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1. B. Bhattacharyya, P.Mulser and Urmi Sanyal (1998).“ Spontaneous Faraday Rotation due to Strong Laser Radiation in a Plasma”. Phys. Lett. A **249**, 324-329.

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3. B.Bhattacharyya, Urmi Sanyal and S. V. Lawande (2000). “Model for Generation of Toroidal and Poloidal Magnetic Fields in a Laser Produced Plasma”. Phys. Rev. E. (In Press).

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Generation of axial and lateral magnetic fields in a laser-produced plasma

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The generation of axial and lateral magnetic fields due to the interaction of intense laser fields with a plasma is investigated analytically. For a CO_2 laser of $10.6\,\mu$m wavelength and 5×10^{14} W cm^{-2} power flux with a plasma of temperature 3 keV, the numerical results show that the magnitude of the lateral magnetic field dominates over the axial field, and the peak value of the lateral field is at less than the critical density, whereas the axial magnetic field peaks at the critical density level. This axial field combined with the lateral field may allow the construction of a new type of tokamak, and may also be important for studying energy transport in laser-fusion schemes.

1. Introduction

Both experimental and theoretical efforts have been directed towards the investigation of self-generated magnetic fields in laser-produced plasmas because of their numerous applications in inertial-confinement fusion (ICF) and other related fields. Various mechanisms have been proposed for the generation of lateral (toroidal) magnetic fields in laser-produced plasmas. Large-scale lateral fields can be produced by the thermoelectric process (Stamper et al. 1971), hot-electron ejection from the focal spot (Raven et al. 1979) and radiation pressure (Mora and Pellat 1981). Sources of small-scale lateral magnetic fields are the dynamo effect (Witalis 1974), resonant absorption (Bezzerides et al. 1977), filamentation (Greek et al. 1978) and Weibel instability (Malte et al. 1987).

Also, there have been reports of the generation of axial (poloidal) magnetic fields of megagauss strength in laser plasmas due to the dynamo effect (Briand et al. 1985), rippled surface irregularities (Kitagawa et al. 1986), ion acoustic turbulence (Dragila 1987), and induced magnetization arising out of the nonlinear optical response of the plasma (Chakraborty et al. 1984, 1988; Bhattacharyya 1994). Axial magnetic fields of gigagauss strength may also be produced (Sudan 1993) owing to electron currents driven by spatial gradients and temporal variations of the ponderomotive force. Stamper (1991) has reviewed the various applications of such magnetic fields in laser-fusion plasmas. From the available literature, it appears that the generation of axial magnetic fields and that of lateral magnetic fields have been reported separately, together with supporting mechanisms. However, no attempt has so far been made to describe the generation of axial and lateral fields simultaneously in laser-produced plasmas.

In this paper, we present a model for the simultaneous generation of axial and

lateral magnetic fields by the interaction of an intense laser beam with an electron plasma. The kinetic energy of the electrons in the presence of the wave is transformed into the energy of the induced d.c magnetic fields in both axial and lateral directions. The term 'd.c.' here means that the fields are unidirectional when averaged over the time period of the wave (i.e. over the fast laser frequency time scale $2\pi/\omega$).

The paper is organized as follows. The formulation of the problem is discussed in Sec. 2. Linearized solutions and dispersion relations are discussed in Sec. 3, and nonlinear solutions and field variables in Sec. 4. In Sec. 5, the nonlinear angular momentum components have been calculated separately in order to estimate the magnitudes of the fields. In Sec. 6, the numerical results along with the graphical representations of variation of fields have been elucidated. In Sec. 7, few remarks and discussions have been added to point out the importance of simultaneous generation of those fields on energy transport in laser fusion schemes and also the possibility of formation of a new type of tokamak out of those fields in future.

2. Formulation of the problem

The plasma is assumed to be a collisionless and hot electron fluid. The mobility of the ion fluid has been ignored. The thermal velocity v_{th} and the Debye length λ_D are small compared with the phase velocity v_φ of the radiation field and the characteristic density scale length L of the plasma respectively. The intensity of the radiation fields should not exceed the threshold power limit for the appearance of self-action effects such as self-focusing, self-trapping and self-phase modulation. The instabilities due to SRS (stimulated Raman scattering) and SBS (stimulated Brillouin scattering) are ignored. Moreover, the width Δx of the conversion (resonance) layer is assume to be much less than the laser wavelength λ_{ls}, and so inhomogeneity due to Landau damping can be neglected.

To describe the interaction of a laser beam with a plasma, we consider the macroscopic behaviour of an electron plasma. Hence the equations of continuity and momentum together with the usual Maxwell equations can be written as

$$\dot{N}+\nabla\cdot(N\dot{\mathbf{r}}) = 0, \tag{1}$$

$$\ddot{\mathbf{r}}+(\dot{\mathbf{r}}\cdot\nabla)\dot{\mathbf{r}}+\frac{e}{m}\mathbf{E}+\frac{e}{mc}(\dot{\mathbf{r}}\times\mathbf{H})+\frac{\nabla P}{mN} = 0, \tag{2}$$

$$c(\nabla\times\mathbf{E})+\dot{\mathbf{H}} = 0, \tag{3}$$

$$c(\nabla\times\mathbf{H})-\dot{\mathbf{E}}+4\pi eN\dot{\mathbf{r}} = 0, \tag{4}$$

$$\nabla\cdot\mathbf{E}+4\pi e(N-N_0) = 0, \tag{5}$$

$$\nabla\cdot\mathbf{H} = 0, \tag{6}$$

where $N_0, \mathbf{E}, \mathbf{H}$ and c are the ion density, electric field, magnetic field and velocity of light respectively, and $\dot{\mathbf{r}}$ is the velocity of the electron of mass m and charge e.

In order to close the above set of equations, one needs to specify the equation of state for electrons. For isothermal process, this can be written as

$$P = \gamma N \kappa T, \tag{7}$$

where γ, κ and T are the specific-heat ratio, Boltzmann's constant and the electron temperature respectively.

For finding the secular-free solutions of the field equations, a perturbation method (Bellman 1964) has been used in which the field variables ϕ (say) can be expressed as

$$\phi = \phi_0 + \varepsilon\phi_1 + \varepsilon^2\phi_2 + \varepsilon^3\phi_3 + \dots, \tag{8}$$

where ϕ_0 represents the unperturbed state of ϕ, and the first-order approximation ϕ_1 represents its linear solution. The nonlinearly excited nth ($n = 2, 3, \dots$)-order approximations can be expressed as ϕ_n, where $\phi_n = \phi_r \phi_{n-r}$ for $r \leqslant n$ and the condition for convergence is $\phi_n/\phi_{n-1} < 1$. The quantity ε is the expansion parameter (Bellman 1964; Ames 1965).

Let us assume that the linearized electric field has the form

$$\mathbf{E}_1 = \frac{m\omega c}{e}(\alpha_\parallel \cos\theta_\parallel, \alpha_\perp \cos\theta_\perp, \beta_\perp \sin\theta_\perp), \tag{9}$$

where

$$\theta_\parallel = k_\parallel x - \omega t, \quad \theta_\perp = k_\perp x - \omega t, \quad \alpha_\parallel = \frac{ea_\parallel}{m\omega c}, \quad (\alpha_\perp, \beta_\perp) = \frac{e(a_\perp, b_\perp)}{m\omega c},$$

subscripts $\parallel$ and $\perp$ indicate the effects of longitudinal and transverse oscillations of the laser fields respectively. In (9), the last two components arise directly from the laser field while the first component arises from the converted mode for a thermal plasma (Kull 1981, 1983). This form for the electric field requires further explanation and justification, which will be given in Sec. 5.

3. Linearized solutions and dispersion relations

Using the relation (8) in (1)–(7), the linearized equations for the electron plasma can be found. On solving these linearized equations, together with (9), we have

$$\mathbf{H}_1 = \frac{m\omega c}{e} n_\perp (0, \beta_\perp \sin\theta_\perp, \alpha_\perp \cos\theta_\perp), \tag{10}$$

$$\mathbf{r}_1 = \frac{c}{X}(\alpha_\parallel \sin\theta, X\alpha_\perp \sin\theta_\perp, X\beta_\perp \cos\theta_\perp), \tag{11}$$

$$N_1 = \frac{N_0 \alpha_\parallel}{X} n_\parallel \sin\theta_\parallel. \tag{12}$$

The linearized dispersion relation for a transverse mode is

$$n_\perp^2 - 1 + X = 0, \tag{13}$$

and that for a longitudinal mode is

$$n_{\parallel}^2 V^2 - 1 + X = 0, \tag{14}$$

where

$$n_{\parallel} = \frac{k_{\parallel} c}{\omega}, \quad n_{\perp} = \frac{k_{\perp} c}{\omega}, \quad X = \frac{\omega_p^2}{\omega^2}, \quad V^2 = \frac{\gamma v_{\text{th}}^2}{2c^2}.$$

It should be noted that the dispersion relations (13) and (14) are independent of each other and free from wave amplitudes. There is no exchange of energy between transverse and longitudinal waves during their propagation in a plasma. Moreover, the dispersion relation (13) is free from thermal effects, whereas the other relation (14) depends on the thermal velocity of the charged particles.

4. Nonlinear solutions and field variables

The second-order field variables can also be obtained easily along with the first-order fields. Hence, by using the first- and second-order field variables, it is found that the nonlinear third-order electric field satisfies

$$\begin{aligned}(c^2 - \tfrac{1}{2}\gamma v_{\text{th}}^2)\,\nabla\nabla\cdot\mathbf{E}_3 + (-c^2\nabla^2 + \omega_p^2)\,\mathbf{E}_3 + \ddot{\mathbf{E}}_3 \\ = 4\pi e N_0 \Big[(\dot{\mathbf{r}}_2\cdot\nabla)\,\dot{\mathbf{r}}_1 - (\dot{\mathbf{r}}_1\cdot\nabla)\,\dot{\mathbf{r}}_2 - \frac{e}{cm}(\dot{\mathbf{r}}_1\times\mathbf{H}_2) - \frac{e}{cm}(\dot{\mathbf{r}}_2\times\mathbf{H}_1) \\ - \frac{\gamma v_{\text{th}}^2}{2N_0^3} n_1^2 \nabla N_1 + \frac{\gamma v_{\text{th}}^2}{2N_0^2} n_1 \nabla N_2 + \frac{\gamma v_{\text{th}}^2}{2N_0^2} N_2 \nabla N_1 + \frac{\partial}{\partial t}\left(\frac{N_2\dot{r}_1 + N_1\dot{r}_2}{N_0}\right)\Big].\end{aligned} \tag{15}$$

The right-hand side of (15) consists of nonlinear terms, which appear exclusively owing to the presence of various nonlinear effects in plasmas. The first two terms arise from the convective derivative, and the next two terms from Lorentz forces, the fifth to seventh terms enforce thermal effects, and the last term gives the plasma current.

Retaining the first-harmonic terms corrected up to third order (Chakraborty et al. 1984), the nonlinear electric field can be found from (15). The nonlinear electron velocity can then be derived as

$$\begin{aligned}\dot{\mathbf{r}}_3 = c[\hat{\mathbf{x}}(\Gamma_{11}\alpha_{\parallel}^3 \sin\theta_{\parallel} + \Gamma_{12}(\alpha_{\perp}^2 - \beta_{\perp}^2)\,\alpha_{\parallel}\sin(2\theta_{\perp} - \theta_{\parallel}) + \Gamma_{13}(\alpha_{\perp}^2 + \beta_{\perp}^2)\alpha_{\parallel}\sin\theta_{\parallel}) \\ + \tilde{\mathbf{y}}(\Gamma_{21}\alpha_{\parallel}^2\alpha_{\perp}\sin\theta_{\perp} + \Gamma_{22}\alpha_{\parallel}^2\alpha_{\perp}\sin(2\theta_{\parallel} - \theta_{\perp}) + \Gamma_{23}(\alpha_{\perp}^2 - \beta_{\perp}^2)\,\alpha_{\perp}\sin\theta_{\perp}) \\ - \hat{\mathbf{z}}(\Gamma_{31}\alpha_{\parallel}^2\beta_{\perp}\cos\theta_{\perp} + \Gamma_{32}\alpha_{\parallel}^2\beta_{\perp}\cos(2\theta_{\parallel} - \theta_{\perp}) + \Gamma_{33}(\alpha_{\perp}^2 - \beta_{\perp}^2)\,\beta_{\perp}\cos\theta_{\perp})].\end{aligned} \tag{16}$$

The expression for the nonlinear displacement turns out to be

$$\begin{aligned}\mathbf{r}_3 = \frac{c}{\omega}[\hat{\mathbf{x}}(\Gamma_{11}\alpha_{\parallel}^3\cos\theta_{\parallel} + \Gamma_{12}(\alpha_{\perp}^2 - \beta_{\perp}^2)\,\alpha_{\parallel}\cos(2\theta_{\perp} - \theta_{\parallel}) + \Gamma_{13}(\alpha_{\perp}^2 + \beta_{\perp}^2)\,\alpha_{\parallel}\cos\theta_{\parallel}) \\ + \hat{\mathbf{y}}(\Gamma_{21}\alpha_{\parallel}^2\alpha_{\perp}\cos\theta_{\perp} + \Gamma_{22}\alpha_{\parallel}^2\alpha_{\perp}\cos(2\theta_{\parallel} - \theta_{\perp}) + \Gamma_{23}(\alpha_{\perp}^2 - \beta_{\perp}^2)\,\alpha_{\perp}\cos\theta_{\perp}) \\ - \hat{\mathbf{z}}(\Gamma_{31}\alpha_{\parallel}^2\beta_{\perp}\sin\theta_{\perp} + \Gamma_{32}\alpha_{\parallel}^2\beta_{\perp}\sin(2\theta_{\parallel} - \theta_{\perp}) + \Gamma_{33}(\alpha_{\perp}^2 - \beta_{\perp}^2)\,\beta_{\perp}\sin\theta_{\perp})],\end{aligned} \tag{17}$$

where

$$\Gamma_{11} = \frac{\tau_{11}+\sigma_{11}}{n_\parallel^2 V^2-1+X}, \qquad \Gamma_{12} = \frac{\tau_{12}+\sigma_{12}}{(2n_\perp - n_\parallel)^2 V^2-1+X},$$

$$\Gamma_{13} = \frac{\tau_{13}}{n_\parallel^2 V^2-1+X}, \qquad \Gamma_{21} = \frac{-\tau_{14}(n_\perp^2-1)+\sigma_{13}}{n_\perp^2-1+X},$$

$$\Gamma_{22} = \frac{\sigma_{14}}{(2n_\parallel - n_\perp)^2-1+X}, \qquad \Gamma_{23} = \frac{\sigma_{15}}{n_\perp^2-1+X},$$

$$\Gamma_{31} = \frac{-\tau_{14}(n_\parallel^2-1)+\sigma_{13}}{n_\perp^2-1+X}, \qquad \Gamma_{32} = -\Gamma_{22}, \qquad \Gamma_{33} = -\Gamma_{23}$$

$$\sigma_{11} = \tfrac{1}{2}(Q_\parallel n_\parallel + S_\parallel), \qquad \sigma_{12} = \tfrac{1}{2}(Q_\perp n_\parallel + S_\parallel),$$

$$\sigma_{13} = \tfrac{1}{2}Q n_\parallel, \qquad \sigma_{14} = \tfrac{1}{2}S_\parallel X, \qquad \sigma_{15} = \tfrac{1}{2}S_\perp X,$$

$$\tau_{11} = \frac{Q_\parallel n_\parallel}{2X} - \frac{V^2}{2X}S_\parallel n_\parallel^2 + \frac{V^2}{2X}(Q_\parallel n_\parallel + S_\parallel)n_\parallel^2 - \frac{V^2}{4X^3}n_\parallel^4,$$

$$\tau_{12} = \frac{Q_\perp n_\perp}{2X} - \frac{V^2}{2X}S_\perp n_\perp n_\parallel + \frac{V^2}{2X}(Q_\perp n_\perp + S_\perp)(2n_\perp - n_\parallel)^2 - \frac{V^2}{2X^3}S_\perp n_\parallel^2,$$

$$\tau_{13} = \tfrac{1}{2}(S - Q n_\perp), \qquad \tau_{14} = \frac{Q(n_\parallel + n_\perp) - S}{2X},$$

$$Q_\parallel = \frac{(4n_\parallel/X^2)(V^2 n_\parallel^2+1)+2n_\parallel/X}{4(V^2 n_\parallel^2-1)+X},$$

$$Q_\perp = \frac{4n_\perp}{4(V^2 n_\perp^2-1)+X}, \qquad Q = \frac{n_\parallel}{2[(n_\parallel+n_\perp)^2-4X]},$$

$$S_\parallel = \frac{2P_\parallel n_\parallel}{X}, \qquad S_\perp = \frac{2P_\perp n_\perp}{X}, \qquad S = \frac{2P(n_\parallel+n_\perp)}{X},$$

$$P_\parallel = -\frac{(n_\parallel/X)[2(V^2 n_\parallel^2-1)-(V^2 n_\parallel^2+1)]}{2[4(V^2 n_\parallel^2-1)+X]},$$

$$P_\perp = \frac{n_\perp X}{2[4(V^2 n_\perp^2-1)+X]}, \qquad P = \frac{n_\parallel}{(n_\parallel+n_\perp)^2-4+X}.$$

5. Nonlinear angular momentum and magnetization

The nonlinearly induced magnetization in a laser-produced plasma can be expressed as

$$\langle \mathbf{M} \rangle = \frac{4\pi e N_0}{c}\langle \mathbf{L} \rangle, \tag{18}$$

where $\langle \mathbf{L} \rangle$ is the angular momentum of electrons averaged over a time period

$2\pi/\omega$, and $\mathbf{L}$ has the form $(2c/e)\boldsymbol{\mu}$, where $\boldsymbol{\mu} = \mathbf{r}\times\mathbf{j}$, and $\mathbf{j} = -e\dot{\mathbf{r}}$. The axial and lateral components of the angular momentum are

$$\langle L_x\rangle = -\frac{c^2}{\omega}[\Gamma_{31}\alpha_\parallel^2+\Gamma_{32}\alpha_\parallel^2+\Gamma_{33}(\alpha_\perp^2-\beta_\perp^2)][\Gamma_{21}\alpha_\parallel^2+\Gamma_{22}\alpha_\parallel^2+\Gamma_{23}(\alpha_\perp^2-\beta_\perp^2)]\,\alpha_\perp\beta_\perp,$$

$$\langle L_y\rangle = \frac{c^2}{\omega}[\Gamma_{31}\alpha_\parallel^2+\Gamma_{32}\alpha_\parallel^2+\Gamma_{33}(\alpha_\perp^2-\beta_\perp^2)]$$
$$\times[\Gamma_{11}\alpha_\parallel^2+\Gamma_{12}(\alpha_\perp^2+\beta_\perp^2)+\Gamma_{23}(\alpha_\perp^2-\beta_\perp^2)]\,\alpha_\parallel\beta_\perp,$$

$$\langle L_z\rangle = -\frac{c^2}{\omega}[\Gamma_{21}\alpha_\parallel^2+\Gamma_{22}\alpha_\parallel^2+\Gamma_{23}(\alpha_\perp^2-\beta_\perp^2)]$$
$$\times[\Gamma_{11}\alpha_\parallel^2+\Gamma_{12}(\alpha_\perp^2+\beta_\perp^2)+\Gamma_{23}(\alpha_\perp^2-\beta_\perp^2)]\,\alpha_\parallel\alpha_\perp.$$

The average x components of the angular momentum, $\langle L_x\rangle$, give rise to the axial (poloidal) magnetic field $\langle M_p\rangle$, which is along the direction of the laser beam. The resultant of the y and z components of the angular momentum produces the lateral (toroidal) magnetic field $\langle M_t\rangle$, which is in the plane perpendicular to the laser beam. It is evident that the average angular momentum and hence the induced magnetization is of sixth order in the amplitudes of the laser fields.

6. Numerical estimations

For a simple numerical estimation, we have taken a CO_2 laser of wavelength 10.6 μm, pulse length 5 ns and power flux 5×10^{14} W cm^{-2}, with a plasma temperature of 3 keV in the region of $0.5N_c$, where N_c is the critical density for a spot radius of 80 μm. These data have been chosen arbitrarily from the available literature. Hence, quantitatively, the parameters are

$$\alpha_\parallel = 0.00109, \qquad \alpha_\perp = 0.154, \qquad \beta_\perp = 0.077,$$

$$V^2 = \frac{\gamma v_{\rm th}^2}{2c^2} = 0.0421, \qquad X = \frac{\omega_p^2}{\omega^2} = 0.492.$$

Equation (16) together with the above values gives the magnitude of the axial magnetic field as

$$M_p = \langle M_x\rangle = 560\ \text{G}.$$

The direction of the axial magnetic field lines here will be away from the target, because the field value M_p is positive. Similarly, the magnitude of the lateral magnetic field is

$$M_t = (\langle M_y\rangle^2+\langle M_z\rangle^2)^{1/2} = 450\ \text{kG}.$$

It follows that the lateral magnetic field is much greater than the axial field. Hence the former dominates over the latter in laser plasmas.

7. Results and discussion

Numerical estimations of the lateral and axial magnetic fields have been obtained for a laser intensity of 5×10^{14} W cm^{-2} and a thermal power flux of $5(1+1/Z)(N\kappa T)(\Delta R/2\tau)$ W cm^{-2}, where Z, κT, τ, N and ΔR are the effective ion

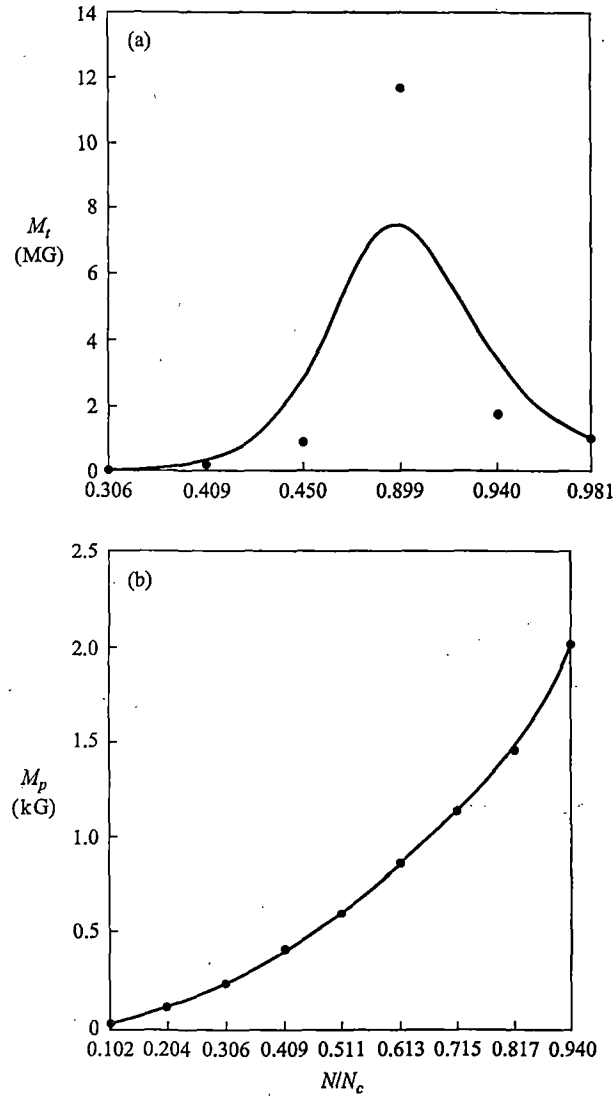

Figure 1. Variation of (a) the lateral field M_t and (b) the axial field M_p with the density ratio N/N_c at $I = 5 \times 10^{14}$ W cm^{-2}, $\tau = 5$ ns and $\lambda = 10.6$ μm. Note the different units (MG and kG) in (a) and (b).

charge for a copper target, the electron temperature in eV, the laser pulse length in ns, the electron density in cm^{-3} and the spot radius in μm respectively. Numerical results show that the axial and lateral magnetic fields increase very slowly towards the critical density as in Figs 1(a) and (b). Figure 1(a) shows that the maximum value of the lateral field is well below the critical density surface, which is consistent with experimental results (Raven et al. 1979) and also with numerically computed results (Boyd et al. 1982). However, Fig. 1(b) shows that the axial magnetic field should have its maximum value at the critical density surface, which is yet to be verified either by experiment or by simulation.

Our results are consequences of the inverse Faraday effect (IFE) (Steiger and Woods 1972; Chakraborty et al. 1990), because in an IFE process the kinetic

energy of the ordered motion of particles in the presence of an electromagnetic wave is transformed into the energy of the induced magnetic field. The field-generation mechanism in our study is a direct process (because, to calculate the induced magnetic fields, we have calculated the average nonlinear angular momentum of electrons via the nonlinear electron velocity and its displacement), whereas the IFE is an indirect process of field generation. At high frequencies, the IFE is also relevant over time scales shorter than twice the oscillation period of the driving wave field, beyond which the wave becomes unstable (Stenflo 1977).

Our results are different from those due to dynamo effects because both toroidal (lateral) and poloidal (axial) fields occur simultaneously rather than acting cyclically (i.e. poloidal helps to produce toroidal, and vice versa). They are also different from those due to the thermoelectric effect ($\nabla N \times \nabla T$) because the temperature gradient in the plasma region of interest has been ignored. One may consider a very long density scale length and uniform temperature when the beam is absorbed in the plasma region, which may be the case in future ICF targets.

The electromagnetic mode of p-polarized laser light can be converted to the electrostatic mode at the critical density N_c, when its electric vector oscillates along the direction of the density gradient, i.e. $\mathbf{E}\cdot\nabla N_e \neq 0$. This effect is known as resonant absorption (Kruer 1987), and also gives rise to a magnetic field in a plasma (Bezzerides et al. 1977). We exclude this effect in our calculation because our interest is in calculating the magnetic field in underdense regions.

Kull (1981, 1983) has shown that mode conversion is possible even in the underdense region for thermal plasmas, and has also pointed out that the width of the conversion layer plays an important role in such conversion. Thus the amplitude of the electromagnetic mode of the laser light will be modified in a thermal plasma. Hence the linearization of the electric fields, as in (9), is justified. Moreover, we have made the following assumptions.

(a) We have taken $(\nu/\omega)(L/\lambda_{ls}) \approx 0.01$, which gives $\Delta x/\lambda_{ls} \ll 1$, where the width of the resonance layer $\Delta x = (\nu/\omega)L$, ν is the collision frequency and L is the density scale length. Hence phenomena occurring at the resonance layer have been ignored.

(b) The inhomogeneity due to Landau damping has also been ignored, since $k_\parallel \lambda_D < 1$, and $k_\perp/k_\parallel \ll 1$, where $k_\parallel = k_0\epsilon^{1/2}/\beta$ is the electrostatic wavenumber, $k_\perp = k_0\epsilon^{1/2}$ is the electromagnetic wavenumber and $k_0 = \omega/c$ is the vacuum wavenumber, $\beta = v_{\rm th}/c \ll 1$ and $\epsilon = 1-\omega_p^2/\omega^2$.

(c) The laser wavelength λ_{ls} is greater than the electrostatic wavelength λ_{es}, and the thermal velocity $v_{\rm th}$ and the Debye length λ_D are small compared with the phase velocity v_φ of the radiation field and the density scale length L of the plasma respectively. The effect of plasma inhomogeneity may therefore be neglected.

Hence, assuming a typical value of the plasma temperature of 3 keV, the dimensionless amplitude of the electrostatic mode $\alpha_\parallel$ can be estimated quantitatively to be of the order of 10^{-3}. This leads us to conclude that about 1% of the laser light is converted here. It should be mentioned that full conversion of laser light is possible, even in an underdense region, through relativistic thermal effects (Kull 1981).

Electrostatic-mode (i.e. the wake-field) generation in an underdense plasma is also of current interest, with the advent of ultrashort-pulse lasers, because such fields play important roles in plasma-based accelerators (Esarey et al. 1994). Such wake fields are also important for the production of magnetic fields in laser plasmas (Sheng et al. 1996). These will be studied elsewhere.

Electrons move along the magnetic field and become trapped in a layer of thickness the order of the Larmor radius. Thus the lateral field enhances lateral energy transport but degrades axial energy propagation (Max 1982). Hence, phenomenologically, it should be stated that the axial field enhances axial energy transport and degrades lateral energy transport. Therefore the rate of energy deposition in conduction regions will increase owing to the presence of the axial field, which enhances the energy transport from a critical surface to an ablation surface. But our results (Fig. 1) dictate that the lateral field dominates over the axial field in laser–plasma interactions. Hence both fields should have great impact on uniform compression of ICF targets.

It may be speculated that the combination of toroidal and poloidal fields set up by the laser may lead to the formation of a magnetic cage that could be used for plasma confinement in a manner similar to tokamaks, toroidal pinches, etc. Such a configuration would be sustained by the laser beams, and may also be heated by them.

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By: B. Bhattacharyya, Urmi Sanyal, and S.V. Lawande

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Spontaneous Faraday rotation due to strong laser radiation in a plasma

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Spontaneous Faraday rotation due to strong laser radiation in a plasma

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Abstract

Theoretical studies on intense laser radiation interacting with plasma reveal an induced nonlinear birefringence which turns out to be spontaneous Faraday rotation. Here, this rotation is called nonlinear Faraday rotation (NFR). The expressions for the nonlinear refractive indices of the laser fields are calculated both in relativistic and non-relativistic limits for a two-component plasma. The NFR angles due to nonlinear induced birefringence are derived in the absence of externally imposed magnetic fields. It is shown that, in the relativistic limit, the electron motion plays an important role in producing NFR and so in the generation of induced magnetic fields. © 1998 Elsevier Science B.V.

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Faraday rotation (FR) is a magneto-optical effect of birefringence. The theory of FR has been developed for a plane polarized wave of very weak field intensity and is used in infinitely small amplitude wave approximation for the linear solution of the field equations in material media including plasmas [1,2]. The magnitude of FR is considerably modified with strong waves. We are not aware of any appropriate theoretical investigation of the modification of the FR (called NFR) effects along the line developed in this paper. The most important factor influencing the value of FR is the appearance of the nonlinearly induced effect of intense laser fields in plasmas. It is shown that the NFR effect exists even in the absence of an ambient magnetic field in the plasma, and dominates due to the electron motion in the relativistic limit of the order of $(m_i/m_e)^3$, where m_e and m_i are the electron and ion masses, respectively.

An important aspect of the Faraday effect in plasmas is the induced magnetization, which is known as the inverse Faraday effect (IFE) [1–3], produced by a circularly polarized wave. It is the consequence of gyration of charged particles, which for left circular polarization is parallel and for right circular polarization is anti-parallel to the direction of propagation. The theory developed in this paper will be useful for the study of (a) the evolution of NFR angles by the method of induced birefringence; and (b) the complimentary effect of IFE and induced magnetization in the plasma.

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To study the magnitude of NFR in a plasma due to nonlinearly induced birefringence of the electromagnetic waves we have assumed the following [4,5]. The waves are sinusoidal i.e. the perturbed field variables are harmonic in nature. The plasma is cold (i.e. $v_{\mathrm{the,i}} \ll c$, where v_{the} and v_{thi} represent the thermal velocities of the electrons and the ions, respectively, and c is the light velocity) and homogeneous with mobile components of electrons and ions. The incident electromagnetic waves are so intense that the motion of electrons and ions becomes relativistic. There is no first harmonic density fluctuation due to the interaction of the waves with the plasma. However, a nonlinearly excited second harmonic density fluctuation exists and its effect on stimulated Brillouin and Raman scattering will be visible only in an order of approximation higher than three, and hence, those effects can also be neglected. The self-action effects arising from ponderomotive forces and thermal instabilities are also neglected because pressure variation and thermal velocities are ignored.

Under the above set of assumptions the fluid motion can be written as

$$\frac{\partial n_{e,i}}{\partial t} + \nabla(n_{e,i} v_{e,i}) = 0, \tag{1}$$

$$\left(\frac{\partial}{\partial t} + (v_{e,i} \cdot \nabla)\right) p_{e,i} = \mp eE \mp (e/c)(v_{e,i} \times H), \tag{2}$$

and the field equations of Maxwell reduce to

$$c^2 \nabla^2 E = \frac{\partial^2 E}{\partial t^2} + 4\pi e \frac{\partial}{\partial t}(n_i v_i - n_e v_e), \tag{3}$$

$$c^2 \nabla^2 H = \frac{\partial^2 H}{\partial t^2} + 4\pi e c[\nabla \times (n_i v_i - n_e v_e)], \tag{4}$$

where the subscripts e and i represent the species of electrons and ions of negative and positive charges respectively; p, v and n stand for their relativistic momentum, velocity and density, respectively; E and H denote the electric and magnetic fields. Vector quantities are easily recognized as such from the context and are not set boldface.

For weak relativistic effects, i.e. when $v_{e,i}^2 \ll c^2$, one has the relativistic momentum for electrons and ions as $p_{e,i} = m_{e,i}(1 - v_{e,i}^2/2c^2) v_{e,i}$.

We start with the linearized wave solution of the electric field of the form [6]

$$E_{\mathrm{l}} = \frac{mc\omega}{2e}[(\hat{y} + \mathrm{i}\hat{z})(\alpha \mathrm{e}^{\mathrm{i}\theta_r} + \overline{\beta} \mathrm{e}^{-\mathrm{i}\overline{\theta}_l}) + (\hat{y} - \mathrm{i}\hat{z})(\alpha \mathrm{e}^{-\mathrm{i}\overline{\theta}_r} + \overline{\beta} \mathrm{e}^{\mathrm{i}\theta_l})], \tag{5}$$

where $\alpha = ea/mc\omega$ and $\beta = eb/mc\omega$ are the dimensionless amplitudes of the two circularly polarized waves of the electric fields; $m = m_e m_i/(m_e + m_i)$ is the reduced mass. For convenience we have chosen the phase of those waves as θ_r $(= k_r \cdot x - \omega t)$ and θ_l $(= k_l \cdot x - \omega t)$, where k_r and k_l are the wave numbers and ω is the wave frequency. The form of those two waves would be chosen such that they reduce to an elliptically polarized wave in an unmagnetized plasma with the same phase (i.e. $\theta_r = \theta_l$). But for a magnetized plasma those will be treated as left and right circularly polarized waves with different phases of θ_r and θ_l, respectively. The quantities with a bar mean the complex conjugate of the corresponding quantities without a bar.

Let a vector ϕ be represented as

$$\phi_{\pm} = \phi_x \pm \mathrm{i}\phi_y. \tag{6}$$

With the aid of relation (6), Eq. (5) can be rewritten as

$$E_{+} = \frac{nc\omega}{e}(\overline{\alpha} \mathrm{e}^{\mathrm{i}\overline{\theta}_r} + \beta \mathrm{e}^{\mathrm{i}\theta_l}), \tag{7}$$

$$E_{-} = \frac{nc\omega}{e}(\alpha \mathrm{e}^{\mathrm{i}\theta_r} + \overline{\beta} \mathrm{e}^{-\mathrm{i}\overline{\theta}_l}). \tag{8}$$

In linear approximation the propagation of two circularly polarized electromagnetic waves in magnetized plasma turns out to be determined by the amplitude-independent dispersion relations

$$n_r^2 = 1 - \frac{X_e + X_i}{(1 - Y_e)(1 + Y_i)}, \tag{9}$$

$$n_l^2 = 1 - \frac{X_e + X_i}{(1 + Y_e)(1 - Y_i)}, \tag{10}$$

where $n_{r,l} = k_{r,l}c/\omega; X_{e,i} = \omega_{pe,i}^2/\omega^2; Y_{e,i} = \Omega_{e,i}/\omega; \Omega_{e,i} = eH_0/cm_{e,i}; \omega_{pe,i}^2 = 4\pi e^2 n_0/m_{e,i}$. Two other dispersion relations of $\overline{n}_r^2$ and $\overline{n}_l^2$ can also be obtained in the same way.

The second-order calculations for E_+ and E_- are as simple as those of the first order. Moreover, the dispersion relations derived from the second-order approximation have not shown any new aspect for interpretation. So, we start with the third-order differential equations for E_+ and E_- in the following,

$$[(D_t^2 - c^2 D_x^2)(D_t - \mathrm{i}\Omega_e)(D_t + \mathrm{i}\Omega_i) + (\omega_{pe}^2 + \omega_{pi}^2)D_t^2]E_+ = (D_t - \mathrm{i}\Omega_e)(D_t + \mathrm{i}\Omega_i)NR_{3+}$$
$$+ 4\pi e n_0 (D_t + i\Omega_i)NE_{3+} - 4\pi e n_0 D_t (D_t + \mathrm{i}\Omega_e)NI_{3+}, \tag{11}$$

$$[(D_t^2 - c^2 D_x^2)(D_t + \mathrm{i}\Omega_e)(D_t - \mathrm{i}\Omega_i) + (\omega_{pe}^2 + \omega_{pi}^2)D_t^2]E_- = (D_t + \mathrm{i}\Omega_e)(D_t - \mathrm{i}\Omega_i)NR_{3-}$$
$$+ 4\pi e n_0 (D_t - i\Omega_i)NE_{3-} - 4\pi e n_0 D_t (D_t + \mathrm{i}\Omega_e)NI_{3-}, \tag{12}$$

where $D_t = \partial/\partial t, D_t^2 = \partial^2/\partial t^2, D_x = \partial/\partial x, D_x^2 = \partial^2/\partial x^2$ and

$$NR_{3\pm} = -4\pi e D_t (n_{i2} v_{i1\pm} - n_{e2} v_{e1\pm}),$$
$$NE_{3\pm} = v_{e2x} D_x v_{e1\pm} \mp (\mathrm{i}e/cm_e) v_{e2x} H_{1\pm} - D_t (v_{e,1}^2 v_{e,1\pm}/2c^2),$$
$$NI_{3\pm} = -v_{i2x} D_x v_{i1\pm} \pm (\mathrm{i}e/cm_i) v_{i2x} H_{1\pm} - D_t (v_{i,1}^2 v_{i,1\mp}/2c^2).$$

The r.h.s. of $NR_{3\pm}$ is due to the plasma current of electrons and ions. The first, second and third terms of the r.h.s. side of $NE_{3\pm}$ are the substantial derivatives of electron momentum, Lorentz force and relativistic effect of electrons, respectively, whereas all terms on the r.h.s. of $NI_{3\pm}$ refer to the ions.

It has already been pointed out that the left and right circularly polarized waves of equal frequencies and different wave numbers are taken when an ambient magnetic field is present in the plasma but they are basically treated as an elliptically polarized wave in an unmagnetized plasma. The various nonlinear effects of the polarized waves can be studied from Eqs. (11) and (12). Solving Eqs. (11) and (12) correctly up to third order [3–5], we obtain the intensity-dependent nonlinear dispersion relations in the following form,

$$nd_l = n_{l(nl)} + n_{l(rl)}, \tag{13}$$

$$nd_r = n_{r(nl)} + n_{r(rl)}, \tag{14}$$

where $n_{r(nl)}$ are the nonlinear terms due to non-relativistic effects, and the nonlinear relativistic terms for the same are $n_{r(rl)}$ and $n_{l(rl)}$, respectively. Their expressions are

$$n_{r(nl)} = -\frac{X}{16 n_r \xi_+}[(C_{22}\overline{C}_4 + C_{11}\overline{C}_2)(n_r + n_l)\xi_+ - 2C_{11}\overline{n}_l(\overline{C}_2 - M_e)(1 + Y_i)$$
$$- 2C_{22}\overline{n}_l(\overline{C}_4 - M_i)(1 - Y_e)]\beta\overline{\beta}\,\mathrm{e}^{\mathrm{i}(\theta_l - \overline{\theta}_l)}, \tag{15}$$

$$n_{r(rl)} = -\frac{X}{8\xi_+ n_r}\{[C_1\overline{C}_2 C_2(1 + Y_i) + C_3\overline{C}_4 C_4(1 - Y_e)]\beta\overline{\beta}\,\mathrm{e}^{\mathrm{i}(\theta_l - \overline{\theta}_l)}$$
$$+ 0.5[C_1^2\overline{C}_1(1 + Y_i) + C_3^2\overline{C}_3(1 - Y_e)]\alpha\overline{\alpha}\,\mathrm{e}^{\mathrm{i}(\theta_l - \overline{\theta}_l)}\}, \tag{16}$$

$$n_{l(nl)} = \frac{X}{16 n_l \xi_-}[(C_{22}\overline{C}_3 + C_{11}\overline{C}_1)(n_r + n_l)\xi_+ - 2C_{11}\overline{n}_r(\overline{C}_1 + M_e)(1 - Y_i)$$
$$- 2C_{22}\overline{n}_r(\overline{C}_3 + M_i)(1 + Y_e)]\alpha\overline{\alpha}\,\mathrm{e}^{\mathrm{i}(\theta_r - \overline{\theta}_r)}, \tag{17}$$

$$n_{l(rl)} = \frac{X}{8\xi_- n_l}\{[C_1\overline{C}_1 C_2(1 - Y_i) + C_3\overline{C}_3 C_4(1 + Y_e)]\alpha\overline{\alpha}\,\mathrm{e}^{\mathrm{i}(\theta_r - \overline{\theta}_r)}$$
$$+ 0.5[C_2^2\overline{C}_2(1 - Y_i) + C_4^2\overline{C}_4(1 + Y_e)]\beta\overline{\beta}\,\mathrm{e}^{\mathrm{i}(\theta_l - \overline{\theta}_l)}\}, \tag{18}$$

where

$$C_1 = -(1 + Y_i)M_e/\xi_+, \qquad C_2 = (1 + Y_i)M_e/\xi_-,$$
$$C_3 = -(1 - Y_e)M_i/\xi_+, \qquad C_4 = (1 + Y_e)M_i/\xi_-,$$
$$C_{11} = [(C_1 n_l - C_2 n_r)(X_i - 4)M_e + (C_3 n_l - C_4 n_r)X_e M_i](4 - X_e - X_i),$$
$$C_{22} = [(C_3 n_l - C_4 n_r)(X_i - 4)M_i + (C_1 n_l - C_2 n_r)X_i M_e](4 - X_e - X_i),$$

$\xi_\pm = (1 \mp Y_e)(1 \pm Y_i)$, $M_{e,i} = m/m_{e,i}$, and also $X = \omega_p^2/\omega^2$, $\omega_p^2 = 4\pi e^2 n_0/m$. Similarly, two other nonlinear dispersion relations for $\overline{nd}_l$ and $\overline{nd}_r$ can also be derived when the bar quantities are used. Since the plasma is undamped, i.e. collision frequencies are ignored, we can drop the bar from all quantities and may treat them as real.

It is evident that two dispersion relations of nd_l and nd_r are coupled by the nonlinear sources of convective derivative, Lorentz force, plasma current and relativistic momentum of charged particles. Moreover, they are intensity dependent. Therefore, there exists a mutual exchange of energy between two circularly polarized waves in the presence of a magnetic field, and an elliptically polarized wave in the absence of a magnetic field, with plasma nonlinearities.

The FR angle Φ can be simply defined as

$$\Phi = \frac{\omega}{2\pi c}(n_l - n_r)L, \tag{19}$$

where n_l and n_r are the refractive indices of the polarized wave or waves, L is the characteristic gradient scale length of the plasma and the other quantities have their usual meanings.

It is evident that if the refractive indices n_l and n_r are linear and relation (19) is used to measure the FR angle Φ, then the corresponding angle would be the linear FR angle. Subsequently, the amount of magnetic field can be estimated easily. In the linear case, the magnitude of the magnetic field would be exactly equal to what was supplied from the outside during the experiment. However, if there is no supplied magnetic field at the beginning then it would be observed that in the linear approximation n_l and n_r are equal which turns out to be zero FR angle. On the other hand, if the refractive indices are nonlinear then the FR angle will be the sum of linear (Φ_{linear}) and nonlinear ($\Phi_{\text{nonlinear}}$) FR angles. Its concomitant H-field should be the combination of the ambient magnetic field (which is equal to the magnitude of the magnetic field taken during the experiment) plus the induced magnetic field (which is spontaneous and a consequence of IFE [3]).

To understand the linear and nonlinear induced birefringence, to calculate the linear (Φ_{linear}) and nonlinear ($\Phi_{\text{nonlinear}}$) FR angles, and also to obtain the H-field for the interaction of high frequency laser fields with a magnetized two-component nondissipative plasma we have performed a systematic study starting from the linear dispersion relations of (9) and (10) to the nonlinear dispersion relations of (13) and (14). Subsequently, it has been shown that even in the absence of a magnetic field the FR angle exists in a high frequency nonlinear plasma phenomenon. High frequency means that higher powers of $X_{e,i}$ and $Y_{e,i}$ can be neglected.

Case (i). In the linear limit, for high frequency laser (i.e., $X_{e,i} \ll 1$ and $Y_{e,i} \ll 1$) in magnetized ($H_0 \neq 0$) and unmagnetized ($H_0 = 0$) plasmas. Simplifying relations (9) and (10), we have for the magnetized plasma

$$n_l = 1 - 0.5(X_e + X_i) + 0.5(X_e + X_i)(Y_e - Y_i), \tag{20}$$

$$n_r = 1 - 0.5(X_e + X_i) - 0.5(X_e + X_i)(Y_e - Y_i), \tag{21}$$

and for the unmagnetized plasma

$$n_l = 1 - 0.5(X_e + X_i), \tag{22}$$

$$n_r = 1 - 0.5(X_e + X_i). \tag{23}$$

Using Eqs. (20) and (21) in relation (19), the linear FR angle Φ_{linear} is

$$\Phi_{\text{linear}} = \frac{\omega X_e Y_e}{2\pi c} L \tag{24}$$

from which follows [6]

$$\Phi_{\text{linear}} = V_{ct} H_0 L \tag{25}$$

where $V_{ct} = 2\pi e^3 n_0 / m_e^2 c^2 \omega^2$ is known as the Verdet constant of the medium [6].

It is evident that Φ_{linear} mainly depends on the behaviour of electrons of the plasma due to the fact that it varies with the electron plasma frequency ω_{pe} and electron cyclotron frequency Ω_e. Moreover, it is independent of the intensity of the waves. On the other hand, if we take Eqs. (22) for n_l and (23) for n_r to study FR it is obvious that $\Phi_{\text{linear}} = 0$ holds because the refractive indices of n_l and n_r in (22) and (23) are exactly equal, i.e., the dispersive rates of the given polarized waves are the same. It follows that the linear FR (Φ_{linear}) angle in the absence of a magnetic field will not exist, in agreement with the FR phenomenon.

We are interested in studying the unmagnetized plasma behaviour and so, our next analysis will be confined to that aspect only.

Case (ii). In the nonrelativistic limit, for high frequency ($X_{e,i} \ll 1$ and $Y_{e,i} \ll 1$) in an unmagnetized ($H_0 = 0$) plasma. Simplifying Eqs. (15) and (17) we have the nonrelativistic dispersion relations of the polarized waves as

$$n_{l(nl)} = \frac{X M_e M_i}{16}(M_e X_i + M_i X_e)\alpha^2, \tag{26}$$

$$n_{r(nl)} = \frac{X M_e M_i}{16}(M_e X_i + M_i X_e)\beta^2. \tag{27}$$

Using Eqs. (26) and (27) in relation (19), we find the nonlinear nonrelativistic FR angle $\Phi_{\text{non-rel}}$ as

$$\Phi_{\text{non-rel}} = \frac{\omega}{2\pi c}\frac{X M_e M_i}{16}(M_e X_i + M_i X_e)(\alpha^2 - \beta^2). \tag{28}$$

It is evident from the above expression that even in the absence of a dc magnetic field a finite FR angle exists for an elliptically polarized wave but it will disappear when circularly polarized waves are considered, i.e., $\alpha = \beta$. It may also be noted that it is intensity dependent and both, electrons and ions, are dominating with the equal order of magnitude because $M_e X_i / M_i X_e = 1$.

Case (iii). In the relativistic limit, for high frequency laser fields ($X_{e,i} \ll 1$ and $Y_{e,i} \ll 1$) in an unmagnetized ($H_0 = 0$) plasma. Simplifying the relativistic dispersion relations (16) and (18) we may write

$$n_{l(rl)} = \frac{X}{8}\{(M_e^3 + M_i^3)(2\alpha^2 + \beta^2)[1 + \tfrac{1}{2}(X_e + X_i)] - 4(M_e^3 Y_e - M_i^3 Y_i)\beta^2\}, \tag{29}$$

$$n_{r(rl)} = -\frac{X}{8}\{(\alpha^2 + 2\beta^2)[1 + \tfrac{1}{2}(X_e + X_i)](M_e^3 + M_i^3)\beta^2 + 4(M_e^3 Y_e - M_i^3 Y_i)\alpha^2\}. \tag{30}$$

From the above two relations we may write the nonlinear relativistic FR angle $\Phi_{\text{rel-FR}}$

$$\Phi_{\text{rel-FR}} = \frac{\omega L X}{16\pi c}\{3(M_e^3 + M_i^3)(\alpha^2 + \beta^2)[1 + \tfrac{1}{2}(X_e + X_i)] + 4(M_e^3 Y_e - M_i^3 Y_i)(\alpha^2 - \beta^2)\}. \tag{31}$$

It is evident from relation (31) that in the relativistic limit the FR angle exists in the absence of magnetic fields. It dominates by electron motion over an order of magnitude $(M_e/M_i)^3$ which is equivalent to $(m_i/m_e)^3$. Moreover, it persists even for circularly polarized waves (i.e. $\alpha = \beta$) and it can be cast into the simplified form

$$\Phi_{\text{rel-FR}} = \frac{3X\omega}{8\pi c}\left(1 + \frac{X_e + X_i}{2}\right)(M_e^3 + M_i^3)\alpha^2 L. \tag{32}$$

For numerical results, in the context of our weak relativistic model [7], we may assume that the laser has an energy level of 100 J with 100 ps (full width at half maximum) pulse focused on a target to a spot radius 40 μm. It produces the laser irradiance $I \simeq 2 \times 10^{16}$ W/cm^2, which yields $\alpha^2 \simeq 0.097$. We also assume that the laser has a 1 μm wavelength, which yields the frequency $\omega \simeq 1.886 \times 10^{15}$/s. Further, we take the plasma density such that $X_e \simeq 0.01$ and the characteristic length L equals twice the spot radius. Then, relation (32) gives approximately the relativistic FR angle ($\Phi_{\text{rel-FR}}$) as 0.27 radians, which turns out to be the angle of rotation $\simeq 15.5°$. It may be measured in the laboratory in future.

In conclusion, the nonlinearly induced birefringence corresponds to a nonlinear FR angle, which enforces one to estimate the order of induced magnetization (i.e. IFE effect) for the propagation of the polarized waves in an unmagnetized plasma. An ambient magnetic field may help to enhance such a magnetization.

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