# CHAPTER-3

# METHODOLOGICAL ISSUES

## 3.1 Introduction :

Macro-Economic variables, which are used in this study, are of time series by nature. These series are not deterministic variables. On the contrary, these are considered to be generated by some underlying stochastic processes. In any time series ($Y_t$), each value of $Y_1, Y_2 \ldots\ldots, Y_t$ is assumed to be drawn randomly from a probability distribution. To be completely general, the observed series $Y_1, Y_2 \ldots\ldots, Y_t$ is assumed to be drawn from a set of jointly distributed random variables. If the underlying probability distribution function of the series could be specified, then one could determine the probability of one or another future values of the variable concerned.

The complete specification of the probability distribution function for any time series is usually impossible. However, it is possible to construct a simplified model for the time series, which explains its randomness in a manner that is useful for econometric studies. This simple model may be a reasonable approximation of the actual and more complicated underlying stochastic process. The usefulness of such a model depends on how closely it captures the true probability distribution and the true random behavior of the series. Consequently, the validity and usefulness of macroeconomic studies with time series like export, economic growth etc. depends upon the nature of underlying stochastic process and upon approximation of the process.

Specification of the underlying stochastic process is preceded by the identification of the nature of the stochastic process. More specifically, it is necessary to know whether the underlying stochastic process is invariant with time or whether it describes a random walk. If the process is non-stationary, it will be difficult to represent time series over past and future intervals of time by an algebraic model. By contrast, if the stochastic process is fixed in time i.e., if it is 'stationary', then one can model the process through an equation with fixed coefficients that can be estimated from the past data. It is analogous to the single equation regression model in which one variable is related to another variable with coefficients that are estimated under the assumption that the structural relationship described by the equation is 'invariant' over time. The probability of a given fluctuation in the process from the mean level is assumed to be the same at any point of time. In other words, the stochastic properties of the stationary process are assumed to be invariant with respect of time. For a stationary process both the joint probability distribution and conditional probability distribution are invariant with respect of time.

### 3.2 The Basic Problem:

In a provocative study Charless R. Nelson and C.I. Plosser (1982) found evidence that macroeconomic variables like GNP, exchange rate, interest rate, employment, money supply, price level etc. behave like random walks. As these series follow 'Random walks', these are not 'trend reverting'. Consequently, these economic variables do not tend to revert back to a long run trend after a shock.

These findings of Nelsion and plosser (1982) posed serious problems for econometric studies with macroeconomic variables. The studies, so far carried out with the macroeconomic variables, were based on the idea that these variables were 'deterministic non-stationary' series. Stationarities in these series were ensured through 'Filtering' like differencing of the series and identifying appropriate Autoregressive Moving Average (ARMA) processes. Findings of Nelson and Plosser (1982) hit the basic idea underlying these studies and the relevance of the studies was threatened consequently.

### 3.3 The Nature Of The Problem:

The reason why Nelson-Plosser findings would threaten the basic approach behind the econometric studies with macroeconometric time series and why random walk process for the time series would limit the use of this series in econometric studies need serious consideration.

First, variance of the random walk processes in the joint probability distribution are no longer constant. Instead, the variances expand out with time and the random walk errors are no longer '*Homoscedastic*'. Consequently, the *Gauss-Markov theorem* would not hold, and *Ordinary Least Squares* (OLS) method would not yield *consistent estimates* of the parameters concerned.

Second, random walk processes fail to possess finite variance. In such case, the regression analysis fails and econometric studies with these series become irrelevant.

Third, detrending the variable before running the regression will not help because even the detrended series still remains non-stationary. Consequently, the random walk process becomes non-deterministic, non-stationary process. In such case detrending fails to ensure stationarity.

Fourth, if a variable follows a random walk, the effects of a temporary shock will not dissipate after several years but instead will be permanent. This occurs because the autocorrelation functions for such variables are 'uniform' by nature and it declines geometrically over time. The random walk process in such case has an infinite memory. The current value of the process depends on all past values and the magnitude of the effect remains unaltered with time. As a result, the effect of a temporary shock will not dissipate after several years but will remain permanent. This further indicates that, in case of the presence of non-stationarity in the series for the variable describing random walks, the series does not revert back to a long run trend after a shock.

### 3.4 Stationarity: Relevance of Unit Root Tests:

Unit Root Test is useful for examining the 'stationarity' of a time series. Given a time series data generating process (DGP), testing for random walk is a test of 'stationarity'. It is also called the 'Unit Root Test'.

Suppose we have a series $\{Y_t\}$ with process such that

$$Y_t = \phi Y_{t-1} + \varepsilon_t \qquad 3.1$$

where $|\phi| < 1$ and $\varepsilon_t \sim$ iid $N(0,\sigma^2_t)$.

We can estimate the parameters in equation (3.1) by OLS. Our estimator is efficient and the series is stationary since $|\phi| < 1$. We could use a t-statistic to test the hypothesis.

$H_0: \phi = 0$
against
$H_1: \phi \neq 0$

This is a legitimate test since the null-hypothesis is a refutable hypothesis, even though the power against a local alternative is negligible.

Now we suppose that the data set was really generated by

$$Y_t = Y_{t-1} + \varepsilon_t$$

Upon recursive substitution this can be written as

$$Y_t = \sum_{j=0}^{t} \varepsilon_{t-j}$$

which is non-stationary since $VAR(Y_t) \rightarrow \infty$ as t gets large. Now we would want to test

$H_0 : \phi = 1$
against
$H_A : \phi < 1$

This is a problem, however, since the centre of mass of the usual estimator would be bounded away from 1. We would tend to error on the side of rejection too many $H_0$.

Under these circumstances 'spurious' regression with a high $R^2$ but near zero Durbin-Watson statistic is found to occur in time series analysis. So the usual tests fail to test 'Stationarity' or random walk process in the time series. It is done through a special test called 'unit Root Test'.

**3.5 Unit Root Test: The Methodology:**

Let us consider the data generating process

$$Y_t = \phi Y_{t-1} + \varepsilon_t \qquad 3.1$$

The associated question is whether $\phi = 1$. Subtracting $Y_{t-1}$ from both sides we get,

$$\Delta Y_t = (\phi - 1) Y_{t-1} + \varepsilon_t$$
$$= \gamma Y_{t-1} + \varepsilon_t$$

$\gamma = 0$ implies that $\phi = 1$ which indicates the presence of a unit root in $\{Y_t\}$.

A drift is allowed by including an intercept

$$Y_t = \alpha_0 + \gamma Y_{t-1} + \varepsilon_t$$

Allowing for linear trend with a drift gives us

$$\Delta Y_t = \alpha_0 + \gamma Y_{t-1} + \alpha_1 t + \varepsilon_t$$

In any event, the test hypothesis is

$H_0 : \gamma = 0$ ( $Y_t$ has a unit root )

against

$H_1 : \gamma \neq 0$ ( $Y_t$ is stationary )

The test statistic $\hat{\gamma} \,/\, \sqrt{Var(\hat{\gamma})}$ is a t-statistic. The critical values come from a set of Tables prepared by Dickey and Fuller. This test is known 'Dickey-Fuller Test'.

The immense literature and diversity of unit root tests can at times be confusing and present a truly daunting prospect for a researcher. The unit root theory has been examined with an emphasis on testing principles. The summary of finding is given below:

*When time series data are used in econometric analysis, the preliminary tests provide information about stationary of the data. Non-stationary data contain unit roots. The main objective of unit root tests is to determine the degree of integration of each individual time series.* Various methods for unit root tests have been applied in the study. Some of them are being explained below.

**3.6 Augmented Dickey Fuller Unit Root Test:**

In order to test for the existence of unit roots, and to determine the degree of differencing necessary to induce stationarity, the Augmented Dickey-Fuller test is used. Dickey and Fuller (1976, 1979), Said and Dickey(1984), Phillips (1987), Phillips and Perron (1988), and others developed modifications of the Dickey-Fuller Test (ADF) determine the the form in which the data should be applied in any econometric analysis. The test is based on the following equations:

$$\Delta y_t = \gamma + \alpha y_{t-1} + \sum_{j=2}^{k} \theta_j \Delta y_{t-j+1} + e_t \qquad 3.2$$

$$\Delta y_t = \gamma + \delta t + \alpha y_{t-1} + \sum_{j=2}^{k} \theta_j \Delta y_{t-j+1} + e_t \qquad 3.3$$

$$\Delta y_t = \alpha y_{t-1} + \sum_{j=2}^{k} \theta_j \Delta y_{t-j+1} + e_t \qquad 3.4$$

where $y_t$ = Modelled Variabes, $\Delta y_t$ = First differenced series of $y_t$

$\Delta y_{t-j+1}$ = First differenced series of $y_t$ at (t-j+1)th lags. ( j=2…k )

The equation (3.2) is related to ADF test with *constant as exogenous*. Equation (3.3) is based on *constant and linear trend as exogenous and* ADF test with *no exogenous* is presented in equation (3.4).

**3.7 The DF- GLS Unit Root Test:**

The DF-GLS test was developed by Elliott, Rothenberg and Stock (1996). The DF-GLS t-test is performed by testing the hypothesis $a_0$ =0 in the regression

$$\Delta y_t^d = a_0 y_t^d + \Delta y_{t-1}^d + ....... + a_p \Delta y_{t-p}^d + u_t \qquad 3.5$$

where $y^d_t$ is the locally de-trended series $y_t$. The local *de-trending* depends on whether we consider a model with drift only or a linear trend.

(i) The model for DF-GLS unit root test without time trends i.e., a model with *drift* only is

$$y_t^\mu = \alpha y_{t-1}^\mu + \sum_{i=1}^{k} \psi_i \Delta y_{t-i}^\mu + u_t \qquad 3.6$$

(ii) The model for DF-GLS unit root test with *time trends* i.e. a model with *linear trend* is

$$y_t^\mu = \alpha y_{t-1}^\mu + \delta t + \sum_{i=1}^{k} \psi_i \Delta y_{t-i}^\mu + u_t \qquad 3.7$$

**3.8 Phillips-Perron unit Root Test:**

Phillips (1987), Phillips and Perron (1988) have generalized the DF tests to situations where disturbance processes $\varepsilon_t$ are serially correlated, without augmenting the initial regression with lagged dependent variables. The PP is intended to add a 'correction factor' to the DF test statistic and the test is designed for examining the presence of any '*structural shift*' in the dataset.

Let the AR(1) model be

$$Y_t = \mu + \varphi_1 Y_{t-1} + \varepsilon_t \ [\, t = 1, \ldots, T \,] \qquad 3.8$$

with Var ($\varepsilon_t$) $\equiv \sigma^2_\varepsilon$

If $\varepsilon_t$ is serially correlated, the ADF approach is to add lagged $\Delta Y_t$ to 'whiten' the residuals. To illustrate the alternative approach, the test statistic $T(\varphi_1 - 1)$ has been considered which is distributed as $\rho_\mu$ from the maintained regression with an intercept but no time trend. The PP modified version is

$$Z\rho_\mu = T(\varphi_1 - 1) - CF \qquad 3.9$$

where the correction factor CF is

$$CF = 0.5(s_{Tl}{}^2 - s_\varepsilon^2)/[\sum_{t=2}^{T}(Y_{t-1} - \bar{Y}_{t-1})^2 / T^2)] \qquad 3.10$$

and, $$s_\varepsilon^2 = T^{-1}\sum_{t=1}^{T}\varepsilon_t^2 \qquad 3.11$$

$$s_{Tl}^2 = s_\varepsilon^2 + 2\sum_{s=1}^{l} W_{sl} \sum_{t=s+1}^{T} \varepsilon_t \varepsilon_{t-s} / T \qquad 3.12$$

and

$$\bar{Y} = \sum_{t=2}^{T} Y_t / (T-1) \qquad 3.13$$

**3.9 Correlogram:**

One of the simple, intuitive and interesting methods of testing 'stationarity' is running a correlogram. Correlogram is nothing but a graphical representation of Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF). The nature of stationarity can also be found almost accurately in most of the cases with the help of Correlogram.

### 3.10 Cointegration:

Macroeconomic variables are usually non-stationary and exhibit random walk processes. Regressing one random variable against the another can lead to spurious results since conventional significance tests will tend to indicate a relationship between the variables when in fact none exists. This is the reason why it is important to test for random walks.

Random walk processes attain stationarity upon first differencing. If a test fails to reject the hypothesis of a random walk, one can difference the series before using them in regression. Since many economic time series follow random walks, variables are subject to first differencing before using them in regression.

However, differencing the data has a cost. The cost arises from the fact that the differncing may result is a loss of information about the long-run relationship between variables concerned. This occurs because the models estimated with differenced data do not have a long run solution. Moreover, the level of a variable and its first difference will typically be very different in terms of mean and variance.

It is of interest, therefore, to ask whether there are formal arguments in favour of or against differencing. One approach to this issue starts by noting that estimation and hypothesis testing, using the least square method, is justified only when the various variables being used are stationary. Differencing would be appealing, therefore, if the first differences of a set of variables were stationary, with the variables themselves being non-stationary.

Engel Granger (1987) hold that there are situations where one can run a regression between two variables even though both variables are random walks. Sometimes two variables follow random walks but linear combinations of those variables will be stationary. In such case the variables are cointegrated. Co-integration provides a method to eliminate the cost of differencing by retaining terms in levels but only in linear combinations, which are stationary.

### 3.11 Cointegration: Definition:

Cointegration is the study concerning the existence of long run equilibrium relationship among variables. The study allows the researcher to describe the existence of an equilibrium or stationary relationship among two or more series, each of which is *individually non-stationary*. According to Engle and Granger (1987) the variables will be cointegrated when the linear combination of non-stationary variables is stationary.

Engle and Granger (1987) provide the following definition of *cointegration.*

The components of the vectors $x_t = (x_{1t}, x_{2t}, \ldots x_{nt})$ are said to be *cointegrated of order d, b,* denoted by $x_t \sim CI\ (d,b)$ if

(i) all the components of $x_t$ are integrated of order d

(ii) there exists a vector $\beta = (\beta_1, \beta_2, \ldots, \beta_n)$ such that the linear combination

$$\beta x_t = \beta_1 x_{1t} + \beta_2 x_{2t} + \ldots + \beta_n x_{nt}$$

is integrated of order (d-b) where b>0.
The vector $\beta$ is called the **cointegrating Vector.**

More specifically, let $\{Y_t\}$ and $\{X_t\}$ be two I(1) series such that

$$Y_t = \alpha + \beta X_t + u_t$$

or, $u_t = Y_t - \alpha - \beta X_t$

Now if $u_t$ is I(0) i.e.,stationary at level, then $Y_t$ and $X_t$ are CI(1,1).

### 3.12 Four Cases for Cointegration:

Here we face four cases for consideration;

**Case 1:** Let $\{Y_t\}$ and $\{X_t\}$ be stationary such that $Y_t \sim I(0)$ and $X_t \sim I(0)$. Consequently, d=0 and there cannot be any b>o so that $Y_t, X_t \sim CI\ (d,b)$.

Under this situation, a long run relationship between $Y_t$ and $X_t$ does already exist and the classical regression model is appropriate.

**Case 2:** Let $\{Y_t\}$ and $\{X_t\}$ sequences are of different orders. Then regression equation using such variables are meaningless.

**Case 3:** The non-stationary $\{Y_t\}$ and $\{X_t\}$ sequences are integrated of the same order but the residual sequence contains a stochastic trend. This is the case in which the regression is spurious.

**Case 4:** The non-stationary $\{Y_t\}$ and $\{X_t\}$ sequences are integrated of the same order and the residual sequence is stationary. In this case $\{Y_t\}$ and $\{X_t\}$ are cointegrated.

### 3.13 Vector Error Correction Modeling:

Vector Error Correction modeling provides important information on the short run relationship between any two cointegrated variables. Vector Error Correction test has provided empirical evidence on the short run causality among variables concerned.

In the present study the Vector Error Correction Model consists of the equations involving GDP and Export.

$$\Delta y_t = \gamma_1 + \rho_1 z_{t-1} + \alpha_1 \Delta y_{t-1} + \alpha_2 \Delta y_{t-2} + \alpha_3 \Delta x_{t-1} + \alpha_4 \Delta x_{t-2} + \varepsilon_{1t} \qquad 3.14$$

$$\Delta x_t = \gamma_2 + \rho_2 z_{t-1} + \beta_1 \Delta x_{t-1} + \beta_2 \Delta x_{t-2} + \beta_3 \Delta y_{t-1} + \beta_4 \Delta y_{t-2} + \varepsilon_{2t} \qquad 3.15$$

where, $\Delta y_t$ = first difference of GDP;

$\Delta x_t$ = first difference of export;

$z_{t-1}$ = first lag of error term of co-integrating equation;

$\varepsilon_{1t}$ and $\varepsilon_{2t}$ are white noise errors;

$\alpha_1$, $\alpha_2$, $\alpha_3$ and $\alpha_4$ ; $\beta_1$, $\beta_2$, $\beta_3$ and $\beta_4$ are the coefficients of lagged variables in the above model.

The focus of the Vector Error Correction analysis is on the lagged $z_t$ terms. These lagged terms are the residuals from the previously estimated cointegrating equations. Lagged $z_t$ terms provide an explanation of short run deviations from the long run equilibrium. Lagging these terms means that disturbance of the last period impacts upon the current time period.

Statistical significance tests are conducted on each of the lagged $z_t$ term in equations (3.14) and (3.15). In general, finding a statistically insignificant coefficient of the $z_t$ term implies that the system under investigation is in short run equilibrium as there are no disturbances present. If the coefficient of the $z_t$ term is found to be statistically significant, then the system is in the state of the short run disequilibrium. In such a case the sign of $z_t$ term gives an indication of the *causal direction* between the two test variables in the short run.

### 3.14 Granger Causality: Introduction:

The study of Cointegration of variables indicates if the variables are related or not. The cointegration procedure stresses upon estimating distributed lag relationship along with the error correction structure. However, the autoregressive structure does not play any significant role in the study of cointegration between variables concerned.

This particular feature of the cointegration equations accounts for the inability of the equation in explaining if the variables concerned are 'exogenous' or 'endogenous'. Engle, Hendry and Richard (1983) define a set of a variable $X_t$ in a parameterized model to be 'weakly exogenous' if the full model can be written in terms of a marginal probability distribution of $X_t$ and a conditional distribution of $Y_t / X_t$ such that estimation of the parameters involve the joint distribution of $Y_t$ and $X_t$. With reference to time series applications, variable $X_t$ is said to be predetermined in the model if $X_t$ is independent of all subsequent structural disturbances $\varepsilon_{t-s}$. Variables that are predetermined in the model can be treated, at least asymptotically, as if they were exogenous in the sense that consistent estimates may be obtained when they appear as regressors.

Cointegrating equations cannot establish if any of the variables is exogenous. Consequently, cointegrating equations cannot be used for forecasting purposes. These equations, therefore, cannot explain if one of the variables could be used for the effective prediction for variations in another variable. This explains why cointegrating relation fails to establish 'Granger' causal relationship between variables concerned. Granger Approach, on the other hand, provides the effective method of determining the nature, direction and pattern of causal relations between variables under study.

### 3.15 Methodology:

Let us consider a jointly covariance stationary stochastic process $y_t$, $x_t$ with $E(x_t) = E(y_t) = 0$ and with a covariance generating function fx(z) , gy(z) and gxy(z). It is assumed that x possesses an autoregressive representation and both y and x are linearly in deterministic.

Then the projection of $x_t$ on past values of x and past values of y is given by

$$X_t = \sum_{j=1}^{\infty} h_j x_{t-j} + \sum_{j=1}^{\infty} v_{t-j} y_{t-j} + u_t \qquad 3.16$$

where the least square residuals $u_t$ obey the orthogonality condition

$E(u_t x_{t-\beta}) = E(u_t y_{t-\beta}) = 0$ for $\beta = 1,2,\ldots..$

Solving (3.16) for $u_t$ permits the orthogonality condition to assume the form of normal equations

$$E\{(x_t - \sum_{j=i}^{\infty} h_j x_{t-j} - \sum_{j=1}^{\infty} v_j y_{t-j}) x_{t-\beta}\} = 0, \qquad 3.17$$

$\beta = 1,2, \ldots\ldots\ldots.$

$$E\{(x_t - \sum_{j=i}^{\infty} h_j x_{t-j} - \sum_{j=1}^{\infty} v_j y_{t-j}) y_{t-\beta}\} = 0, \qquad 3.18$$

$\beta = 1,2, \ldots\ldots\ldots.$

These equations can be written as

$$C_x(\beta) = \sum_{j=i}^{\infty} h_j c_x(\beta - j) + \sum_{j=1}^{\infty} v_j c_{yx}(\beta - j) \qquad 3.19$$

$$C_x(\beta) = \sum_{j=i}^{\infty} h_j c_{xy}(\beta - j) + \sum_{j=1}^{\infty} v_j c_y(\beta - j) \qquad 3.20$$

These equations hold only for positive integer

$\beta = 1,2, \ldots\ldots\ldots.$

Multiplying both sides of (3.19) and (3.20) by $z^{\beta}$ and summing over all $\beta$, we get the following equations in terms of z transformation

$$g_{xy}(z) + m(z) = h(z) g_x(z) + v(z) g_{yx}(z) \qquad 3.21$$

$$g_{xy}(z) + n(z) = h(z) g_{xy}(z) + v(z) g_y(z) \qquad 3.22$$

where m(z) and n(z) are unknown series in non-positive power of z only. That m(z) and n(z) series are non-positive powers of z is equivalent with equations (3.19) and (3.20) holding only for $\beta>1$. Equations (3.21) and (3.22) are the normal equations for h(z) and v(z).

Following Weiner, Granger (1969) has proposed that '*y causes x*' *whenever v(z)≠0. That is, y is said to cause x if, given all past values of x, past values y help predict x.* The conditions under which v(z) does or does not equal to zero turn out to be of substantial interest to econometrician and macro-economists.

Let us consider the projection of $y_t$ on the entire x process

$$y_t = \sum_{j=1}^{\infty} b_j x_{t-j} + \varepsilon_t \qquad 3.23$$

where $E(\varepsilon_t x_{t-j}) = 0$ for all j

Let $x_t$ have the 'World Moving Average' presentation such that

$$X_t = d(L)\eta_t$$

$$\eta_t = x_t - P[x_t / x_{t-1}, x_{t-2}, \ldots\ldots\ldots]$$

$$\sum_{j=1}^{\infty} d_j^2 < \infty$$

Then $gx(z)=\sigma^2_n d(z)d(z^{-1})$ 3.24

It is assumed that x possesses an autoregressive representation so that $[d(z^{-1})]$ is one sided square summable in non-negative power of z. It is always possible to uniquely factor the cross covariance generating function as

$$g_{xy}(z) = \alpha(z)\varphi(z^{-1}) \qquad 3.25$$

where $\alpha(z)$ and $\varphi(z)$ are one sided in non-negative power of z.

Substituting (3.24) and (3.25) into the usual relation

$$b(z) = g_{yx}(z) / g_x(z) \qquad 3.26$$

we have

$$b(z) = \alpha(z)\varphi(z^{-1}) / \sigma_{\eta}^{2} d(z)d(z^{-1}) \qquad 3.27$$

Evidently, b(z) is one sided in non-negative powers of z if and only if $\varphi(z^{-1}) = kd(z^{-1}) = kd(z^{-1})$, where k is a constant. Under this condition (3.26) becomes

$$b(z) = k\alpha(z) / \sigma_{\eta}^{2} d(z) \qquad 3.28$$

Here α(z) has an inverse that is one sided in non-negative power of z.

Now if b(z) is one sided in non-negative power of z, the equation (3.21) and (3.22) are both satisfied with v(z)=0 and

$$b(z) = z[d(z)/z] + d(z^{-1}) \qquad 3.29$$

Consequently, equation (3.22) becomes

$$\phi(z)\alpha(z^{-1}) + n(z) = z[d(z)/z] + \frac{1}{d(z)}\alpha(z^{-1})\phi(z) \qquad 3.30$$

Dividing both sides of equation (3.30) by $z\alpha(z^{-1})$ gives

$$\phi(z)/z + n(z)/z\alpha(z^{-1}) = [d(z)/z] + \varphi(z)/d(z) \qquad 3.31$$

where $n(z)/z\alpha(z^{-1})$ involves only negative powers of z. Since the right hand side involves only non-negative power of z, (3.31) implies

$$d(z)[\varphi(z)/z] = [d(z)/z]\varphi(z) \qquad 3.32$$

This equation (3.32) can be satisfied if $\varphi(z) = kd(z)$ where k is a constant.

Now let $(x_t, y_t)$ be a jointly covariance stationary, strictly indeterministic process with zero mean. Then $\{y_t\}$ fails to Granger cause $\{x_t\}$ if and only if there exists a vector moving average representation

$$\begin{bmatrix} x_t \\ y_t \end{bmatrix} = \begin{bmatrix} c(L)_{11} 0 \\ c(L)_{21} c(L)_{22} \end{bmatrix} \begin{bmatrix} \varepsilon_t \\ u_t \end{bmatrix} \qquad 3.33$$

where $\varepsilon_t$ and $u_t$ are serially uncorrelated processes with means zero and $E(\varepsilon_t u_s)=0$ for all t and where the one-step-ahead errors

$[x_t - p(px_t / x_{t-1},......., y_{t-i},....)] and [y_t / y_{t-1},....., y_{t-1},......)]$ are each linear combination of $\varepsilon_t$ and $u_t$.

Under these situations Sims (1972) explains the concept of causality through the following theorem

*"$Y_t$ can be expressed as a distributed lag of current and past x's (with no further x's) with a disturbances process that is orthogonal to past, present and future x's if and only y does not Granger cause x".*

Consequently, the test involves estimating the following regressions:

$$y_t = \sum_{i==1}^{n} \alpha_i x_{t-j} + \sum_{j=1}^{n} \beta_j y_{t-j} + u_{1t} \qquad 3.34$$

$$x_t = \sum_{i==1}^{n} \alpha_i y_{t-j} + \sum_{j=1}^{n} \delta_j x_{t-j} + u_{2t} \qquad 3.35$$

where it is assumed that the disturbances $u_{1t}$ and $u_{2t}$ are uncorrelated.

Equation (3.34) postulates that current $y_t$ is related to past values of $y_t$ itself as well as of $x_t$ and (3.35) postulates a similar behavior for $x_t$. Four cases then can be distinguished.

**(i) Unidirectional Causality From x to y :**

*It is indicated if the estimated coefficients on the lagged x in (3.34) are statistically different from zero as a group (i.e, $\sum\alpha_i \neq 0$) and the set of estimated coefficients on the lagged y in (3.35) is not statistically different from zero (i.e, $\sum\delta_j=0$).*

**(ii) Unidirectional causality from y to x :**

*It exists if the set of lagged x coefficients in (3.34) is not statistically different from zero (i.e,=0) and the set of the lagged y coefficient in (3.35) is statistically different from zero (i.e, $\sum\delta_j \neq 0$).*

**(iii) Feedback or Bilateral Causal:**

*It is suggested when the sets of x and y coefficient are statistically significant in both the regressions.*

**(iv) Independence:**

*It is suggested when the sets of x and y coefficients are not statistically significant in both the regressions.*

**3.16 Conventional Granger Causality Test:**

The model for Conventional Granger Causality test is based on the following equations:

$$Y_t = \sum_{j=1}^{m} a_j X_{t-j} + \sum_{j=1}^{m} b_j Y_{t-j} + \varepsilon_t \qquad 3.36$$

$$X_t = \sum_{j=1}^{m} a_j Y_{t-j} + \sum_{j=1}^{m} b_j X_{t-j} + \eta_t \qquad 3.37$$

where $Y_t$ and $X_t$ represent first difference of GDP and Export series respectively.

**3.17 'Window Finding' of Structural Changes: Methodology:**

The choice of sub-periods objectively involves the identification of structural changes through 'Window Finding'. The basic procedure is described below.

Sometimes researcher seeks to investigate the stability of the coefficient estimates as the sample size increases. Sometimes researcher also wants to find out whether the estimate will be different in enlarged samples and whether these will remain stable over time. Working with a sample, a researcher may produce a regression which is too closely tailored to his sample by experimenting with too many formulations of his model. In this case, he is not contained whether the estimated function will perform equally well outside the sample of data which has been used for the estimation of coefficients. Furthermore, there may have occurred events which change the structure of the relationship like changes in taxation law, introduction of birth control measures and so on. If such structural changes occur, the co-efficient may not be stable. These may again be sensitive to the changes in the sample compositions.

Testing for structural stability calls for the use of additional observations besides the sample that are used to estimate a given model. Procedures for testing structural stability are given by Rao (1960) and Chow (1952).

The econometric method which involves "Window Finding" uses Chow test to identify the sub-periods. Here equality between two regressions co-efficient concerning the relationship over two different periods is tested. This is done by F-test. Let us consider two samples with $n_1$ and $n_2$ observations respectively and the general model for data set is –

$$Y = X\beta + u \qquad 3.38$$

$$Y \rightarrow n \times 1$$

$$X \rightarrow n \times k$$

$$\beta \rightarrow n \times k$$

$$n \rightarrow n_1 + n_2$$

Let us rewrite the model for these two individual samples such as

$$Y_1 = (z_1 w_1)\begin{pmatrix} \gamma_1 \\ \delta_1 \end{pmatrix} + u_1 \qquad 3.39$$

$$Y_2 = (z_2 w_2)\begin{pmatrix} \gamma_2 \\ \delta_2 \end{pmatrix} + u_2 \qquad 3.40$$

where $Y_1 \rightarrow n_1 \times 1$

$Y_2 \rightarrow n_2 \times 1$

$Z_1 \rightarrow n_1 \times 1$

$Z_2 \rightarrow n_2 \times 1$

$W_1 \rightarrow n_1 \times m$

$W_2 \rightarrow n_2 \times m$

$\gamma_1 \rightarrow 1 \times 1$

$\gamma_2 \rightarrow 1 \times 1$

$\delta_1 \rightarrow m \times 1$

$\delta_2 \rightarrow m \times 1$

By combining (3.39) and (3.40) we have

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \begin{pmatrix} z_1 0 w_1 0 \\ 0 z_2 0 w_2 \end{pmatrix} \begin{pmatrix} \gamma_1 \\ \gamma_2 \\ \delta_1 \\ \delta_2 \end{pmatrix} + \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \qquad 3.41$$

And the null hypothesis of interest is

$H_0 : \gamma_1 = \gamma_2$ (=$\beta$ say)

Under the null hypothesis, the model is

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \begin{pmatrix} z_1 w_1 0 \\ z_2 0 w_2 \end{pmatrix} \begin{pmatrix} \beta \\ \delta_1 \\ \delta_2 \end{pmatrix} + \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \qquad 3.42$$

The L.S estimate of the efficient vector in (3.42) is

$$\begin{pmatrix} \beta \\ \hat{\delta}_1 \\ \hat{\delta}_2 \end{pmatrix} = \left[ \begin{pmatrix} z_1 w_1 0 \\ z_2 0 w_2 \end{pmatrix} \begin{pmatrix} z_1 w_1 0 \\ z_2 0 w_2 \end{pmatrix} \right]^{-1} \begin{bmatrix} z_1 \; w_1 0 \\ z_2 0 w_2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \qquad 3.43$$

If we fit (3.39) and (3.40) individually, their LS estimates of the coefficients will be

$$\begin{pmatrix} c_1 \\ \delta_1 \end{pmatrix} = \left[ (z_1 w_1)' (z_1 w_1) \right]^{-1} (z_1 w_1)' y_1 \qquad 3.44$$

$$\begin{pmatrix} c_1 \\ \delta_2 \end{pmatrix} = \left[ (z_1 w_1)' (z_1 w_1) \right]^{-1} (z_1 w_1)' y_1 \qquad 3.45$$

where $c_1$ is the estimate of $\gamma_1$. The sum of squares necessary for computing test statistics can then be obtained by using the results in (3.43), (3.44) and (3.45). The sum of squares measures the distance of individual observations from the common regression plane is

$$Q_1 = \left[ \begin{pmatrix} c_1 \\ \delta_1 \end{pmatrix} - \begin{pmatrix} z_1 w_1 0 \\ z_2 0 w_2 \end{pmatrix} \begin{pmatrix} b \\ \delta_1 \\ \delta_2 \end{pmatrix} \right]' \left[ \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} - \begin{pmatrix} z_1 w_1 0 \\ z_2 0 w_2 \end{pmatrix} \begin{pmatrix} b \\ \delta_1 \\ \delta_2 \end{pmatrix} \right] \qquad 3.46$$

Here $Q_1 / \delta_2$ has $\chi^2$ distribution with (n-2m-1) degrees of freedom where we assume that $u_1$ and $u_2$ have a common variance $\delta_2$. Now $Q_1$ can be decomposed into two sum squares $Q_2$ and $Q_3$. $Q_2$ will measures the distance of the individual estimated regression planes, and $Q_3$ measures the distance of the individual estimated plane from the common regression plane. Thus,

$$Q_3 = \left[y_1 - (z_1 w_1)\begin{pmatrix} c_1 \\ \delta_1 \end{pmatrix}\right]^{\prime} y_1 - (z_1 w_1)\begin{bmatrix} c_1 \\ \delta_1 \end{bmatrix} + \left[y_2 - (z_2 w_2)\begin{pmatrix} c_2 \\ \delta_2 \end{pmatrix}\right]^{\prime} y_2 - (z_2 w_2)\begin{bmatrix} c_2 \\ \delta_2 \end{bmatrix} \qquad 3.47$$

and $Q_3 = Q_1 - Q_2$. Here $Q_2/\delta_2$ has a $\chi^2$ distribution with (n-2m-1) degrees of freedom.

Again,

$$Q_3 = \left[(z_1 w_1)\begin{pmatrix} c_1 \\ \delta_1 \end{pmatrix} - (z_1 w_1)\begin{pmatrix} b \\ \delta_1 \end{pmatrix}\right]^{\prime}\left[(z_2 w_2)\begin{pmatrix} c_1 \\ \delta_1 \end{pmatrix} - (z_1 w_1)\begin{pmatrix} b \\ \delta_1 \end{pmatrix}\right]$$
$$+\left[(z_2 w_2)\begin{pmatrix} c_2 \\ d_2 \end{pmatrix} - (z_2 w_2)\begin{pmatrix} b \\ d_2 \end{pmatrix}\right]^{\prime}\left[(z_2 w_2)\begin{pmatrix} c_2 \\ d_2 \end{pmatrix} - (z_2 w_2)\begin{pmatrix} b \\ d_2 \end{pmatrix}\right]$$

It may be noted that $c_1$ is the estimate of $\gamma_1$ obtained from the first regression and that $d_2$ is the estimate of $\delta_2$, obtained from pooled regression plane. So the ratio is

$$F = \frac{Q_3 / l}{Q_2 /(n - 2m - 2l)} \qquad 3.48$$

So, we have an F-distribution with (l, n-2m-2l) digress of freedom. Here $Q_3$ is the restricted sum of squares and that $Q_2$ is the unrestricted sum of squares.

If, however, the new observations $n_2$ are fewer than the number of parameters in the function we may proceed as follows. First, from the augmented sample we obtain the regression equation.

$$Y = \hat{\beta}_0 + \hat{\beta}_1 + \ldots\ldots\ldots\ldots + \hat{\beta}_k X_k \qquad 3.49$$

From which we calculate the residual sum of squares

$$\sum e^2 = \sum y^2 - \sum \hat{y}^2 \qquad 3.50$$

with ($n_1 + n_2 - k$) degrees of freedom.

Second, from the original sample of $n_1$ we have

$$Y = \hat{\beta}_0 + \hat{\beta}_1 + \hat{\beta}_k X_k \qquad 3.51$$

from which the unexplained sum of squares is

$$\sum e^2 = \sum y^2 - \sum \hat{y}^2 \qquad 3.52$$

with $n_1 - k$ degrees of freedom.

Third, subtracting the two sums of residuals we find

$$\sum e^2 = \sum c_1{}^2 \qquad 3.53$$

with $(n_1 + n_2 - k) = n_2$ degrees of freedom, where $n_2$ are the additional observations. Further, we form F* ratio where

$$F^* = \frac{\sum e_2 - \sum e_{12} / n_2}{\sum e_{12} / (n_1 - k)} \qquad 3.54$$

The null hypotheses are

$H_0 : b_i = \beta_i$ $(i = 0,1,2..........k)$

$H_0 : b_i \neq \beta_i$

The F* ratio is compared with the theoretical value of obtained from the F-table with $v_i = n_2$ and $v_2 = (n-k)$ degrees of freedom.

If F* ratio exceeds the table value of F, we reject the null hypothesis i.e, we accept that the structural coefficient are unstable. This indicates that their values are values are changing in extended sample period.

### 3.18 Vector Autoregressive (VAR) Model:

Economic theories sometimes suggest a relationship between two variables, $y_t$ and $z_t$. In that case modeling each series involves an autoregression of $y_t$ on lagged values of $y_t$ and an autoregression of $z_t$ on lagged values of $z_t$. However such a separate approach would not capture any interaction between the variables concerned.

However, such interactions between the variable are captured through a Vector Autoregression (VAR) model where the time path of $\{y_t\}$ is affected by the current and past realizations of $\{z_t\}$ sequence and the time path of $\{z_t\}$ sequence is allowed to be affected by current and past realizations of $\{y_t\}$ sequence . *In a VAR $y_t$ is related not just to its own lagged values but also those of z and similarity, $z_t$ is related to its own lagged values and those of $y_t$, such that*

$$y_t = b_1 - b_{11}y_{t-1} + b_{12}z_{t-1} + \varepsilon_{1t} \qquad 3.55$$

$$z_t = b_2 - b_{21}y_{t-1} + b_{22}z_{t-1} + \varepsilon_{2t} \qquad 3.56$$

The VAR model, consisting of the equations (3.55) and (3.56) can be written as

$$\begin{pmatrix} y_t \\ z_t \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} + \begin{bmatrix} b_{11}b_{12} \\ b_{21}b_{22} \end{bmatrix} \begin{bmatrix} y_{t-1} \\ z_{t-1} \end{bmatrix} + \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix}$$

or $x_t = b - \pi_1 x_{t-1} + \varepsilon_t$ 3.57

where, b=($b_1$ $b_2$) is the vector of constants usually known as drift

$\varepsilon_t = (\varepsilon_{1t}\varepsilon_{2t})$ are innovations relative to information
set $x_{t-1} = (z_{t-1}\ \ z_{t-1})$

The equation (3.57) define a VAR (1,2) Model where order (p)=1 and k (number of variables)=2.

This form of the VAR is in its 'Reduced Form' in the sense that no current dated values of the $y_t$ and $z_t$ appear in any of the equations. The 'Reduced Form' VAR could serve as the solution in a dynamic simultaneous equation model. For example, let us consider a VAR system with contemporaneous relationship between two variables such that

$$y_t = b_{10} + b_{12}z_t + \gamma_{11}y_{t-1} + \gamma_{12}z_{t-1} + \varepsilon_{yt} \qquad 3.58$$

$$x_t = b_{20} + b_{21}y_t + \gamma_{21}y_{t-1} + \gamma_{22}z_{t-1} + \varepsilon_{zt} \qquad 3.59$$

where it is assumed that

(i) both $y_t$ and $z_t$ are stationary, and

(ii) $\varepsilon_{yt}$ and $\varepsilon_{zt}$ are white noise disturbances

such that

$\varepsilon_{yt} \sim \text{iidN}(0,\sigma^2_y)$

$\varepsilon_{zt} \sim \text{iidN}(0,\sigma^2_z)$

Now the VAR system consisting of equations (3.58) and (3.59) can be written as

$$y_t - b_{12}z_t = b_{10} + \gamma_{11}y_{t-1} + \gamma_{12}z_{t-1} + \varepsilon_{yt}$$

$$x_t - b_{21}y_t = b_{20} + \gamma_{21}y_{t-1} + \gamma_{22}z_{t-1} + \varepsilon_{zt}$$

or, $$\begin{bmatrix} 1 & b_{12} \\ b_{21} & 1 \end{bmatrix}\begin{bmatrix} y_t \\ z_t \end{bmatrix} = \begin{bmatrix} b_{10} \\ b_{20} \end{bmatrix} - \begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{bmatrix}\begin{bmatrix} y_{t-1} \\ z_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{2t} \end{bmatrix}$$

or, $Bx_t = \Gamma_0 + \Gamma_1 x_{t-1} + \varepsilon_t$ 3.60

where

$$\text{B} = \begin{bmatrix} 1 & b_{12} \\ b_{21} & 1 \end{bmatrix}$$

$$x_t = \begin{bmatrix} y_t \\ x_t \end{bmatrix}$$

$$\Gamma_0 = \begin{bmatrix} b_{10} \\ b_{20} \end{bmatrix}$$

$$\Gamma_1 = \begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{bmatrix}$$

$$\varepsilon_t = \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{zt} \end{bmatrix}$$

Pre-multiplying equation (3.60) by $B^{-1}$ we have

$$X_0 = A_0 + A_1 x_{t-1} + e_t \qquad 3.61$$

where $A_0 = B^{-1}\Gamma_0$

$A_1 = B^{-1}\Gamma_1$

$\varepsilon_t = B^{-1}\varepsilon_t$

Now 'equivalent form' of (2.47) is

$$y_t = a_{10} + a_{11}y_{t-1} + a_{12}z_{t-1} + e_{yt} \qquad 3.62$$

$$z_t = a_{20} + a_{21}y_{t-1} + a_{22}z_{t-1} + e_{yt} \qquad 3.63$$

Thus the 'Structural VAR' constituted by equations (3.58) and (3.59) is converted to the 'Standard Form' constituted by equation (3.62) and (3.63)

Here

$$\varepsilon_{1t} = (\varepsilon_{yt} - b_{12}\varepsilon_{zt})/(1 - b_{12}b_{21})$$

$$\varepsilon_{2t} = (\varepsilon_{zt} - b_{21}\varepsilon_{yt})/(1 - b_{12}b_{21})$$

Thus $e_{1t}$ and $e_{2t}$ are the composites of the two shocks $\varepsilon_{yt}$ and $\varepsilon_{zt}$.

**3.19 Stability of the VAR Model:**

The first order VAR model of (3.61) defines a first order difference equation which can be iterated backward to obtain

$$x_t = A_0 + A_1(A_0 + A_1 x_{t-1} + e_{t-1}) + e_t$$

$$= (I + A_1)(A_0 + A_1^{\,2} x_{t-2} + A_1 e_{t-1} + e_t$$

where I = 2*2 identity matrix

After n iterations, we have

$$x_t = (1 + A_1 + \ldots + A_1^n)A_0 + \sum_{i=0}^{n} A_1^1 e_{t-1} + A_1^{n-1} x_{t-n-1} \qquad 3.64$$

It is observed that the convergence requires that the expression $A_1^n$ vanish as n→$\alpha$ . Consequently, the stability of the VAR model requires that the roots of ($1-a_{11}L$) ($1-a_{22}L$)-($a_{12}a_{21}$ $L^2$) i.e. outside the unit circle. The stability conditions holds iff

(i) the $\{y_t\}$ and $\{z_t\}$ sequences are jointly covariance stationary

(ii) each sequence has a finite and time-invariant mean and a finite time-invariant variance.

**3.20 Impulse Response Functions:**

If the stability condition is met, the particular solution for $x_t$ (3.64) can be written as

$$x_t = \mu + \sum_{i=0}^{n} A_1^1 e_{t-1} \qquad 3.65$$

where $\mu = \left(\bar{y}, \bar{z}\right)'$

and $\bar{y} = [a_{10}(1 - a_{22}) + a_{12}a_{20}]/\Delta$

$$\bar{z} = [a_{20}(1 - a_{11}) + a_{21}a_{10}]/\Delta$$

$$\Delta = (1 - a_{11})(1 - a_{22}) - a_{12}a_{21}$$

Equation (3.65) is Vector Moving Average (VMA) representation of (3.61) in that the variables, $y_t$ and $z_t$ are represent in terms of the current and past values of two types of shocks (ie., $e_{1t}$ and $e_{2t}$). Again equation (3.65) can further be simplified as

$$x_t = \mu + \sum_{t=0}^{\alpha} \phi_i \varepsilon_{t-i} \qquad 3.66$$

where $\phi_i = \dfrac{A}{(1 - b_{12}b_{22})} \begin{bmatrix} 1 & b_{12} \\ -b_{21} & 1 \end{bmatrix}$

Consequently, we have for (3.66)

$$\begin{bmatrix} y_{12} \\ z_{21} \end{bmatrix} = \begin{bmatrix} \bar{y} \\ \bar{z} \end{bmatrix} + \sum_{i=0}^{\infty} \begin{bmatrix} \phi_{11(i)} & \phi_{12(i)} \\ \phi_{21(i)} & \phi_{22(i)} \end{bmatrix} \begin{bmatrix} \varepsilon_{yt-i} \\ \varepsilon_{zt-i} \end{bmatrix}$$

The four sets of coefficients $\phi_{11(i)}, \phi_{12(i)}, \phi_{21(i)} and \phi_{22(i)}$ are called the 'Impulse Response Functions'. Plotting coefficients of $\square_{jk(i)}$ against I is a practical way to visually represent the behavior of the $\{y_t\}$ and $\{z_t\}$ series in response to various shocks.

**3.21 Variance Decomposition:**

Given the equation (3.66), we have for n$^{th}$ period

$$x_{t-n} = \mu + \sum_{t=0}^{n} \varepsilon_{t-n-i} \qquad 3.67$$

$E(x_{t+n}) = \mu$ represents the unconditions n period ahead forecast error such that

$$x_{t-n} - E(x_{t-n}) = \sum_{i=0}^{n-1} \varepsilon_{t-n-1} \qquad 3.68$$

Using (3.68) we can find one-period ahead, two period ahead and thus n period ahead forecast errors. Each of the forecast errors would have variances. It is possible to decompose the n-step ahead forecast error variance owing to each shocks in $\{y_t\}$ and $\{z_t\}$ sequences.

Thus the 'Forecast Error Variance Decomposition' indicates the proportion of variation in a sequence owing to its 'own shock' versus shocks to other variables.

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