CHAPTER-10

SPECTRAL ANALYSIS FOR FURTHER CONFIRMATION OF THE NATURE OF GRANGER CAUSALITY

10.1 Introduction:

The appearance of 'Spectral Analysis' in the study of macroeconomic time series data from the middle of 1960s, motivated by the requirement of a more insightful knowledge of the series structure and supported by the comtemporaneous progress in spectral estimation and computation. The first work on *Spectral Analysis,* undertaken by Nerlove (1964), focused on the problem of seasonal adjustment procedure and on the general spectral structure of economic data. Granger and Hatanaka (1964) emphasized on use of the *'Cross Spectral Methods'* as being important tools for discovering and interpreting the relationship between economic variables. After the early years, the range of application of such analysis was extended to the study of other econometric issues, like the trade cycle separation, the problem of business cycle extraction and the analysis of co-movements among series. It has been comovement clear from the beginning that *'Spectral Analysis'* is a powerful tool for inspecting cyclical phenomenon and highlighting lead-lag relations among series. It also provides a rigorous and versatile way to define formally and quantitatively each series components and by means of filtering, it provides a reliable extraction method. In particular, *'Cross Spectral Analysis'* allows a detailed study of the correlation among series.

Spectral Analysis may sound strange initially, but it can actually relate to daily life. When we look at some monochromatic light source, we feel its 'strength' and 'color'. The color of the light source reflects the frequency of the light emitting from the source. In this instance, our eyes behave as crude 'spectrometer', namely, a meter for measuring 'spectra'. In simple terms, it can differentiate different 'colors' of light. When the light source emitting white light, our eyes, due to its crudeness, cannot see these different 'colors' contained in it . We need some other better tools, like, a prism to see the frequency composition of light. 'Spectral Analysis' plays the role of a prism when we try to look at the frequency composition of a time series.

We seek to examine the causal relation between Economic Growth and Export Growth in this chapter through the *'Spectral Analysis'.* This will enable us to re-examine the nature and direction of *'Granger Causality'*, as obtained in chapters 5 through 9, between the variables concerned.

10.2 Spectral Estimation: Methodology:

10.2.1 Fourier Transformation:

Given a function h(t) of a real variable t, the *Fourier Transformation* of h(t) can be defined as

$$H(w) = \int_{-\infty}^{+\infty} h(t)e^{-iwt}\,dt \qquad 10.1$$

provided the integral exists for real w.

A sufficient condition for H(w) to exist is

$$\int_{-\infty}^{+\infty} h(t)dt < \infty$$

If (10.1)is regarded as an integral equation for h(t) given H(w), then a simple inversion formula exists of the form

$$h(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} H(w)e^{iwt}\,dw \qquad 10.2$$

and h(t) is called the *Fourier Transform* of H(w)

In time series the discrete form of the *Fourier Transform* is used when h(t) is only defined for integral values of t.

Then we have

$$H(w) = \sum_{-\infty}^{+\infty} h(t)e^{-iwt} \qquad -\pi \le w \le \pi$$

is the *Fourier Transform* of h(t). Here H(w) is defined only in the interval $[-\pi, \pi]$. The Inverse Fourier Transform is given by

$$h(t) = \frac{1}{2\pi} \int_{-\pi}^{+\pi} H(w) e^{iwt} dw .$$

10.2.2 Peridogram And Auto- Spectrum:

Let us consider a finite series u(j) of length T = N Δt
where N = number of data

Δt = the sampling periodicity

v_k = the frequency= k/(N Δt)

t_j = the time = j Δt

The Discrete Fourier Transform (DFT) U(k) of u(j) and its inverse (IDFT) for finite series are

$$U(k) = \frac{1}{N} \sum_{J=0}^{N-t} u(j) e^{-i2\pi Jk/N} \qquad 10.3$$

$$U(j) = \frac{1}{N} \sum_{k=[N/2]}^{[N-1]/2} u(k) e^{-i2\pi jk/N} \qquad 10.4$$

where k Є [-[N/2], [(N-1)/2] and j = 0,..., N-1

Equation (10.3) can only be an approximation of the corresponding real quantity, since it provides only for finite set of discrete frequencies. The quantity

$P_{u(k)}$ = $|U(k)|^2$ is the *Power Spectrum* and its estimators is *'Schuster's Periodogram'*

$$P_u(k) = \Delta t \sum_{J=-[N-1]}^{[N-1]} \gamma_{uu}(J) e^{-i2\pi Jk/N}$$

$$= \Delta t \sum_{J=-[N-1]}^{[N-1]} \gamma_{uu}(J) Cos \frac{2\pi Jk}{N} \qquad 10.5$$

where, $\gamma_{uu}(-J) = \gamma_{uu}(J)$

$$N^{-1} \sum_{J=-(N-J)}^{(N-J)} [\{u(J) - \bar{u}\}\{u(J+J) - \bar{u}\}]$$

and $\gamma_{uu}(J)$ is the *'Standard Sample Estimation'* at Lag J of the *'auto covariance Function'*

The technique of 'Windowing' is applied for building a *'Spectral Estimation'* which has a smaller variance than P_u (k). The result of *'Windowing'* is the *'Smoothed Spectrum'*.

$$\hat{S}_u(k) = \Delta t \sum_{J=-(N-1)}^{N-1} w_M(J) \gamma_{uu}(J) Cos \frac{2\pi Jk}{N} \qquad 10.6$$

Since $P_u(k)$ and $\gamma_u(J)$ are related by DFT, equation(10.6) can also be written as

$$\hat{S}_u(k) = \Delta t \sum_{k'=-[N/2]}^{[N-1/2]} P_u(k') W_{M'}(k-k') \qquad 10.7$$

Here the *Convolution* of the *Periodogram* $P_u(k)$ with Fourier Transformation of $W_M(J)$ is the *'Spectral Window'* $W_M(k)$ of width $M' = M^{-1}$.

Thus the *'Smoothed Spectrum'* is nothing but the *Periodogram* seen through a window opened on a coverient interval around k.

10.2.3 Cross Spectrum:

Cross Spectrum is obtained by substituting the *Cross-Covariance Function* in equation (10.6) for the *Autocovarience Function*. Thus, if we have two time series $u_1(J)$ and u_2 (J) and their *Cross Covariance Function* $\gamma_{22}(J) = \gamma_{22}(-J)$, the *Cross Spectrum* is

$$\hat{S}_{12}(k) = \Delta t \sum_{J=-(N-1)}^{N-1} w\,(J)\gamma_{12}(J)e^{-i2\pi Jk/N}$$

$$= \hat{C}_{12}(k) - i\hat{Q}_{12}(k) \qquad 10.8$$

The real part $\hat{C}_{12}(k)$ is the *'Cospectrum'* and the imaginary $\hat{Q}_{12}(k)$ the *'Quadrature Spectrum"*
Here the *Coherence Spectrum* is

$$\hat{K}_{12}(k) = \frac{\left|\hat{S}_{12}(k)\right|}{\sqrt{\hat{S}_1(k)}} \qquad 10.9$$

and the '*Phase Spectrum*' is

$$\hat{Q}_{12}(k) = \arctan\left(-\frac{\hat{Q}_{12}(k)}{\hat{C}_{12}(k)}\right) \qquad 10.10$$

Again the '*Gain Spectrum*' is

$$\hat{G}_{12}(k) = \frac{\left|\hat{S}_{12}(k)\right|}{\hat{S}_1(k)} \qquad 10.11$$

10.3 Univariate Periodograms Nature and Significance:

The *Univariate Periodograms* for income growth (Y) and Export Growth (X) are given by the Figures 10.1-10.2. The Figures 10.3-10.4 provide the corresponding 'Univariate Periodograms' by periods.

Figure-10.1

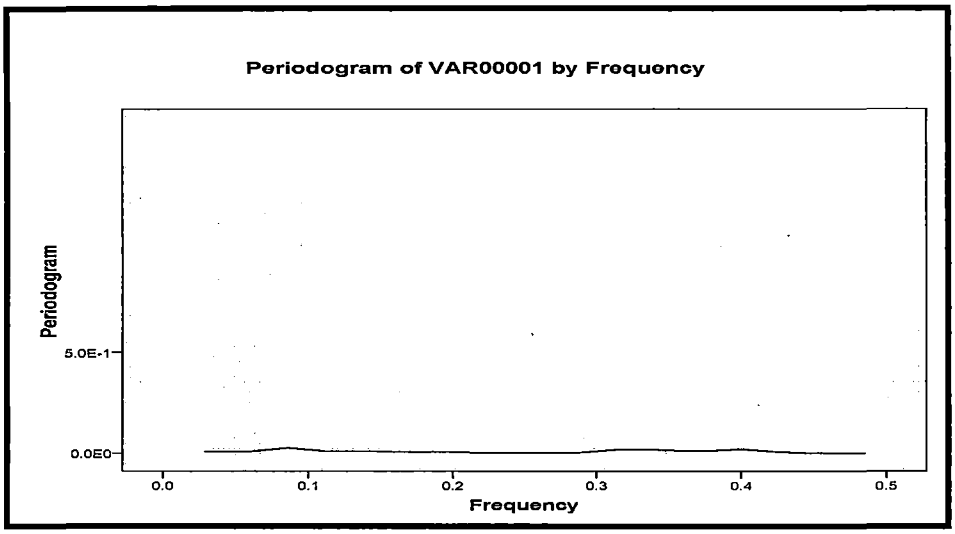


Figure-10.2

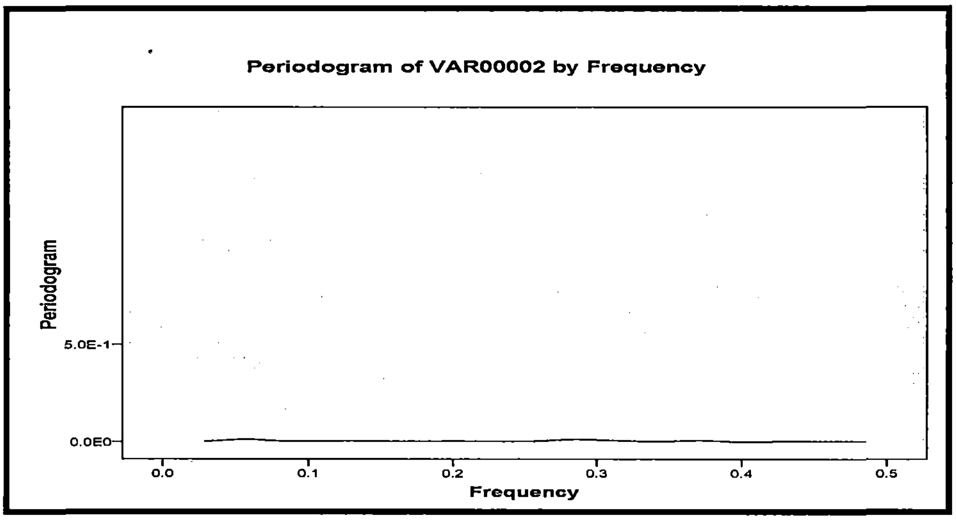


Figure-10.3

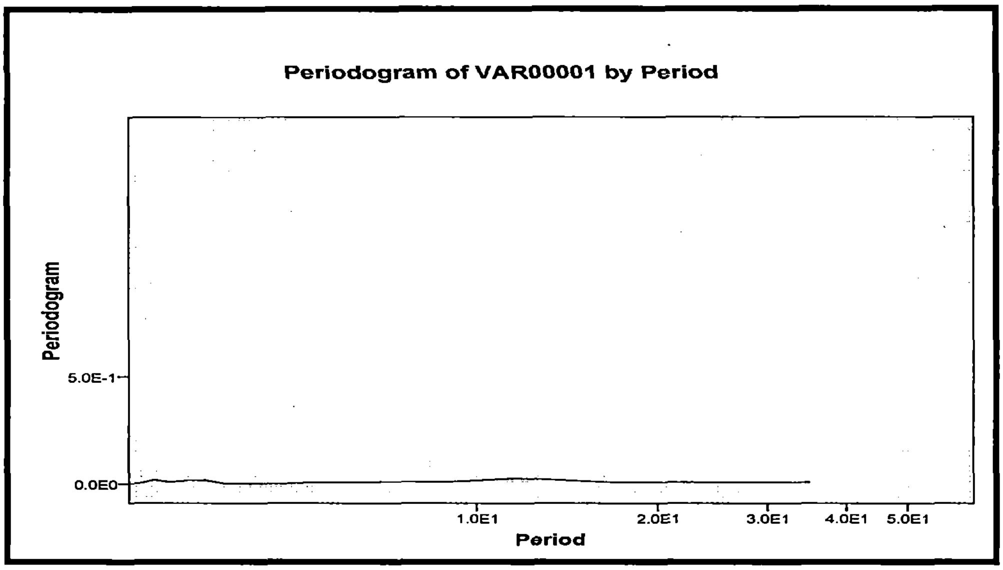


Figure-10.4

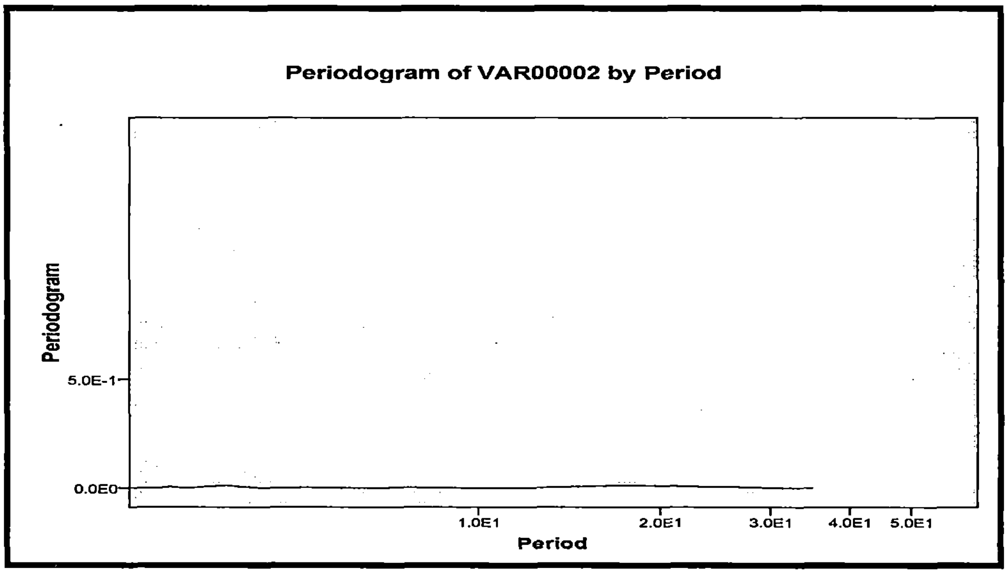


Figure-10.5

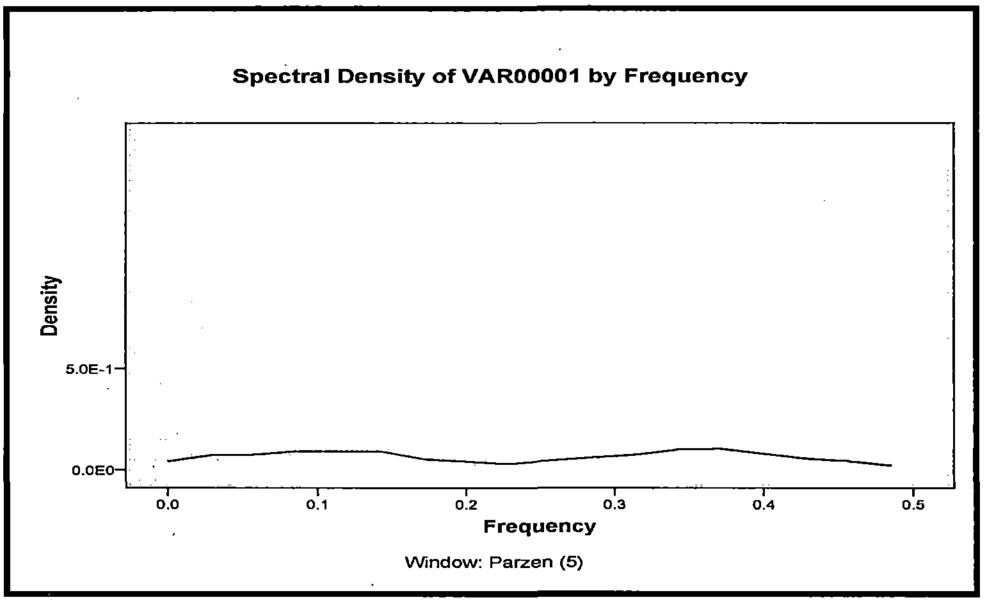
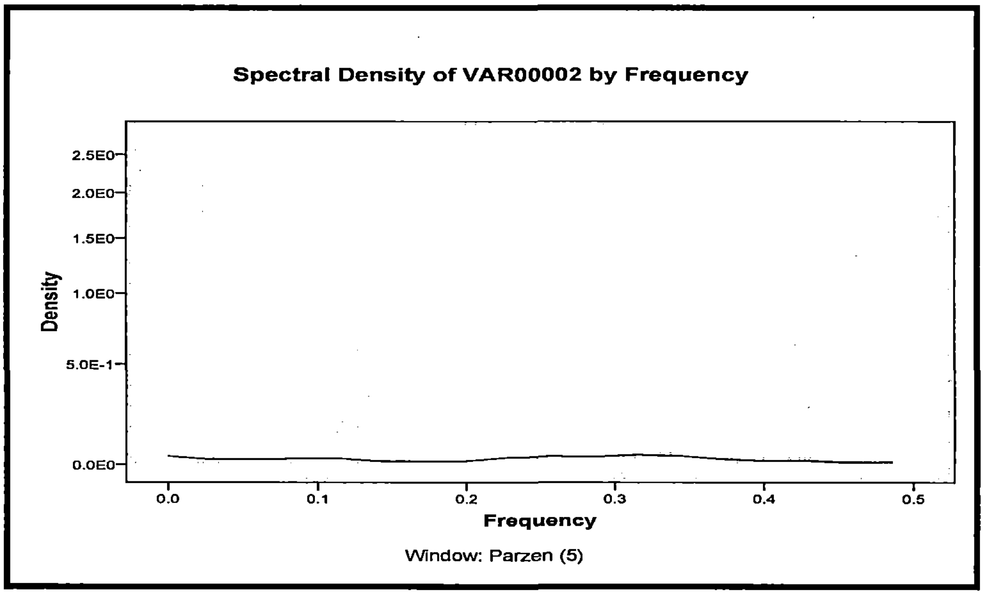


Figure-10.6



Figures 10.1-10.2 indicate that

the periodograms are almost horizontal straight lines without any noticeable periodicity. These periodograms, therefore, testify for 'almost uniform' decomposition of variance in Y and X across different frequencies.

These periodograms, in turn, also indicate almost unform' decomposition o variances in Y and X across different periods. This is substantiated by the nature of periodograms of Y and X (across periods) as given by the Figures 10.3-10.4.. These Figures 10.5-10.6, therefore, testify for 'stationarity' of Y and X in view of the fact that variances of the variables concerned remained constant over time (time invariant variance) during the period of over study. These exist no 'deterministic' or 'stochastic' trends in the variables concerned.

10.4 Spectral Density Representations: Nature and Significance:

Figures 10.5-10.6 present the *'Spectral Density by Frequency'* for Y and X respectively. These figures also indicate that ***each Spectral Density' is free from any noticeable 'periodicity. Absence of sharp peak in the spectral densities concerned indicates that the univariate spectral structural of the variables (y and X) do not exhibit any cyclical behavior.***

Figure-10.7

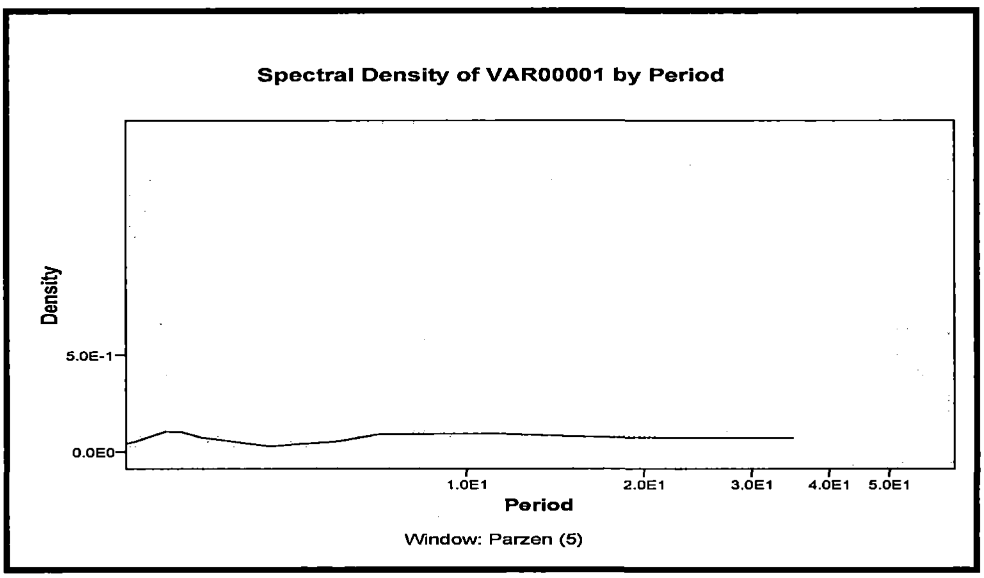
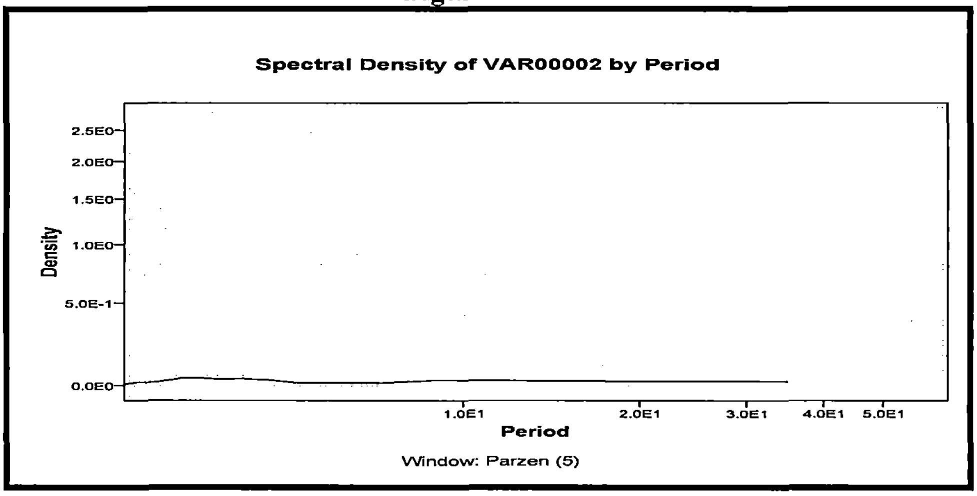


Figure-10.8

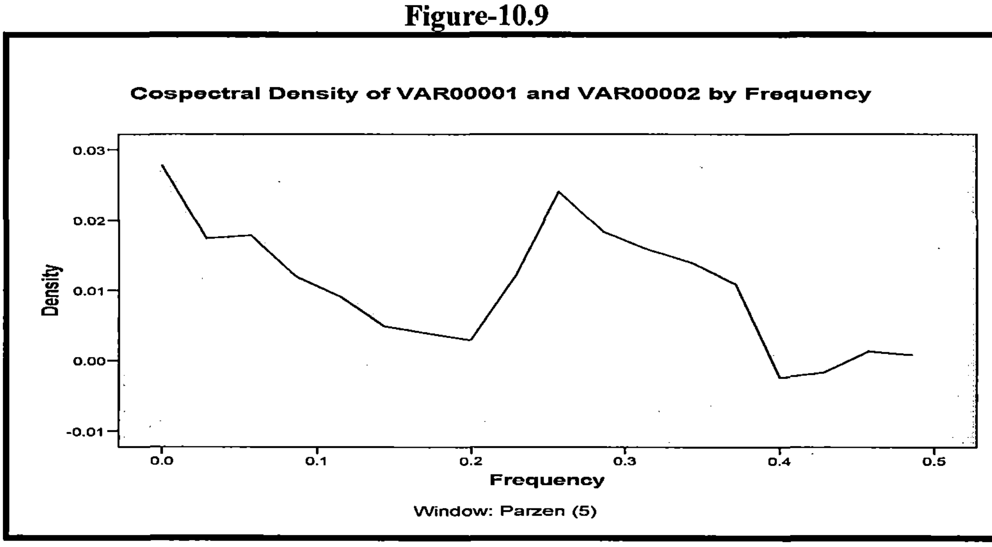


The corresponding 'Spectral Densities by period' for Y and X are given by the Figures 10.7 and 10.8. These figures also exhibit absence of 'periodicity' in the 'Spectral Densities' concerned. These findings confirm the incidence of ***statistically insignificant auto-regressive structures for the endogenous variables (Y and X) in the VAR Model studied in chapter-5.***

10.5 The 'Cospectral Density by Frequency' For Income Growth (Y) and Export Growth(X):

The '*Cospectral Density by Frequency*' for Y and X is given by the Figure-10.9

Figure-10.9



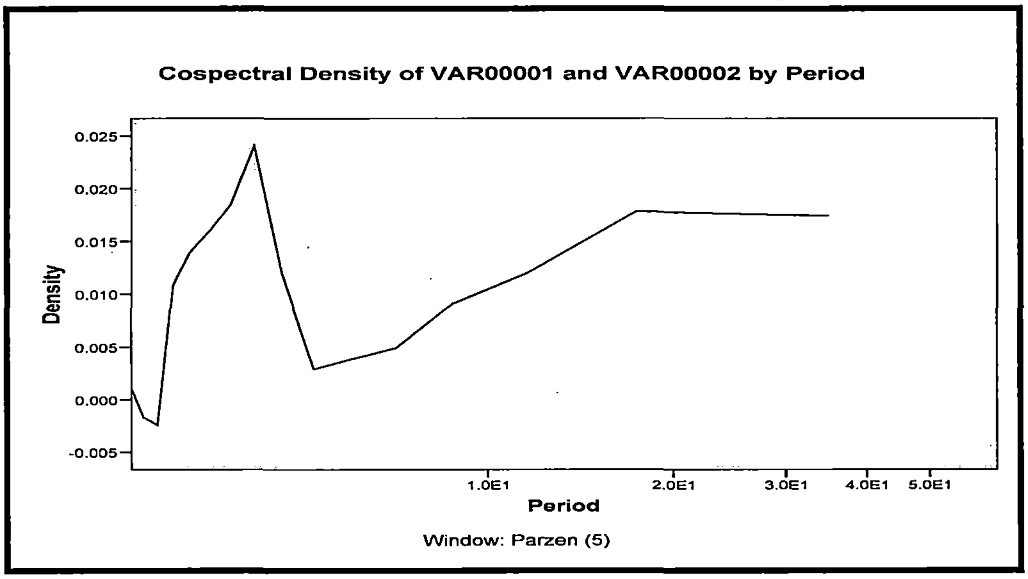
The '*Cospectral Density*'

(i) is not a horizontal straight line

(ii) is marked by the presence of structural ups and downs

(iii) contains a singular sharp peak at frequency 0.25 (approx)

Figure-10.10



The *Cospectral Density* by period for these variables as given by the Figure-10.10 also exhibits these features. However, in this *Cospectrum* the sharp peak appears at period 3 (approx).

These observations indicate that

(i) there did exist significant co-movement of Y and X over the period of time

(ii) the co-movement is marked by some 'periodicities'

(iii) ther did exist a dominant periodicity at period 3 (approx).

All these observations confirm that ***Y and X are 'cointegrated' and the long-run relationship between these variables is 'Stable'.***

10.6 'Coherency Spectrum' For Y and X:

The 'Coherency Spectrum' by Frequency between Y and X is being presented through Figure-10.11 while the Figure-10.12 presents the corresponding 'Coherency Spectrum' by period. The 'Spectrum' in Figure-10.11 shows that

Figure-10.11

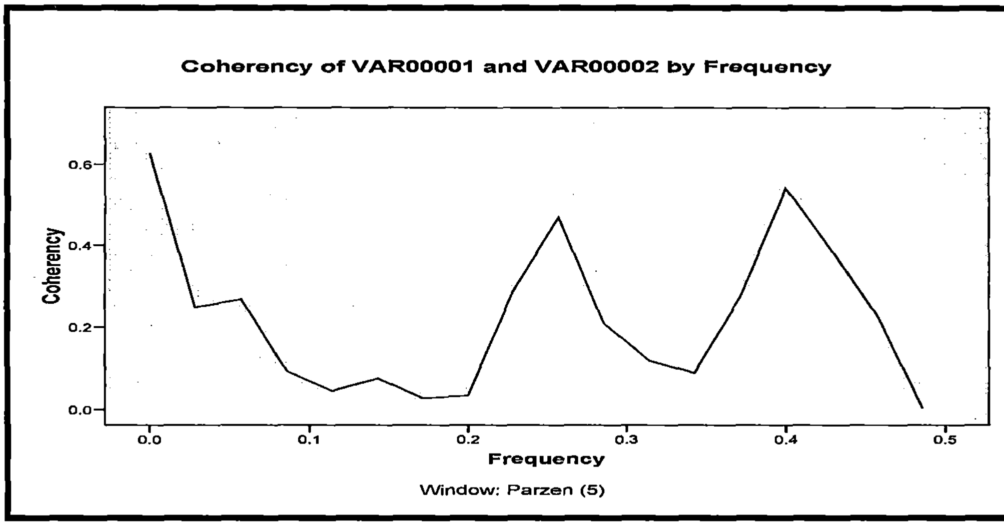
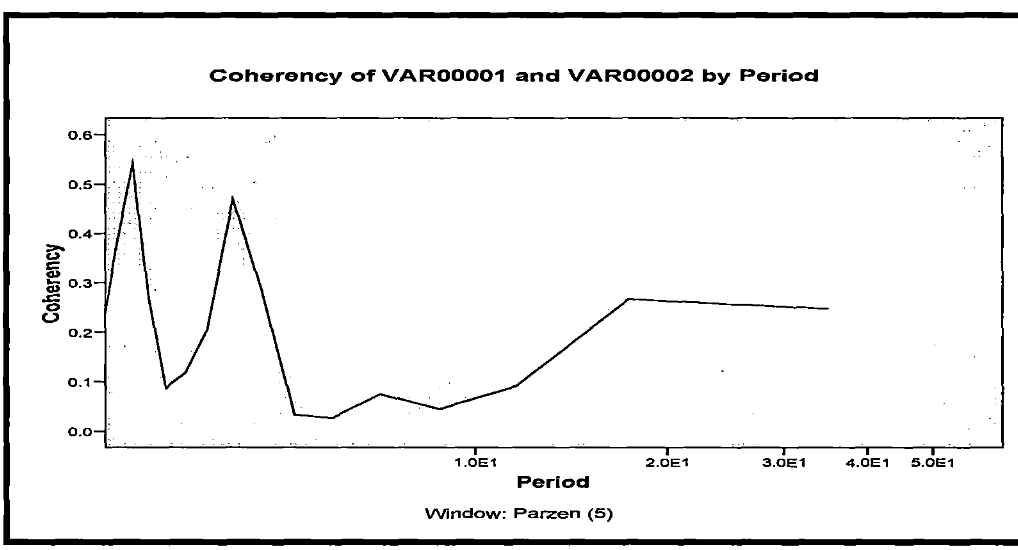


Figure-10.12



(i) the 'coherency' for the variables Y and X is as high as 0.55 for at frequency 0.4

(ii) the 'Coherency' is 0.5 (approx) at frequency-0.25

The 'Spectrum' in Figure-10.12 correspondingly shows that

(i) the 'Coherency' is as high as 0.55 at period 2 and

(ii) the 'Coherency' is 0.5 (approx) at period 3.

All these observations confirm that

(i) there did exist high intensity of co-movements of the variables concerned.

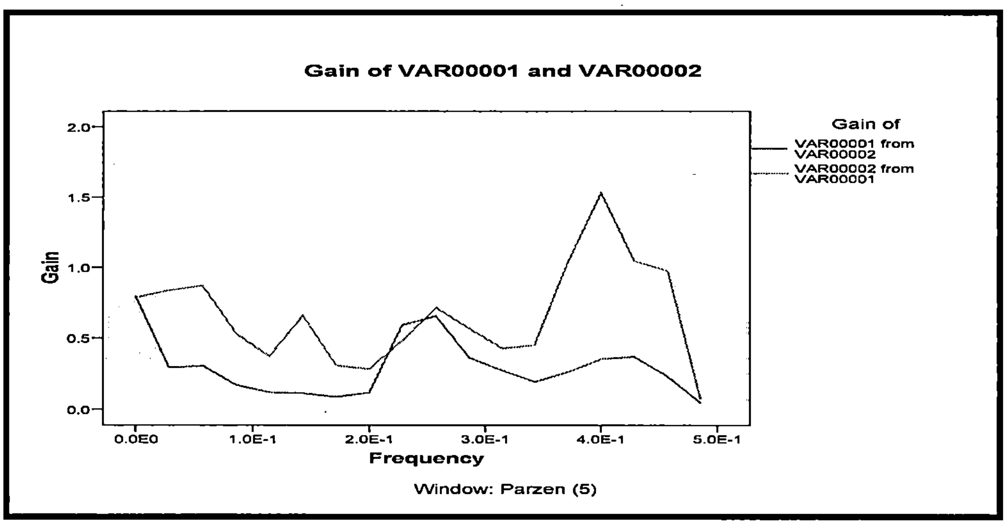
(ii) there did exist a 'stable' relationship between the variables concerned

(iii) there did exist significant the 'periodicity' at frequency 0.25 or at period 3.

10.7 'Gain Spectrum' For Y and X:

The 'Gain Spectrum' for Y and X by frequency is given by the Figure-10.13.

Figure-10.13



The 'Gain Spectrum' shows that

(i) the 'Gains' of Y (Income Growth) from X (Export Growth) from Y (Income Growth) lies Over the 'Gains' of X from Y across all the frequency levels.

(ii) the 'Gain' of Y from X is also very significant at frequency 0.40

All these observations confirm that

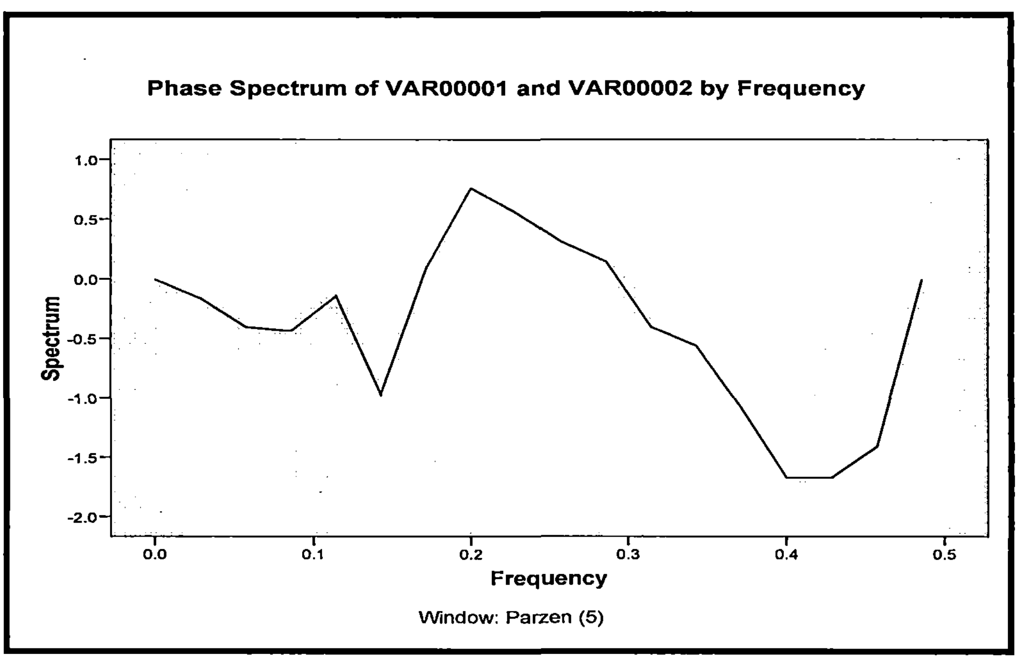
(i) the contribution of variations in X (Export Growth) to variations in Y (Income Growth) was higher than that of Y (Income Growth) to X (Export Growth) at all frequency levels and therefore,

(ii) there did exist 'Uni-directional Granger Causality' from Export Growth to Income Growth over the period of our study across all the frequency levels.

10.8 'Phase Spectrum' for Y and X :

The 'Phase Spectrum' for Y and X is being presented through the Figure-10.14.

Figure-10.14



The 'Phase Spectrum' shows that

the phase difference $Q_{12}(k)$ is negative over almost all the frequency levels (barring frequency range 0.1-0.3). 'Export Growth', therefore, was in 'lead' positions and 'Income Growth' was in lag position across almost all the frequency levels. However, the lagged position of Economic Growth implies that Export Growth favorably affected Economic Growth.

These observation indicates that variations in Export Growth occurred first and such variations then led to variations in Income Growth. Consequently, the 'Unidirectional Causality' from 'Export Growth' to 'Income Growth' is further established.

10.9 Overview of Findings from the Spectral Analysis:

In this 'Spectral Analysis

(i) 'Univariate Periodograms' for Y and X, confirm that Y and X are 'Stationary' i.e, $Y \sim I(0)$ and $X \sim I(0)$

(ii) 'Auto Spectra' confirm absence of periodicity in Y and X across all frequency levels and the incidence of statistically insignificant auto-regressive structure for the endogenous variables in the VAR model studied in chapter-5

(iii) the 'Cospectrum' for Y and X exhibit 'periodicity' at frequency 0.25. This confirms that Y and X are 'Cointegrated' and the long-run relationship between Y and X is 'Stable'

(iv) the 'Coherence Spectrum' for Y and X confirms that there did exist strong 'Coherence' in their comovements over the period of study

(v) the 'Gain Spectrum' for the variables confirms the existence of 'Unidirectional Granger Causality' from Export Growth to Economic Growth over the period of study.

(vi) the 'Phase Spectrum' for the variables further confirms that Export Growth 'Granger Caused' Economic Growth in the economy of Sri Lanka over the period of study.