APPENDIX A

\[ \delta \left( \frac{1}{\delta} \right)_u = \text{constant} = L \]  \hspace{1cm} (A1)

It is evident from

\[ \delta \left( \frac{u_x^2}{\delta} \right)_u + \delta \left( \frac{1}{\delta} \right) v_y^2 = (v_y^2)_v \]

that

\[ \delta \left( \frac{u_x^2}{\delta} \right)_u = \text{constant} = M \text{ (say)} \]  \hspace{1cm} (A2)

and \( (v_y^2)_v = Lv_y^2 + M \) \hspace{1cm} (A3)

From (A1), we get,

\[ \delta = Ne^{-Lu} \]  \hspace{1cm} (A4)

where

\[ N = \text{constant} \neq 0 \]

In the following it will be shown that the above equations are not satisfied simultaneously and hence \( \delta \left( \frac{1}{\delta} \right)_u = \text{constant} \) is not permissible.

Case(i): \( L \neq 0 \)

From (A2) using (A1), we get,

\[ \delta \left[ \left( \frac{u_x^2}{\delta} \right)_u \frac{1}{\delta} + u_x^2 \left( \frac{1}{\delta} \right)_u \right] = M \]

\[ \left( u_x^2 \right)_u = M - Lu_x^2 \]

Integrating the above expression with the help of \( u = \int e^{k\beta} \frac{\alpha^2}{\alpha^2} dX \) we get
\[
\frac{e^{2 \beta \xi}}{\alpha^4} = \frac{1}{P} \{Q - e^{-Lu}\}
\]  \hspace{1cm} (A5)

where \( P = \text{constant} \neq 0, \ Q = \text{constant} \)

\[\phi_x = \phi_x X_u = \alpha^2\]  \hspace{1cm} (A6)

Using \( e^\beta = f_x^2 + \phi^2 + \alpha^2 \) and (A5) we get,

\[
\frac{\left(f_x^2 + \phi^2 + \alpha^2\right)^2}{\alpha^4} = \frac{1}{P} \{Q - e^{-Lu}\}
\]

Let \( \alpha^2 = f_x^2 + \phi^2 \)

We get from above

\[
\alpha^2 = \left[\frac{1}{2k^2} \left\{ \frac{Q - e^{-Lu}}{P} \right\}^{1/2} \right]^\frac{1}{k-1}
\]  \hspace{1cm} (A7)

Now,

\[\phi_x = \alpha^2\]

\[
\phi_x = \left[\frac{1}{2k^2} \left\{ \frac{Q - e^{-Lu}}{P} \right\}^{1/2} \right]^\frac{1}{k-1}
\]

\[
\phi = \int \left[\frac{1}{2k^2} \left\{ \frac{Q - e^{-Lu}}{P} \right\}^{1/2} \right]^\frac{1}{k-1} \, du
\]  \hspace{1cm} (A8)

\[
\frac{1}{\alpha} = \frac{1}{\left[\frac{1}{2k^2} \left\{ \frac{Q - e^{-Lu}}{P} \right\}^{1/2} \right]^\frac{1}{k-1}}
\]  \hspace{1cm} (A9)

Further considering (A4) in

\[
\delta(X) = \frac{\alpha_{xx} - k' \alpha_x \beta_x}{\alpha} \neq 0, \text{ we get}
\]
\[ \frac{\alpha_{xx} - k^2 \alpha_x \beta_x}{\alpha} = Ne^{-Lu} \]

which may be written as

\[ \frac{e^{i\beta}}{\alpha} \left( e^{-i\beta} \alpha_x \right) = Ne^{-Lu} \]

With the change of variable from \( X \) to \( u \) with the help of \( u = \int \frac{e^{i\beta}}{\alpha^2} dX \), the above is reduced to

\[ \frac{e^{2i\beta}}{\alpha^4} \left( \frac{1}{\alpha} \right)_{uu} + \frac{1}{\alpha} Ne^{-Lu} = 0 \]

which may be re-written with the help of (A5) as

\[ \left\{ \frac{Q - e^{-Lu}}{\alpha} \right\} \left( \frac{1}{\alpha} \right)_{uu} + \frac{1}{\alpha} P. Ne^{-Lu} = 0 \]

\[ (A10) \]

\[ \frac{1}{\alpha} \] can be determined from A9. Elimination of \( \frac{1}{\alpha} \) from (A10) with the help of (A9) indicates as if tangent of an angle is expressible in terms of a rational function in amalgamation with roots. This is not permissible in a physical situation.

Case II: \( L = 0 \)

Taking \( L = 0 \), we similarly arrive at a equation which indicates as if the tangent of an angle is expressible in terms of a rational function in amalgamation with roots. This is not possible in a physical situation.