlinear excited second-order electric field $\mathbf{E}_2$ is

$$\mathbf{E}_2 = i(Mc\omega/2e)[\hat{x}(\xi_\parallel \alpha_\parallel^2 e^{2i\theta_\parallel} + \xi_\perp(\alpha_\perp^2 - \beta_\perp^2)e^{2i\theta_\parallel}) + \xi(\hat{y}\alpha_\perp - i\hat{z}\beta_\perp)\alpha_\parallel e^{i(\theta_\perp + \theta_\parallel)}] - c.c.$$

... ... (3.5.11)

where $\xi_\parallel = -2(P_{1\parallel}/Q_{1\parallel})$, $\xi_\perp = -2(P_{1\perp}/Q_{1\perp})$, $\xi = 2(P_1/Q_1)$ and also

$$P_{1\parallel} = X G_1 [(V_i^2 n_\parallel^2/2) - 1][(V_e^2 n_\parallel^2/2) - 1] + (X/4)[(V_e^2 n_\parallel^2/2) - 1]\tau_{i\parallel}$$
$$+ (X/4)[(V_i^2 n_\parallel^2/2) - 1]\tau_{e\parallel}$$

$$Q_{1\parallel} = 4\,[(V_i^2 n_\parallel^2/2) - 1][(V_e^2 n_\parallel^2/2) - 1] + X_i[(V_e^2 n_\parallel^2/2) - 1] + X_e[(V_i^2 n_\parallel^2/2) - 1]$$

$$P_{1\perp} = -(X/4)[(V_e^2 n_\perp^2/2) - 1]\tau_{i\perp} + (X/4)[(V_i^2 n_\perp^2/2) - 1]\tau_{e\perp}$$

$$Q_{1\perp} = 4\,[(V_i^2 n_\perp^2/2) - 1][(V_e^2 n_\perp^2/2) - 1] + X_i[(V_e^2 n_\perp^2/2) - 1] + X_e[(V_i^2 n_\perp^2/2) - 1]$$

$$P_1 = X\,G, \qquad Q_1 = (n_\parallel + n_\perp)^2 - 4 + X_e + X_i,$$

$$G_1 = (1/2)(M_i G_e D_e + M_e G_i D_i), \qquad G = (1/2)(M_i G_e + M_e G_i)$$

$$\tau_{e\parallel} = \sigma_e n_\parallel + G_i D_i V_e^2 M_e n_\parallel^2, \quad \tau_{e\perp} = M_e^2 n_\perp$$

$$\tau_{i\parallel} = \sigma_i n_\parallel + G_e D_e V_i^2 M_i n_\parallel^2, \quad \tau_{i\perp} = M_i^2 n_\perp$$

$$\sigma_e = M_e^2 D_i^2 - V_e^2 G_i^2/2, \qquad \sigma_i = M_i^2 D_e^2 - V_i^2 G_e^2/2$$

The expression for the second order magnetic field $\mathbf{H}_2$ is,

$$\mathbf{H}_2 = -(Mc\omega/2e)[(\hat{y}\beta_\perp - i\hat{z}\alpha_\perp)\xi\alpha_\parallel e^{i(\theta_\perp+\theta_\parallel)}] \mp \text{c.c.} \tag{3.5.12}$$

Also the velocities of electrons ($\dot{\mathbf{r}}_{e2}$) and of ions ($\dot{\mathbf{r}}_{i2}$) have the forms

$$\dot{\mathbf{r}}_{e2} = (c/2)[\hat{x}(\eta_\parallel \alpha_\parallel^2 e^{2i\theta_\parallel} + \eta_\perp(\alpha_\perp^2 - \beta_\perp^2)e^{2i\theta_\perp}) + \eta(\hat{y}\alpha_\perp - i\hat{z}\beta_\perp)\alpha_\parallel e^{i(\theta_\perp+\theta_\parallel)}] + \text{c.c.} \tag{3.5.13}$$

and

$$\dot{\mathbf{r}}_{i2} = (c/2)[\hat{x}(\zeta_\parallel \alpha_\parallel^2 e^{2i\theta_\parallel} + \zeta_\perp(\alpha_\perp^2 - \beta_\perp^2)e^{2i\theta_\perp}) + \zeta(\hat{y}\alpha_\perp - i\hat{z}\beta_\perp)\alpha_\parallel e^{i(\theta_\perp+\theta_\parallel)}] + \text{c.c.} \tag{3.5.14}$$

where $\eta_\parallel = (1/4)(P_{e\parallel}/Q_{1\parallel})$, $\eta_\perp = (1/4)(P_{e\perp}/Q_{1\perp})$, $\eta = (1/4)(P_e/Q_1)$, and

$\zeta_\parallel = (1/4)(P_{i\parallel}/Q_{1\parallel})$, $\zeta_\perp = (1/4)(P_{i\perp}/Q_{1\perp})$, $\zeta = (1/4)(P_i/Q_1)$ and also

$$P_e = -4X_e G, \qquad P_i = -4X_i G$$

$$P_{e\parallel} = -[4\{(V_i^2 n_\perp^2/2) - 1\} + X_i]\tau_{e\parallel} + X_e\tau_{i\parallel} + 4X_e[(V_i^2 n_\parallel^2/2) - 1]G_1$$

$$P_{e\perp} = -[4\{(V_i^2 n_\perp^2/2) - 1\} + X_i]\tau_{e\perp} + X_e\tau_{i\perp}$$

$$P_{i\parallel} = -[4\{(V_e^2 n_\perp^2/2) - 1\} + X_e]\tau_{i\parallel} + X_i\tau_{e\parallel} + 4X_i[(V_e^2 n_\parallel^2/2) - 1]G_1$$

$$P_{i\perp} = -[4\{(V_e^2 n_\perp^2/2) - 1\} + X_e]\tau_{i\perp} + X_i\tau_{e\perp}$$

The excited electron and ion densities are

$$N_{e2} = (N_0/2)[S_{e\parallel}\alpha_\parallel^2 e^{2i\theta_\parallel} + S_{e\perp}(\alpha_\perp^2 - \beta_\perp^2)e^{2i\theta_\perp}) + c.c. \tag{3.5.15}$$

and $$N_{i2} = (N_0/2)[S_{i\parallel}\alpha_\parallel^2 e^{2i\theta_\parallel} + S_{i\perp}(\alpha_\perp^2 - \beta_\perp^2)e^{2i\theta_\perp}) + c.c. \tag{3.5.16}$$

where $S_{e\parallel} = (\eta_\parallel - M_e G_i D_i/2)n_\parallel$, $\quad S_{e\perp} = \eta_\perp n_\perp$

$S_{i\parallel} = (-\zeta_\parallel + M_i G_e D_e/2)n_\parallel$, $\quad S_{i\perp} = -\zeta_\perp n_\perp$.

It is evident that the linearized solutions (given in Sec. 3.3) contain first harmonic terms. But the second order solutions (derived in this section) contribute only second harmonic terms. Hence, in absence of the first harmonic terms the nonlinearly excited second order fields do not contribute d.c. magnetic field. Moreover, in our calculations, plasma inhomogeneity, field fluctuation, and collisional effects have been ignored. Thus, the second-order solenoidal wave field does not generate a magnetic field. Hence, the nonlinearly excited third order field variables are to be explored for possible magnetic field generation by our mechanism.

3.6 Third order field equations and solutions correct upto first order

Using perturbation technique, the third order equations of continuity for electron and ion become

$$\dot{n}_{e3} + n_0(\nabla.\dot{\mathbf{r}}_{e3}) = -\nabla.(n_{e1}\dot{\mathbf{r}}_{e2}) - \nabla.(n_{e2}\dot{\mathbf{r}}_{e1}) \tag{3.6.1}$$

$$\dot{n}_{i3} + n_0(\nabla.\dot{\mathbf{r}}_{i3}) = -\nabla.(n_{i1}\dot{\mathbf{r}}_{i2}) - \nabla.(n_{i2}\dot{\mathbf{r}}_{i1}) \tag{3.6.2}$$

Similarly, the third order equations of momentum for electron and ion are

$$\ddot{\mathbf{r}}_{e3}+\frac{e}{m_e}\mathbf{E}_3+\frac{v_{the}^2}{2n_0}\nabla n_{e3}=-(\dot{\mathbf{r}}_{e1}.\nabla)\dot{\mathbf{r}}_{e2}-(\dot{\mathbf{r}}_{e2}.\nabla)\dot{\mathbf{r}}_{e1}-\frac{e}{m_e c}(\dot{\mathbf{r}}_{e1}\times\mathbf{H}_2)$$
$$-\frac{e}{m_e c}(\dot{\mathbf{r}}_{e2}\times\mathbf{H}_1)-\frac{v_{the}^2}{2n_0^3}n_{e1}^2\nabla n_{e1}+\frac{v_{the}^2}{2n_0^2}n_{e1}\nabla n_{e2}+\frac{v_{the}^2}{2n_0^2}n_{e2}\nabla n_{e1} \quad (3.6.3)$$

$$\ddot{\mathbf{r}}_{i3}-\frac{e}{m_i}\mathbf{E}_3+\frac{v_{thi}^2}{2n_0}\nabla n_{i3}=-(\dot{\mathbf{r}}_{i1}.\nabla)\dot{\mathbf{r}}_{i2}-(\dot{\mathbf{r}}_{i2}.\nabla)\dot{\mathbf{r}}_{i1}+\frac{e}{m_i c}(\dot{\mathbf{r}}_{i1}\times\mathbf{H}_2)$$
$$+\frac{e}{m_i c}(\dot{\mathbf{r}}_{i2}\times\mathbf{H}_1)-\frac{v_{thi}^2}{2n_0^3}n_{i1}^2\nabla n_{i1}+\frac{v_{thi}^2}{2n_0^2}n_{i1}\nabla n_{i2}+\frac{v_{thi}^2}{2n_0^2}n_{i2}\nabla n_{i2}\nabla n_{i1} \quad (3.6.4)$$

The third order Maxwell equations to be written as

$$\nabla\times\mathbf{E}_3=-(\frac{\dot{\mathbf{H}}_3}{c}) \quad (3.6.5)$$

$$\nabla\times\mathbf{H}_3=\frac{\dot{\mathbf{E}}_3}{c}+\frac{4\pi e n_0}{c}(\dot{\mathbf{r}}_{i3}-\dot{\mathbf{r}}_{e3})+\frac{4\pi e}{c}(n_{i1}\dot{\mathbf{r}}_{i2}-n_{e1}\dot{\mathbf{r}}_{e2})+\frac{4\pi e}{c}(n_{i2}\dot{\mathbf{r}}_{i1}-n_{e2}\dot{\mathbf{r}}_{e1}) \quad (3.6.6)$$

$$\nabla.\mathbf{E}_3=4\pi e(n_{i3}-n_{e3}) \quad (3.6.7)$$

$$\nabla.\mathbf{H}_3=0 \quad (3.6.8)$$

Also the third order equations of state for electron and ion may also be expressed, by taking the operator ∇ on both sides, as

$$\nabla\frac{p_{e3}}{n_0}=k_B T_0\frac{n_{e1}\nabla n_{e2}}{n_0^2}+k_B T_0\frac{n_{e2}\nabla n_{e1}}{n_0^2}-k_B T_0\frac{n_{e1}^2\nabla n_{e1}}{n_0^3} \quad (3.6.9)$$

and

$$\nabla\frac{p_{i3}}{n_0}=k_B T_0\frac{n_{i1}\nabla n_{i2}}{n_0^2}+k_B T_0\frac{n_{i2}\nabla n_{i1}}{n_0^2}-k_B T_0\frac{n_{i1}^2\nabla n_{i1}}{n_0^3} \quad (3.6.10)$$

From equations (3.6.1) to (3.6.10) we get three basic partial differential equations as

$$(c^2\nabla^2-c^2\mathrm{graddiv}-D_t^2)\mathbf{E}_3-4\pi e\, n_0 D_t^2\mathbf{r}_{i3}+4\pi e\, n_0 D_t^2\mathbf{r}_{e3}=\mathbf{NEE} \quad (3.6.11)$$

$$(D_t^2-\frac{v_{thi}^2}{2}\mathrm{graddiv})D_t\mathbf{r}_{i3}-\frac{e}{m_i}D_t\mathbf{E}_3=\mathbf{NLI} \quad (3.6.12)$$

$$(D_t^2-\frac{v_{the}^2}{2}\mathrm{graddiv})D_t\mathbf{r}_{e3}+\frac{e}{m_e}D_t\mathbf{E}_3=\mathbf{NLE} \quad (3.6.13)$$

where **NEE, NLE, NLI** have nonlinear forms as:

$$\mathbf{NEE}=4\pi e D_t[(n_{i1}\dot{\mathbf{r}}_{i2}-n_{e1}\dot{\mathbf{r}}_{e2})+(n_{i2}\dot{\mathbf{r}}_{i1}-n_{e2}\dot{\mathbf{r}}_{e1})] \quad (3.6.14)$$

$$\begin{aligned}\mathbf{NLE}= & -D_t[(\dot{\mathbf{r}}_{e1}.\nabla)\dot{\mathbf{r}}_{e2}+(\dot{\mathbf{r}}_{e2}.\nabla)\dot{\mathbf{r}}_{e1}+\frac{e}{M_e c}(\dot{\mathbf{r}}_{e1}\times\mathbf{H}_2)+\frac{e}{M_e c}(\dot{\mathbf{r}}_{e2}\times\mathbf{H}_1)]+\\ & +\frac{v_{the}^2}{2n_0}\mathrm{graddiv}(n_{e1}\dot{\mathbf{r}}_{e2}+n_{e2}\dot{\mathbf{r}}_{e1})+D_t\frac{v_{the}^2}{2n_0^2}[-\frac{n_{e1}^2}{n_0}\nabla n_{e1}+n_{e2}\nabla n_{e1}+n_{e1}\nabla n_{e2}]\end{aligned} \quad (3.6.15)$$

$$\begin{aligned}\mathbf{NLI}= & -D_t[(\dot{\mathbf{r}}_{i1}.\nabla)\dot{\mathbf{r}}_{i2}+(\dot{\mathbf{r}}_{i2}.\nabla)\dot{\mathbf{r}}_{i1}-\frac{e}{M_i c}(\dot{\mathbf{r}}_{i1}\times\mathbf{H}_2)-\frac{e}{M_i c}(\dot{\mathbf{r}}_{i2}\times\mathbf{H}_1)]+\\ & +\frac{v_{thi}^2}{2n_0}\mathrm{graddiv}(n_{i1}\dot{\mathbf{r}}_{i2}+n_{i2}\dot{\mathbf{r}}_{i1})+D_t\frac{v_{thi}^2}{2n_0^2}[-\frac{n_{i1}^2}{n_0}\nabla n_{i1}+n_{i2}\nabla n_{i1}+n_{i1}\nabla n_{i2}]\end{aligned} \quad (3.6.16)$$

The terms in the right hand side of **NEE** are due to plasma current for electrons and ions. The first two terms in the right hand side of **NLE** and **NLI** are coming from convective derivatives of momentum equations of electrons and ions respectively, the third and fourth terms of **NLE** and **NLI** represent the Lorentz force for electrons and ions respectively. The fifth terms of **NLE** and **NLI** are the respective derivatives of the continuity equations for electrons and ions. The last three terms of **NLE** and **NLI** are due to pressure gradient of electrons and ion respectively. Solving (3.6.11), (3.6.12) and (3.6.13) by Cramer's rule and retaining only first harmonic terms, the following expression for nonlinear velocity of the electron ($\dot{\mathbf{r}}_{e3}$) is as

$$\dot{\mathbf{r}}_{e3} = i(c/2)[\hat{x}\{R_{11}\alpha_{\parallel}^{3} e^{i\theta_{\parallel}} + R_{12}(\alpha_{\perp}^{2} - \beta_{\perp}^{2})\alpha_{\parallel} e^{i(2\theta_{\perp}-\theta_{\parallel})} + R_{13}(\alpha_{\perp}^{2} + \beta_{\perp}^{2})\alpha_{\parallel} e^{i\theta_{\parallel}}\}$$

$$+\hat{y}\{R_{21}\alpha_{\parallel}^{2}\alpha_{\perp} e^{i(2\theta_{\parallel}-\theta_{\perp})} + R_{22}(\alpha_{\perp}^{2} - \beta_{\perp}^{2})\alpha_{\perp} e^{i\theta_{\perp}} + R_{23}\alpha_{\parallel}^{2}\alpha_{\perp} e^{i\theta_{\perp}}\}$$

$$-i\hat{z}\{R_{31}'\alpha_{\parallel}^{2}\beta_{\perp} e^{i(2\theta_{\parallel}-\theta_{\perp})} + R_{32}(\alpha_{\perp}^{2} - \beta_{\perp}^{2})\beta_{\perp} e^{i\theta_{\perp}} + R_{33}\alpha_{\parallel}^{2}\beta_{\perp} e^{i\theta_{\perp}}\} + c.c. \quad (3.6.17)$$

and that of the ion ($\dot{\mathbf{r}}_{i3}$)

$$\dot{\mathbf{r}}_{i3} = i(c/2)[\hat{x}\{T_{11}\alpha_{\parallel}^{3} e^{i\theta_{\parallel}} + T_{12}(\alpha_{\perp}^{2} - \beta_{\perp}^{2})\alpha_{\parallel} e^{i(2\theta_{\perp}-\theta_{\parallel})} + T_{13}(\alpha_{\perp}^{2} + \beta_{\perp}^{2})\alpha_{\parallel} e^{i\theta_{\parallel}}\}$$

$$+\hat{y}\{T_{21}\alpha_{\parallel}^{2}\alpha_{\perp} e^{i(2\theta_{\parallel}-\theta_{\perp})} + T_{22}(\alpha_{\perp}^{2} - \beta_{\perp}^{2})\alpha_{\perp} e^{i\theta_{\perp}} + T_{23}\alpha_{\parallel}^{2}\alpha_{\perp} e^{i\theta_{\perp}}\}$$

$$-i\hat{z}\{T_{31}\alpha_{\parallel}^{2}\beta_{\perp} e^{i(2\theta_{\parallel}-\theta_{\perp})} + T_{32}(\alpha_{\perp}^{2} - \beta_{\perp}^{2})\beta_{\perp} e^{i\theta_{\perp}} + T_{33}\alpha_{\parallel}^{2}\beta_{\perp} e^{i\theta_{\perp}}\} + c.c.$$

--- --- --- (3.6.18)

where values of all R's and T's are written in the Appendix 3.A , and we will drop the subscript 3 in the subsequent sections. It is a fact that the nondissipative plasma approximation demands the real values of wavenumber (k) and frequency (ω) of the radiation fields which entrusts to avoid field fluctuations and collisional effects in our calculations.

3.7 Nonlinear angular momentum and magnetization

The total nonlinear magnetic moments for electrons and ions turn out as

$$\boldsymbol{\mu} = \boldsymbol{\mu}_e + \boldsymbol{\mu}_i \tag{3.7.1}$$

where $\boldsymbol{\mu}_e$ and $\boldsymbol{\mu}_i$ are the nonlinear electronic and ionic magnetic moments respectively and can be expressed as

$$\boldsymbol{\mu}_e = (\mathbf{r}_e \times \mathbf{j}_e)/2c \text{ and } \boldsymbol{\mu}_i = (\mathbf{r}_i \times \mathbf{j}_i)/2c \tag{3.7.2}$$

where $\mathbf{j}_e$ and $\mathbf{j}_i$ are the nonlinear current vectors due to electrons and ions respectively. The expressions of $\mathbf{j}_e$ and $\mathbf{j}_i$ are given by

$$\mathbf{j}_e = -e\dot{\mathbf{r}}_e \text{ and } \mathbf{j}_i = e\dot{\mathbf{r}}_i \tag{3.7.3}$$

The nonlinear electron and ion velocities $\dot{\mathbf{r}}_e$ and $\dot{\mathbf{r}}_i$ are given in (3.6.17) and (3.6.18) respectively and their displacements $\mathbf{r}_e$ and $\mathbf{r}_i$ are defined in (3.A-1) and (3.A-2). The nonlinear angular momentum for two-component plasmas is given by

$$\mathbf{L} = (2c/e)(\boldsymbol{\mu}_i - \boldsymbol{\mu}_e) \tag{3.7.4}$$

It is evident that the nonlinear angular momentum $\mathbf{L}$ comprises of $\mathbf{L}_e[=-(2c/e)\,\boldsymbol{\mu}_e)]$ due to the electrons and $\mathbf{L}_i[=(2c/e)(\boldsymbol{\mu}_i)]$ due to the ions.

So, the nonlinear induced magnetization in laser plasmas, averaged over the fast laser frequency time scale (i.e.$2\pi/\omega$) can be expressed as

$$\langle M \rangle = (4\pi e N_0/c)(\langle \mu_i \rangle - \langle \mu_e \rangle) = \langle M_i \rangle + \langle M_e \rangle \tag{3.7.5}$$

where $\langle M_e \rangle$ and $\langle M_i \rangle$ are the averaged induced magnetization for electrons and ions respectively.

The total averaged induced poloidal magnetic field for electrons and ions in the x-direction (i.e., the direction of the wave propagation or, in other words, the direction of the laser beam) can be expressed as

$$M_p = (\langle M_{ex} \rangle + \langle M_{ix} \rangle) \tag{3.7.6}$$

where, $\langle M_{ex} \rangle = -(4\pi e N_0/c)\langle \mu_{ex} \rangle = (2\pi e^2 N_0/c^2)\langle L_{ex} \rangle$ and

$\langle M_{ix} \rangle = (4\pi e N_0/c)\langle \mu_{ix} \rangle = (2\pi e^2 N_0/c^2)\langle L_{ix} \rangle$, and also where, the average value of the x-component of $\mathbf{L}_e$ $<L_{ex}>$ can be expressed as

$$\langle L_{ex}\rangle = r_{ey}\dot{r}_{ez} - r_{ez}\dot{r}_{ey}$$

$$= -\frac{c^2}{\omega}[R_{21}^2\alpha_\parallel^4\alpha_\perp\beta_\perp + R_{21}R_{22}\alpha_\parallel^2(\alpha_\perp^2-\beta_\perp^2)\alpha_\perp\beta_\perp + R_{21}R_{23}\alpha_\parallel^4\alpha_\perp\beta_\perp + R_{22}R_{21}\alpha_\parallel^2(\alpha_\perp^2-\beta_\perp^2)\alpha_\perp\beta_\perp] +$$

$$R_{22}^2(\alpha_\perp^2-\beta_\perp^2)^2\alpha_\perp\beta_\perp + R_{22}R_{23}\alpha_\parallel^2(\alpha_\perp^2-\beta_\perp^2)\alpha_\perp\beta_\perp + R_{23}R_{21}\alpha_\parallel^4\alpha_\perp\beta_\perp +$$

$$R_{23}R_{22}\alpha_\parallel^2(\alpha_\perp^2-\beta_\perp^2)\alpha_\perp\beta_\perp + R_{23}^2\alpha_\parallel^4\alpha_\perp\beta_\perp]$$

and the average value of the x-component of $\mathbf{L}_i$ denoted by $<L_{ix}>$ can be written as

$$\langle L_{ix}\rangle = r_{iy}\dot{r}_{iz} - r_{iz}\dot{r}_{iy}$$

$$= -\frac{c^2}{\omega}[T_{21}^2\alpha_\parallel^4\alpha_\perp\beta_\perp + T_{21}T_{22}\alpha_\parallel^2(\alpha_\perp^2-\beta_\perp^2)\alpha_\perp\beta_\perp + T_{21}T_{23}\alpha_\parallel^4\alpha_\perp\beta_\perp + T_{22}T_{21}\alpha_\parallel^2(\alpha_\perp^2-\beta_\perp^2)\alpha_\perp\beta_\perp] +$$

$$T_{22}^2(\alpha_\perp^2-\beta_\perp^2)^2\alpha_\perp\beta_\perp + T_{22}T_{23}\alpha_\parallel^2(\alpha_\perp^2-\beta_\perp^2)\alpha_\perp\beta_\perp + T_{23}T_{21}\alpha_\parallel^4\alpha_\perp\beta_\perp +$$

$$T_{23}T_{22}\alpha_\parallel^2(\alpha_\perp^2-\beta_\perp^2)\alpha_\perp\beta_\perp + T_{23}^2\alpha_\parallel^4\alpha_\perp\beta_\perp]$$

The resultant of both y- and z-components of induced magnetization for electrons and ions turns out to be the total toroidal magnetic fields. It will be in a plane perpendicular to the direction of laser beams. The resultant toroidal magnetic field can be expressed as

$$M_t = [(<M_{iy}>)^2 + (<M_{iz}>)^2 + (<M_{ey}>)^2 + (<M_{ez}>)^2]^{\frac{1}{2}} \qquad (3..7.7)$$

where, $<M_{ey}> = -(4\pi e N_0/c)<\mu_{ey}> = (2\pi e^2 N_0/c^2)<L_{ey}>$,

$<M_{ez}> = -(4\pi e N_0/c)<\mu_{ez}> = (2\pi e^2 N_0/c^2)<L_{ez}>$ and also

$<M_{iy}> = (4\pi e N_0/c)<\mu_{iy}> = (2\pi e^2 N_0/c^2)<L_{iy}>$,

$$<M_{iz}> = (4\pi e N_0/c)<\mu_{iz}> = (2\pi e^2 N_0/c^2)<L_{iz}>$$

where, we also have

$$\langle L_{ey}\rangle = r_{ez}\dot{r}_{ex} - r_{ex}\dot{r}_{ez}$$

$$= \frac{c^2}{\omega}[R_{11}R_{31}\alpha_\parallel^5\beta_\perp + R_{12}R_{31}\alpha_\parallel^3(\alpha_\perp^2-\beta_\perp^2)\beta_\perp + R_{13}R_{31}\alpha_\parallel^3(\alpha_\perp^2+\beta_\perp^2)\beta_\perp + R_{32}R_{11}\alpha_\parallel^3(\alpha_\perp^2-\beta_\perp^2)\beta_\perp] +$$

$$R_{32}R_{12}(\alpha_\perp^2-\beta_\perp^2)^2\alpha_\parallel\beta_\perp + R_{32}R_{13}(\alpha_\perp^4-\beta_\perp^4)\alpha_\parallel\beta_\perp + R_{33}R_{11}\alpha_\parallel^5\beta_\perp +$$

$$R_{33}R_{12}\alpha_\parallel^3(\alpha_\perp^2-\beta_\perp^2)\beta_\perp + R_{33}R_{13}\alpha_\parallel^3(\alpha_\perp^2+\beta_\perp^2)\beta_\perp]$$

and

$$<L_{ez}> = r_{ex}\dot{r}_{ey} - r_{ey}\dot{r}_{ex}$$

$$= \frac{c^2}{\omega}[R_{11}R_{21}\alpha_\parallel^5\alpha_\perp + R_{12}R_{21}\alpha_\parallel^3(\alpha_\perp^2-\beta_\perp^2)\alpha_\perp + R_{13}R_{21}\alpha_\parallel^3(\alpha_\perp^2+\beta_\perp^2)\alpha_\perp + R_{11}R_{22}\alpha_\parallel^3(\alpha_\perp^2-\beta_\perp^2)\alpha_\perp] +$$

$$R_{22}R_{12}(\alpha_\perp^2-\beta_\perp^2)^2\alpha_\parallel\alpha_\perp + R_{22}R_{13}(\alpha_\perp^4-\beta_\perp^4)\alpha_\parallel\alpha_\perp + R_{23}R_{11}\alpha_\parallel^5\alpha_\perp +$$

$$R_{23}R_{12}\alpha_\parallel^3(\alpha_\perp^2-\beta_\perp^2)\alpha_\perp + R_{23}R_{13}\alpha_\parallel^3(\alpha_\perp^2+\beta_\perp^2)\alpha_\perp]$$

$$<L_{iy}> = r_{iz}\dot{r}_{ix} - r_{ix}\dot{r}_{iz}$$

$$= -\frac{c^2}{\omega}[T_{11}T_{31}\alpha_\parallel^5\beta_\perp + T_{12}TR_{31}\alpha_\parallel^3(\alpha_\perp^2-\beta_\perp^2)\beta_\perp + T_{13}T_{31}\alpha_\parallel^3(\alpha_\perp^2+\beta_\perp^2)\beta_\perp + T_{32}T_{11}\alpha_\parallel^3(\alpha_\perp^2-\beta_\perp^2)\beta_\perp] +$$

$$T_{32}T_{12}(\alpha_\perp^2-\beta_\perp^2)^2\alpha_\parallel\beta_\perp + T_{32}T_{13}(\alpha_\perp^4-\beta_\perp^4)\alpha_\parallel\beta_\perp + T_{33}T_{11}\alpha_\parallel^5\beta_\perp +$$

$$T_{33}T_{12}\alpha_\parallel^3(\alpha_\perp^2-\beta_\perp^2)\beta_\perp + T_{33}T_{13}\alpha_\parallel^3(\alpha_\perp^2+\beta_\perp^2)\beta_\perp]$$

and

$$L_{iz} = r_{i3x}\dot{r}_{i3y} - r_{i3y}\dot{r}_{i3x}$$

$$= \frac{c^2}{\omega}[T_{11}T_{21}\alpha_{\|}^5\alpha_{\perp} + T_{12}T_{21}\alpha_{\|}^3(\alpha_{\perp}^2 - \beta_{\perp}^2)\alpha_{\perp} + T_{13}T_{21}\alpha_{\|}^3(\alpha_{\perp}^2 + \beta_{\perp}^2)\alpha_{\perp} + T_{11}T_{22}\alpha_{\|}^3(\alpha_{\perp}^2 - \beta_{\perp}^2)\alpha_{\perp}] +$$

$$T_{22}T_{12}(\alpha_{\perp}^2 - \beta_{\perp}^2)^2\alpha_{\|}\alpha_{\perp} + T_{22}T_{13}(\alpha_{\perp}^4 - \beta_{\perp}^4)\alpha_{\|}\alpha_{\perp} + T_{23}T_{11}\alpha_{\|}^5\alpha_{\perp} +$$

$$T_{23}T_{12}\alpha_{\|}^3(\alpha_{\perp}^2 - \beta_{\perp}^2)\alpha_{\perp} + T_{23}T_{13}\alpha_{\|}^3(\alpha_{\perp}^2 + \beta_{\perp}^2)\alpha_{\perp}]$$

3.8 Numerical results and graphical illustration

The numerical estimations of both the toroidal and poloidal magnetic fields have been made for the Nd-glass laser with wavelengths in μm, pulse lengths in nsec, and intensities in W/cm^2. The thermal power flux is $5(1+1/Z)'(Nk_BT)(\Delta R/\tau)$ W/cm^2, where Z, k_BT, τ, N and R are the effective ionic charge number, the plasma temperature in eV, the laser pulse lengths in nsec, the density in cm^{-3} and the spot radius R in μm, respectively. The region of our interest has been assumed to be composed of slabs, and the length ΔR of each slab is so chosen such that the temperature in that slab is fairly constant and the mean value of the density can reasonably be taken. The induced magnetic fields have been calculated on the basis of those values of temperature and density. Numerical results show that the induced toroidal magnetic field is not maximum at the critical density but well below it, as in Fig.3.1(a). These results are consistent with the experimental results [Raven etal., (1979)] and numerically computed results [Boyd etal., (1982)]. But, Fig.3.1(b) tells us that the position of the peak value of the poloidal fields is near the critical density, which is yet to be verified either by experiment or by simulation. The magnitudes of toroidal and poloidal fields are different in different plasma expansion regions. So, their Larmor radius effects on the rate of energy deposition in conduction regions would be different and hence, the effect of energy transport from a critical surface to an ablation surface is not uniform. More studies are needed for energy transport due to such fields. Fig. 3.2(a) shows that the toroidal field decreases with increasing pulse length but the poloidal field has no change as shown in Fig. 3.2(b). Fig. 3.3(a) and 3.3(b) show that magnetic fields will increase with increasing laser intensities. These results agree with earlier results [Stamper etal., (1972), Mora and Pellat (1981), Yabe etal., (1983), Raven etal., (1979), Bezzerides etal., (1977), Briand etal., (1985), Bychenkov etal., (1984), Chakraborty etal., (1988), Bhattacharyya (1994), Bhattacharyya and Sanyal (1999)]. Fig. 3.4(a) points out that toroidal fields increase very slowly with increase in the wavelength, but poloidal fields increase exponentially as shown in Fig. 3.4(b). A rough sketch in Fig.3.5 shows the region of the underdense plasma where our model for generation of magnetic fields may be valid. It is better to point out here that out of 80μm density scale length the thickness of the resonance layer is of the order of one micrometer and the length of influence of Landau damping is approximately 24 μm. Hence, the typical length of the region of interest for field generation is about 55 μm only. These results have been discussed in the Appendix 2.A of Chapter-2. In fact, one can modify our results by incorporating the effects of resonance layer and Landau damping.

3.9 Few important remarks

This mechanism spontaneously generates both the axial and lateral magnetic fields simultaneously in a two component, nondissipative, homogeneous plasma where no temperature gradient exists. Here also kinetic energy of the charged particles (electrons and ions) is transformed into the energy of the induced magnetic field in plasma. This is caused by the interaction of intense laser with a plasma. Without applying any initial magnetic field the field is generated here. So, it is called self-generated magnetic field. Such fields are unidirectional over fast frequency time scale ($2\pi/\omega$). But in laboratory, we require longer time to measure this field. Longer time scale means the laser pulse length or hydrodynamic time scale, where this time scale is L/v_{thi} ,where L is the characteristic length, v_{thi} is thermal velocity of ion.

Since moderate laser intensity is assumed here and ion contribution is also considered, the magnetic field value does not change much even over the longer time scale.

We might compare our mechanism with the inverse Faraday effect (IFE). In IFE, energy of the electromagnetic wave is converted into the energy of the induced magnetic field. IFE is an indirect process of field generation because in IFE, the field of circularly polarized waves cause electrons and ions to gyrate in orbits whose radii depend on radiation intensity, the wave frequency and plasma density. This gyration of charged particles induce a magnetic moment. For left circularly polarized waves, magnetic moment is parallel and for right circularly polarized waves, magnetic moment is antiparallel. When the dispersion rates of two components (left and right) differ, a residual induced magnetization occurs. This field is due to IFE. But our mechanism of magnetic field generation is a direct process because here magnetic field is calculated from average nonlinear magnetic moment of electron and ion through their velocities and displacements. In dynamo effect, magnetic fields are produced cyclically, one field after another. But our mechanism produces both the axial and lateral magnetic fields simultaneously. Temperature gradient is not considered here and so, thermoelectric effect is neglected. Plasma is assumed to be of uniform temperature in our case. Inhomogeneities due to Landau damping is ignored.

Uniform compression is necessary for inertial confinement fusion. Self-generated magnetic fields may have serious effect on heat flux which has major role in energy transport and hence compression of target. Ions and electrons get trapped in a layer of radius of the order of Larmor radii. Larmor radius for ions are greater than that for electrons. Strong toroidal field enhances lateral energy transport, but degrades axial energy transport, which affects energy deposition on the conversion layer and implosion of ICF target is affected by it. But axial magnetic field similarly can enhance axial energy transport, which helps in energy deposition on conversion layer. But for that, strong axial magnetic field must be generated. In our mechanism poloidal magnetic field is generated without applying any external field. In our case, axial field is very weak in comparison to lateral field. But we have seen that axial field can be increased appreciably by increasing the laser wavelength and the laser intensity and the density of the plasma.

But increase in intensity of laser produces inhomogeneities in plasma which cause many complications like self -focussing, self -trapping and self-phase modulation which are undesirable. Increasing laser intensity without producing self-action effects is another field of future research. Density cannot be increased indefinitely, because after critical density laser cannot penetrate the plasma. Laser wavelength can be increased, because it does not harm our mechanism. So, increasing the laser wavelength, we can produce large axial magnetic field. This field enhances the energy transport from the critical surface to the ablation surface. Hence, the energy distribution in conduction region due to the magnetic field is important. The self- generated fields in both the directions may produce a magnetic cage to provide insulation in ICF scheme similar to spheromak.

In our case only 3.8% of the incident laser radiation is converted into induced magnetic field energy. This conversion should be improved for producing larger induced magnetic field. It might be interesting to investigate the possibility of increasing the conversion rate of Kull's mechanism.

3-A APPENDIX - First harmonic terms correct up to third order

Integrating (3.6.17) and (3.6.18) the expressions for the displacements of electron and ion can be written, by retaining only first harmonic terms correct up to third order, as the following

$$r_{e3} = i\left(\frac{c}{2\omega}\right)[\hat{x}\{R_{11}\alpha_{\parallel}^3 e^{i\theta_{\parallel}} + R_{12}(\alpha_{\perp}^2 - \beta_{\perp}^2)\alpha_{\parallel} e^{i(2\theta_{\perp}-\theta_{\parallel})} + R_{13}(\alpha_{\perp}^2 + \beta_{\perp}^2)\alpha_{\parallel} e^{i\theta_{\parallel}}\}$$

$$+\hat{y} R_{21}\alpha_{\parallel}^2\alpha_{\perp} e^{i(2\theta_{\parallel}-\theta_{\perp})} + R_{22}(\alpha_{\perp}^2 - \beta_{\perp}^2)\alpha_{\perp} e^{i\theta_{\perp}} + R_{23}\alpha_{\parallel}^2\alpha_{\perp} e^{i\theta_{\perp}}\}$$

$$-i\hat{z}\{R_{31}\alpha_{\parallel}^2\beta_{\perp} e^{i(2\theta_{\parallel}-\theta_{\perp})} + R_{32}(\alpha_{\perp}^2 - \beta_{\perp}^2)\beta_{\perp} e^{i(2\theta_{\perp}-\theta_{\perp})} + R_{33}\alpha_{\parallel}^2\beta_{\perp} e^{i\theta_{\perp}}\}] + \text{c.c.} \qquad (3.A\text{-}1)$$

and

$$r_{i3} = i\left(\frac{c}{2\omega}\right)[\hat{x}\{T_{11}\alpha_{\parallel}^3 e^{i\theta_{\parallel}} + T_{12}(\alpha_{\perp}^2 - \beta_{\perp}^2)\alpha_{\parallel} e^{i(2\theta_{\perp}-\theta_{\parallel})} + T_{13}(\alpha_{\perp}^2 + \beta_{\perp}^2)\alpha_{\parallel} e^{i\theta_{\parallel}}\}$$

$$+\hat{y}\{T_{21}\alpha_{\parallel}^2\alpha_{\perp} e^{i(2\theta_{\parallel}-\theta_{\perp})} + T_{22}(\alpha_{\perp}^2 - \beta_{\perp}^2)\alpha_{\perp} e^{i\theta_{\perp}} + T_{23}\alpha_{\parallel}^2\alpha_{\perp} e^{i\theta_{\perp}}\}$$

$$-i\hat{z}\{T_{31}\alpha_{\parallel}^2\beta_{\perp} e^{i(2\theta_{\parallel}-\theta_{\perp})} + T_{32}(\alpha_{\perp}^2 - \beta_{\perp}^2)\beta_{\perp} e^{i\theta_{\perp}} + T_{33}\alpha_{\parallel}^2\beta_{\perp} e^{i\theta_{\perp}}\}] + \text{c.c.} \qquad (3.A\text{-}2)$$

where $R_{11} = P_{11}/Q_{11}$, $R_{12} = P_{12}/Q_{12}$, $R_{13} = P_{13}/Q_{13}$

$$P_{11} = [A_{e1}\{n_{\parallel}^2(V_i^2/2) - 1 + X_i\} + X_e A_{i1} - M_e(\sigma_{11} + \Gamma_{11})\{n_{\parallel}^2(V_i^2/2) - 1\}]$$

$$P_{12} = [A_{e2}\{(2n_{\perp} - n_{\parallel})^2(V_i^2/2) - 1 + X_i\} + X_e A_{i2} - M_e(\sigma_{21} + \Gamma_{21})\{(2n_{\perp} - n_{\parallel})^2(V_i^2/2) - 1\}]$$

$$P_{13} = [A_{e3}\{n_{\perp}^2(V_i^2/2) - 1 + X_i\} + X_e A_{i3}]$$

$$Q_{11} = [\{n_{\parallel}^2(V_i^2/2) - 1\}\{n_{\parallel}^2(v_e^2/2) - 1\} + X_e\{n_{\parallel}^2(V_i^2/2) - 1\} + X_i\{n_{\parallel}^2(V_e^2/2) - 1\}]$$

$$Q_{12} = [\{n_\parallel^2(V_i^2/2)-1\}\{n_\parallel^2(V_e^2/2)-1\} +$$
$$X_e\{n_\parallel^2(V_i^2/2)-1\} + X_i\{n_\parallel^2(V_e^2/2)-1\}],$$
$$Q_{13} = [\{n_\parallel^2(V_i^2/2)-1\} + \{n_\parallel^2(V_e^2/2)-1\} +$$
$$X_e[\{n_\parallel^2(V_i^2/2)-1\} + X_i\{n_\parallel^2(V_e^2/2)-1\}],$$

and $R_{21} = P_{21}/Q_{21}$, $R_{22} = P_{22}/Q_{22}$, $R_{23} = P_{23}/Q_{23}$,

$$P_{21} = [M_e\sigma_{31}],\; P_{22} = [M_e\sigma_{41}] \quad ,$$
$$Q_{21} = [n_\parallel^2 - 1 + X_i + X_e],\; Q_{22} = [n_\perp^2 - 1 + X_i + X_e],$$
$$Q_{23} = [n_\perp^2 - 1 + X_i + X_e],$$
$$R_{31} = P_{31}/Q_{31},\quad R_{32} = P_{32}/Q_{32},\quad R_{33} = P_{33}/Q_{33},$$
$$P_{31} = P_{21},\; P_{32} = P_{22},\; P_{33} = -P_{23};\quad Q_{31} = Q_{21},\; Q_{32} = Q_{22},\; Q_{33} = Q_{23},$$

also we have

$$T_{11} = S_{11}/L_{11},\qquad T_{12} = S_{12}/L_{12},\qquad T_{13} = S_{13}/L_{13}$$
$$S_{11} = [A_{i1}\{n_\parallel^2(V_e^2/2)-1+X_e\} + X_iA_{e1} +$$
$$M_i(\sigma_{11}+\Gamma_{11})\{n_\parallel^2(V_e^2/2)-1\}],$$
$$S_{12} = [A_{i2}\{(2n_\perp-n_\parallel)^2(V_e^2/2)-1+X_e\} + X_iA_{e2} +$$
$$M_i(\sigma_{21}+\Gamma_{21})\{(2n_\perp-n_\parallel)^2(V_e^2/2)-1\}],$$
$$S_{13} = [A_{i3}\{n_\parallel^2(V_e^2/2)-1+X_e\} + X_iA_{e3}]$$
$$L_{11} = [\{n_\parallel^2(V_i^2/2)-1\}\{n_\parallel^2(V_e^2/2)-1\} +$$
$$X_e\{n_\parallel^2(V_i^2/2)-1\} + X_i\{n_\parallel^2(V_e^2/2)-1\}],$$
$$L_{12} = [\{(2n_\perp-n_\parallel)^2(V_i^2/2)-1\}\{(2n_\perp-n_\parallel)^2(V_e^2/2)-1\} +$$
$$X_e\{(2n_\perp-n_\parallel)^2(V_i^2/2)-1\} + X_i\{(2n_\perp-n_\parallel)^2(V_e^2/2)-1\}],$$

$$L_{13}=L_{11},\quad T_{21}=S_{21}/L_{21},\quad T_{22}=S_{22}/L_{22},\quad T_{23}=S_{23}/L_{23}\ ,$$

$$S_{21}=[M_i\sigma_{31}],\ S_{22}=[M_i\sigma_{41}],\quad S_{23}=[\rho_{i4}(n_\parallel^2-1+X_e)+X_i\rho_{e4}-M_i\Gamma_{31}],$$

$$L_{21}=[n_\parallel^2-1+X_i+X_e],\ L_{22}=[n_\perp^2-1+X_i+X_e],\quad L_{23}=[n_\perp^2-1+X_i+X_e],$$

$$T_{31}=S_{31}/L_{31},\quad T_{32}=S_{32}/L_{32},\quad T_{33}=S_{33}/L_{33},$$

$$S_{31}=S_{21},\ S_{32}=S_{22},\ S_{33}=-S_{23};\quad L_{31}=L_{21},\ L_{32}=L_{22},\ L_{33}=L_{23}.$$

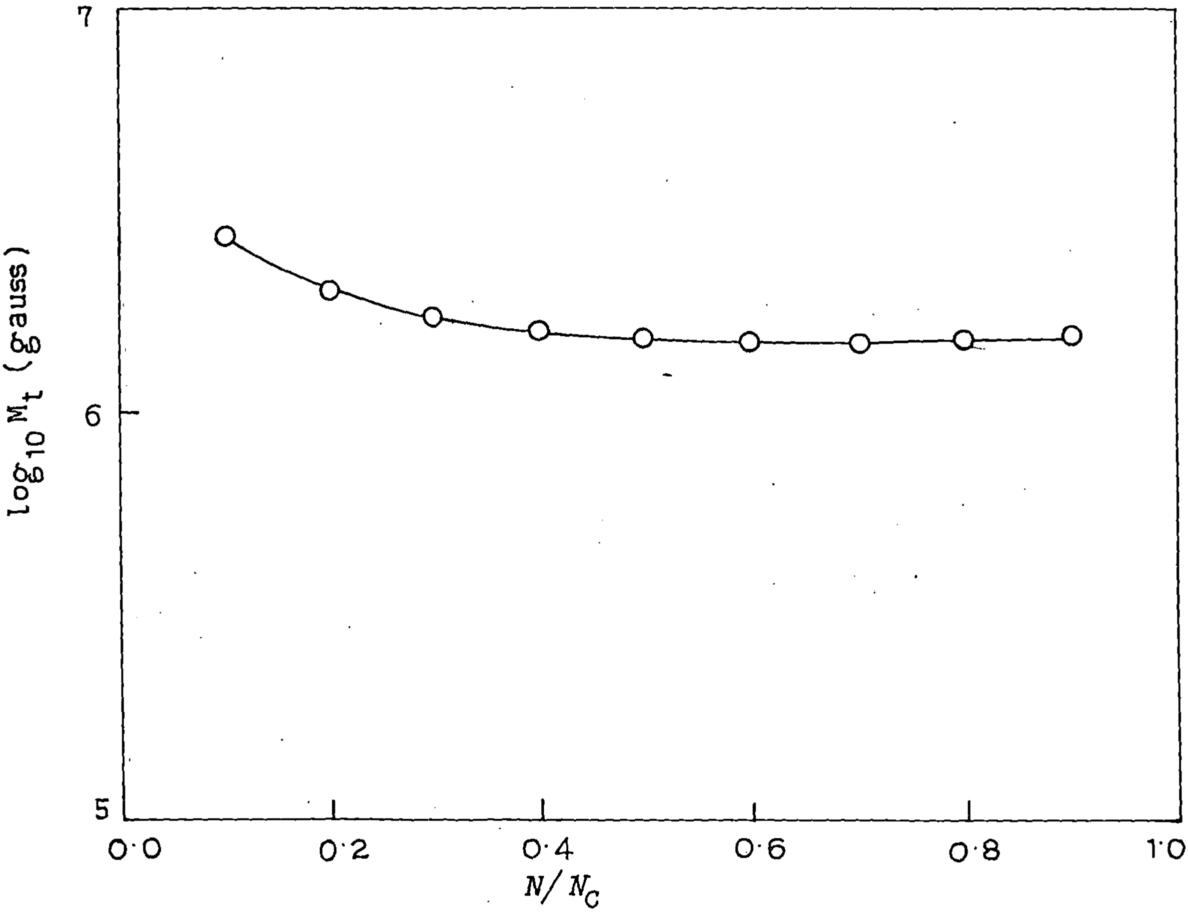

Fig. 3.1(a). The variation of the logarithmic values of the toroidal magnetic fields in gauss ($\log_{10} M_t$) with the density ratio (N/N_c); at $\tau = 5 nsec$, $\lambda = 1.06\ \mu m$, $I = 10^{15}\ W/cm^2$.

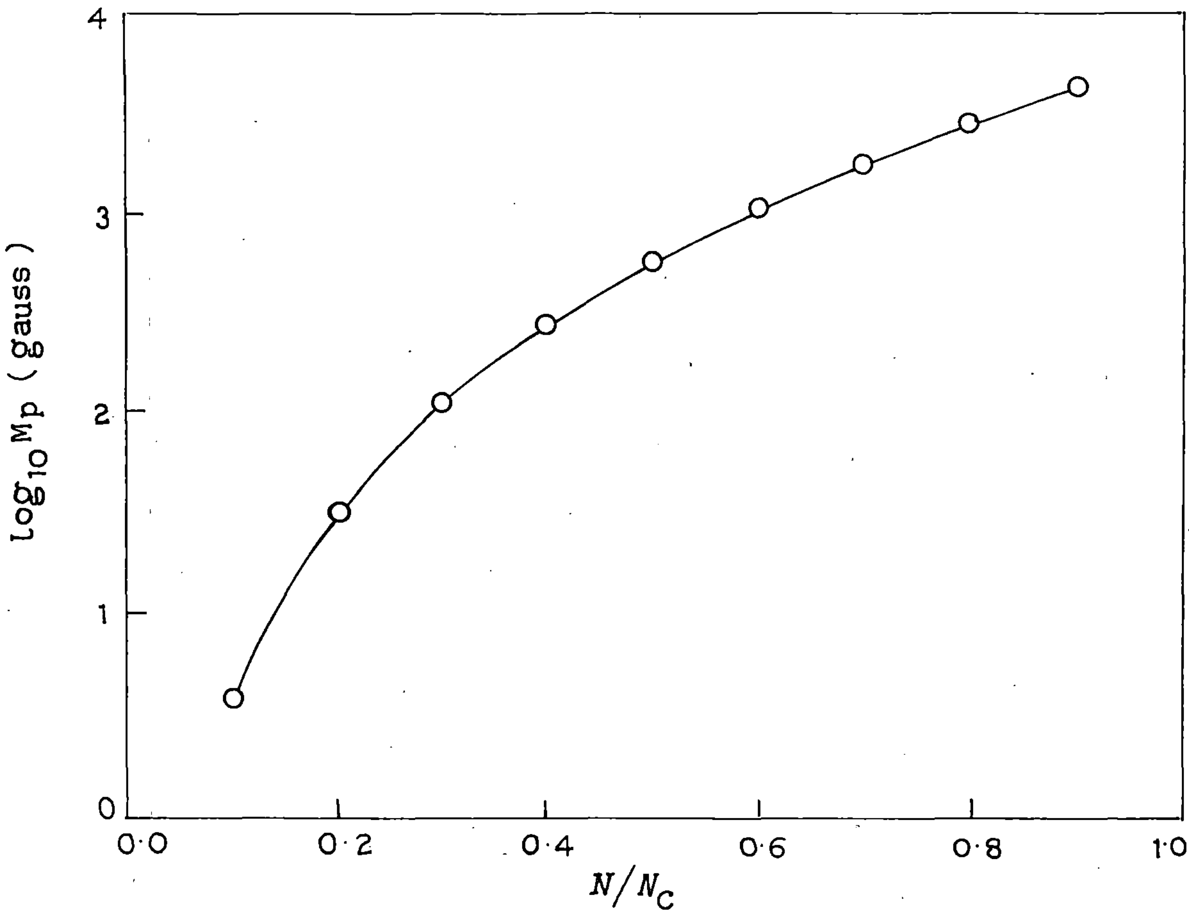

Fig. 3.1(b) The variation of the logarithmic values of the poloidal magnetic fields in gauss ($\log_{10} M_p$) with the density ratio (N/N_C); at $\tau = 5nsec$, $\lambda = 1.06\,\mu m$, $I = 10^{15}\ W/cm^2$.

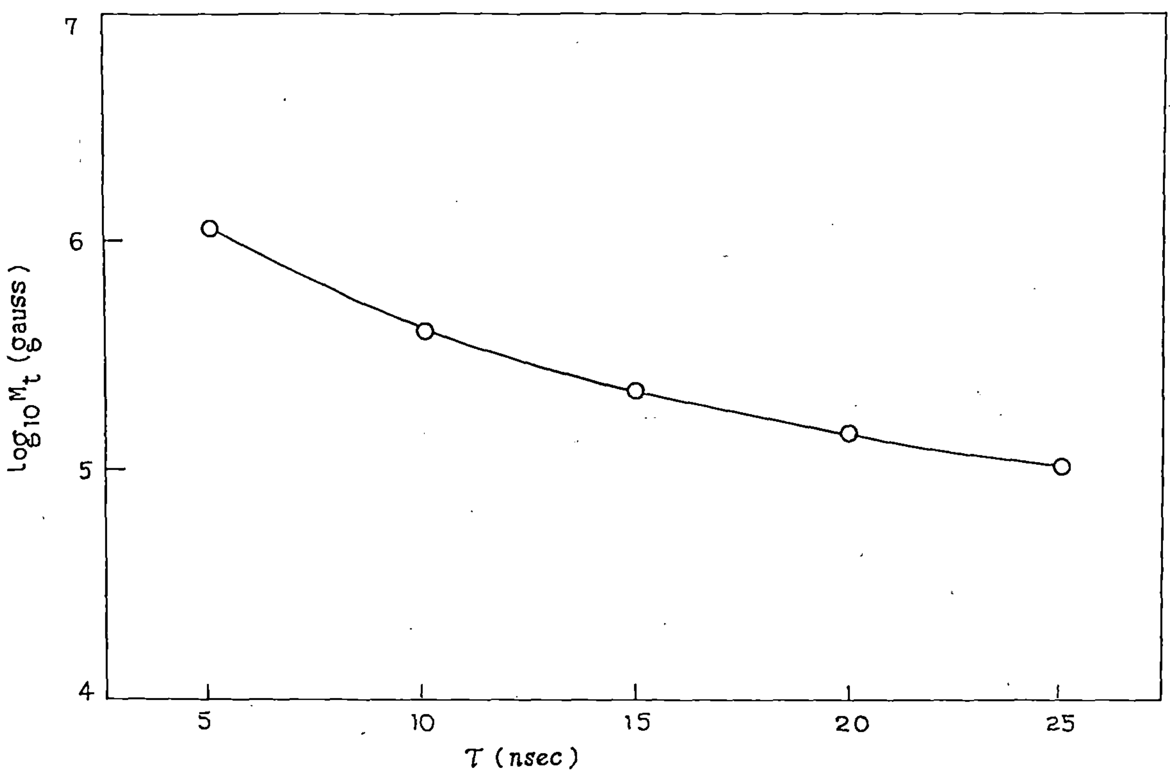

Fig. 3.2(a). The variation of the logarithmic values of the toroidal magnetic fields in gauss ($\log_{10} M_p$) with the pulse length (τ) in *nsec*; at $(N/N_c) = 0.3$, $\lambda = 1.06\,\mu m$, $I = 10^{15}\ W/cm^2$.

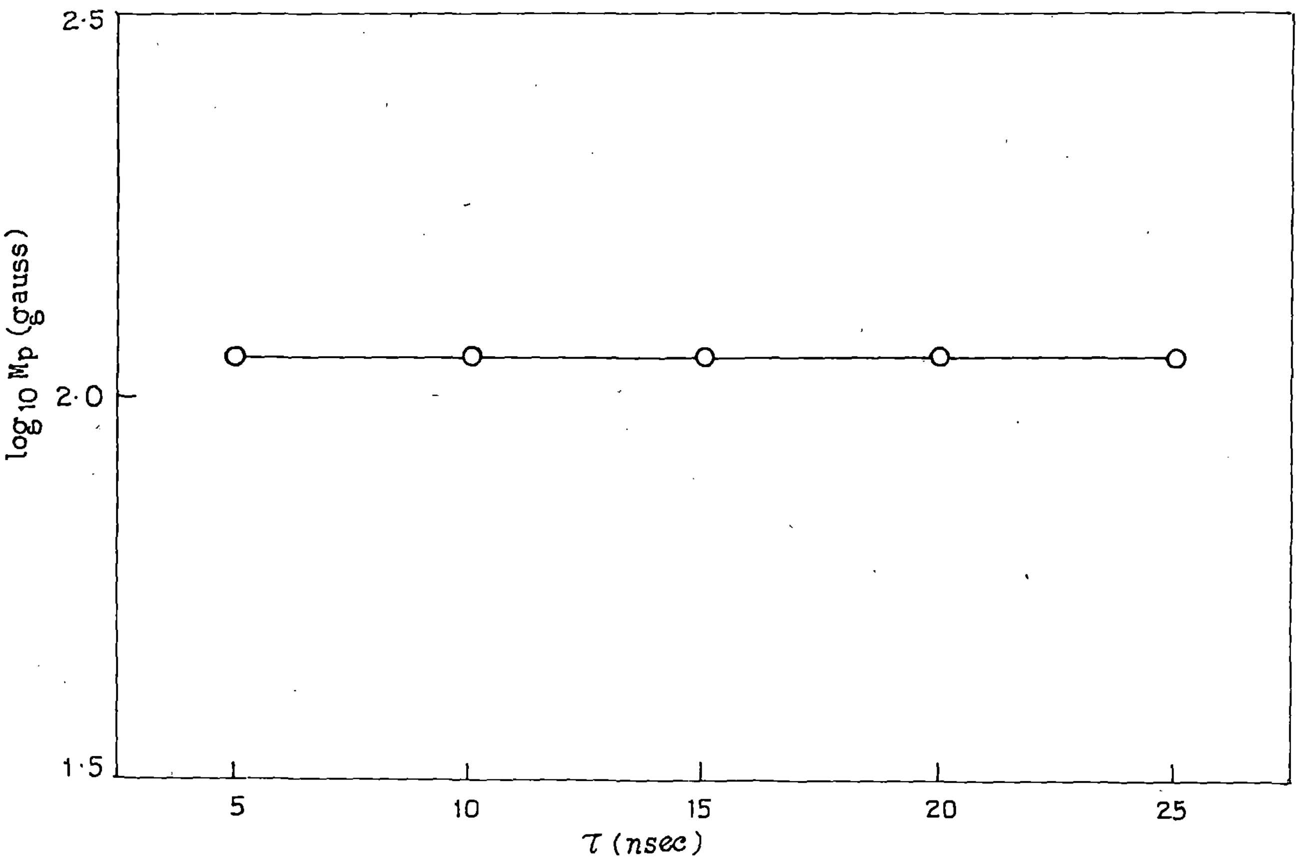

Fig. 3.2 (b). The variation of the logarithmic values of the poloidal magnetic fields in gauss ($\log_{10} M_p$) with pulse length (τ) in *nsec*; at $(N/N_c) = 0.3$, $\lambda = 1.06\ \mu m$, $I = 10^{15}\ W/cm^2$.

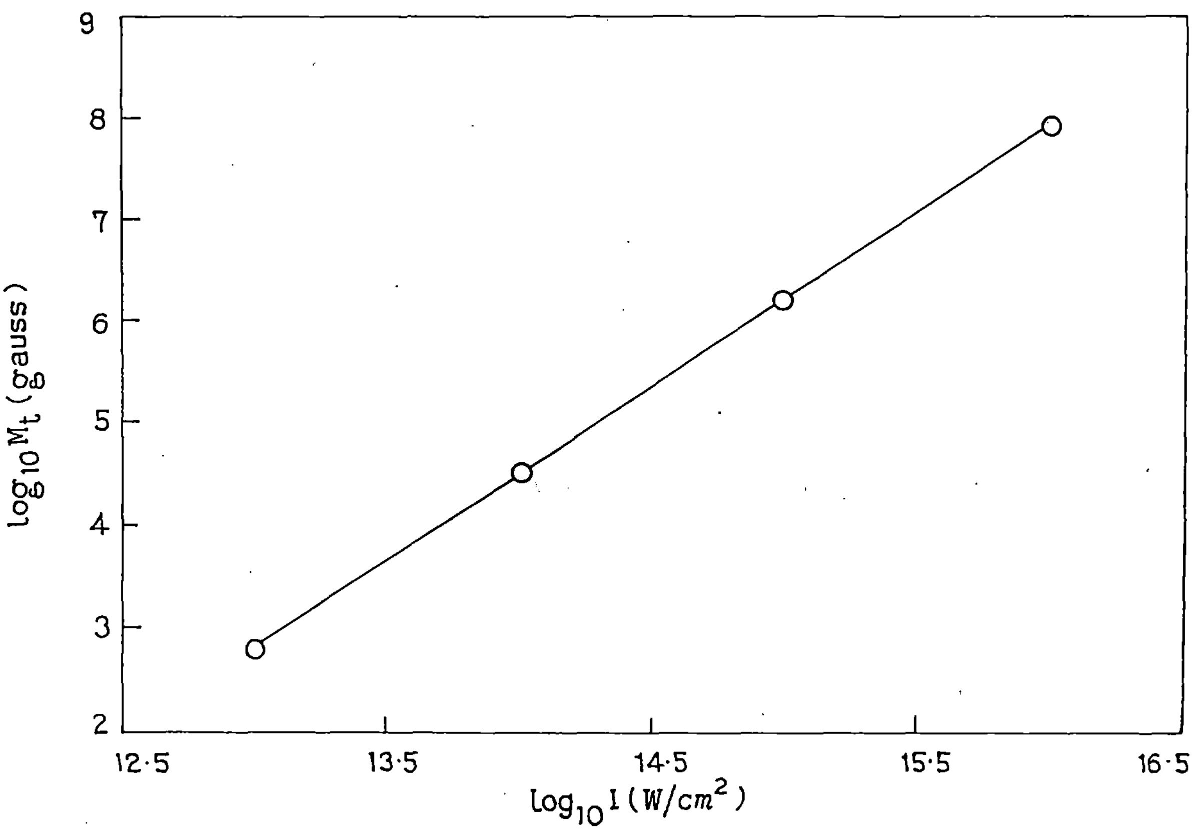

Fig. 3·3(a). The variation of the logarithmic values of the toroidal magnetic fields in gauss ($\log_{10} M_t$) with the logarithmic laser intensity (I) in W/cm^2; at τ=5 nsec, (N/N_c)=0·3, λ=1·06 μm .

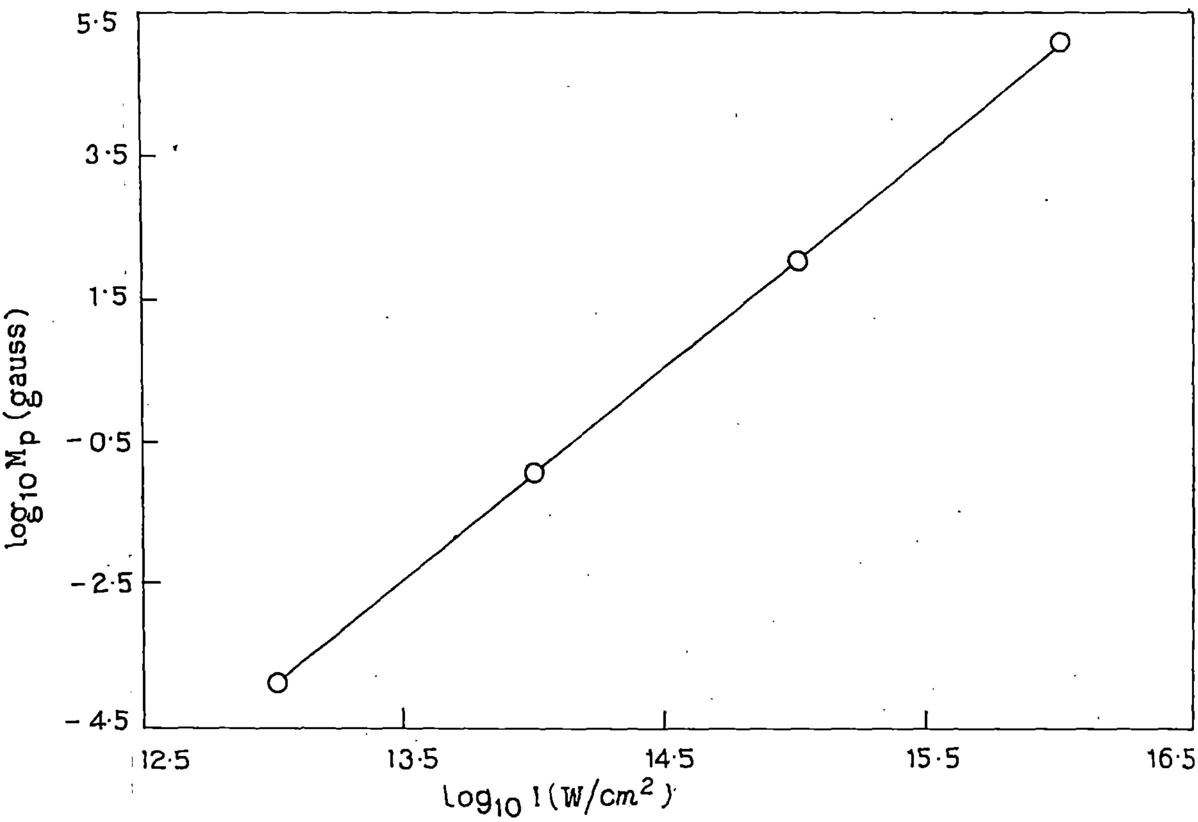

Fig. 3.3(b). The variation of the logarithmic values of the poloidal magnetic fields in gauss ($\log_{10} M_p$) with the logarithmic laser intensity (I) in W/cm^2; at $\tau = 5$ nsec, $(N/N_c) = 0.3$, $\lambda = 1.06\ \mu m^2$.

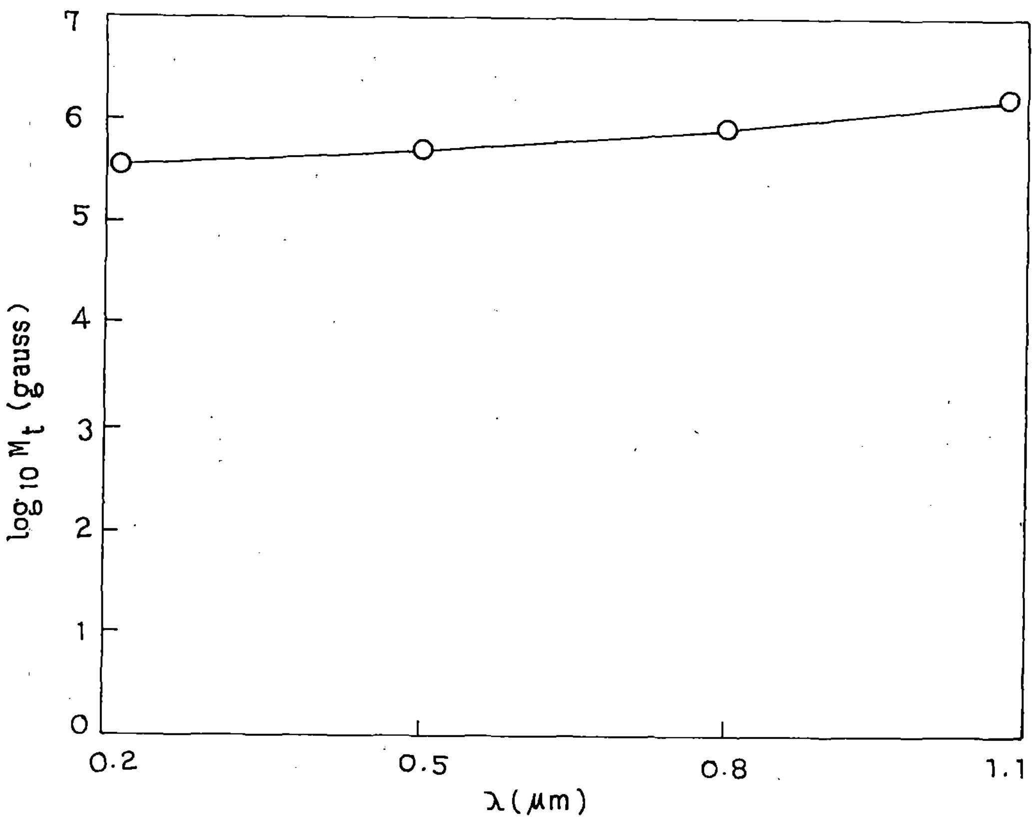

Fig. 3.4(a). The variation of the Logarithmic values of the toroidal magnetic fields in gauss ($\log_{10} M_t$) with the laser wavelength (λ) in μm : at $(N/N_c) = 0.3$, $\tau = 5$nsec, $I = 10^{15}$ W/cm^2 .

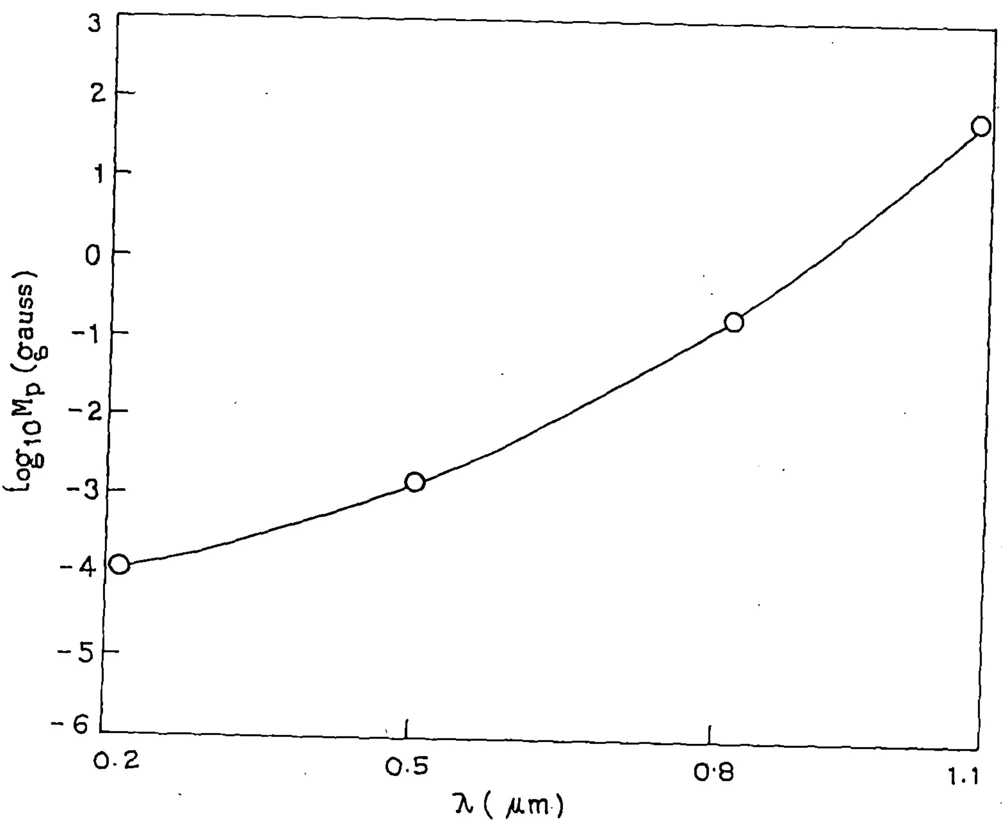

Fig. 3.4(b). The variation of the logarithmic values of the poloidal magnetic fields in gauss ($\log_{10} M_p$) with the laser wavelength (λ) in μm: at $(N/N_c) = 0.3$, $\tau = 5$nsec, $I = 10^{15}$ W/cm^2 .

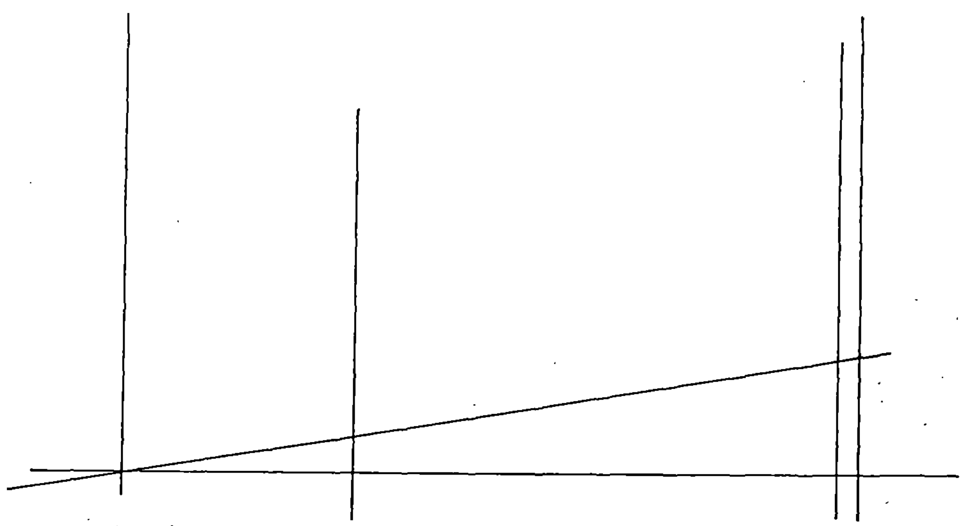

Fig. 3.5. A rough sketch for different regions out of the characteristic length (L)80μm