CHAPTER - 8

COVERED INTEREST RATE ARBITRAGE AND THE LONG–RUN RELATION BETWEEN SPOT AND FORWARD RATES IN THE FOREIGN EXCHANGE (RUPEE/DOLLAR) MARKET IN INDIA – FURTHER STUDY ON MARKET EFFICIENCY WITH ARIMA (p, d, q) FORECASTING.

8.1 Introduction

Studies on UIRAP usually concentrate on the relation between spot rate and forward rate. However, some studies focus on the relation between spot rate and one-period lagged forward rate. In these studies spot rates have been reported to be closely following one-period lagged forward rates. Some other studies concentrate on the relation between spot rate and contemporaneous forward rates. It is held in these studies that spot rates closely follow the contemporaneous forward rates.

Moreover, the studies do not establish clearly which of the variables (spot rate or lagged forward rate or contemporaneous forward rate) maintains an independent cointegrating relation with the other. Consequently, the direction of Granger Causality remains obscure.

8.2 Objective of Study

The study in this chapter (Chapter 8) is devoted to examining the relation between spot and forward rate in Indian foreign exchange (rupee/dollar) market. More specifically, we seek to examine

(i) how spot rate and one-period lagged forward rate is related in the market

(ii) which variable (spot rate or lagged forward rate) maintains an independent cointegrating relation with the other

(iii) the nature of Granger causality in the relation between the rates

(iv) how spot and contemporaneous forward rates are related

(v) if the usually observed high correlation between these two variables is apparent or real

(vi) if risk premium exists in the market

(vii) how far the capital market is efficient.

8.3 Theoretical issues:

The uncovered interest rate arbitrage parity (UIRAP) doctrine holds that domestic interest rate (i_t) must be higher (lower) than the foreign interest rate (i_t^*) at any period t by an amount equal to the expected depreciation $\dot{s}_{t+1}^e$ of the domestic currency at period $(t+1)$ such that

$$i_t = i_t^* + \dot{s}_{t+1}^e \qquad (8.1)^1$$

$$\text{where } \dot{s}_{t+1}^e = \frac{s_{t+1}^e - s_t}{s_t}$$

Equation (8.1) indicates, that

$$i_t - i_t^* = \dot{s}_{t+1}^e$$

which implies that, positive interest rate differentia ($i_t > i_t^*$) in favor of the domestic country under UIRAP, must equal in equilibrium with the expected depreciation of the domestic currency.

UIRAP doctrine is based on the proposition that investors are risk-neutral in the sense that they consider both the domestic and foreign bonds equally risky. However, investors may consider foreign bonds more risky since depreciation of foreign currency reduces the gain from foreign bond interest earnings. In such event investors may become 'risk averse' and take resort to 'hedging' in order to minimize such loss of earning following foreign currency depreciation. Hedging activities involve spot purchase of foreign currency for investment and simultaneously off-setting forward sale of foreign currency obtained at the time of maturity of investments in order to cover foreign exchange risk. Let tF_{t+1} be the forward rate quoted at period t for the expected spot rate at $(t+1)$ period. Then tF_{t+1} may be substituted for s_{t+1}^ein the UIRP equation (8.1) such that in equilibrium as given in equation (8.2)

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[1] In equation (8.1) $i_h = i_t$ and $i_f = i_t^*$. Thus equation (8.1) is a modified form of the equation (1.5).

$$i_t - i_t^* = \frac{tF_{t+1} - s_t}{s_t}$$

$$i_t - i_t^* = \frac{tF_{t+1}}{s_t} - 1 \tag{8.2}$$

$$i_t - i_t^* = f_{t+1}$$

where, f_{t+1} is the forward premium (discount) indicating the proportion by which a country's forward exchange rate exceeds (falls below) its spot rate[2]. Thus the normal formulation of the Covered Interest Rate Arbitrage Parity (CIRAP) doctrine states that, in equilibrium under hedging operations, forward premium (discount) equals positive (negative) interest rate differential in favor of the domestic economy.

In CIRAP tF_{t+1} is substituted for s_{t+1}^e and f_{t+1} becomes a proxy for $\dot{s}_{t+1}^e$. Thus the forward rate tF_{t+1} serves as an 'Unbiased Predictor' of the future spot rate s_{t+1}^e such that

$$tF_{t+1} = s_{t+1}^e \tag{8.3}$$

s_{t+1}^e represents the market's subjective expectation for the spot rate to prevail at $(t+1)$ period. This expectation becomes '*rational*' iff s_{t+1}^e equals the conditional expectation of s_{t+1} derived on the basis of the set of information available at period t such that

$$s_{t+1}^e = E\,[s_{t+1}/\Omega_t] = E_t s_{t+1} \tag{8.4}$$

$$\text{where } \Omega_t = \{s_{t-i};\ \ i = 0,1,2,3,\ldots\ldots\}$$

Thus $s_{t+1}^e = E_t s_{t+1}$ represents the series of one-period ahead ARIMA (p, d, q) forecast at period t for spot rate prevailing at period $(t-1)$.

If $E_t s_{t+1}$ is a Minimum Mean Square Error (MMSE) forecast for s_{t+1}, then

$$s_{t+1} = E_t s_{t+1} + u_{t+1} \tag{8.5}$$

$$\text{where } \ u_{t+1} \sim \text{iid } N(0, \sigma^2 u) \text{ and}$$

$u_{t+1} = s_{t+1} - E_t s_{t+1}$ represents the '*white noise*' forecast error. Again from equations (8.3) and (8.4)

[2]Equation (8.2) represents the equation (1.8).

$$tF_{t+1} = s^{e}_{t+1} = E_t s_{t+1} \tag{8.6}$$

Then equation (8.5) implies that

$$s_{t+1} = tF_{t+1} + u_{t+1} \tag{8.7}$$

where u_{t+1} is GWN (0, σ^2u)

Lagging one-period gives

$$s_t = (t-1)F_t + u_t \tag{8.8}$$

where $u_t = s_t - (t-1)F_t$ is GWN (0,σ^2u)

However, equation (8.8) can also be rearranged as

$$(t-1)F_t = s_t - u_t = s_t + \omega_t \tag{8.9}$$

where $\omega_t = -u_t$ and ω_t is GWN (0,σ^2u).

If both s_t and $(t-1)F_t$ are I (1) variables with $u_t \sim I(0)$, equations (8.8) and (8.9) would represent cointegrating relations between s_t and $(t-1)F_t$.

However, for $s_t \sim I(1)$ and $(t-1)F_t \sim I(1)$ variables, there can exist at most one independent cointegrating equation. It is, therefore, an empirical interest to identify which one of equations (8.8) and (8.9) would serve as the truly independent cointegrating relation between s_t and $(t-1)F_t$. Equation (8.8), if identified as that equation, would imply how s_t maintain a long-run relation with $(t-1)F_t$. In that event, equation (8.8) is the cointegrating transfer function relation for s_t with $(t-1)F_t$. On the other hand, equation (8.9), if identified as the independent cointegrating relation, would represent the cointegrating transfer function relation that $(t-1)F_t$ maintains with s_t .

It may be noted that $tF_{t+1}$is quoted by the Reserve Bank of India at any period t for (t+1) period. More specifically, one-month, three-month or sixth-month forward premia/discounts are being quoted at any date t. Therefore, $tF_{t+1}$is derived on the basis of the CIRAP equation (8.2) such that

$$tF_{t+1} = s_t[1 + (i_t - i^*_t)] \tag{8.10}$$

Banks quote these forward rates for forward buying and selling of currencies. Thus computations of tF_{t+1} involve algebraic operations.

However, s^e_{t+1} represents market agents (like speculators, arbitrageurs and hedgers) expectation at period t for the spot rate prevailing at (t+1) period. This expectation formation is independent of the computation of tF_{t+1}, since s^e_{t+1} is formed on the basis of the information available at period t. Usually, s^e_{t+1} is formed as the one-period ahead forecast on the basis of ARIMA (p, d, q) structure of spot rate s_t.

Thus tF_{t+1} and s^e_{t+1} represent independent forecast series for $(t+1)$ period spot rate. There exists, therefore, no economic rationality, to establish equality between tF_{t+1} and s^e_{t+1} at the outset. Yet any divergence between tF_{t+1} and s^e_{t+1} would trigger arbitrage activities resulting in the equality between these two forecasts in the long run and thereby an equilibrium in the currency market.

However, the equality between tF_{t+1} and s^e_{t+1}, as given by the equation (8.3), gets modified when the investors are risk-averse such that they need some additional return i.e., risk premium for holding risky currencies. Let us assume that risk-averse investors hold that

$$E_t s_{t+1} = s^e_{t+1} > tF_{t+1}$$

i.e, market agent's expected spot price of dollar for (t+1) period exceeds the quoted forward price of dollar. Speculators will take a long-position in the currency market by purchasing dollar forward. If the expectation is realized, investors receive dollars at the contracted forward price on the delivery date $(t+1)$ and them immediately resale the acquired dollars at the prevailing spot exchange rate. Thus they reap some speculative profit per dollar which equals $s^e_{t+1} - tF_{t+1}$. In such case, speculators, with an access to sufficient fund, will engage in swapping activities leading to a large scale rise in the demand for forward dollar. As a result, forward dollar price will be bid up and speculative profit will dwindle. At some point speculative profit will no longer be enough to compensate for the risk of being wrong. Speculation will cease and the currency market will reach equilibrium when the speculative profit is just equal to the required risk premium (ρ_t) such that

$$s^e_{t+1} - tF_{t+1} = \rho_t \tag{8.11}$$

or, $$tF_{t+1} = s^e_{t+1} - \rho_t \tag{8.11a}$$

$$\text{or,}\qquad s^e_{t+1} = tF_{t+1} + \rho_t \qquad (8.11b)$$

Thus equation (8.11a) modifies the equation (8.3) when investors are risk averse. Equation (8.11a) represents '*efficient market equilibrium'* since forward rate tF_{t+1} in it reflects publicly available information embodied in $s^e_{t+1} = E_t s_{t+1}$, the rational expectations for s_{t+1} and also the market attitude to risk in the form of risk premium(ρ_t).

In further transformation of equation (8.11)

$$s_{t+1} - tF_{t+1} = s_{t+1} - E_t s_{t+1} + \rho_t$$

$$\text{or, } s_{t+1} - tF_{t+1} = u_{t+1} + \rho_t$$

[since $s_{t+1} = E_t s_{t+1} + u_{t+1}$ in equation (8.5), where $u_{t+1} \sim \text{GWN}(0, \sigma^2_u)$]

$$\text{or, } s_{t+1} = \rho_t + tF_{t+1} + u_{t+1} \qquad (8.12)$$

Lagging one-period back we have

$$s_t = \rho_{t-1} + (t-1)F_t + u_t \qquad (8.13)$$

$$\text{where } u_t \sim \text{GWN}(0, \sigma^2_u)$$

Equation (8.13) replaces the equation (8.12), as the equation for spot rate s_t when investors are risk averse and in it spot rate consists of

(i) previous period quoted forward rate,

(ii) previous period risk premium in the market, and

(iii) an unpredictable error in the market expectations for the spot rate at time t.

If ρ_t is assumed to be a constant such that $\rho_{t-1} = \rho_t = \rho^*$, or a stationary [I(0)] process, then equation (8.13) defines a cointegrating equation for s_t and $(t-1)F_t$ with cointegrating vector (1, 1), given that both s_t and $(t-1)F_t$ are I (1) variables.

8.4 ARIMA Forecast For one - period ahead Future Exchange Rate:

Univariate stochastic structure for s_t has been identified as ARIMA (4, 1, 0) such that

$$s_t = [(1-L)(1-\emptyset)L^4]^{-1}\epsilon_t; \quad \epsilon_t \sim \text{iid } N(0, \sigma^2_\epsilon)$$

The estimated equation as given in equation (7.2) is

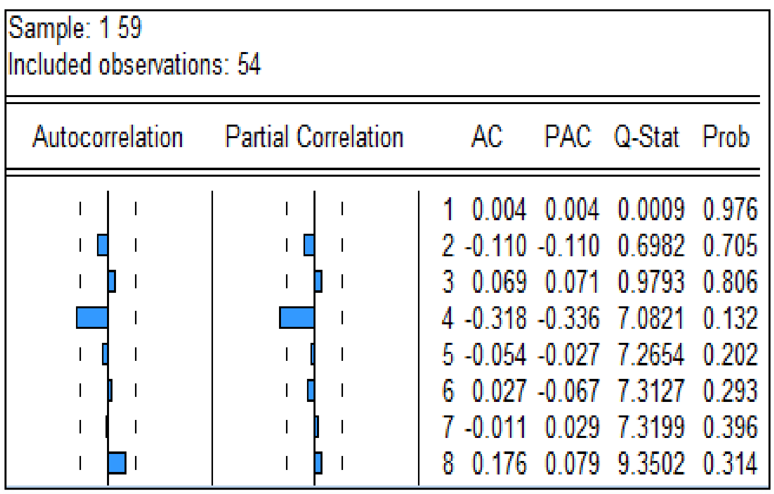
$$(1 - L)s_t = 0.003033 - 0.272036\,(1\text{-}L)\,L^4 s_t \qquad (7.2)$$

SE	(0.001836)	(0.129645)
t	[1.651731]	[-2.098308]
Prob	(0.0986)	(0.0359)

R-squared = 0.089, Adjusted R-squared = 0.071, S.E. of regression = 0.012, Sum squared resid = 0.007, Log likelihood = 161.831, D.W stat = 1.972, Mean dependent var = 0.002, S.D. dependent var = 0.0127, Akaike info criterion = -5.919, Schward criterion = -5.845, F-stat = 5.1104, Prob(F-stat) = 0.0279.

Residuals (ϵ_t) of the estimated equation are white noise as evidenced from its correlogram presented through Figure -7.1

Figure-7.1[3]
Correlogram of residuals from Equation (7.2)



Sample: 1 59
Included observations: 54

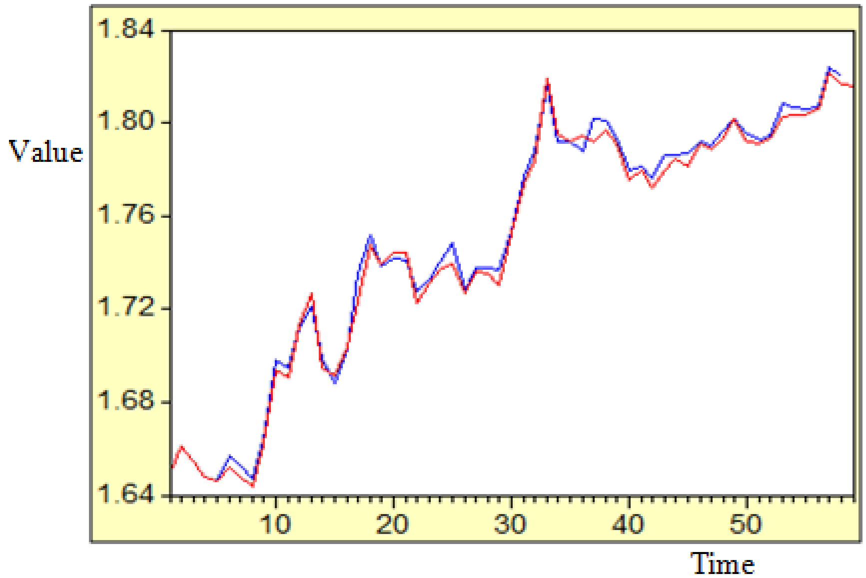
Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.004	0.004	0.0009	0.976
		2	-0.110	-0.110	0.6982	0.705
		3	0.069	0.071	0.9793	0.806
		4	-0.318	-0.336	7.0821	0.132
		5	-0.054	-0.027	7.2654	0.202
		6	0.027	-0.067	7.3127	0.293
		7	-0.011	0.029	7.3199	0.396
		8	0.176	0.079	9.3502	0.314

The estimated ARIMA (4, 1, 0) structure for s_t as given by the equation (7.2) may be used for generating one – period ahead forecast for s_{t+1} the forecast series is s^e_{t+1}. Time plots of s_{t+1} series along with s^e_{t+1} series are being given by Figure – 7.2 below.

[3] Figures (7.1) and (7.2) as well as Table 7.1 are being reproduced here for ready reference.

Figure – 7.2

Time Plots of (s_{t+1}____) and (s^{e}_{t+1}____) series



It appears from the Figure – (7.2) that one – period ahead forecast for s_{t+1} almost coincide with the corresponding s^{e}_{t+1} actual. Basic statistics of the forecast error [$s_{t+1} - s^{e}_{t+1}$] are given in the Table – 7.1.

Table – 7.1: Statistics of the Forecast Errors (e_{t+1})

Observations 54	
Mean	8.57E-16
Median	-0.000156
Maximum	0.008847
Minimum	-0.009469
Std. Dev.	0.003476
Skewness	0.063991
Kurtosis	3.712757
Jarque-Bera	1.179906
Probability	0.554353

Mean value of (e_{t+1}) almost collapses on zero and the very small standard deviation of (e_{t+1}) indicate that s^{e}_{t+1} series represents the *Minimum Mean Square Error* (MMSE) forecast for s_{t+1}. s^{e}_{t+1} series has been subject to ADF unit root test and the table – 7.2 presents the results of such test

Table 7.2 shows that $s^e_{t+1} = E_t s_{t+1}$ series is I(1), while in figure 7.2 the time path of s^e_{t+1} series virtually mimics that of $s_{t+1} \sim I(1)$ series.

Again since the forecast residuals (ϵ_{t+1}) are white noise, no more information could be ploughed back from the forecast residuals in order to improve upon the forecast. Consequently,

$$s_{t+1} = s^e_{t+1} + e_{t+1};\ \epsilon_{t+1} \sim GWN(0, \sigma^2_\epsilon) \qquad (8.14)$$

Thus s^e_{t+1} virtually emerges as the 'rational expectation' forecast for s_{t+1}.

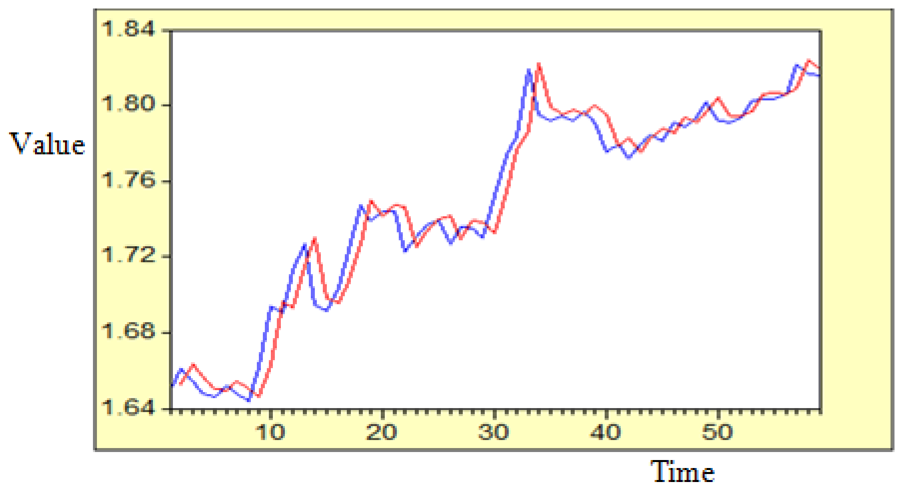
Again lagging one period, $$s_t = s^e_t + \epsilon_t\ ;\ \epsilon_t \sim GWN(0, \sigma^2_\epsilon) \qquad (8.15)$$

8.5 Graphical Analysis:

Spot rate (s_t) has been plotted against the lagged one –month forward rate $(t-1)F_t$ in Figure 8.1.

Figure 8.1

Time plots of Spot rate (s_t -----) and Lagged Forward rate $[(t-1)F_t$ ----]



The graph superimposes two line diagrams. It is observed that

(i) the forward rate $(t-1)F_t$ appears to track the spot rate s_t

(ii) spot rate leads the change of direction either downward or upward. Forward rate follows that change only at the next observation. It means that change of direction in spot rate at period t is being completely missed by $(t-1)$ period forward rate and it is only reflected in t-period forward rate.

Figure - 8.2 Time plots of Spot rate (s_t -----) and Contemporaneous Forward rate (tF_{t+1}-----)

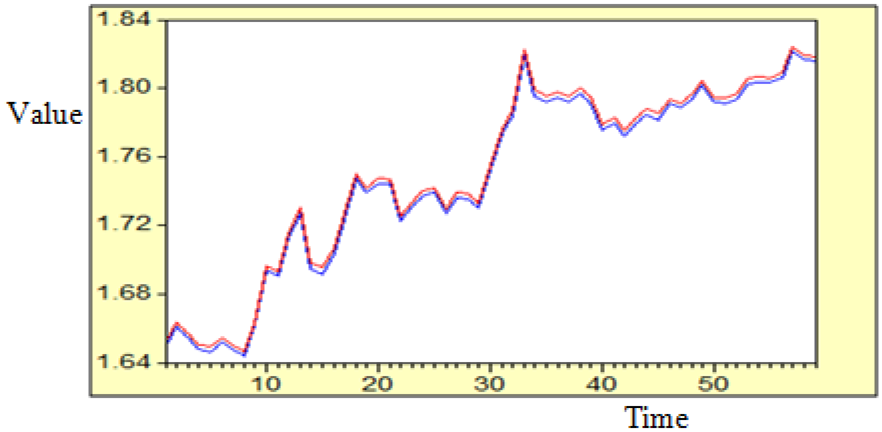


Figure-8.2 presents the graph in which spot rates s_t are being plotted against the contemporaneous forward rates tF_{t+t}. It appears that spot rates are more closely linked to the contemporaneous rather to lagged forward rates. These two lines are completely indistinguishable.

It may, however, be noted that change in market sentiment leads to change in the direction of both spot and forward rates simultaneously. This is evident from the computation of (t+1) period forward rate at time t as given in the equation (8.10)

$$tF_{t+1} = s_t[1 + (i_t - i_t^*)] \tag{8.10}$$

Given interest rate differential $(i_t - i_t^*)$, tF_{t+1} is directly related to s_t . Market sentiments, which produce variations in spot rate s_t at time t, affect one-period ahead forward rate tF_{t+1}. Thus causality appears to run from spot rate s_t to forward rate tF_{t+1} and this assertion requires empirical verification.

8.6 Cointegration between Current Spot rate s_t and Lagged Forward rate $(t-1)F_t$

Rearrangement of the equation (8.13) gives

$$(t-1)F_t = -\rho_{t-1} + s_t - u_t \quad [\text{given that } \rho_{t-1} = \rho_t] \tag{8.16}$$

The estimable form of the equation (8.16) is

$$(t-1)F_t = \propto + \beta s_t + \omega_t \tag{8.17}$$

where it is held that $\propto = -\rho_{t-1} = -\rho^*$

and $\beta = 1$

The estimated equation is

$$(t-1)\hat{F}_t = 0.015683 + 0.991053\ s_t \tag{8.18}$$

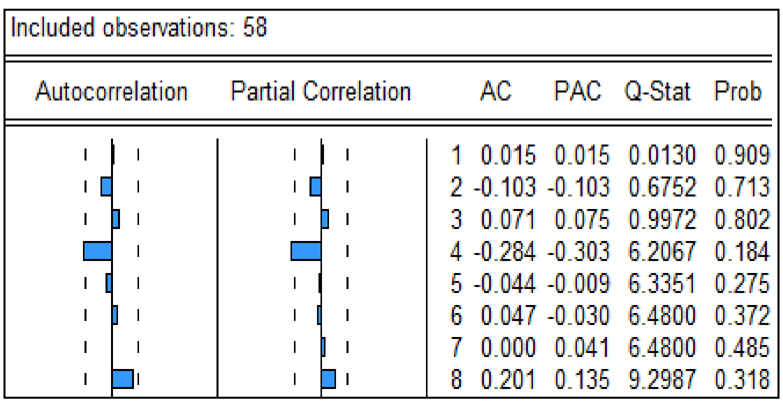
S.E	(0.055724)	(0.031838)
t	[0.281445]	[31.12799]
Prob	0.7794	0.0000

R^2 =0.945363, $\bar{R}^2$ =0.944388, DW = 1.960304, AIC = -5.859010, SIC = -5.787960, F = 968.9515, Probability (F) = 0.0000, n = 59

The ADF Test statistics for $\hat{\omega}_t$, the residual series of the equation (8.18), is -7.337389 with probability = 0.0000. It indicates that $\hat{\omega}_t$, the residuals series of the equation (8.18), is stationary. The correlogram of $\hat{\omega}_t$, as given in figure (8.3) below, shows that $\hat{\omega}_t$ is white noise.

Figure: 8.3

Correlogram of Residuals ($\hat{\omega}_t$) of the Equation (8.18)



Included observations: 58

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.015	0.015	0.0130	0.909
		2	-0.103	-0.103	0.6752	0.713
		3	0.071	0.075	0.9972	0.802
		4	-0.284	-0.303	6.2067	0.184
		5	-0.044	-0.009	6.3351	0.275
		6	0.047	-0.030	6.4800	0.372
		7	0.000	0.041	6.4800	0.485
		8	0.201	0.135	9.2987	0.318

Jarque-Berra statistic, as given in the Table – 8.1, indicates normality for $\hat{\omega}_t$ such that $\hat{\omega}_t \sim$ GWN (0, 0.12) where $\sigma\hat{\omega} = 0.012$

Table: 8.1

Basic Statistics for $\hat{\omega}_t$

Observations 58	
Mean	-8.97E-16
Median	0.001221
Maximum	0.034383
Minimum	-0.031646
Std. Dev.	0.012598
Skewness	-0.103656
Kurtosis	3.812286
Jarque-Bera	1.698400
Probability	0.427757

In the estimated equation (8.18)

(i) $\hat{\propto}$, which proxies for the risk premium (ρ^*), is not significantly different from zero even at 5% level. This indicates the absence of risk premium in the market implying '*risk neutrality'* of market agents.

(ii) $\hat{\beta}$, the coefficient of s_t, is significant even at 1% level.

(iii) the coefficient $\hat{\beta}$ is not statistically different from unity even at 1% level[4].

Thus given $\hat{\propto} = 0$ and $\hat{\beta} = 1$, the estimated equation (8.18) virtually becomes

$$(t-1)\hat{F}_t = s_t \tag{8.19}$$

such that the estimable equation (8.17) becomes

$$(t-1)F_t = s_t + \omega_t \tag{8.20}$$

$$\text{where } \omega_t \sim \text{GWN}(0, \sigma_u^2)$$

The equation (8.20) shows that the forward rate quoted at any time $(t-1)$ for the period t is actually an optimal forecast of the next period's spot rate, in the sense that it will be proved wrong, only to the extent of a '*white noise*' random error. This further means that the forecast cannot be improved upon by using publicly available data. Thus the lagged one-period ahead forward rate is the *'unbiased predictor'* of the spot rate implying that forward rate cannot be bettered as a forecast.

[4] Please see it in appendix

8.7 VEC Model for $(\mathbf{t-1})\mathbf{F_t}$ and $(\boldsymbol{s_t})$: Stability of the Long-run Relation

The estimated equation (8.18) represents the equation of cointegration for $(\mathrm{t}-1)\mathrm{F_t}$ with $\mathrm{s_t}$ which indicates that there exists a long-run relation between lagged forward rates $(\mathrm{t}-1)\mathrm{F_t}$ and the current spot rate $\mathrm{s_t}$. Stability of this long-run relationship needs to be examined through the estimation of an appropriate VEC Model for these two variables. The estimated VEC Model is being presented through the Table: 8.2 below.

Table: 8.2
Estimated VEC Model for the Lagged Forward Rate $(\mathbf{t-1})\mathbf{F_t}$ and Current Spot Rate $(\boldsymbol{s_t})$

Findings and

variable	$d(t-1)F_t$			(ds_t)		
	Coefficient	S.E	t-Statistics	Coefficient	S.E	t-Statistics
Z_{t-1}	-0.5748	(0.1328)	[-4.3282]	-1.8514	(4.4499)	[-0.4160]
$L^{-1}[d(t-1)F_t$	0.0037	(0.0043)	[0.8548]	-0.1393	(0.1465)	[-0.4160]
$L^{-1}[d(s_t)]$	0.4182	(0.1342)	[3.1152]	-1.8693	(4.4978)	[-0.4156]
Constant	0.0016	(0.0003)	[4.2164]	0.0085	(0.0127)	[0.6657]
	$R^2 = 0.999131$, $\bar{R}^2 = 0.999081$, F statistic = 19936.82 Log Likelihood = 363.1378, AIC = -12. 82635, SIC = -12.68168			$R^2 = 0.017241$, $\bar{R}^2$ =-0.039457, F statistic = 0.304079, Log Likelihood = 166.483 AIC = -5.802987, SIC = -5.658319		

8.7.1 Findings and Explanations:-

It is observed that in the Table 8.2 that

(a) the coefficient of the error correction term (Z_{t-1}) in $d(t-1)F_t$ equation is negative and significant even at 1% level while that in the ds_t equation is not significantly different from zero even at 5% level.

(b) this finding confirms that it is the lagged one-period ahead forward rate $(t-1)F_t$ which maintains a cointegrating (i.e., long-run) relation with the current spot rate (s_t) over the period of study, and

(c) any deviation of the lagged one-period ahead forward rate $(t-1)F_t$ from its long run equilibrium level at any period is corrected at the next period through the countervailing movement of $(t-1)F_t$ alone since the error correction term (Z_{t-1}) in

$d(t-1)F_t$ equation is statistically significant and its value is negative. Again spot rate plays no role in this adjustment process since the error correction term in ds_t equation is not statistically significant.

(d) the coefficient of the first lag of ds_t i.e, $L^{-1}[d(s_t)]$ in the $d[(t-1)F_t]$ equation is significant at 5% level while that of the $L^{-1}[d(t-1)F_t]$ in the ds_t equation is not significant even at 10% level. These findings imply that, in the short-run dynamic path of adjustment, shocks transmitted through current spot rate profile *Granger Cause* variations in lagged forward rate.

(e) the absolute value of the coefficient of the error correction term (Z_{t-1}) in $d[(t-1)F_t]$ equation is less than unity. It implies that the dynamic path of adjustment of the lagged one-period ahead forward rate towards equilibrium is non-oscillatory and nor explosive by nature.

8.8 Intervention Analysis through the Study of Impulse Response Functions and Variance Decompositions of Forecast Errors

8.8.1 Impulse Response Functions Study

Estimated VEC model shows that shocks, transmitted through spot rate s_t channel, affect the dynamic movement of lagged forward rate $(t-1)F_t$ significantly. The extent of such effect can be better understood through the study of the respective Impulse Response Functions of the endogenous variables [s_t and $(t-1)F_t$] concerned. Figure 8.4 through Figure 8.7 present these functions with the Cholesky Ordering $(t-1)F_t$ and s_t.

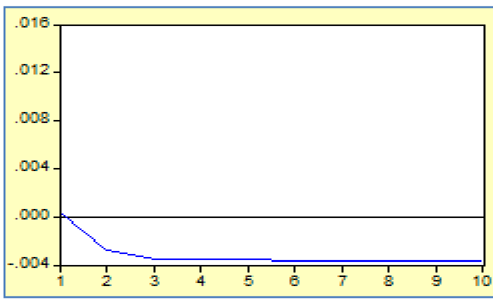


Figure: 8.4
Response of $(t-1)F_t$ to $(t-1)F_t$

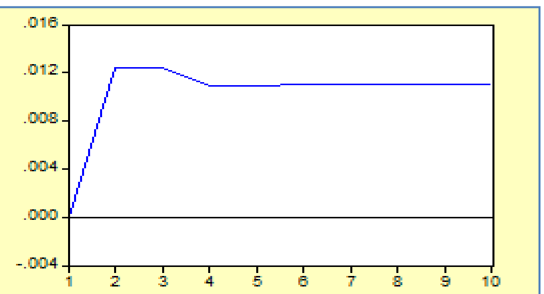


Figure: 8.5
Response of $(t-1)F_t$ to s_t

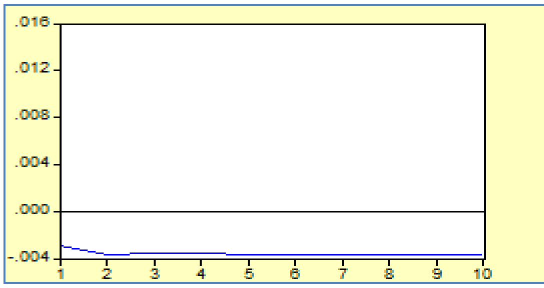


Figure: 8.6
Response of s_t to $(t-1)F_t$

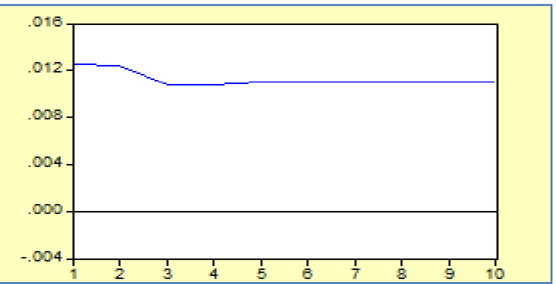


Figure: 8.7
Response of s_t to s_t

Figure 8.5 shows that shocks, transmitted through the spot rate channel, bring in significant change in lagged forward rate profile significantly over time. As a matter of fact, spot rate shocks have altered the long-run level of lagged forward rate profile.

Figure 8.6 shows that shocks, transmitted through lagged forward rate channel, fail to exert any discernible effect on the spot rate profile. Figure 8.7 shows that spot rate shocks play the most dominant role in constituting its own long-run profile. Figure 8.4 indicates that forward rate shocks play a minor role in constituting its own long-run profile.

8.8.2 Variance Decomposition Study

Variance Decomposition of forecast errors of the endogenous variables concerned are being presented through the Tables 8.3 and 8.4 below with Cholesky Ordering $(t-1)F_t$, s_t.

Period	S.E.	FRDL1	SPTL
1	0.000383	100.0000	0.000000
2	0.012747	4.726963	95.27304
3	0.018139	6.248268	93.75173
4	0.021435	7.200090	92.79991
5	0.024295	7.770986	92.22901
6	0.026936	8.148089	91.85191
7	0.029343	8.422436	91.57756
8	0.031559	8.628179	91.37182
9	0.033629	8.785898	91.21410
10	0.035580	8.910350	91.08965

Table: 8.3
Variance Decomposition of $(t-1)F_t$,

Period	S.E.	FRDL1	SPTL
1	0.012845	5.183654	94.81635
2	0.018192	6.613811	93.38619
3	0.021459	7.498937	92.50106
4	0.024302	8.013876	91.98612
5	0.026929	8.348562	91.65144
6	0.029323	8.592006	91.40799
7	0.031527	8.774678	91.22532
8	0.033587	8.914644	91.08536
9	0.035530	9.025099	90.97490
10	0.037371	9.114532	90.88547

Tables: 8.4
Variance Decomposition of s_t

Table 8.3 shows that shocks, transmitted through spot rate channel, account for more than 90% of forecast error variances of lagged forward rate. Only about 10% of such variances are being explained by its own shocks. Table 8.4 shows that more than 90% of forecast error variances of spot rate (s_t) are being explained by shocks transmitted through spot rate channel. Only 9% of such variances are due to shocks transmitted through lagged forward rates channel.

The intervention analysis through the study of respective impulse response functions and variance decomposition of forecast error variances testify for the observed Granger Causality running from spot rate to lagged forward rate. This finding supports the direction and nature of Granger Causality as observed in the study with the VEC model.

8.9 Relation Between Current Spot Rate (s_t) and Contemporaneously Formed One-Period Ahead Forward Rate (tF_{t+1})

It may be relevant here to examine how spot rates are closely linked to contemporaneously formed forward rates. Figure-8.2 plots the spot rates against the contemporaneous forward rates and the two lines in the figure are so close together as to be completely indistinguishable. In statistical terms, the correlation coefficient between the spot rate and the contemporaneously formed forward rate is 0.999936 as compared with 0.945363 for the lagged forward rates. It may however be noted that the match is more apparent than real as revealed through the results of Cointegration study. The study is carried through the estimation of the equation (8.17) upon transformation involving substitution of tF_{t+1} for $(t-1)F_t$ in it. The transformed model is

$$tF_{t+1} = \delta + \theta s_t + u_{t+1} \tag{8.21}$$

It is assumed that $\delta = 0$ and $\theta = 1$ in parity with the assumption maintained in case of the equation (18.18) given that $u_{t+1} \sim \text{iid } N(0, \sigma^2 u)$. The estimated equation is

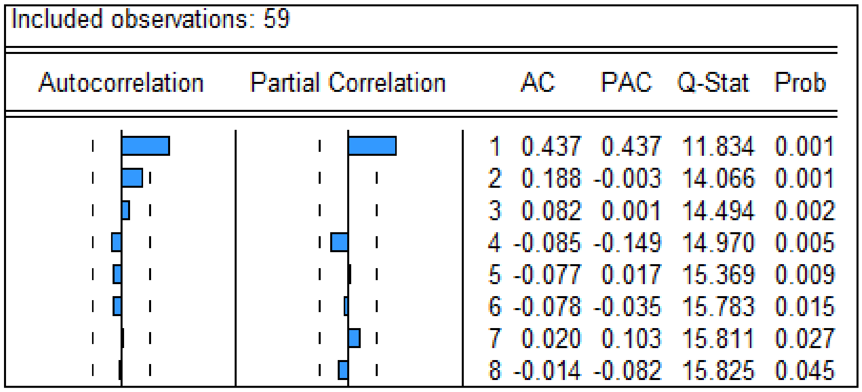
$$t\hat{F}_{t+1} = -0.003068 + 1.003406\,(s_t) \tag{8.22}$$

S.E	(0.001865)	(0.001066)
t	[-1.645335]	[940.9350]
Prob	0.1054	0.0000

R^2 = 0.999936, $\bar{R}^2$ = 0.999934, DW = 1.071349, F = 885358.5, Prob (F-Stat) = 0.0000, AIC = -12.5913, SIC = -12.52271, Log Likelihood = 373.4974, n = 59, df = 57.

DW = 1.0771349 indicates that residuals $(\hat{u}_{t+1})$ of the estimated equation (8.22) suffer from serial correlation. The correlogram of the residuals $(\hat{u}_{t+1})$, as given by the Figure-(8.8), testifies for the significant first order serial correlation in the residuals $(\hat{u}_{t+1})$.

Figure 8.8: Correlogram of $(\hat{u}_{t+1})$, Residuals from the Equation (8.21)



Included observations: 59

	AC	PAC	Q-Stat	Prob
1	0.437	0.437	11.834	0.001
2	0.188	-0.003	14.066	0.001
3	0.082	0.001	14.494	0.002
4	-0.085	-0.149	14.970	0.005
5	-0.077	0.017	15.369	0.009
6	-0.078	-0.035	15.783	0.015
7	0.020	0.103	15.811	0.027
8	-0.014	-0.082	15.825	0.045

Results of ADF test on these residuals, as given in the Table 8.5, indicate that these residuals $(\hat{u}_{t+1})$ are free from '*unit roots*' even at 1% level. Thus these residuals are stationary, though not '*white-noise*'.

Table 8.5: Results of ADF Unit root Test on $\hat{u}_{t+t}$

Variable	Null Hypothesis	Lag Length*	ADF Test Statistics	Prob.	Mac-Kinnon Critical Value**		
					1%	5%	10%
$(\hat{u}_{t+1})$	$(\hat{u}_{t+1})$has unit root Exogenous: Constant	0	-4.38255	0.000	-3..548	-2.912	-2.594
	$(\hat{u}_{t+1})$has unit root Exogenous: Constant, Linear Trend	0	-4.1242	0.004	-4.124	-3.489	-3.173
	$(\hat{u}_{t+1})$has unit root Exogenous: None	0	-4.427	0.000	-2.605	-1.946	-1.613

**MacKinnon (1996) one-side p-values. *Based on SIC, Max Lag = 1

Presence of first order auto-correlation in residuals $(\hat{u}_{t+1})$ indicates that

(i) $t\hat{F}_{t+1}$ is not cointegrated with (s_t) and the equation (8.22) is not the cointegrating equation for tF_{t+1} vis-a-vis (s_t), the spot rate.

(ii) the equation (8.22) suffers from '*mis-specification error'* and the presence of first order auto-correlation in ($\hat{u}_{t+1}$) is the 'outcome' of such 'mis-specification'. Absence of cointegration between tF_{t+1} and (s_t) spot rate has been confirmed by the '*Johansen Test'* results, as presented through the Table-8.6 below.

Table: 8.6 Johansen Cointegration Tests for Equation (8.22) with Restricted Intercepts and Trends

Trace Test				
Null	Alternative	Statistic	5% critical value	1% critical value
$r = 0$	$r = 1$	16.03904	19.96	24.60
$r \leq 1$	$r = 2$	3.995105	9.24	12.97
Maximum Eigen Value Test				
Null	Alternative	Statistic	5% critical value	1% critical value
$r = 0$	$r = 1$	12.04393	15.67	20.20
$r \leq 1$	$r = 2$	3.95105	9.24	12.97

**MacKinnon (1996) one-side p-values. *Based on SIC, Max Lag = 1

Both the Trace Test and Maximum Eigen Value Test unanimously suggest the number of cointegrating Vectors (r) as r = 0. This implies that both the tests confirm absence of cointegration between tF_{t+1} and s_t even at 5% level. Thus equation (8.22) falls short of capturing cointegrating relation between contemporaneously formed forward rate and spot rate over the period of study.

8.10 VEC Model for tF_{t+1} ans s_t

Cointegration between tF_{t+1} and s_t has further been studied through the estimation of VEC Model for these variables where lag length has been selected through the AIC and SIC criteria. Table-8.7 presents the estimated VEC Model concerned.

Table: 8.7

Estimation of VEC Model for tF_{t+1} and s_t

variable	$d(tF_{t+1})$			(ds_t)		
	Coefficient	S.E	t-Statistics	Coefficient	S.E	t-Statistics
Z_{t-1}	-5.862345	4.84503	-1.20997	-5.39004	4.86689	-1.10750
$L^{-1}d(tF_{t+1})$	-2.328966	4.37592	-0.53222	-2.294191	4.39567	-0.52192
$L^{-1}[d(s_t)]$	2.263510	4.40143	0.51427	2.223363	4.42129	0.50288
Constant	0.002903	0.00171	1.70206	0.002920	0.00171	1.70284
	$R^2 = 0.061122$ $\bar{R}^2 = 0.007978$ F statistic = 1.151028 Log Likelihood = 171.0041 AIC = -5.859794 SIC = -5.716422			$R^2 = 0.053563$ $\bar{R}^2$ =-0.000009 F statistic = 0.999835 Log Likelihood = 170.7475 AIC = -5.850791 SIC = -5.707419		

8.10.1 Findings and Explanations:

a. The coefficients of cointegrating terms Z_{t-1} in both $d(tF_{t+1})$ and $d(s_t)$ equations are not different from zero even at 5% level and this finding confirms the absence of Cointegration between tF_{t+1} and s_t.

b. Coefficients of $L^{-1}d(tF_{t+1})$ and $L^{-1}d(s_t)$ terms in both the equations are not statistically (at even 10% level) significant. This finding suggests for the block exogeniety for both tF_{t+1} and s_t in the VEC Model concerned.

Therefore, the conjecture, that spot rates are more closely linked to contemporaneous rather than to lagged forward rates, turns out to be more apparent than real. These two series fail to share common trends and thus emerge '*non-cointegrated*' implying absence of any meaningful long-run relation between them.

8.11 Summary of the Study in Chapter 8

(A) The graphical study shows that

(i) one-period lagged forward rate $(t-1)F_t$ tracks the spot rate (s_t) closely but spot rate leads change of direction upward or downward in every case. Change of direction in spot rate at any period t is completely missed by the lagged forward rate and is only reflected at (t+1) period lagged forward rate.

(ii) spot rate (s_t) appears to be more closely linked to contemporaneous forward rate (tF_{t+1}) in view of the fact that time plots of spot rates (s_t) against that of contemporaneous forward rates (tF_{t+1}) in a graph are so close together that they appear completely indistinguishable. These observations indicate that spot rates are apparently more closely linked to contemporaneous forward rate than to one-period lagged forward rate. This further hint at the existence of a cointegrating relation between spot rate and contemporaneous forward rate. This possibility has been verified through the study of '*cointegration relation'* of spot rate (s_t) with the contemporaneous forward rate (tF_{t+1}) and also with one period lagged forward rate $(t-1)F_t$.

(B) The cointegration study shows that

(i) there exists no cointegrating equation between spot rate s_t and contemporaneous forward rate tF_{t+1}. The estimated equation for these variables suffers from significant first order autocorrelation indicating the *'errors of mis-specification'* in it. Johansen Cointegration tests also confirm the absence of cointegration between spot rate and contemporaneous forward rate tF_{t+1}.

(ii) one period lagged forward rate $(t-1)F_t$ and spot rate s_t are '*cointegrated*'. The residuals of '*cointegrating equation'* are '*white noise'*. Johansen Cointegration Tests also confirm the existence of cointegrating between $(t-1)F_t$ and s_t.

(iii) the conjecture that spot rates are more closely linked to the contemporaneous forward rate tF_{t+1} rather than to one-period lagged forward rate $(t-1)F_t$ is more apparent than real. In the estimated VEC model for tF_{t+1} and s_t, the cointegrating terms in dtF_{t+1} and ds_t equations are not statistically different from zero implying absence of cointegration between tF_{t+1} and s_t.

(C) The estimated VEC model for $(t-1)F_t$ and s_t

(i) further testifies for the existence of cointegration between the variables concerned. The estimated model further confirms that it is $(t-1)F_t$ which maintains an independent '*cointegrating*' relation with s_t. The cointegrating term in ds_t equation, which is statistically insignificant (even at 10% level), nullifies the assertion that s_t maintains an independent cointegrating relation with $(t-1)F_t$.

(ii) shows that the long-run relationship between these variables is '*stable*' and the dynamic path of adjustment is non-oscillatory but damped over time.

(iii) also indicates that '*causality'*, in *Granger sense*, runs from spot rate s_t to lagged forward rate $(t-1)F_t$. This funding is in conformity with the resolution drawn from the CIRP relation, given by the equation (8.10).

(D)

(i) In the estimated Cointegrating equation (8.18) for $(t-1)F_t$ and s_t the coefficient $(\widehat{\alpha})$ is not statistically different from zero (at 5% only) and the coefficient $(\widehat{\beta})$ of s_t is not statistically different from unity even at 1% level. $\widehat{\alpha} = 0$ indicates the absence of *'risk premium'* in the market implying '*risk neutrality'* for market agents.

Given $\widehat{\beta} \simeq 1$ with $\widehat{\alpha} = 0$, the estimated equation (8.18) virtually becomes, as given by the equation (8.19), $(t-1)\widehat{F}_t = s_t$ implying that the lagged one-period ahead forward rate $(t-1)F_t$ equals the spot rate s_t in equilibrium. This testifies for the absence of '*arbitrage profit*' in the capital market in equilibrium.

(ii) Equation (8.20)

$$(t-1)F_t = s_t + \omega_t$$

$$\text{where } \omega_t \sim \mathrm{GWN}(0, \sigma_u^2)$$

entails '*white noise'* residuals (ω_t) , and it implies that forward rate cannot be bettered as forecast.

Again we have

$$(t-1)F_t = s_t^e + \epsilon_t + \omega_t$$

where $\in_t$ and ω_t are '*white noise'* residuals

$$\text{or, } (t-1)F_t = s_t^e + \eta_t$$

where $\eta_t = (\in_t + \omega_t)$ and η_t is *white noise*.

Thus one-period lagged forward rate $(t-1)F_t$ equals the MMSE forecast for s_t (i.e. s_t^e) except for some white noise error in the Indian foreign exchange market. This further implies the absence of unexpected arbitrage profit in the market, and thereby the '*efficiency*' of the market concerned.

(E) Impulse response functions of the endogenous variables indicate that

(i) shocks, transmitted through spot rate channel, affect the lagged forward rate profile significantly, while

(ii) shocks, transmitted through lagged forward rate channel, exhibit no discernable effect on the long-run profile of spot rate.

(F) Variance Decomposition[5] study shows that

(i) spot rate shocks play the most dominant role in explaining forecast error variances for lagged forward rate.

(ii) forward rate shocks play an insignificant role in explaining the forecast error variances for spot rate.

Thus the intervention analysis testifies for the existence of Granger Causality, running from spot rate to lagged forward rate, as was evident from the estimated VEC Model for the endogenous variables concerned.

8.12 Conclusions and Inferences of the Study in Chapter 8.

The study, therefore, shows that, in the Indian foreign exchange (rupee/dollar) market, over the period 3rd January, 2011- 2nd November, 2015.

(i) one-period lagged forward rate maintains an independent long-run stable relation with spot rate

(ii) there exists no long-run relation between spot rate and contemporaneous forward rate.

[5] Please see it in appendix

(iii) Granger Causality runs from spot rate to lagged forward rate.

(iv) there exists no risk premium in the foreign exchange market.

(v) CIRAP holds such that lagged forward exchange rate $(t-1)F_t$ on average equals the spot rate s_t, in the long run and

(vi) there exists, therefore, no scope for reaping arbitrage profit arising out of the difference between forward rate and corresponding spot rate. This testifies for the '*efficiency*' of Indian foreign exchange (rupee/dollar) market over the period of study.