IF BELIEF CLOSURE FAILS THEN KNOWLEDGE CLOSURE FAILS

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Knowledge Closure, Belief Closure and Modus Ponens:

Modus Ponens (MP) is the name of a valid form of argument in propositional logic. The form of the argument is this.

p implies q

p

Therefore, q.

Its corresponding conditional statement is that if it is true that p implies q, and it is true that p, then, it is true that q.

More than one hundred years ago, Lewis Carroll[1] argued that we have no option but uncritically accept MP because no inference is possible without it; every inference without an uncritical acceptance of MP engenders an infinite regression. The regression involved is roughly this. Infer B from A, your inference presupposes X1 (i.e., If A then B, it is A, therefore, B). Be critical about this presupposition and take it for a hypothesis, not a rule of inference. Name it hypothesis C. Then, you are inferring B from A and C, not from A alone. Consequently, your inference presupposes X2 (i.e., If A and C then B, it is A and C, therefore, B). As you moved from X1 to X2, you can move from X2 to X3 (i.e., If A, C and D then, B, it is A, C and D, therefore, B). Likewise, you can go on ad infinitum unless you stop at a point and uncritically accept MP.

We know that MP is a rule of inference. One might not know that it is a rule of inference though, barring the cases of mental disorders (lunatics) and mental immaturity (babies), we find that everyone with the ability to infer one belief from another does use MP. However, knowing of MP may be understood as to know that MP is a valid form of argument and, hence, in accordance to this understanding, it is not just the knowing how to use MP that becomes sufficient for knowing MP. In this sense of knowing MP, it is quite plausible that some people do know MP and some

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[1] Of course, one may try to doubt on the the uniqueness of MP on the ground that, every other rule of inference has the same epistemic status, hence, as MP needs to be uncritically accepted, so also every other rule of inference. We will not try to enter into this problem, namely, whether MP has epistemic priority over all other rules of inference or not.

people do not. Therefore, we can claim that either S knows that MP or S does not know that MP. That is, either S knows that MP is a valid form of argument or S does not know that MP is a valid form of argument, despite the fact that every S uses MP in the course of inferring certain beliefs from some other beliefs. We present this truth, in short, as T1: K(MP) or ~K(MP).

In our argument, we employ T1 as a tautology. It is no less than a rule of inference. We employ T1 to validly infer certain truths from the truth of some premise(s). Parallel to MP, the Principle of Knowledge Closure (KC) can be stated in the form of an argument:

S knows that p implies q

S knows that p

Therefore, S knows that q.

Its corresponding conditional statement is that if S knows that p implies q and knows that p, then, S knows that q.

When S knows that p implies q, the implication from p to q becomes one of S's known implications. By the principle of KC, knowledge of the consequent follows from the knowledge of the antecedent of a known implication. It is parallel to that the truth of a consequent logically follows from the truth of the antecedent of an implication that happens to be true. We can understand KC as the principle that knowledge is closed under known implications. Of course, in epistemology, KC's validity is in dispute. [2]

Similar to the statement of the principle of KC, we can state the principle of Belief Closure (BC). The principle of BC means that, like knowledge, belief is closed under known implications. We can express BC in an argument form as

S knows that p implies q

S believes that p

Therefore, S believes that q.

Its corresponding conditional statement is that if S knows that p implies q, and believes that p, then, S believes that q.

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[2] Dretske, F. (1970), is famous for his refutation of the KC. Stine, G. (1976), makes an attempt to counter Dretske, F. (1970). Dretske (2005) is an improved version of his 1970 paper; he attempts to reply some of his critics and provides a more plausible explanation against KC.

Following Dretske (2005), one may ask, Why should K be valid when R is not (when R and K stand for the following two arguments)?

S regrets that p (e.g., p: S's son could not get first class)

S knows that p implies q (e.g., q: Someone could not get first class)

Therefore, S regrets that q.

S knows that p (e.g., p: S is watching a game)

S knows that p implies q (e.g., q: S is not a brain in a vat)

Therefore, S knows that q.

Negation of KC amounts to that S knows that p and knows that p implies q but does not know that q. If a context confirms this negation, we count that context as an instance of the Failure of Knowledge Closure (FKC). Similarly, a context confirming to the denial of BC is an instance of the Failure of Belief Closure (FBC). Let us define FBC and FKC as D1 and D2, respectively, by using "Kx" for S knows that x, and "Bx" for S believes that x.

D1: FBC if and only if a context confirms Bp & K(p implies q) & ~Bq.

D2: FKC if and only if a context confirms Kp & K(p implies q) & ~Kq.

The Assumptions of the Argument:

Our argument depends on the following four assumptions.

A1: {K(MP)&{[Bp&B(p implies q)]& ~Bq}} implies B(MP fails with respect to p and q)

A2: {B(MP fails with respect to p and q) or S doubts on (that Mp works with respect to p and q)} implies ~K[MP works with respect to p and q)]

A3: ~K[MP works with respect to p and q] implies ~{Kp & K(p implies q) implies Kq}

A4: If S does not know MP then Kp and K(p implies q) is not sufficient for Kq.

When S has the knowledge of equivalence between p and ~ ~p and, at the same time, believes that Q but does not believe that ~~Q, then, S believes that Q is an exception to the said equivalence. Similarly, if S has the knowledge of the implication that p implies (p or q) and, at the same time, believes that P and does not believe that P or Q, then, S believes that the said implication fails with respect to P and Q. Again, similarly, if S knows MP and, at the same time, believes that P and believes that P implies Q but does not believe that Q, then, S does believe that MP fails with respect to P and Q.

Assume that a subject S knows a logical truth T and that T is in conditional form. If S encounters a proposition x and, though x is in the form of T and S believes the antecedent part of x, S does not believe the consequent part of x. In such a context, we assume that, somehow, S believes that T does not work with respect to x. Accordingly, if we put forth MP for that x, [p & (p implies q)] for the antecedent of x and q for the consequent of x, we assume that it is A1.

Now, if S believes that MP does not work with respect to P and Q, then, his knowing that P and his knowing that P implies Q are not sufficient for his knowing that Q. Moreover, with the belief that MP does not work with respect to P and Q, S must believe that it is ~Q, hence, S does not know that Q, even if S knows that P and knows that P implies Q.

We assume that S does not know that a logical truth T works with respect to p and q, even if S has the knowledge of T, if either S believes that T fails or S doubts that T succeeds with respect to p and q. Therefore, it is A2.

Secondly, if S does not know that T works with respect to p and q, then, even if S satisfies a part of T2 that results from a reformulation of T in terms of some epistemic operators on p and q, this is not sufficient for the claim that S satisfies T2. Therefore, it is A3.

When it is (1) S believes that p and believes that p implies q but not that q and (2) S knows that MP, we find that S believes that MP fails with respect to p and q. Now, when it is (1) and (2) and, thereby, S believes that MP fails with respect to p and q, S's knowledge that p plus his knowledge that p implies q is not sufficient for his knowledge that q. Because, the very reason, namely, an application of MP to p and p implies q, which we may cite for the sufficiency of [p and (p implies q)] for q is not believed by S.

Thus we conclude that, when it is (1) and (2), it is (3) ~ {[Kp & K(p implies q)] implies Kq}. Also, we obtain (3) even when it is (2)* S does not know that MP. For, when it is (2)*, to claim that {[Kp & K(p implies q)] implies Kq} amounts to the claim that S knows the consequent of a proposition even if S does not know the very reason we normally use to infer that consequent. Therefore, it is A4.

If it is (1), then, whether (2) or (2)*, it is (3). As (1) expresses FBC and (3) expresses FKC, we conclude that FBC is sufficient for FKC.

A formal presentation of our argument that FBC implies FKC:

(1) FBC

(2) For some p and q, {[Bp & K(p → q)] & ~Bq} From 1, by D1

(3) K(MP) or ~K(MP) T1

(4) [Bp & K(p → q)] & ~Bq From 2, by EI

(5) {K(MP) or ~K(MP)} & {[Bp & K(p → q)] & ~Bq} From 3 and 4 by Conjunction

(6) {{K(MP) & {[Bp & K(p → q)] & ~Bq}} or
{~K(MP) & {[Bp & K(p → q)] & ~Bq}} From 5, by Distribution

(7) K(MP)& {[Bp & K(p → q)] & ~Bq} Assumption

(8) B(MP fails with respect to p and q) From 7, by A1

(9) ~K(MP works with respect to p and q) From 8, by A2

(10) ~{[Kp & K(p → q)] → Kq} From 9, by A2

(11) [Kp & K(p → q)] & ~Kq From 10, by Implication

(12) {K(MP)& {[Bp & K(p → q)] & ~Bq}} →
{[Kp & K(p→ q)] & ~Kq} From 7-11, by Conditional Proof

(13) ~K(MP) & {[Bp & K(p→ q)] & ~Bq} Assumption

(14) ~K(MP) From 13, by Simplification

(15) ~{[Kp & K(p→ q)] → Kq} From 14, by A4

(16) [Kp & K(p→ q)] & ~Kq From 15, by Implication

(17){~K(MP)&{[Bp&K(p→ q)]&~Bq}}
→ {[Kp & K(p→ q)] & ~Kq} From 13-16, by Conditional Proof

(18) {[Bp&K(p→ q)]&~Bq}→{[Kp & K(p→ q)] & ~Kq} From 12 and 17, by Tautology: {[(X & Y) → Z] & [(~X & Y) → Z]} → (Y→ Z)

(19) FBC → FKC From 18, by D1 and D2

Thus, we prove that if Belief Closure fails then Knowledge Closure fails. That is, an instance of the failure of Belief Closure is an instance of the failure of Knowledge Closure.

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