APPENDIX A

$$\delta\left(\frac{1}{\delta}\right)_u = \text{constant} = L \qquad \text{(A1)}$$

It is evident from

$$\delta\left(\frac{u_x^2}{\delta}\right)_u + \delta\left(\frac{1}{\delta}\right)v_Y^2 = \left(v_Y^2\right)_v$$

that

$$\delta\left(\frac{u_x^2}{\delta}\right)_u = \text{constant} = M \text{ (say)} \qquad \text{(A2)}$$

and $\left(v_Y^2\right)_v = Lv_Y^2 + M$ (A3)

From (A1), we get,

$$\delta = Ne^{-Lu} \qquad \text{(A4)}$$

where

$N = \text{constant} \neq 0$

In the following it will be shown that the above equations are not satisfied simultaneously and hence $\delta\left(\frac{1}{\delta}\right)_u = \text{constant}$ is not permissible.

Case(i): $L \neq 0$

From (A2) using (A1), we get,

$$\delta\left[\left(u_x^2\right)_u \frac{1}{\delta} + u_x^2\left(\frac{1}{\delta}\right)_u\right] = M$$

$$\left(u_x^2\right)_u = M - Lu_x^2$$

Integrating the above expression with the help of $u = \int \frac{e^{k''\beta}}{\alpha^2} dX$ we get

$$\frac{e^{2\beta k''}}{\alpha^4} = \frac{1}{P}\left\{Q - e^{-Lu}\right\} \qquad \text{(A5)}$$

where $P = \text{constant} \neq 0$, $Q = \text{constant}$

$$\phi_u = \phi_X X_u = \alpha^2 \qquad \text{(A6)}$$

Using $e^{\beta} = f_\pi^2 + \phi^2 + \alpha^2$ and (A5) we get,

$$\frac{\left(f_\pi^2 + \phi^2 + \alpha^2\right)^{2k''}}{\alpha^4} = \frac{1}{P}\left\{Q - e^{-Lu}\right\}$$

Let $\alpha^2 = f_\pi^2 + \phi^2$

We get from above

$$\alpha^2 = \left[\frac{1}{2k''}\left\{\frac{Q - e^{-Lu}}{P}\right\}^{1/2}\right]^{\frac{1}{k''-1}} \qquad \text{(A7)}$$

Now,

$$\phi_u = \alpha^2$$

$$\phi_u = \left[\frac{1}{2k''}\left\{\frac{Q - e^{-Lu}}{P}\right\}^{1/2}\right]^{\frac{1}{k''-1}}$$

$$\phi = \int\left[\frac{1}{2k''}\left\{\frac{Q - e^{-Lu}}{P}\right\}^{1/2}\right]^{\frac{1}{k''-1}} du \qquad \text{(A8)}$$

$$\frac{1}{\alpha} = \frac{1}{\left[\frac{1}{2k''}\left\{\frac{Q - e^{-Lu}}{P}\right\}^{1/2}\right]^{\frac{1}{k''-1}}} \qquad \text{(A9)}$$

Further considering (A4) in

$\delta(X) = \dfrac{\alpha_{XX} - k''\alpha_X\beta_X}{\alpha} \neq 0$, we get

$$\frac{\alpha_{XX} - k''\alpha_X \beta_X}{\alpha} = Ne^{-Lu}$$

which may be written as

$$\frac{e^{k''\beta}\left(e^{-k''\beta}\alpha_X\right)_X}{\alpha} = Ne^{-Lu}$$

With the change of variable from X to u with the help of $u = \int \frac{e^{k''\beta}}{\alpha^2} dX$, the above is reduced to

$$\frac{e^{2\beta k''}}{\alpha^4}\left(\frac{1}{\alpha}\right)_{uu} + \frac{1}{\alpha} Ne^{-Lu} = 0$$

which may be re-written with the help of (A5) as

$$\left\{Q - e^{-Lu}\right\}\left(\frac{1}{\alpha}\right)_{uu} + \frac{1}{\alpha} P.Ne^{-Lu} = 0 \qquad \text{(A10)}$$

$\frac{1}{\alpha}$ can be determined from A9. Elimination of $\frac{1}{\alpha}$ from (A10) with the help of (A9) indicates as if tangent of an angle is expressible in terms of a rational function in amalgamation with roots. This is not permissible in a physical situation.

Case II: $L = 0$

Taking $L = 0$, we similarly arrive at a equation which indicates as if the tangent of an angle is expressible in terms of a rational function in amalgamation with roots. This is not possible in a physical situation.