Chapter -2

An Analytical Study of Spontaneous Generation of Magnetic Fields in a Laser Produced Plasma *

2.1 Introduction

The generation of spontaneous magnetic field in laser-produced plasmas is an active area of research. Theoreticians and experimentalists are busy in finding out different mechanisms for producing self-generated magnetic fields in laser-plasma interaction problems because of its immense importance for designing targets in inertial confinement fusion (ICF). The spontaneous generation of lateral magnetic fields of different magnitudes are reported through different mechanisms in the laser produced plasmas. Similarly, the generation of axial magnetic fields is also reported through other different mechanisms. So, both the axial and lateral magnetic fields give evidence for their simultaneous occurrence in laser produced plasmas [Stamper (1991)]. But, so far, no such mechanism exists, other than the mechanism proposed by us [*], for evaluating the occurrence of simultaneous as well as spontaneous generation of axial and lateral magnetic fields in laser produced plasmas. In this Chapter, our motivation is to explain the mechanism, which was first reported by us [*], more clearly for spontaneous production of both the axial and lateral magnetic fields at a time in a very simple case for a CO_2 gas laser interacting with a one-component thermal plasma. The study for simultaneous as well as spontaneous generation of axial and lateral magnetic fields in the laser produced plasmas would be important to understand the energy transport and therefore, the implosion physics of the ICF. This axial field combined with the

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lateral field may allow the construction of a new hybrid type of tokamak [Hasegawa etal., (1986)]. Such magnetic fields due to many component plasmas interacting with different lasers are to be studied in the next Chapter.

2.2 Basic assumptions and equations

A one-component plasma means that the plasma contains electrons only. The mobility of ions is very low because of its heavy mass and so it provides the neutralizing, static, uniform background of positive charges [Kaw and Dawson (1970)]. Therefore, the equation of momentum transfer and conservation of charged particles for electrons would be taken for our study. This plasma is assumed to be hot but the thermal velocity ($v_{th} = \sqrt{\frac{2k_B T}{m}}$, where k_B is the Boltzmann constant, T is the electron temperature and m is the electron mass) of electron is small compared with the phase velocity ($v_\phi = \omega/k$, where ω and k represent the frequency and wave number respectively) of the laser field. We also assume that the Debye length ($\lambda_D = \sqrt{\frac{k_B T}{4\pi n_0 e}}$ where e is the electronic charge and n_0 is the background ion density) is small compared to the density scale length ($L = \frac{\nabla n}{n_0}$, where n is the electron density) of the plasma. High frequency and small Debye length allows us to assume that the laser produced plasma is a one-component fluid. Moreover, the plasma is assumed to be collisionless (i.e., the collision frequency (ν) is small relative to the wave frequency (ω)). The incident light wave can parametrically excite electromagnetic waves in the plasma leading to a stimulated scattering of the laser [Liu etal., (1974), Forsulund etal., (1975), Nishikawa and Liu (1976)]. Stimulated Brillouin scattering (SBS) involves parametric coupling of incident light with an ion acoustic wave and a back scattered electromagnetic wave. The stimulated Raman scattering (SRS) involves coupling of incident light wave with an electron plasma wave and a back scattered electromagnetic wave. The phenomena of SRS and SBS are omitted in our studies [Shen (1976) and Kruer (1988)] and hence the effects of their instabilities are ignored in our calculations. Laser intensity should not exceed the threshold power limit so that powerful electromagnetic waves (i.e., lasers) cannot change the form of amplitude. Hence, the self-action effects such as self-focussing, self-trapping etc., can be ignored [Max (1976), Shen (1976), Sun etal., (1987), Kruer (1988), Esarey etal., (1994)]. The width of the resonance layer is also assumed to be much less than the laser wavelength so that the inhomogeneity arises at the resonance layer can be ignored [Kull (1981, 1983)], and also the inhomogeneity due to Landau damping has been neglected because of exclusion of the regions of resonance and Landau damping in our calculations. The thickness of the resonance layer or conversion layer and the length of the Landau damping region are also estimated in Appendix 2.A for a given laser and plasma parameters, and subsequently, the region of interest for magnetic field generation is identified.

Under these assumptions, we start with the following basic equations:

The equation of continuity is:

$$\dot{n} + \nabla.(n\,\dot{\mathbf{r}}) = 0 \tag{2.2.1}$$

The equation of momentum is:

$$\ddot{\mathbf{r}} + (\dot{\mathbf{r}}.\nabla)\dot{\mathbf{r}} + \frac{e}{m}\mathbf{E} + \frac{e}{m}(\dot{\mathbf{r}}\times\mathbf{H}) + \frac{\nabla p}{mn} = 0 \tag{2.2.2}$$

Also the four Maxwell's equations are:

$$\nabla.\mathbf{E} = -4\pi e(n - n_0) \tag{2.2.3}$$

$$\nabla.\mathbf{H} = 0 \tag{2.2.4}$$

$$\nabla\times\mathbf{E} = -\frac{\dot{\mathbf{H}}}{c} \tag{2.2.5}$$

$$\nabla\times\mathbf{H} = \frac{\dot{\mathbf{E}}}{c} - (4\pi e/c)n\,\dot{\mathbf{r}} \tag{2.2.6}$$

where **E** and **H** are the electric and the magnetic fields respectively. $\dot{\mathbf{r}}$ is velocity of the electron of mass m and charge e, n and n_0 are the electron and background ion densities respectively, c is the velocity of light. The single dot and double dots represent the first and second order derivatives with respect to time t respectively.

The laser plasma interaction process is so fast that the process may be assumed to be isothermal. So, the equation of state can be written as

$$p = nk_B T \tag{2.2.7}$$

where p and T represent the plasma pressure and temperature respectively, and k_B is the Boltzmann's constant.

2.3 Perturbation scheme

The perturbation technique [Bellman (1964), Ames (1965)] is used to find out the solutions of these basic equations (2.2.1) to (2.2.7). In this technique, any variable ϕ can be expressed as

$$\phi = \phi_0+\epsilon\phi_1+\epsilon^2\phi_2+\epsilon^3\phi_3+\text{----------} \tag{2.3.1}$$

where ϕ_0 represents the value of ϕ in unperturbed state, the first order (or linear) approximation of ϕ gives it's linear (or first order) solution and bears the subscript 1. The nonlinearly excited fields of second and third order approximations bear the subscripts 2 and 3 respectively and so on. In this scheme, the nth order approximation ϕ_n (for n = 1, 2, 3, . . .) can be expressed as $\phi_r\phi_{n-r}$, $r \leq n$ and the condition for convergence is $(\phi_r/\phi_{n-r}) < 1$. The parameter ϵ is solely a mathematical artifice and that allows us to compare the degree of approximation in a convenient way. Using this perturbation approximation, all the physical variables used in the basic equations of (2.2.1) to (2.2.7) can be expressed as

$$\begin{aligned}
&\mathbf{E}=0+\epsilon\mathbf{E}_1+\epsilon^2\mathbf{E}_2+\epsilon^3\mathbf{E}_3+\ldots\ldots\ldots\\
&\mathbf{H}=0+\epsilon\mathbf{H}_1+\epsilon^2\mathbf{H}_3+\epsilon^3\mathbf{H}_3+\ldots\ldots\ldots\\
&\dot{\mathbf{r}}=0+\epsilon\dot{\mathbf{r}}_1+\epsilon^2\dot{\mathbf{r}}_2+\epsilon^3\dot{\mathbf{r}}_3+\ldots\ldots\ldots\\
&n=n_0+\epsilon n_1+\epsilon^2 n_2+\epsilon^3 n_3+\ldots\ldots\ldots\\
&T=T_0+\epsilon T_1+\epsilon^2 T_2+\epsilon^3 T_3+\ldots\ldots\ldots\\
&p=p_0+\epsilon p1+\epsilon^2 p_2+\epsilon^3 p_3+\ldots\ldots\ldots
\end{aligned} \tag{2.3.2}$$

where $\epsilon = (v_{osc}/c)$ is the ratio of the electron quiver velocity (v_{osc}) to the velocity of light c and is also known as an expansion parameter. Quiver velocity is an important parameter that characterizes the plasma's response to the incident laser because with this velocity the electron oscillates in the laser wave's electric field and is expressed as $v_{osc}=(eE/m\omega)$, where ω denotes wave frequency and e stands for the magnitude of the laser electric field **E.**

2.4 Linearized field equations and solutions

Let the linearized solution of electric field propagating in the direction of x-axis be

$$\mathbf{E}_1=(mc\omega/2e)[(\hat{x}\alpha_{\|}e^{i\theta_{\|}}+\hat{y}\alpha_{\perp}e^{i\theta_{\perp}}-i\hat{z}\beta_{\perp}e\theta_{\perp})]+c.c \quad (2.4.1)$$

where $\alpha_{\|}=(ea_{\|}/mc\omega)$, $\alpha_{\perp}=(ea_{\perp}/mc\omega)$, $\beta_{\perp}=(eb_{\perp}/mc\omega)$ are dimensionless amplitudes of the field in longitudinal (bearing the subscript $\|$) and transverse (bearing the subscript $\perp$) directions, $\theta_{\|} = k_{\|}x - \omega t$ and $\theta_{\perp}= k_{\perp}x - \omega t$ are phases in longitudinal and transverse directions respectively. $k_{\|}$ and $k_{\perp}$ are the wavenumbers of waves of frequency single ω in longitudinal and transverse directions respectively. Also $\hat{x}$, $\hat{y}$, $\hat{z}$ are the respective unit vectors along the three coordinate axes and c.c. represents their respective complex conjugates.

It is evident that the last two components arise directly from the laser field while the first component arises from the converted mode for thermal plasma [Kull (1981, 1983)]. In fact, the electromagnetic mode of p-polarized light can be converted into the electrostatic mode at the critical density n_c, when its electric vector oscillates along the direction of the density gradient, i.e., $\mathbf{E}.\nabla n \neq 0$. This effect is known as resonant absorption [Kruer (1987)], and also produces a magnetic field in a plasma [Thompson etal., (1977), Bezzerides etal., (1977), Speziale and Catto (1978), Mora and Pellat (1981)]. We exclude this effect in our calculation because our interest is to calculate the magnetic field in underdense plasma regions. In the laser produced plasmas, Kull [(1981), (1983)] has shown that the mode conversion is possible even in the underdense region if the plasma be treated as thermal . He has also pointed out that the width of the conversion layer plays an important role in such conversion. Thus, the amplitude of the electromagnetic mode of the laser light will be modified in thermal plasma. Hence, the form of the linearized electric field (2.4.1) is properly written. Further, we add the following three assumptions that lead to the clear physical idea of using the electric field of the form written in the equation (2.4.1).

(I) Let $(\nu/\omega)(L/\lambda_{ls}) \approx 0.01$, which gives $\Delta x << 1$, where the width of the resonance layer is denoted by $\Delta x = (\nu/\omega)L$. Here, ν, L, ω and λ_{ls} are the collision frequency, the density scale length, the frequency of the wave and the laser wavelength respectively. Hence, phenomena occurring at the resonance layer have been ignored.

(ii) The homogeneity due to Landau damping has also been ignored, since $k_{\parallel}\lambda_D < 1$, and $k_{\perp}/k_{\parallel} << 1$, where $k_{\parallel} = k_0\epsilon^{1/2}/\beta$ is the electrostatic wave number, $k_{\perp} = k_0\,\epsilon^{1/2}$ is the electromagnetic wavenumber and $k_0 = \omega/c$ is the vacuum wavenumber, the velocity ratio $\beta = v_{th}/c << 1$ and

the dielectric constant of the plasma $\epsilon = 1 - \omega_p^2/\omega^2$, where ω_p is the plasma frequency.

(iii) The laser wavelength λ_{ls} is greater than the electrostatic wavelength λ_{es}, and the thermal velocity v_{th} and the Debye length λ_D are small compared with the phase velocity v_ϕ of the radiation field and the density scale length L of the plasma respectively. The effect of plasma inhomogeneity may therefore be neglected.

Hence, using a typical value of the plasma temperature of 3 keV, the dimensionless amplitude of the electrostatic mode $\alpha_{\parallel}$ can easily be estimated, quantitatively, of the order of 10^{-3}. This leads us to conclude that about 3.8% of the laser light is to be converted here. It may also be mentioned that full conversion of laser light is possible, even in underdense region, through relativistic thermal effects [Kull (1983)]. Moreover, the justification for ignoring Landau damping has been explained in Appendix 2-A.

Applying perturbation technique, as stated in (2.3.2), in (2.2.1) to (2.2.7), we have the following first order equations for continuity as:

$$\dot{n}_1 + n_0(\nabla . \dot{\mathbf{r}}_1) = 0 \qquad (2.4.2),$$

and for momentum as

$$\ddot{\mathbf{r}}_1 + \frac{e}{m}\mathbf{E}_1 + \frac{\nabla p_1}{m n_0} = 0 \tag{2.4.3}$$,

and then for Maxwell's equations as

$$\nabla.\mathbf{E}_1 = -4\pi e\, n_1 \tag{2.4.4}$$

$$\nabla.\mathbf{H}_1 = 0 \tag{2.4.5}$$

$$\nabla\times\mathbf{E}_1 = -\frac{\dot{\mathbf{H}}_1}{c} \tag{2.4.6}$$

$$\nabla\times\mathbf{H}_1 = \frac{\dot{\mathbf{E}}_1}{c} - (4\pi e n_0 / c)\,\dot{\mathbf{r}}_1 \tag{2.4.7}$$

and finally for the equation of state as

$$p_1 = k_B T_0 n_1 + k_B n_0 T_1 \tag{2.4.8}$$

It is assumed that the laser plasma interaction process is isothermal. So, the perturbed temperature T_1 is to be treated as zero i.e., there is no change of temperature in plasma during interaction process. Hence, the first order equation of state (2.4.8) can be written as

$$p_1 = k_B T_0 n_1 \tag{2.4.9}$$

Multiplying the operator ∇ on both sides of the equation (2.4.9), we have

$$\nabla p_1 = k_B T_0 \nabla n_1 \tag{2.4.10}$$

Using (2.4.10) in (2.4.3), we have

$$\ddot{\mathbf{r}}_1 + \frac{e}{m}\mathbf{E}_1 + \left(\frac{K_B T_0}{m}\right)\left(\frac{\nabla n_1}{n_0}\right) = 0 \tag{2.4.11}$$

Using equation (2.4.1) in the equation (2.4.6), we have the solution of the first order magnetic field as

$$\mathbf{H}_1 = \frac{mc\omega}{2c}[\hat{y}(n_\perp \beta_\perp e^{i(k_\perp x-\omega t)} - \overline{n_\perp}\,\overline{\beta_\perp} e^{-i(\overline{k_\perp}x-\omega t)}) + \hat{z}(n_\perp \alpha_\perp e^{i(k_\perp x-\omega t)} - \overline{n_\perp}\,\overline{\alpha_\perp} e^{-i(\overline{k_\perp}x-\omega t)})] \quad (2.4.12)$$

From equation (2.4.4), we get the first order electron density (n_1) as

$$n_1 = -(in_0/2X)[n_\| \alpha_\| e^{i(k_\| x-\omega t)} - \overline{n_\|}\,\overline{\alpha_\|} e^{-i(\overline{k_\|}x-\omega t)})] \quad (2.4.13)$$

where, $X = \frac{\omega_p^2}{\omega^2}$, and $\omega_p = \sqrt{\frac{4\pi e^2 n_0}{m}}$ is the plasma frequency.

Taking the cross product of the operator ∇ with (2.4.12) and also using the time derivative of (2.4.1), the equation (2.4.7) can be solved easily. We have the first order velocity of electron ($\dot{\mathbf{r}}_1$) as

$$\dot{\mathbf{r}}_1 = \hat{i}x\{\frac{\alpha_\|}{X} e^{i(k_\| x-\omega t)} - \frac{\overline{\alpha_\|}}{X} e^{-i(\overline{k_\|}x-\omega t)}\} + i\hat{y}\{\alpha_\perp e^{i(k_\perp x-\omega t)} - \overline{\alpha_\perp} e^{-i(\overline{k_\perp}x-\omega t)}) + \hat{z}\{\beta_\perp e^{i(k_\perp x-\omega t)} + \overline{\beta_\perp} e^{-i(\overline{k_\perp}x-\omega t)}\} \quad (2.4.14)$$

2.5 Linear dispersion relation

Combining the equations (2.4.6) and (2.4.7),we get

$$-c^2\nabla\times\nabla\times\mathbf{E}_1 = \ddot{\mathbf{E}}_1 - 4\pi e n_0 \ddot{\mathbf{r}}_1 \quad (2.5.1)$$

Rewriting the equation (2.4.11), we have

$$\ddot{\mathbf{r}}_1 = -\frac{e}{m}\mathbf{E}_1 - \frac{v_{th}^2}{2}(\frac{\nabla n_1}{n_0}) \quad (2.5.2)$$

where $v_{th} = \sqrt{\frac{2k_B T_0}{m}}$

Combining the equations (2.5.1), (2.5.2) together with the equation (2.4.4), we have

$$\ddot{\mathbf{E}}_1 - c^2\nabla^2\mathbf{E}_1 + \omega_p^2\mathbf{E}_1 + (c^2 - \frac{v_{th}^2}{2})\nabla\nabla.\mathbf{E}_1 = 0 \tag{2.5.3}$$

where $\omega_p = \sqrt{\dfrac{2k_B T_0}{m}}$, the operator $\nabla\nabla$ = grad div. , $\nabla^2 = \nabla.\nabla$.

Using the equation (2.4.1) in (2.5.3) and then equating the coefficients of $e^{i\theta_\perp}$ for one of the transverse components [say, y component of (2.5.3)] we have

$$n_\perp^2 - 1 + X = 0 \tag{2.5.4}$$

and also taking the coefficients of $e^{i\theta_\parallel}$ for its longitudinal component, i.e., x-component of (2.5.3), we get

$$n_\parallel^2 V^2 - 1 + X = 0 \tag{2.5.5}$$

where $n_\perp = (k_\perp c/\omega)$, $n_\parallel = (k_\parallel c/\omega)$, and $V^2 = (v_{th}^2/2c^2)$

Equations (2.5.4) and (2.5.5) are known as the linearized dispersion relations for transverse and longitudinal modes respectively. It is to be noted that the dispersion relations (2.5.4) and (2.5.5) are independent from their wave amplitudes. So, there is no exchange of energy between the longitudinal and transverse modes. Hence, they are independent of each other in linear case. It may also be noted that the longitudinal mode depends on V, i.e., on the thermal velocity and consequently on plasma temperature. But, the transverse mode is independent of thermal velocity and hence of plasma temperature. Both the modes depend on plasma frequency (ω_p), that means they depend on the density of the plasma.

2.6 Second order equations and solutions

Using the perturbation approximations (2.3.2), we write, similarly, the second order equations for continuity, momentum and Maxwell as the following

$$\dot{n}_2 + n_0(\nabla.\dot{\mathbf{r}}_2) + \nabla.(n_1\dot{\mathbf{r}}_1) = 0 \tag{2.6.1}$$

$$\ddot{\mathbf{r}}_2 + (\dot{\mathbf{r}}_1.\nabla)\dot{\mathbf{r}}_1 + \frac{e}{m}\mathbf{E}_2 + \frac{e}{mc}(\dot{\mathbf{r}}_1 \times \mathbf{H}_1) + \frac{\nabla p_2}{m n_0} - \frac{n_1 \nabla p_1}{m n_0^2} = 0 \tag{2.6.2}$$

$$\nabla.\mathbf{E}_2 = -4\pi e\, n_2 \tag{2.6.3}$$

$$\nabla.\mathbf{H}_2 = 0 \tag{2.6.4}$$

$$\nabla\times\mathbf{E}_2 = -\frac{\dot{\mathbf{H}}_2}{c} \tag{2.6.5}$$

$$\nabla\times\mathbf{H}_2 = \frac{\dot{\mathbf{E}}_2}{c} - \frac{4\pi e n_0}{c}\dot{\mathbf{r}}_2 - \frac{4\pi e}{c} n_1 \dot{\mathbf{r}}_1 \tag{2.6.6}$$

The terms containing perturbed temperatures T_1 and T_2 in the second order equation of state are neglected because the process is treated as isothermal i.e., the change of temperature be zero. Hence, we have the second order equation of state as

$$p_2 = k_B T_0 n_2 \tag{2.6.7}$$

From (2.6.5) and (2.6.6) we have

$$(D^2 + c^2 \nabla\nabla. - c^2\nabla^2)\mathbf{E}_2 - 4\pi e n_0 D\dot{\mathbf{r}}_2 = \mathbf{SE}_2 \tag{2.6.8}$$

where $\mathbf{SE}_2 = 4\pi e D(n_1\dot{\mathbf{r}}_1)$, $D = \frac{\partial}{\partial t}$, $D^2 = \frac{\partial^2}{\partial t^2}$ and $\nabla\nabla. =$ grad div

Using the relations (2.4.9) and (2.6.7), the equation (2.6.2) can be written as

$$\ddot{\mathbf{r}}_2 + \frac{e}{m}\mathbf{E}_2 + \frac{v_{th}^2}{2n_0}\nabla n_2 = [-(\dot{\mathbf{r}}_1.\nabla)\dot{\mathbf{r}}_1 - \frac{e}{mc}(\dot{\mathbf{r}}_1 \times \mathbf{H}_1) + \frac{v_{th}^2}{2}(\frac{n_1 \nabla n_1}{n_0^2})] \tag{2.6.9}$$

Eliminating n_2 from (2.6.9), by using (2.6.1), we get

$$(D^2 - v_{th}^2/2\ \nabla\nabla.\ )\dot{\mathbf{r}}_2 + (e/m)D\mathbf{E}_2 = \mathbf{SR_2} \tag{2.6.10}$$

where $\mathbf{SR_2} = D\left[-(\dot{\mathbf{r}}_1.\nabla)\dot{\mathbf{r}}_1 - \frac{e}{m\ c}(\dot{\mathbf{r}}_1 \times \mathbf{H}_1) + \frac{v_{th}^2}{2n_0^2} n_1(\nabla n_1)\right] + \frac{v_{th}^2}{2n_0}\nabla\nabla.\ (n_1\ \dot{\mathbf{r}}_1)$

Solving equations (2.6.8) and (2.6.10), we have the second order electric field ($\mathbf{E_2}$) is, in one hand, as

$$\mathbf{E_2} = (1/\Delta 2)[\{D^2 - (v_{th}^2/2)\nabla\nabla.\ \}\mathbf{SE_2} + 4\pi e n_0 D\ \mathbf{SR_2}] \tag{2.6.11}$$

and, on the other hand, the second order electron velocity($\dot{\mathbf{r}}_2$) is

$$\dot{\mathbf{r}}_2 = (1/\Delta 2)[D^2 + c^2\nabla\nabla. - c^2\nabla^2)\mathbf{SR_2} - (e/m)\ D\mathbf{SE_2}] \tag{2.6.12}$$

where $\Delta 2$ denotes an operator and has the form

$$\Delta 2 = (D^2 + c^2\nabla\nabla. - c^2\nabla^2)(D^2 - v_{th}^2/2\ \nabla\nabla) + \omega_p^2 D^2$$

Simplifying (2.6.11), we get the expressions for second order field variables as

$$\mathbf{E_2} = i\left(\frac{mc\omega}{2e}\right)\Big[\hat{x}\Big\{P_\parallel \alpha_\parallel^2 e^{2i(k_\parallel x-\omega t)} - \overline{P}_\parallel \overline{\alpha}_\parallel^2 e^{-2i(\overline{k}_\parallel x-\omega t)} + P_\perp(\alpha_\perp^2 - \beta_\perp^2)e^{2i(k_\perp x-\omega t)} -$$

$$\overline{P}_\perp(\overline{\alpha}_\perp^2 - \overline{\beta}_\perp^2)e^{-2i(\overline{k}_\perp x-\omega t)}\Big\} + \hat{y}\Big\{P\alpha_\parallel\alpha_\perp e^{i(\overline{k}_\parallel + \overline{k}_\perp x - 2\omega t)} - \overline{P}\ \overline{\alpha}_\parallel\overline{\alpha}_\perp e^{-i(\overline{k}_\parallel + \overline{k}_\perp)x-\omega t)}\Big\}$$

$$- i\hat{z}\Big\{P\alpha_\parallel\beta_\perp e^{i(\overline{k}_\parallel + \overline{k}_\perp x - 2\omega t)} + \overline{P}\ \overline{\alpha}_\parallel\overline{\beta}_\perp e^{-i(\overline{k}_\parallel + \overline{k}_\perp)x-\omega t)}\Big\}\Big] \tag{2.6.13}$$

where $P_\parallel = \dfrac{-\left(\frac{n_\parallel}{X}\right)\{2(V^2 n_\parallel^2 - 1) - (V^2 n_\parallel^2 + 1)\}}{2\{4(V^2 n_\parallel^2 - 1) + X\}}$,

$P_\perp = \dfrac{n_\perp X}{2\{4(V^2 n_\perp^2 - 1) + X\}}$, and $$P = \frac{n_\parallel}{\{(n_\parallel + n_\perp)^2 - 4 + X\}},$$

also where, $\overline{P_\parallel}$, $\overline{P_\perp}$, and $\overline{P}$ are the complex conjugate of $P_\parallel$, $P_\perp$ and P respectively.

And also simplifying equation (2.6.11) we get the second order electron velocity

$$\dot{\mathbf{r}}_2 = \left(\frac{c}{2}\right)\Big\{\hat{x}\,[Q_\parallel \alpha_\parallel^2 e^{2i(k_\parallel x-\omega t)} + \overline{Q_\parallel}\,\overline{\alpha_\parallel^2}\, e^{-2i(\overline{k_\parallel} x-\omega t)}) + Q_\perp(\alpha_\perp^2 - \beta_\perp^2)e^{2i(k_\perp x-\omega t)} +$$

$$\overline{Q_\perp}(\overline{\alpha_\perp^2} - \overline{\beta_\perp^2})e^{-2i(\overline{k_\perp} x-\omega t)}] + \hat{y}\,[Q\alpha_\parallel \alpha_\perp e^{i(\overline{k_\parallel + k_\perp} x-2\omega t)} + \overline{Q}\,\overline{\alpha_\parallel}\,\overline{\alpha_\perp}\, e^{-i(\overline{k_\parallel}+\overline{k_\perp})x-\omega t)}]$$

$$- i\hat{z}\,[Q\alpha_\parallel \beta_\perp e^{i(\overline{k_\parallel + k_\perp} x-2\omega t)} - \overline{Q}\,\overline{\alpha_\parallel}\,\overline{\beta_\perp}\, e^{-i(\overline{k_\parallel}+\overline{k_\perp})x-\omega t)}]\Big\} \qquad (2.6.14)$$

where $$Q_\parallel = \left(\frac{n_\parallel}{X}\right)\frac{\{(\frac{2}{X})(V^2 n_\parallel^2 + 1)\}}{2\{4(V^2 n_\parallel^2 - 1) + X\}},$$

$$Q_\perp = \frac{n_\perp}{\{4(V^2 n_\perp^2 - 1) + X\}},$$

and $$Q = \frac{n_\parallel}{2\{(n_\parallel + n_\perp)^2 - 4 + X\}}.$$

also where, $\overline{Q_\parallel}$, $\overline{Q_\perp}$, and $\overline{Q}$ are the complex conjugate of $Q_\parallel$, $Q_\perp$, and Qrespectively.

Solving (2.6.5) together with (2.6.13), we get the second order magnetic field($\mathbf{H}_2$) as

$$\mathbf{H}_2 = -(mc\omega/2e)\,\hat{y}\,[(S\alpha_\parallel \beta_\perp e^{i(\overline{k_\parallel} + \overline{k_\perp} x-2\omega t)} + \overline{S}\,\overline{\alpha_\parallel}\,\overline{\beta_\perp}\, e^{-i(\overline{k_\parallel}+\overline{k_\perp})x-\omega t)}]$$

$$[i\hat{z}\,\{S\alpha_\parallel \alpha_\perp e^{i(\overline{k_\parallel} + \overline{k_\perp} x-2\omega t)} - \overline{S}\,\overline{\alpha_\parallel}\,\overline{\alpha_\perp}\, e^{-i(\overline{k_\parallel}+\overline{k_\perp})x-\omega t)}\}] \qquad (2.6.15)$$

where $S = P\frac{(n_{\|}+n_{\perp})}{2}$ also where $\overline{S}$ are the complex conjugate of S.

Solving (2.6.3) along with (2.6.13), we get the second order electron density(n_2) as

$$n_2=(n_0/2)[\, S_{\|}\,\alpha_{\|}^2 e^{2i(k_{\|}x-\omega t)}+\overline{S}_{\|}\,\overline{\alpha}_{\|}^2 e^{-2i(\overline{k}_{\|}x-\omega t)}+S_{\perp}(\alpha_{\perp}^2-\beta_{\perp}^2)e^{2i(k_{\perp}x-\omega t)}+\overline{S}_{\perp}(\overline{\alpha}_{\perp}^2-\overline{\beta}_{\perp}^2)e^{-2i(\overline{k}_{\perp}x-\omega t)}] \tag{2.6.16}$$

where $S_{\|} = \frac{2P_{\|}n_{\|}}{X}$, and $S_{\perp} = \frac{2P_{\perp}n_{\perp}}{X}$.

also where, $\overline{S_{\|}}$ and $\overline{S_{\perp}}$ are the complex conjugate of $S_{\|}$ and $S_{\perp}$ respectively.

It is evident that the linearized solutions (stated in section 2.4) contain the first harmonic terms. But, the second order solutions given above contribute only second harmonic terms. Hence, in absence of the first harmonic terms the nonlinearly excited second order fields do not contribute d.c.magnetic field. Moreover, in our calculations, we have ignored plasma inhomogeneity, field fluctuations and collisional effects. Thus, second order solenoidal wave fields do not generate any d.c. magnetic field. Hence, the nonlinearly excited third order fields with secular free first harmonic solution correct upto third order [Chiao and Godine (1964), Chakraborty etal., (1984)] are to be explored for possible magnetic field generation in our studies here.

2.7. Third order equations and solutions

Similar to the secs. 2.5 and 2.6 of this chapter, we have third order equations of continuity for electron as

$$\dot{n}_3+n_0(\nabla.\dot{\mathbf{r}}_3)=-\nabla.(n_1\dot{\mathbf{r}}_2)-\nabla.(n_2\dot{\mathbf{r}}_1) \tag{2.7.1}$$

and the same order equation of momentum as

$$\ddot{\mathbf{r}}_3+\frac{e}{m}\mathbf{E}_3+\frac{V_{th}^2}{2n_0}\nabla n_3=-(\dot{\mathbf{r}}_2.\nabla)\dot{\mathbf{r}}_1-(\dot{\mathbf{r}}_1.\nabla)\dot{\mathbf{r}}_2-\frac{e}{m}\frac{}{c}(\dot{\mathbf{r}}_1\times\mathbf{H}_2)-\frac{e}{m}\frac{}{c}(\dot{\mathbf{r}}_2\times\mathbf{H}_1)-$$

$$\frac{1}{mn_0^3}n_1^2\nabla p_1+\frac{1}{mn_0^2}n_1\nabla p_2+\frac{1}{mn_0^2}n_2\nabla p_1 \tag{2.7.2}$$

and also the set of Maxwell's equations of third order is as follows

$$\nabla.\mathbf{E}_3 = -4\pi e\, n_3 \tag{2.7.3}$$

$$\nabla.\mathbf{H}_3 = 0 \tag{2.7.4}$$

$$\nabla\times\mathbf{E}_3 = -\frac{\dot{\mathbf{H}}_3}{c} \tag{2.7.5}$$

$$\nabla\times\mathbf{H}_3 = \frac{\dot{\mathbf{E}}_3}{c} - (4\pi e n_0/c)\,\dot{\mathbf{r}}_3 - (4\pi e/c)\,(n_1\dot{\mathbf{r}}_2 + n_2\dot{\mathbf{r}}_1) \tag{2.7.6}$$

Finally, the third order equation of state can be written in a modified form as

$$p3 = k_B T_0 n_3 \tag{2.7.7}$$

The perturbed temperatures T_1, T_2, and T_3 lead to zero values because our interest is to study the isothermal process only.

From the above equations (2.7.5) and (2.7.6), we get

$$(D^2 + c^2\nabla\nabla. - c^2\nabla^2)\mathbf{E}_3 - 4\pi e n_0 D\,\dot{\mathbf{r}}_3 = \mathbf{SE}_3 \tag{2.7.8}$$

where $\mathbf{SE}_3 = 4\pi e D\,(n_1\dot{\mathbf{r}}_2 + n_2\dot{\mathbf{r}}_1)$ is the source of nonlinear plasma current.

Using (2.4.10), (2.6.7), and (2.7.7) in the equation (2.7.2) can be rewritten as

$$\ddot{\mathbf{r}}_3 + \frac{e}{m}\mathbf{E}_3 + \frac{v_{th}^2}{2n_0}\nabla n_3 = -(\dot{\mathbf{r}}_2.\nabla)\dot{\mathbf{r}}_1 - (\dot{\mathbf{r}}_1.\nabla)\dot{\mathbf{r}}_2 - \frac{e}{m\,c}(\dot{\mathbf{r}}_1\times\mathbf{H}_2) - \frac{e}{m\,c}(\dot{\mathbf{r}}_2\times\mathbf{H}_1) - \frac{v_{th}^2}{2n_0^3}n_1^2\nabla n_1 + \frac{v_{th}^2}{2n_0^2}n_1\nabla n_2 + \frac{v_{th}^2}{2n_0^2}n_2\nabla n_1 \tag{2.7.9}$$

Now, eliminating n_3 from the equations (2.7.8) and (2.7.1)we have

$$\{D^2 - (v_{th}^2/2)\nabla\nabla.\}\,\dot{\mathbf{r}}_3 + \frac{e}{m}\dot{\mathbf{E}}_3 = \mathbf{SR}_3 \tag{2.7.10}$$

where,

$$\mathbf{SR}_3 = D[(-\dot{\mathbf{r}}_2\nabla)\dot{\mathbf{r}}_1 - (\dot{\mathbf{r}}_1.\nabla)\dot{\mathbf{r}}_2 - \frac{e}{m\,c}(\dot{\mathbf{r}}_1\times\mathbf{H}_2) - \frac{e}{m\,c}(\dot{\mathbf{r}}_2\times\mathbf{H}_1) - \frac{v_{th}^2}{2n_0^3}n_1^2\nabla n_1 + \frac{v_{th}^2}{2n_0^2}n_1\nabla n_2 + \frac{v_{th}^2}{2n_0^2}n_2\nabla n_1.$$

The first two terms of $\mathbf{SR_3}$ are the sources of nonlinear convective derivative of momentum, the third and fourth terms of $\mathbf{SR_3}$ are due to Lorentz forces but, the last three terms of the same are the nonlinear sources arising out of the pressure gradient.

Solving these two equations (2.7.8) and (2.7.10), we get the expression for third order electron velocity as

$$\dot{\mathbf{r}}_3=(1/\Delta 3)[\{D^2+c^2\ \nabla\nabla\,.\ -c^2\nabla^2\}(\mathbf{SR_3})-(e/m)\,D(\mathbf{SE_3})] \tag{2.7.11}$$

where, the operator $\Delta 3=(D^2+c^2\ \nabla\nabla\,.\ -c^2\nabla^2)\ \{D^2-(v_{th}{}^2/2)\ \nabla\nabla\,.\ \}+\omega_p{}^2D^2]$.

Simplifying all the terms of (2.7.11), we get the third order electron velocity as

$$\begin{aligned}
\dot{\mathbf{r}}_3 = \frac{ic}{2}[\hat{x}(&\Gamma_{11}\, \alpha_\parallel^2 \bar{\alpha}_\parallel e^{i[(2k_\parallel-\bar{k}_\parallel)x-\omega t]} - \bar{\Gamma}_{11}\, \bar{\alpha}_\parallel^2 \alpha_\parallel e^{-i[(2\bar{k}_\parallel-k_\parallel)x-\omega t]} \\
&+\Gamma_{12}\,(\alpha_\perp^2-\beta_\perp^2)\bar{\alpha}_\parallel e^{i[(2k_\perp-\bar{k}_\parallel)x-\omega t]} - \bar{\Gamma}_{12}\,(\bar{\alpha}_\perp^2-\bar{\beta}_\perp^2)\alpha_\parallel e^{-i[(2\bar{k}_\perp-k_\parallel)x-\omega t]} \\
&+\left\{\Gamma_{13}\,\alpha_\parallel e^{i[(k_\parallel+k_\perp-\bar{k}_\parallel)x-\omega t]} - \bar{\Gamma}_{13}\,\bar{\alpha}_\parallel e^{-i[(\bar{k}_\parallel+\bar{k}_\perp-k_\perp)x-\omega t]}\right\}(\alpha_\perp\bar{\alpha}_\perp+\beta_\perp\bar{\beta}_\perp) \\
+\hat{y}(&\Gamma_{21}\,\bar{\alpha}_\parallel\alpha_\parallel\alpha_\perp e^{i[(k_\parallel+k_\perp-\bar{k}_\parallel)x-\omega t]} - \bar{\Gamma}_{21}\,\alpha_\parallel\bar{\alpha}_\parallel\bar{\alpha}_\perp e^{-i[(\bar{k}_\parallel+\bar{k}_\perp-k_\parallel)x-\omega t]} \\
&+\Gamma_{22}\,\alpha_\parallel^2\bar{\alpha}_\perp e^{i[(2k_\parallel-\bar{k}_\perp)x-\omega t]} - \bar{\Gamma}_{22}\,\bar{\alpha}_\parallel^2\alpha_\perp e^{-i[(2\bar{k}_\parallel-k_\perp-k_\parallel)x-\omega t]} \\
&+\Gamma_{23}\,(\alpha_\perp^2-\beta_\perp^2)\bar{\alpha}_\perp e^{i[(2k_\perp-\bar{k}_\perp)x-\omega t]} - \bar{\Gamma}_{23}\,(\bar{\alpha}_\perp^2-\bar{\beta}_\perp^2)\alpha_\perp e^{-i[(2\bar{k}_\perp-k_\perp)x-\omega t]}) \\
-i\,\hat{z}(&\Gamma_{31}\,\bar{\alpha}_\parallel\alpha_\parallel\beta_\perp e^{i[(k_\parallel+k_\perp-\bar{k}_\parallel)x-\omega t]} + \bar{\Gamma}_{31}\,\alpha_\parallel\bar{\alpha}_\parallel\bar{\beta}_\perp e^{-i[(\bar{k}_\parallel+\bar{k}_\perp-k_\parallel)x-\omega t]} \\
&+\Gamma_{32}\,\alpha_\parallel^2\bar{\beta}_\perp e^{i[(2k_\parallel-\bar{k}_\perp)x-\omega t]} + \bar{\Gamma}_{32}\,\bar{\alpha}_\parallel^2\beta_\perp e^{-i[(2\bar{k}_\parallel-k_\perp-k_\parallel)x-\omega t]} \\
&+\Gamma_{33}\,(\alpha_\perp^2-\beta_\perp^2)\bar{\beta}_\perp e^{i[(2k_\perp-\bar{k}_\perp)x-\omega t]} + \bar{\Gamma}_{33}\,(\bar{\alpha}_\perp^2-\bar{\beta}_\perp^2)\beta_\perp e^{-i[(2\bar{k}_\perp-k_\perp)x-\omega t]})]
\end{aligned}$$

$$+\ \text{all other third harmonic terms} \tag{2.7.12}$$

where $$\Gamma_{11}=\frac{(\tau_{11}+\sigma_{11})}{\{(2n_{\parallel}-\bar{n}_{\parallel})^2V^2-1+X\}},$$

$$\Gamma_{12}=\frac{(\tau_{12}+\sigma_{12})}{\{(2n_{\perp}-\bar{n}_{\parallel})^2V^2-1+X\}},$$

$$\Gamma_{13}=\frac{\tau_{13}}{\{(n_{\parallel}+n_{\perp}-\bar{n}_{\perp})^2V^2-1+X\}},$$

$$\Gamma_{21}=\frac{[-\tau_{14}\{(n_{\parallel}+n_{\perp}-\bar{n}_{\parallel})^2-1\}+\sigma_{13}]}{\{(n_{\parallel}+n_{\perp}-\bar{n}_{\parallel})^2-1+X\}},$$

$$\Gamma_{22}=\frac{\sigma_{14}}{\{(2n_{\parallel}-\bar{n}_{\perp})^2-1+X\}},$$

$$\Gamma_{23}=\frac{\sigma_{15}}{\{(2n_{\perp}-\bar{n}_{\perp})^2-1+X\}},$$

$$\Gamma_{31}=\frac{[-\tau_{14}\{(n_{\parallel}+n_{\perp}-\bar{n}_{\parallel})^2-1\}-\sigma_{13}]}{\{(n_{\parallel}+n_{\perp}-\bar{n}_{\parallel})^2-1+X\}}.$$

$$\Gamma_{32}=-\Gamma_{22},\quad \Gamma_{33}=-\Gamma_{23},$$

$$\tau_{11}=\frac{Q_{\parallel}n_{\parallel}}{X}-\frac{Q_{\parallel}\bar{n}_{\parallel}}{2X}-\frac{V^2}{4X^3}(2n_{\parallel}-\bar{n}_{\parallel})n_{\parallel}^2\bar{n}_{\parallel}-\frac{V^2}{X}S_{\parallel}n_{\parallel}\bar{n}_{\parallel}+\frac{V^2}{2X}S_{\parallel}\bar{n}_{\parallel}^2+\frac{V^2}{2X}(Q_{\parallel}\bar{n}_{\parallel}+S_{\parallel})(2n_{\parallel}-\bar{n}_{\parallel})^2,$$

$$\tau_{12}=\frac{Q_{\perp}n_{\perp}}{X}-\frac{Q_{\perp}\bar{n}_{\parallel}}{2X}+\frac{V^2}{X}S_{\perp}\bar{n}_{\parallel}^2-\frac{V^2}{X}S_{\perp}n_{\perp}\bar{n}_{\parallel}+\frac{V^2}{2X}(Q_{\perp}\bar{n}_{\parallel}+S_{\perp})(2n_{\perp}-\bar{n}_{\parallel})^2,$$

$$\tau_{13}=\frac{S}{2}-\frac{Q\bar{n}_{\perp}}{2},\quad \tau_{14}=\frac{Q(n_{\parallel}+n_{\perp})}{2X}-\frac{S}{2X},$$

$$\sigma_{11}=\frac{1}{2}(Q_{\parallel}\bar{n}_{\parallel}+S_{\parallel}),\qquad \sigma_{12}=\frac{1}{2}(Q_{\perp}\bar{n}_{\parallel}+S_{\perp}),\qquad \sigma_{13}=\frac{\bar{Q}_{\parallel}r}{2},\quad \sigma_{14}=\frac{S_{\parallel}X}{2},\ \sigma_{15}=\frac{S_{\perp}X}{2},$$

It may be pointed out that third order electron velocity is combination of first as well as third harmonic terms. These third harmonics have a very weak nuclear response [Chiao and Godine (1964) and Chakraborty etal., (1984)] in the nonlinear phenomena. So, we neglect all third harmonic terms in our subsequent studies for the angular momentum of electrons. Since, the plasma is nondissipative (i.e., the collision frequency ν is very much less than the wave frequency ω), we then drop the bar from our calculations and writing them in terms of trigonometric functions, the simplified form of electron velocity with first harmonic terms correct upto third order(i.e., retaining only the first harmonic terms and neglecting third harmonic terms as stated above) can be written as

$$\dot{\mathbf{r}}_3 = \hat{x}\dot{r}_{3x} + \hat{y}\dot{r}_{3y} + \hat{z}\dot{r}_{3z}$$

$$= \hat{x}[c\{\Gamma_{11}\alpha_{\|}^3 \mathrm{Sin}\ \theta_{\|} + \Gamma_{12}(\alpha_{\perp}^2 - \beta_{\perp}^2)\alpha_{\|}\mathrm{Sin}\ (2\theta_{\perp} - \theta_{\|}) + \Gamma_{13}(\alpha_{\perp}^2 + \beta_{\perp}^2)\alpha_{\|}\mathrm{Sin}\ \theta_{\|}]$$

$$+ \hat{y}[\Gamma_{21}\alpha_{\|}^2\alpha_{\perp}\mathrm{Sin}\ \theta_{\perp} + \Gamma_{22}\alpha_{\|}^2\alpha_{\perp}\mathrm{Sin}\ (2\theta_{\|} - \theta_{\perp}) + \Gamma_{23}(\alpha_{\perp}^2 - \beta_{\perp}^2)\alpha_{\perp}\mathrm{Sin}\ \theta_{\perp}]$$

$$+ \hat{z}[-\{\Gamma_{31}\ \alpha_{\|}^2\beta_{\perp}\mathrm{Cos}\ \theta_{\perp} + \Gamma_{32}\alpha_{\|}^2\beta_{\perp}\mathrm{Cos}\ (2\theta_{\|} - \theta_{\perp}) + \Gamma_{33}(\alpha_{\perp}^2 - \beta_{\perp}^2)\beta_{\perp}\mathrm{Cos}\ \theta_{\perp}\}] \qquad (2.7.13)$$

Integrating the equation (2.7.13, we get the electron displacement in terms of first harmonic , correct upto the third order as

$$\mathbf{r}_3 = \hat{x}r_{3x} + \hat{y}r_{3y} + \hat{z}r_{3z}$$

$$= \hat{x}[(c/\omega)\{\Gamma_{11}\alpha_{\|}^3\ \mathrm{Cos}\ \theta_{\|} + \Gamma_{12}(\alpha_{\perp}^2 - \beta_{\perp}^2)\alpha_{\|}\mathrm{Cos}\ (2\theta_{\perp} - \theta_{\|}) + \Gamma_{13}(\alpha_{\perp}^2 + \beta_{\perp}^2)\alpha_{\|}\ \mathrm{Cos}\ \theta_{\|}]$$

$$+ \hat{y}[\Gamma_{21}\alpha_{\|}^2\alpha_{\perp}\mathrm{Cos}\ \theta_{\perp} + \Gamma_{22}\alpha_{\|}^2\alpha_{\perp}\mathrm{Cos}\ (2\theta_{\|} - \theta_{\perp}) + \Gamma_{23}(\alpha_{\perp}^2 - \beta_{\perp}^2)\alpha_{\perp}\mathrm{Cos}\ \theta_{\perp}]$$

$$+ \hat{z}[-\{\Gamma_{31}\ \alpha_{\|}^2\beta_{\perp}\mathrm{Sin}\ \theta_{\perp} + \Gamma_{32}\alpha_{\|}^2\beta_{\perp}\mathrm{Sin}\ (2\theta_{\|} - \theta_{\perp}) + \Gamma_{33}(\alpha_{\perp}^2 - \beta_{\perp}^2)\beta_{\perp}\ \mathrm{Sin}\ \theta_{\perp}\}] \qquad (2.7.12)$$

2.8 Angular momentum and magnetization

We know that the angular momentum can be expressed as

$$\mathbf{L} = (2c/e)\ \boldsymbol{\mu}, \qquad (2.8.1)$$

where $\mu = (1/2c)(\mathbf{r}\times\mathbf{j})$ (2.8.2)

The relation (2.8.2)is known as nonlinear angular momentum of electron motion. We drop here the subscript 3 from this and subsequent calculations. Also, **j** represents the electron current density and can be expressed as

$$\mathbf{j} = -e\,\dot{\mathbf{r}} \tag{2.8.3}$$

Hence, combining (2.8.2) and (2.8.3), we have the nonlinear angular momentum as

$$\mathbf{L} = -\,(\mathbf{r}\times\dot{\mathbf{r}}) \tag{2.8.4}$$

Putting the values of **r** and $\dot{\mathbf{r}}$, from the equations (2.7.11) and (2.7.12) respectively, in the equation (2.8.4), we have

$$\mathbf{L} = \hat{x}L_x + \hat{y}L_y + \hat{z}L_z$$
$$= \hat{x}(r_y\dot{r}_z - \dot{r}_y r_z) + \hat{y}(r_z\dot{r}_x - \dot{r}_z r_x) + \hat{z}(r_x\dot{r}_y - \dot{r}_x r_y). \tag{2.8.5}$$

Therefore, the nonlinear induced magnetization can be written as

$$\mathbf{M} = (4\pi e n_0/c)\,\mathbf{L}. \tag{2.8.6}$$

If we make average of angular momentum **L** of electrons over fast frequency time scale $(2\pi/\omega)$, we get average value of **L** as $\langle L\rangle$.

It is evident that the x-component of angular momentum is $(4\pi e n_0/c)\langle L_x\rangle$ that gives rise to the average axial or poloidal magnetic field in the direction of wave propagation as $\langle M_p\rangle$. So,

$$\langle M_p\rangle = \frac{4\pi e n_0}{c}\langle L_x\rangle \tag{2.8.7}$$

where

$$\langle L_x\rangle = r_y\dot{r}_z - \dot{r}_y r_z$$
$$= -(c^2/\omega)[\Gamma_{31}\alpha_\parallel^2 + \Gamma_{32}\alpha_\parallel^2 + \Gamma_{33}(\alpha_\perp^2 - \beta_\perp^2)].\ [\Gamma_{21}\alpha_\parallel^2 + \Gamma_{22}\ \alpha_\parallel^2 + \Gamma_{23}(\alpha_\perp^2 - \beta_\perp^2)]\ \alpha_\perp\beta_\perp,$$

The resultant of the y-component $\langle L_y\rangle$ and the z-component $\langle L_z\rangle$ of angular momentum **L** gives rise to the value of the nonlinear angular momentum in a plane perpendicular to the direction of a laser beam. This resultant angular momentum produces the toroidal (lateral) magnetic field which is denoted by $\langle M_t \rangle$. So,

$$\langle M_t \rangle = \frac{4\pi e n_0}{c} \sqrt{\langle L_y \rangle^2 + \langle L_z \rangle^2} \tag{2.8.8}$$

where

$$\begin{aligned} \langle L_y \rangle &= r_z \dot{r}_x - \dot{r}_z r_x \\ &= (c^2/\omega)[\Gamma_{31}\alpha_\parallel^2 + \Gamma_{32}\alpha_\parallel^2 + \Gamma_{33}(\alpha_\perp^2 - \beta_\perp^2)].\ [\Gamma_{11}\alpha_\parallel^2 + \Gamma_{12}(\alpha_\perp^2 - \beta_\perp^2) + \Gamma_{13}(\alpha_\perp^2 + \beta_\perp^2)]\ \alpha_\parallel \beta_\perp, \end{aligned}$$

and

$$\begin{aligned} \langle L_z \rangle &= r_x \dot{r}_y - \dot{r}_x r_y \\ &= -(c^2/\omega)[\Gamma_{21}\alpha_\parallel^2 + \Gamma_{22}\alpha_\parallel^2 + \Gamma_{23}(\alpha_\perp^2 - \beta_\perp^2)].\ [\Gamma_{11}\alpha_\parallel^2 + \Gamma_{12}(\alpha_\perp^2 - \beta_\perp^2) + \Gamma_{13}(\alpha_\perp^2 + \beta_\perp^2)]\ \alpha_\parallel \alpha_\perp. \end{aligned}$$

It is observed that without applying any external magnetic field , we get finite M_p and M_t values in the interaction of the laser with the plasma. This magnetic field must have generated within the system. So, our mechanism produces both the axial and lateral fields spontaneously and simultaneously. This mechanism is a direct process to get induced magnetic fields, because here the field is calculated from the nonlinear angular momentum via nonlinear electron velocity and displacement.

2.9 A simple numerical example

For a simple numerical estimation, we have taken a CO_2 laser of wavelength 10.6 μm, pulse length 5 nsec and power flux 5×10^{14} W/cm², with a plasma of temperature 3keV and density 0.5 N_c (N_c=critical density) for a spot radius of 80μm. These data have been chosen arbitrarily from the available literature. Hence, quantitatively, the parameters are $\alpha_\parallel$=0.00109, $\alpha_\perp$=0.154, $\beta_\perp$=0.077,

$V^2 = v_{th}^2/(2c^2) = 0.0421$, $X = \omega_p^2/\omega^2 = 0.492$.

The magnitude of the axial magnetic field is found out to be

$M_p = \langle M_x \rangle = 560$ gauss. (2.9.1)

The direction of the axial magnetic field lines here will be away from the target, because the field value M_p is positive. Similarly, the magnitude of the lateral magnetic fields

$M_t = (<M_y>^2 + <M_z>^2)^{1/2} = 450$ KG (2.9.2)

It follows that the lateral magnetic field is much greater than the axial field. Hence, the former dominates over the latter in laser plasmas. More results have also been elucidated with illustrations in the next section.

2.10 Results and discussions

Numerical estimations of the lateral and axial magnetic fields have been obtained for a laser intensity of 5×10^{14} W cm^{-2} and a thermal power flux of $5(1+z)nkT(\Delta R/2\tau)$ W/cm^2 for a one-component plasma of temperature 3keV and density 0.5 N_c (where N_c is the critical density), where z= effective ion charge for a copper target, kT = electron temperature in eV, τ= laser pulse length in nsec, n= density of plasma in cm^{-3} and ΔR = spot radius in μm. Numerical results show that axial magnetic field increases with plasma density which is shown in fig. 2.1(a) . So, this field must have the maximum value at the critical density surface which is the future scope of experimentalist to prove it experimentally or by simulation. The variation of toroidal field with the laser intensity is shown in fig. 2.2(b) which shows that the maximum value of the lateral field is well below the critical density surface, which is consistent with experimental results [Stamper etal., (1971), Raven etal., (1979)] and also with numerical results [Boyd etal., (1982)].

Kull [1981, 1983] has shown that the mode conversion is possible even in the underdense region for thermal plasmas and width of the conversion layer plays an important role in such mode conversion. Then amplitude of the laser light is modified in a thermal plasma. Hence, our assumption of linear electric field of equation (2.4.1) is justified.

The electromagnetic mode of a p- polarized laser can be converted to the electrostatic mode at critical density when its electric vector oscillates along the direction of density gradient. This effect is known as resonance absorption [Kruer(1987)]. Resonance absorption gives rise to magnetic field in a laser produced plasma [Thompson etal., (1975), Bezzerides etal., (1977), Woo etal., (1978), Speziale and Catto (1978), Kolodner etal., (1979), Mora and Pellat (1981)]. We consider an underdense plasma to estimate the magnitude of the magnetic fields. So, our mechanism is different from resonance absorption. However, this mechanism plays an important role in production of magnetic fields in laser plasmas only when the effect of the resonance layer [Kull (1981)] is taken into our studies.

Our results are consequences of the inverse Faraday effect (IFE) [Pomeau and Quemada (1967), Stieger and Woods (1972), Talin etal.,(1975), Abdullaev and Frolov (1981)] because in an IFE process the kinetic energy of the ordered motion of particles in the presence of an electromagnetic wave is transformed into the energy of the induced magnetic field. The field generation mechanism in our case is a direct process, because to calculate the induced magnetic fields, we have calculated the average nonlinear angular momentum of electrons via the nonlinear electron velocity and its displacement, whereas the IFE is an indirect process of field generation. At high frequencies, the IFE is also relevant over time scales shorter than twice the oscillation period of the driving wave field, beyond which the wave becomes unstable [Stenflo (1977)].

Dynamo [Parker (1979), Dragila (1987), Zeldovich etal., (1983)] effect gives rise to the axial and the lateral fields in a cyclic manner, but not simultaneously. But our method is to calculate both the fields generated simultaneously. So, it is different from dynamo effect.

Field generated by our method is also different from that due to the thermoelectric (i.e., $\nabla N \times \nabla T$) effect [Stamper etal., (1971), Stamper (1991)], because the temperature gradient in the plasma region of interest has been ignored. One may consider a very long density scale length and uniform temperature when the beam is absorbed in the plasma region, which may be the case in future ICF targets.

The electrostatic mode (i.e., the wake field) generation in underdense plasma is also of current interest at the advent of ultra short pulse lasers because such field has an important role to produced a plasma-based accelerator [Gorbunov etal., (1996), Dubinov etal., (1996), Esarey etal., (1997)]. Such wake field is also important for the production of magnetic fields in laser plasmas. All these will be studied elsewhere.

The electrons move along the magnetic field and get trapped in a layer of the order of Larmor radius. So, the lateral field enhances lateral energy transport but degrades axial energy propagation [Mead etal., (1984), Max etal., (1978), Bernstain etal., (1978), Nuckolls (1974), Max (1982)]. Hence, phenomenologically, it should be stated that the axial field enhances axial energy transport and degrades lateral energy transport. Therefore, the rate of energy deposition in conduction regions would increase by the axial field which enhances the energy transport from a critical surface to an ablation surface. But our results, and also the figs. 2.1(a) and 2.1(b), dictates that the lateral field dominates over the axial field in laser plasma interactions. Hence, both the fields should have great impact on uniform compression of ICF targets. Therefore, it is the time to study thoroughly to understand which field inhibits and which one accelerates energy transport processes for effective implosion and also to study to what extent these fields are important in ICF research.

It may be speculated that the combination of toroidal and poloidal fields set up by the laser may lead to the formation of a magnetic cage [Eliezer etal., (1992)] which could be used for plasma confinement in a manner similar to tokamaks, toroidal pinches etc. The configuration would be sustained by the laser beams and may also be heated by them [Hasegawa etal., (1988)].

2.A. APPENDIX : Electrostatic wave and Landau damping [Kull (1981 and 1983)]

It is fact that the fluid description neglects Landau damping of an electrostatic wave in laser plasmas, but in Vlasov-Maxwell theory, it can be shown easily from the dispersion relation that the Landau damping exists outside the resonance layer (Δx) and the distance over which the electrostatic wave can freely propagate in a reasonable order of magnitude. By taking the dispersion relation [Krall and Trivelpiece (1973)]# in the following form

$$D(\omega,k) \equiv 1 - \Sigma\,(\frac{\omega_{pe,i}^2}{k^2})\int \frac{(\partial F_{e,i}/\partial u)}{u-\omega/k} du = 0 \tag{2.A-1}$$

all symbols have their usual meanings [Krall and Trivelpiece (1973)].

Generally, (2.A:1) deals with complex number such that

$$D(\omega,k) = D_r(\omega,k) + iD_i(\omega,k) \tag{2.A-2}$$

where the subscripts r and i represent the real and imaginary part of the dispersion relation (2.A-1) respectively. Let the complex wavenumber and complex frequency be

$$k = k_r + ik_i \quad \text{and} \quad \omega = \omega_r + \omega_i \tag{2.A-3}$$

Assuming the imaginary parts to be small i.e.

$$\omega_i/\omega_r \ll 1 \quad \text{and} \quad k_i/k_r \ll 1 \tag{2.A-4}$$

and expanding (2.A-2) and then separating the real and imaginary parts we get

$$D_r(\omega_r,k_r) = 0 \tag{2. A-5}$$

$$\partial_{\omega_i} D_r(\omega_r,k_r)\omega_i + \partial_{k_r} D_r(\omega_r,k_r)k_i + D_i(\omega_r,k_r) = 0 \tag{2.A-6}$$

where

$$D_r(\omega,k) \equiv 1 - \Sigma\,(\frac{\omega_{pe,i}^2}{k^2})\int \frac{(\partial F_{e,i}/\partial u)}{u-\omega_r/k_r} du \tag{2.A-7}$$

c

$$D_i(\omega,k) = -\pi\Sigma\left(\frac{\omega_{pe,i}^2}{k^2}\right)\left(\frac{\partial F_{e,i}(\omega_r/k_r)}{\partial u}\right) \qquad (2.A\text{-}8)$$

where c is the Landau contour [Krall and Trivelpiece (1973)].

For mode conversion analysis we take the frequency as

$$\omega_r = \omega; \quad \omega_i = 0 \qquad (2.A\text{-}9)$$

and also assuming the distribution function is Maxwellian then we have

$$f_{e,i} = \Sigma\left(\frac{m_{e,i}}{2\pi K T_{e,i}}\right)^{3/2}\exp\left(-\frac{m_{e,i}u^2}{2KT_{e,i}}\right) \qquad (2.A\text{-}10)$$

and neglecting ion terms, which are small by m_e/m_i, the integral of (2.A-7) may be evaluated explicitly for fluid approximation as

$$D_r = \epsilon - \frac{v_{the}^2\, k_r^2}{\omega^2} \qquad (2.A\text{-}11)$$

$$D_i = 3^{3/2}(\pi/2)^{1/2}\epsilon^{-3/2}\exp\left(-\frac{3}{2}\frac{1}{\epsilon}\right) \qquad (2.A\text{-}12)$$

where (2.A-11) is the real dispersion relation which follows directly from the equation (2.5.5), if the ion distribution is dropped, and (2.A-12) gives the dispersion relation for imaginary effect.

One then readily obtains the expression for real wavenumber as

$$k_r = \frac{k_0\sqrt{\epsilon}}{V_e} \qquad (2A\text{-}13)$$

and for imaginary wavenumber as

$$k_i = \frac{-1}{V_e\sqrt{\epsilon}}\omega_{pe}\sqrt{\frac{\pi}{8}}\left[\left(\frac{3}{\epsilon}\right)\exp\left(-\frac{1}{\epsilon}\right)\right]^{3/2} \qquad (2.A\text{-}14)$$

where k_0 is the vacuum wavelength and $\epsilon = 1 - \frac{\omega_{pe}^2}{\omega^2}$ and $V_e = v_{the}/c$.

Therefore, the wave amplitude then decreases by the factor $\exp(-\psi)$ where the damping argument ψ is of the form

$$\psi = \int K_i \, dz \tag{2.A-15}$$

For a linear profile $\epsilon = z/L$, we have the damping argument as

$$\psi = \sqrt{\frac{3}{2}\pi(\frac{k_0 L}{V_e})}\exp\left[-\frac{3}{2}\frac{1}{\epsilon}\right] \tag{2.A-16}$$

which is to be a linear function of $(\frac{k_0 L}{V_e})$ only when the density is fixed.

Moreover, for the same profile we have the following

$$\int k_r dz = (\frac{2}{3})\xi^{3/2} \tag{2.A-17}$$

Let n be the number of the electrostatic wavelength, which can be expressed easily as

$$n = (1/3\pi)\xi^{3/2} \tag{2.A-18}$$

For typical numerical estimation as used in sec.2.9; we have the thickness of the resonance layer $\Delta x \approx 0.8\mu m$ and the number of electrostatic wavelength be $n \approx 2$. Hence, for the linear profile $\epsilon \approx 0.2$: the length of the region of Landau damping be approximately $24\,\mu m$ microns whereas the magnetic field region would be $55\ \mu m$. Hence, our result would be modified if plasma inhomogeneity due to resonance and Landau damping effects are included i.e. if the entire region of plasma are considered. All these will be studied elsewhere.

The equation (2.A-1) is similar to the equation (8.3.11) if the electrostatic case be retained only and the function $F_{e,i}$ be the integral of the distribution function of $f_{e,i}$ defined in Krall and Trivelpiece (1973). Moreover, the equations (8.6.11) and (8.6.12) may be expressed as (2.A-7) and (2.A-8) respectively for real (ω_r) and real (k_r).

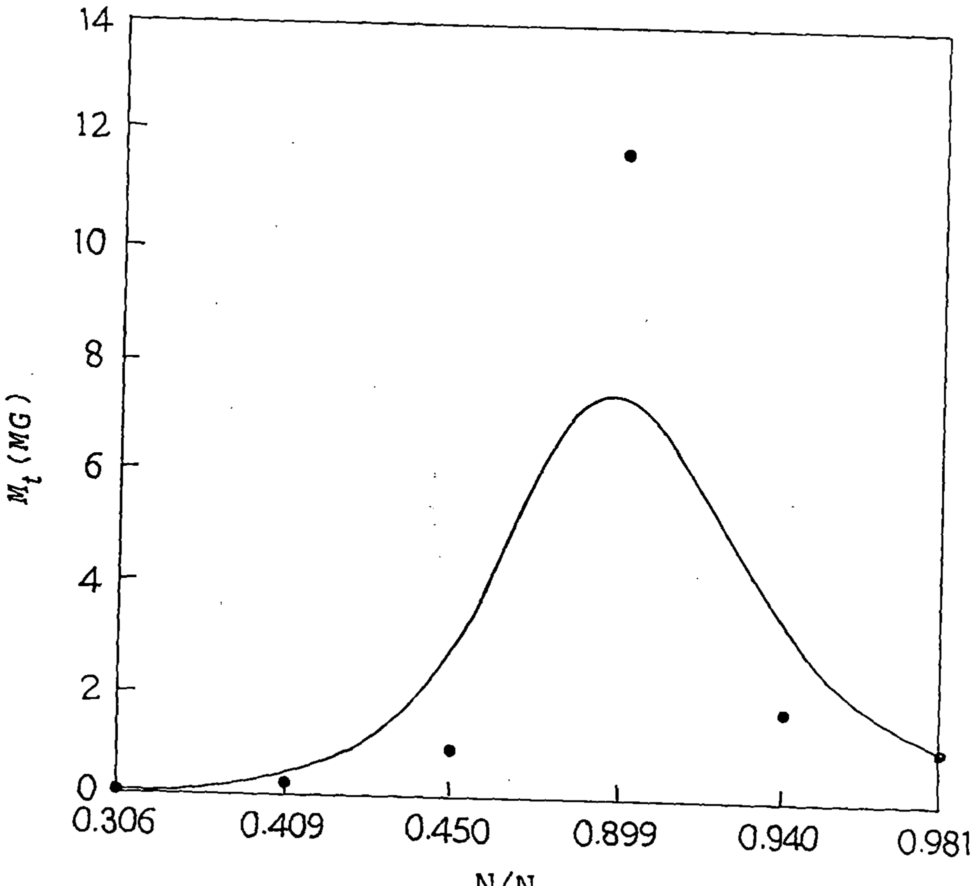

Fig. 2·1(a). The variation of the lateral magnetic field in MG with the density ratio N/N_C: at $I = 5\times10^{14}$ W/cm^2, $\mathcal{T}$=5nsec and $\lambda = 10{\cdot}6$ μm.

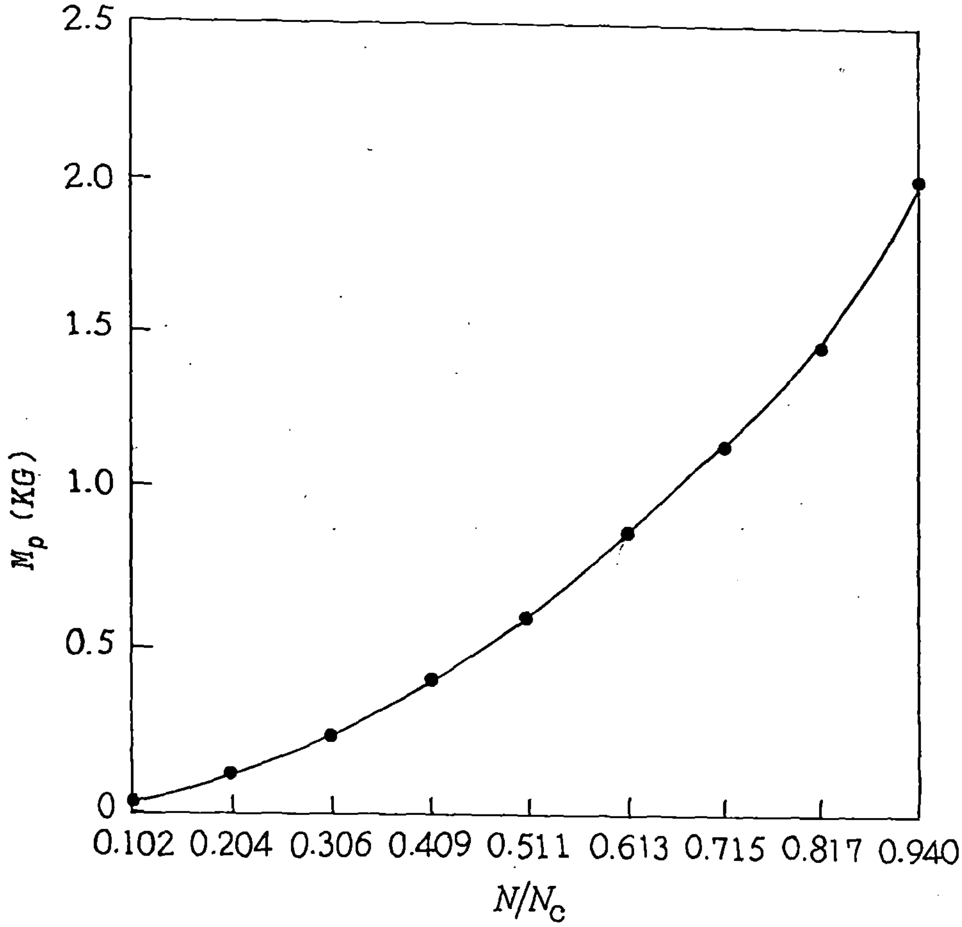

Fig. 2·1(b). The variation of the axial magnetic field in kG with the density ratio N/N_c at $I=5\times10^{14}$ W/cm^2, $\tau=5$ nsec and $\lambda=10.6\ \mu$m.