CHAPTER NINE

FORECASTING BY VECTOR AUTOREGRESSION MODELS

9.1 Vector Autoregressive (VAR) Models

Vector Autoregression (VAR) models were introduced by the macro- econometrician Christopher Sims (1980) to model the joint dynamics and causal relations among a set of macroeconomic variables. “The vector autoregression (VAR) is commonly used for forecasting systems of interrelated time series and for analyzing the dynamic impact of random disturbances on the system of variables. The VAR approach sidesteps the need for structural modeling by treating every endogenous variable in the system as a function of the lagged values of all of the endogenous variables in the system”. (QMS, 2009: Eviews 7:User’s Guide II)

“The vector autoregression (VAR) model is one of the most successful, flexible, and easy to use models for the analysis of multivariate time series. It is a natural extension of the univariate autoregressive model to dynamic multivariate time series. The VAR model has proven to be especially useful for describing the dynamic behavior of economic and financial time series and for forecasting. It often provides superior forecasts to those from univariate time series models and elaborate theory-based simultaneous equations models. Forecasts from VAR models are quite flexible because they can be made conditional on the potential future paths of specified variables in the model”. (QMS, 2009: Eviews 7:User’s Guide II)

In addition to data description and forecasting, the VAR model is also used for structural inference and policy analysis. In structural analysis, certain assumptions about the causal structure of the data under investigation are imposed, and the resulting causal impacts of unexpected shocks or innovations to specified variables on the variables in the model are summarized. These causal impacts are usually summarized with impulse response functions and forecast error variance decompositions.

Though VAR models in economics were made popular by Sims (1980), the definitive technical reference for VAR models is Lutkepohl (1991), and updated surveys of VAR techniques are given in Watson (1994) and Lutkepohl (1999) and Waggoner and Zha (1999). Applications of VAR models to financial data are given in Hamilton (1994), Campbell, Lo and MacKinlay (1997), Cuthbertson (1996), Mills (1999) and Tsay (2001).

The mathematical representation of a VAR is:

Let $Y_t = (y_{1t}, y_{2t}, \ldots\ldots y_{nt})$ denote an $(n \times 1)$ vector of time series variables. The basic $\rho -$ lag vector autoregressive (VAR (P)) model has the form:

$$Y_t = c + \pi_1 Y_{t-1} + \pi_2 Y_{t-2} + \cdots \ldots\ldots\ldots\ldots\ldots\ldots\ldots + \pi_P Y_{t-P} + \varepsilon_t,\ t = 1, \ldots . T \qquad (9.1)$$

Where, π_i are $(n \times n)$ coefficient matrices and ε_t is an $(n \times 1)$ unobservable zero mean white noise vector process (serially uncorrelated or independent) with time invariant covariance matrix Σ. For example, a bivariate $VAR(2)$ model by equation has the form:

$$\begin{pmatrix} y_{1t} \\ y_{2t} \end{pmatrix} = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} + \begin{pmatrix} \pi_{11}^1 & \pi_{12}^1 \\ \pi_{21}^1 & \pi_{22}^1 \end{pmatrix} \begin{pmatrix} y_{1t-1} \\ y_{2t-1} \end{pmatrix} + \begin{pmatrix} \pi_{11}^2 & \pi_{12}^2 \\ \pi_{21}^2 & \pi_{22}^2 \end{pmatrix} \begin{pmatrix} y_{1t-2} \\ y_{2t-2} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix} \qquad (9.2)$$

$$Or, y_{1t} = c_1 + \pi_{11}^1 y_{1t-1} + \pi_{12}^1 y_{2t-1} + \pi_{11}^2 y_{1t-2} + \pi_{12}^2 y_{2t-2} + \varepsilon_{1t} \qquad (9.3)$$

$$y_{2t} = c_2 + \pi_{21}^1 y_{1t-1} + \pi_{22}^1 y_{2t-1} + \pi_{21}^2 y_{1t-2} + \pi_{22}^2 y_{2t-2} + \varepsilon_{2t} \qquad (9.4)$$

In the similar fashion, a multi-variate $VAR(P)$ model by equation has the form:

$$\begin{pmatrix} y_{1t} \\ y_{2t} \\ \vdots \\ y_{nt} \end{pmatrix} = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} + \begin{pmatrix} \pi_{11}^1 & \pi_{12}^1 \cdots & \pi_{1P}^1 \\ \pi_{21}^1 & \pi_{22}^1 & \pi_{2P}^1 \\ \vdots & \vdots & \vdots \\ \pi_{P1}^1 & \pi_{P2}^1 \ldots . & \pi_{PP}^1 \end{pmatrix} \begin{pmatrix} y_{1t-1} \cdots . y_{1t-p} \\ y_{2t-1} \cdots . y_{2t-P} \\ \vdots \\ y_{nt-1} \cdots . y_{nt-P} \end{pmatrix} +$$

$$\begin{pmatrix} \pi_{11}^2 & \pi_{12}^2 \cdots & \pi_{1P}^2 \\ \pi_{21}^2 & \pi_{22}^2 & \pi_{2P}^2 \\ \vdots & \vdots & \vdots \\ \pi_{P1}^2 & \pi_{P2}^2 \ldots . & \pi_{PP}^2 \end{pmatrix} \begin{pmatrix} y_{1t-1} \cdots . y_{1t-p} \\ y_{2t-1} \cdots . y_{2t-P} \\ \vdots \\ y_{nt-1} \cdots . y_{nt-P} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \vdots \\ \varepsilon_{nt} \end{pmatrix}$$

Where, $\mathrm{cov}(\varepsilon_{1t}, \varepsilon_{2s}) = \sigma_{12}$ for t = s; 0 otherwise. Notice that each equation has the same regressors - lagged values of y_{1t} and y_{2t}. Hence, the $VAR(P)$ model is just a seemingly unrelated regression (SUR) model with lagged variables and deterministic terms as common regressors.

In lag operator notification, the $VAR(P)$ is written as:

$$\Pi(L)\mathrm{Y_t} = \mathrm{c} + \varepsilon_\mathrm{t} \qquad (9.5)$$

Where, $\Pi(L) = \mathrm{I_n} - \Pi_1 \mathrm{L} - \Pi_\mathrm{P} \mathrm{L}^\mathrm{P}$

The $VAR(P)$ is stable if the roots of $det\left(\mathrm{I_n} - \Pi_{1^{Z-\cdots-}}\Pi_{\mathrm{p}Z^\mathrm{P}}\right) = 0$ lie outside the complex unit circle (have modulus greater than one), or equivalently if Eigen values of the companion matrix

$$F = \begin{pmatrix} \Pi_1 & \Pi_2 & \cdots & \Pi_n \\ I_n \ddots \ 0 & 0 & \cdots & 0 \\ 0 \ \cdots & 0 & 0 & 0 \end{pmatrix}$$ have modulus less than one.

9.2 Study on Narrow Money Supply and Price Level by VAR Model

To analyze the relationship between narrow money supply and price level by VAR technique, we need the stationary series of the concerned variables. The stationary series of the variables narrow money supply and price level are $dLnM_{1t}$ and $dLnCPI_t$ respectively. Lags 5 are the appropriate lags (according to Schwarz Criterion) of the variables $dLnM_{1t}$ and $dLnCPI_t$ to apply VAR model. Using 5 lags for each endogenous variable, the VAR models with $dLnCPI_t$ as dependent variable and lagged $dLnCPI_t$ and $dLnM_{1t}$ as independent variables is given by equation (9.6), and $dLnM_{1t}$ as dependent variable and lagged $dLnCPI_t$ and $dLnM_{1t}$ as independent variables is given by equation (9.7).

$$\begin{aligned} dLnCPI_t = \gamma_1 &+ \alpha_1 dLnM_{1_{t-1}} + \alpha_2 dLnM_{1_{t-2}} + \alpha_3 dLnM_{1_{t-3}} + \alpha_4 dLnM_{1_{t-4}} \\ &+ \alpha_{5_d} LnM_{1_{t-5}}\, \beta_1 dLnCPI_{t-1} + \beta_2 dLnCPI_{t-2} + \beta_3 dLnCPI_{t-3} \\ &+ \beta_4 dLnCPI_{t-4} + \beta_5 dLnCPI_{t-5} + \varepsilon_{1t} \end{aligned} \tag{9.6}$$

$$\begin{aligned} dLnM_{2t} = \ & \gamma_2 + \delta_1 dLnM_{2_{t-1}} + \delta_2 dLnM_{2_{t-2}} + \delta_3 dLnM_{2_{t-3}} + \delta_4 dLnM_{2_{t-4}} + \\ & \delta_5 dLnM_{2_{t-5}} + \ \theta_1 dLnCPI_{t-1} + \theta_2 dLnCPI_{t-2} + \theta_3 dLnCPI_{t-3} + \theta_4 dLnCPI_{t-4} \ + \\ & \theta_5 dLnCPI_{t-5} + \varepsilon_{2t} \end{aligned} \tag{9.7}$$

The results from VAR model according to equation (9.6) are presented through Table-9.1.

Table-9.1: Results from VAR with $dLnCPI_t$ as Dependent Variable (Lag:1 to 5)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
$Constant$	$\gamma_1 = 0.0118$	0.0055	2.1455	0.0338
$dLnM_{1t-1}$	$\alpha_1 = 0.0937$	0.0617	1.51880	0.1312
$dLnM_{1t-2}$	$\alpha_2 = 0.1239$	0.0557	2.2236	0.0279
$dLnM_{1t-3}$	$\alpha_3 = 0.0448$	0.0595	0.7532	0.4527
$dLnM_{1t-4}$	$\alpha_4 =$ -0.1185	0.0575	-2.0601	0.0414
$dLnM_{1t-5}$	$\alpha_5 = 0.0363$	0.0635	0.5718	0.5684
$dCPI_{t-1}$	$\beta_1 = 0.0996$	0.0855	1.1650	0.2461
$dCPI_{t-2}$	$\beta_2 = 0.0194$	0.0797	0.2433	0.8081
$dCPI_{t-3}$	β_3 = -0.1169	0.0792	-1.4764	0.1422
$dCPI_{t-4}$	$\beta_4 = 0.3502$	0.0767	4.5636	0.0000
$dCPI_{t-5}$	$\beta_5 =$ -0.2152	0.0804	-2.6750	0.0084

R^2=0.5358 Adj. R^2=0.4960 S.E. of regression= 0.0182
Log likelihood=370.4535 Durbin-Watson stat=1.9881

With $dLnCPI_t$ as the dependent variable of VAR model based on equation (9.6), the coefficient of $dLnM_{1_{t-2}}$ is $\alpha_2 = 0.1239$, which is positive and significant at 5% level. This clearly indicates that $dLnCPI_t$ is Granger caused by the lagged $dLnM_{1_t}$preceding two periods. That is, current price level is directly caused by the M_1 money supply of two periods/quarters back. This confirms that a ten percent rise of change in M_1 money supply at two quarters back causes the change in price level at current time by 1.23%. This implies that as money supply increases at 2 quarters back, there is expansion of credit. As the commercial banks expand more money, market rate of interest falls. At low rate of interest, people do not want to keep money in bank, rather they withdraw money from banks to purchase bond when its price falls. This further causes money supply to increase in the society. As non-bank public have more money in hands, they are ready to pay still higher and higher price to have the commodities and thereby price further rises and inflation arises.

The coefficient of $dLnM_{1_{t-4}}$ is significant at 10% level but it is negative, which is not desirable in accordance with Quantity Theory of Money. The coefficients of other lagged $dLnM_{1_t}$are not significant. The coefficient of $dLnCPI_{t-4}$ is $\beta_4 = 0.3502$, which is positive and significant at less than 1% level. This implies that $dLnCPI_t$ is not only caused by lagged $dLnM_{1_t}$ but also by own lagged term of $dLnCPI_t$. It means inflation of four periods back has the influence on current price level positively. For example, if price was high at four quarters back, the suppliers create artificial scarcity of goods with the expectation of further rise in price to earn more profit. As commodities become scarce, price level further rises. In this way, past inflation causes present inflation and same has happened in the economy of Nepal during the study period.

Since the Adjusted R^2 of VAR equation (9.6) is 0.4960, the dependent variable $dLnCPI_t$ is explained approximately 50% by the variation of independent variables the model is not poorly fitted. The value of standard error of regression, log likelihood ratio and D-W statistic also confirm that the estimated VAR model of equation (9.6) possesses the characteristics of goodness of fit.

The results from VAR model based on equation (9.7) are presented through Table-9.2, where $dLnM_{1_t}$ is taken as dependent variable and lagged (1 to 5 lags) $dLnM_{1_t}$ and $dLnCPI_t$ are taken as independent variables.

Table-9.2: Results from VAR with $dLnM_{1t}$ as Dependent Variable (Lag:1 to 5)

Variable	Coefficient	Std. Error	t-statistic	Prob.
$Constant$	γ_2 = 0.0260	0.0078	3.3287	0.0011
$dLnM_{1t-1}$	δ_1 = 0.0198	0.0871	0.2275	0.8204
$dLnM_{1t-2}$	δ_2 = -0.3221	0.0788	-4.0833	0.0001
$dLnM_{1t-3}$	δ_3 = -0.0340	0.0841	-0.4047	0.6863
$dLnM_{1t-4}$	δ_4 = 0.4203	0.0813	5.1696	0.0000
$dLnM_{1t-5}$	δ_5 = 0.0487	0.0897	0.5434	0.5877
$dLnCPI_{t-1}$	θ_1 = 0.2939	0.1214	2.4196	0.0169
$dLnCPI_{t-2}$	θ_2 = 0.1114	0.1131	0.9852	0.3263
$dLnCPI_{t-3}$	θ_3 = 0.1589	0.1118	1.4214	0.1576
$dLnCPI_{t-4}$	θ_4 = -0.2760	0.1087	-2.5385	0.0123
$dLnCPI_{t-5}$	θ_5 = -0.0288	0.1138	-0.2530	0.8007

R^2=0.6919 Adj. R^2=0.6680 S.E. of regression= 0.0257
Log likelihood=319.6362 Durbin-Watson stat=1.9895

In the Table-9.2, the coefficient of $dLnCPI_{t-1}$ is $\theta_1 = 0.2939$, which is positive and significant at 5% level. It means the change in M_1 money supply at current time 't' is caused by the change in price level of preceding time period 't-1'.when there is inflation in the economy, supply of money should be increased to cope with the inflationary pressure. The increase in money supply further causes the price level to rise. The rising price requires increasing money supply further to bring adjustment between demand and supply and rise in money supply as a result of price level is a short term relief from inflationary pressure. In this way, there is circular relationship between price level and money supply. The Quantity Theory of Money states that rise in money supply causes price level to increase, but the VAR model of equation (9.7) states that not only rise in money supply causes price to rise but rise in price also causes money supply to rise. Thus, the coefficient of $dLnCPI_{t-1}$ implies that a 10% rise of change in price level at preceding time period requires to increase the change of M_1 money supply at current time by 2.93%.

Next, the coefficient of $dLnCPI_{t-4}$ is $\theta_4 =$ -0.2760, which is negative though it is significant. This negative coefficient is not desirable to analyze the relationship between M_1 money supply and price level. The coefficient of $dLnM_{1_{t-4}}$ is $\delta_4 =$ 0.4203, which is positive and significant at less than 1% level implying that change in current money supply is affected not only by price level but also by the change in money supply of four periods back. This clearly indicates that rise in money supply in the previous periods brings about the rise in money supply at current time. For example, when there is increase in money supply to cope with inflation the credit expands in the society. Non-bank public has more money in hands. The consumers are ready to pay higher price to have commodity rather than go without commodity. Consumers afford more and more not only for necessities but also for comforts and luxuries because they have no crisis of money. The quantity of money is increasing but value of money continues to fall. The falling value of money requires to increase quantity of money further. Hence, the increase in money supply in the past periods causes money supply to increase in the current time as well.

So far as the robustness of the VAR model of equation (9.7) is concerned, the Adjusted R^2 is 0.6680, which explains 66% of the dependent variable $dLnM_{1t}$ with the variation of independent variables. This model represents goodness of fit satisfactorily. Likewise, the standard error of regression, log likelihood ratio and D-W statistic also prove that the estimated VAR model of equation (9.7) is claimed to be robust.

9.3 Residuals Diagnostic of VAR Models

($dLnCPI_t$ Regressed on Lagged $dLnM_{1t}$)

In order to examine the consistency of estimated VAR model of equation (9.6), we have tested Residual Diagnostic through Correlogram Squared Residuals, Serial Correlation LM Test and Heteroscedasticity Test. With the help of these tests, we can conclude whether or not estimated regression equation (9.6) is consistent.

9.3.1 Correlogram of Squared Residuals

The Correlogram-Q-statistic of the residuals squared of estimated regression equation (9.6) is presented through Table-9.3. The ACFs and PACFs of correlogram of the residual squared are nearly zero at all lags and the Q-statistics at all lags are not significant with large p-values. This indicates that there is no evidence of rejecting the null hypothesis of no serial correlation. This implies that the residuals of the estimated VAR model of equation (9.6) are not correlated with their own lagged values. Hence, there is strong evidence of goodness of fit of the estimated VAR model of equation (9.6).

Table-9.3: Correlogram of Squared Residuals of Equation (9.6)

Lag	AC	PAC	Q-Stat	Prob	Lag	AC	PAC	Q-Stat	Prob
1	0.125	0.123	2.6276	0.622	9	0.014	0.002	3.8352	0.986
2	0.023	0.034	2.7051	0.745	10	-0.026	-0.029	3.9408	0.992
3	-0.044	-0.041	2.9904	0.810	11	-0.023	-0.011	4.0263	0.995
4	0.003	0.004	2.9914	0.886	12	-0.097	-0.103	5.5399	0.986
5	0.054	0.041	3.4379	0.904	13	-0.029	-0.049	5.6765	0.991
6	-0.003	-0.008	3.4395	0.944	14	0.125	0.123	2.6276	0.622
7	-0.046	-0.037	3.7586	0.958	15	0.023	0.034	2.7051	0.745
8	0.017	0.017	3.8042	0.975	16	-0.044	-0.041	2.9904	0.810

9.3.2 Breusch-Godfrey Lagrange Multiplier Test for Serial Correlation

The results of Breusch-Godfrey Lagrange Multiplier test for serial correlation have been presented through Table-9.4. As reported by F-statistic and $T \times R^2$ value and their corresponding probabilities of B-G LM test of Table-9.4, the null hypothesis of no autocorrelation cannot be rejected. The B-G LM test implies that residuals are not serially correlated. Due to the non-presence of serial correlation, the estimated VAR model of equation (9.6) is considered as the consistent model for representing the relationship between M_1 money supply and price level.

Table-9.4: Breusch-Godfrey Serial Correlation LM Test

F-statistic	0.3516	Prob. F(1,129)	0.5542
$T \times R^2$	0.3832	Prob. Chi-Square(1)	0.5359

9.3.3 Residuals Heteroscedasticity Test

The null hypothesis of White's (1980) is H_0: there is homoscedasticity in the residuals. The null hypothesis is not rejected if the F-statistic and χ^2statistic are not significant. No rejection of null hypothesis confirms that residuals of estimated regression do not suffer from heteroscedasticity problem and estimated regression is claimed to be consistent. Table-9.5 presents the Residual Heteroscedasticity test of estimated VAR model of equation (9.6).

From the Table-9.5, it is observed that F-statistic and $T \times R^2$ value are not significant as reported by the probability values of the fourth column. There is no evidence of rejecting the null hypothesis that residuals are homoscedastic. It means, the residuals of estimated VAR model of equation (9.6) do not suffer heteroscedastic problem. Hence, it is claimed that the estimated VAR model of equation (9.6) representing the relationship between narrow money supply and price level is consistent model.

Table-9.5: White Heteroscedasticity of Residuals of Equation (9.6)

Test Summary	Value	Degree of Freedom	Probability
F-statistic	1.0911	Prob. F(65,75)	0.3562
$T \times R^2$	68.5309	Prob. Chi-suare(65)	0.3585

9.4 Stability Test of Estimated VAR Models

($dLnCPI_t$ Regressed on Lagged $dLnM_{1t}$)

To examine the stability of the estimated regression equation (9.6), we apply some test such as Inverse Roots of Characteristic Polynomials; Ramsey's RESET test and Coefficient Variance Decomposition (coefficient diagnostic) test.

9.4.1 Inverse Roots of Characteristic Polynomials

The Inverse Roots of Characteristic Polynomials of Estimated VAR model of equation (9.6) is presented through Table-9.6. In the table, the modulus value of all roots does not exceed one, which means no root lies outside the unit circle. Thus, the estimated VAR model of equation (9.6) satisfies the stability condition, i.e. the estimated VAR model is stable.

Table-9.6: Inverse Roots of Characteristic Polynomials of Equation (9.6)

Root	Modulus
0.0012 - 0.9657i	0.9657
0.0012 + 0.9657i	0.9657
-0.9333	0.9333
0.7341	0.7341
-0.0600 - 0.7060i	0.7086
-0.0600 + 0.7060i	0.7086
0.5757 - 0.2502i	0.6277
0.5757 + 0.2502i	0.6277
-0.6058	0.6058
-0.1192	0.1192

9.4.2 Ramsey's RESET Test

Table-9.7 demonstrates the results from Ramsey’s RESET test of estimated VAR model of equation of equation (9.6).

In the upper part of the Table-9.7, F-statistic, t-statistic and likelihood ratio are not significant as reported by the corresponding probability values. The null hypothesis ‘correct specification is linear’ is not rejected even if the variable $Fitted^2$ term is included in to the regression equation (9.6). It means the estimated VAR model of equation (9.6) does not avoid the property of linearity. Likewise, in lower part of Table-9.7, H_0: $\gamma = 0$, is not rejected as reported by the t-statistic. Hence, the Ramsey’s RESET test implies that the estimated regression equation (9.6) is stable model containing the properties of linearity and it is not misspecified model.

Table-9.7: Ramsey's RESET Test of VAR Equation (9.6)

Test-statistic	Value	Degree of Freedom	Probability
t-statistic	0.0229	129	0.9817
F-statistic	0.0005	(1, 129)	0.9817
Likelihood ratio	0.0005	1	0.9808

Unrestricted Test Equation

Variable	Coefficient	Std. Error	t-Statistic	Prob.
$Constant$	γ_1 = 0.0118	0.0055	2.1334	0.0338
$dLnM_{1t-1}$	α_1 = 0.0943	0.0664	1.4192	0.11582
$dLnM_{1t-2}$	α_2 = 0.1247	0.0663	1.8798	0.0624
$dLnM_{1t-3}$	α_3 = 0.0454	0.0651	0.6981	0.4863
$dLnM_{1t-4}$	α_4 = -0.1193	0.0662	-1.8001	0.0742
$dLnM_{1t-5}$	α_5 = 0.0365	0.0641	0.5690	0.5703
$dCPI_{t-1}$	β_1 = 0.1001	0.0880	1.1373	0.2575
$dCPI_{t-2}$	β_2 = 0.0198	0.0820	0.2415	0.8095
$dCPI_{t-3}$	β_3 = -0.1178	0.0876	-1.3437	0.1814
$dCPI_{t-4}$	β_4 = 0.3523	0.1175	2.9960	0.0033
$dCPI_{t-5}$	β_5 = -0.2160	0.0875	-2.4669	0.0149
$Fitted^2$	ρ =-0.1057	4.6019	-0.0229	0.9817

9.4.3 Coefficient Variance Decomposition

Intriligator[37](1978) opines that since most aggregate economic time series are highly correlated with their own previous values and with present and past values of other time series, multi-colinearity can become a serious problem as more and more series and lagged values of series are added to the model. As the system expands, it can become very difficult to separate the effects of the explanatory variables, and the parameter estimates can become highly sensitive to the combination of variables used in the model.

37 Intriligator, Michael D. (1978). *Econometric Models, Techniques, and Applications.* Englewood Cliffs, NJ: Prentice- Hall

Therefore, it is necessary to examine whether or not any colinearity exists in the estimated VAR model. If colinearity is detected, the estimated coefficients are not reliable and they do not represent the concrete relationship between dependent variable and lagged regressors in the VAR model.

The Coefficient Variance Decomposition view of an equation provides information on the Eigen vector decomposition of the coefficient covariance matrix. This decomposition is a useful tool to help diagnose potential colinearity problems amongst the regressors. The decomposition calculations follow those given in Belsley, Kuh and Welsch (BKW) (1980). Note that although BKW use the singular-value decomposition as their method to decompose the variance-covariance matrix, since this matrix is a square positive semi-definite matrix, using the Eigen value decomposition will yield the same results. (QMS, 2009: Eviews 7: User's Guide II)

For simple linear least squares regression, the coefficient variance-covariance matrix can be decomposed as given by:

$$Var(\hat{\beta}) = \sigma^2(X'X)^{-1} = \sigma^2 VS^{-1}V' \quad (9.8)$$

Where S is the diagonal matrix containing the Eigen values of $X'X$, and V is a matrix whose columns are equal to the corresponding Eigen vectors.

The variance of an individual coefficient estimate is then:

$$Var(\hat{\beta}_i) = \sigma^2 \sum_j v_{ij}^2 \quad (9.9)$$

where, μ_i is the j[th] Eigen values and v_{ij} is the ('i j')[th] condition number of the covariance matrix, k_j:

$$k_j = \frac{min\,(\mu_m)}{\mu_j} \quad (9.10)$$

If we let,

$$\phi_{ij} = \frac{v_{ij}^2}{\mu_j} \quad (9.11)$$

and

$$\phi_i = \sum_j \phi_{ij}' \quad (9.12)$$

$$\pi_{ji} = \frac{\phi_{ij}}{\phi_i} \quad (9.13)$$

(Source: QMS, 2009: Eviews 7: User's Guide II)

These proportions, together with the condition numbers, can then be used as a diagnostic tool for determining colinearity between each of the coefficients.

Belsley, Kuh and Welsch recommend the following procedure:

Check the condition numbers of the matrix. A condition number smaller than 1/900 (0.001) could signify the presence of colinearity. Note that BKW use a rule of any number greater than 30, but base it on the condition numbers of X , rather than $X^{'}X^{-1}$ .If there are one or more small condition numbers, then the variance-decomposition proportions should be investigated. Two or more variables with values greater than 0.5 associated with a small condition number indicate the possibility of colinearity between those two variables. (QMS,2009: Eviews 7: User's Guide II)

The results from Coefficient Variance Decomposition based on VAR model of equation (9.6) are presented through Annex-IV. The top line of the Annex-IV shows the Eigen values, sorted from largest to smallest, with the condition numbers below. Note that the final condition number is always equal to 1. None of the ten Eigen values have condition numbers smaller than 0.001, with the smallest condition number being: 0.023767, which would indicate no colinearity between and among the regressors.

The second section of the table displays the decomposition proportions. The proportions associated with the smallest condition number are located in the first column. Only two of these ten values are larger than 0.5, but they are not very close to 1. This indicates that there is more or less no colinearity between those ten regressors.

Since the regressors are not collinear, the estimated coefficients of VAR model of equation (9.6) are claimed to be reliable and stable that bear consistent positive relationship between M_1 money supply and price level in the economy of Nepal during the study period.

9.5 Study on Broad Money Supply and Price Level by VAR Model

To analyze the relationship between broad money supply and price level by VAR technique, we need the stationary series of the concerned variables. The stationary series of the variables broad money supply and price level are $dLnM_{2t}$ and $dLnCPI_t$

respectively. Lags 4 are the appropriate lags (according to Schwarz Criterion) of the variables $dLnM_{2t}$ and $dLnCPI_t$ to apply VAR model. Using 4 lags for each endogenous variable, the VAR models with $dLnCPI_t$ as dependent variable and lagged $dLnCPI_t \; and \; dLnM_{2t}$ as independent variables is given by equation (9.14), and $dLnM_{2t}$ as dependent variable and lagged $dLnCPI_t \; and \; dLnM_{2t}$ as independent variables is given by equation (9.15).

$$dLnCPI_t = \gamma_1 + \alpha_1 dLnM_{2_{t-1}} + \alpha_2 dLnM_{2_{t-2}} + \alpha_3 dLnM_{2_{t-3}} + \alpha_4 dLnM_{2_{t-4}} + dLnCPI_{t-1} + \beta_2 dLnCPI_{t-2} + \beta_3 dLnCPI_{t-3} + \beta_4 dLnCPI_{t-4} + \varepsilon_{1t} \tag{9.14}$$

$$dLnM_{2t} = \gamma_2 + \delta_1 dLnM_{2_{t-1}} + \delta_2 dLnM_{2_{t-2}} + \delta_3 dLnM_{2_{t-3}} + \delta_4 dLnM_{2_{t-4}} + \theta_1 dLnCPI_{t-1} + \theta_2 dLnCPI_{t-2} + \theta_3 dLnCPI_{t-3} + \theta_4 dLnCPI_{t-4} + \varepsilon_{2t} \tag{9.15}$$

The results from VAR model based on equation (9.14) are presented through Table-9.8. With $dLnCPI_t$ as the dependent variable of VAR model based on equation (9.14), the coefficient of $dLnM_{2_{t-2}}$ is $\alpha_2 = 0.1265$, which is positive and significant at 5% level. This clearly indicates that $dLnCPI_t$ is Granger caused by the lagged $dLnM_{2_t}$ of preceding two periods. That is, current price level is directly caused by the M_2 money supply of two periods/quarters back. This confirms that a ten percent rise of change in M_2 money supply at two quarters back causes the change in price level at current time by 1.26%. This implies that as money supply increases at 2 quarters back, there is expansion of credit. As money supply of commercial banks increases, market rate of interest falls. At low rate of interest, people do not want to keep money in bank, rather they withdraw money from banks to purchase bond when its price falls. This further causes money supply to increase in the society. As non-bank public have more money in hands, they are ready to pay still higher and higher price to have the commodities and thereby price further rises and inflation arises.

Table-9.8: Results from VAR with $dLnCPI_t$ as Dependent Variable (Lag:1 to 4)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
$Constant$	$\gamma_1 = 0.0129$	0.0054	2.3924	0.0181
$dLnM_{2t-1}$	$\alpha_1 = 0.0844$	0.0590	1.4307	0.1548
$dLnM_{2t-2}$	$\alpha_2 = 0.1265$	0.0597	2.1169	0.0361
$dLnM_{2t-3}$	$\alpha_3 = -\ 0.0388$	0.0610	-0.6366	0.5255
$dLnM_{2t-4}$	$\alpha_4 = -0.1312$	0.0576	-2.2768	0.0244
$dLnCPI_{t-1}$	$\beta_1 = -0.0336$	0.0715	-0.4704	0.6388
$dLnCPI_{t-2}$	$\beta_2 = -0.0450$	0.0710	-0.6341	0.5270
$dLnCPI_{t-3}$	$\beta_3 = -0.1007$	0.0677	-1.4865	0.1395
$dLnCPI_{t-4}$	$\beta_4 = 0.5042$	0.0675	7.4618	0.0000

R^2=0.4607 Adj. R^2=0.4282 S.E. of regression= 0.0195
Log likelihood=362.0368 Durbin-Watson stat=1.625

The coefficient of $dLnM_{1_{t-4}}$ is significant at 5% level but it is negative, which is not desirable in accordance with Quantity Theory of Money. The coefficients of other lagged $dLnM_{2_t}$are not significant. The coefficient of $dLnCPI_{t-4}$ is $\beta_4 = 0.5042$, which is positive and significant at less than 1% level. This implies that $dLnCPI_t$ is not only caused by lagged $dLnM_{2_t}$ but also by own lagged term of $dLnCPI_t$. It means inflation of four periods back has the influence on current price level positively. For example, if price was high at four quarters back, the suppliers create artificial scarcity of goods with the expectation of further rise in price to earn more profit. As commodities become scarce, price level further rises. In this way, past inflation causes present inflation and same has happened in the economy of Nepal during the study period.

Since the Adjusted R^2 of VAR equation (9.14) is 0.4282, the dependent variable $dLnCPI_t$ is explained approximately 42% by the variation of independent variables, the model is not so satisfactorily fitted. The value of standard error of regression and log likelihood ratio indicate that the VAR model is appropriately fitted. However, D-W statistic is only 1.625886, which implies that there is the possibility of positive serial correlation.

The results from VAR model based on equation (9.15) are presented through Table-9.9, where $dLnM_{2_t}$ is taken as dependent variable and lagged (1 to 4 lags) $dLnM_{2_t}$ and $dLnCPI_t$ are taken as independent variables.

Table-9.9: Results from VAR with $dLnM_{2t}$ as Dependent Variable (Lag:1 to 4)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
$Constant$	γ_2 =0.0265	0.0077	3.3992	0.0009
$dLnM_{2t-1}$	δ_1=0.1379	0.0849	1.6244	0.1067
$dLnM_{2t-2}$	δ_2= -0.2068	0.0860	-2.4024	0.0177
$dLnM_{2t-3}$	δ_3=0.0148	0.0877	0.1696	0.8655
$dLnM_{2t-4}$	δ_4=0.2311	0.0829	2.7870	0.0061
$dLnCPI_{t-1}$	θ_1=0.1844	0.1031	1.7891	0.0759
$dLnCPI_{t-2}$	θ_2=0.1660	0.1022	1.6236	0.1068
$dLnCPI_{t-3}$	θ_3=0.1225	0.0975	1.2565	0.2111
$dLnCPI_{t-4}$	θ_4=-0.0375	0.0973	-0.3853	0.7006

In the Table-9.9, the coefficient of $dLnCPI_{t-1}$ is θ_1 = 0.1844, which is positive and significant at 10% level. It means the change in M_2 money supply at current time 't' is caused by the change in price level of preceding time period 't-1'.when there is inflation in the economy, supply of money should be increased to cope with the inflationary pressure. The increase in money supply further causes the price level to rise. The rising price requires increasing money supply further to bring adjustment between demand and supply and rise in money supply as a result of price level is a short term relief from inflationary pressure. In this way, there is circular relationship between price level and money supply. The Quantity Theory of Money states that rise in money supply causes price level to increase, but the VAR model of equation (9.15) states that not only rise in money supply causes price to rise but rise in price also causes money supply to rise. Thus, the coefficient of $dLnCPI_{t-1}$ implies that a 10% rise of change in price level at preceding time period requires to increase the change of M_2 money supply at current time by 1.84%.

Next, the coefficients of $dLnM_{2_{t-2}}$ is: δ_2= -0.2068, which is negative though significant at 5% level. This negative coefficient is not desirable for direct relationship between money supply and price level. The coefficient $dLnM_{2_{t-4}}$ is δ_4=

0.2311, which is positive and significant at less than 1% level implying that change in current money supply is affected not only by price level but also by the change in money supply at 4 periods back. This clearly indicates that rise in money supply in the previous four-period back brings about the rise in money supply at current time. For example, when there is increase in money supply to cope with inflation the credit expands in the society. Non-bank public has more money in hands. The consumers are ready to pay higher price to have commodity rather than go without commodity. Consumers afford more and more not only for necessities but also for comforts and luxuries because they have no crisis of money. The quantity of money is increasing but value of money continues to fall. The falling value of money requires increasing quantity of money further. Hence, the increase in money supply in the past periods causes money supply to increase in the current time as well.

9.6 Residuals Diagnostic of VAR Models ($dLnCPI_t$ Regressed on Lagged $dLnM_{2t}$)

In order to examine the consistency of estimated VAR model of equation (9.14), we have tested Residual Diagnostic through Correlogram Squared Residuals, Serial Correlation LM Test and Heteroscedasticity Test. With the help of these tests, we can conclude whether or not estimated regression equation (9.14) is consistent.

9.6.1 Correlogram of Squared Residuals

The Correlogram-Q-statistic of the squared residuals of estimated regression equation (9.14) is presented through Table-9.10. The ACFs and PACFs of correlogram of the residual squared are nearly zero at all lags and the Q-statistics at all lags are not significant with large p-values. This indicates that there is no evidence of rejecting the null hypothesis of no serial correlation. This implies that the residuals of the estimated VAR model of equation (9.14) are not correlated with their own lagged values. Hence, there is strong evidence of goodness of fit of the estimated VAR model of equation (9.14).

Table-9.10: Correlogram of Squared of Residuals of Equation (9.14)

Lag	AC	PAC	Q-Stat	Prob	Lag	AC	PAC	Q-Stat	Prob
1	-0.046	-0.046	0.3134	0.576	9	-0.032	-0.035	3.3897	0.947
2	-0.013	-0.015	0.3384	0.844	10	-0.024	-0.041	3.4784	0.968
3	0.005	0.003	0.3414	0.952	11	-0.063	-0.062	4.0949	0.967
4	0.023	0.023	0.4168	0.981	12	-0.027	-0.043	4.2073	0.979
5	0.125	0.128	2.7449	0.739	13	-0.042	-0.055	4.4870	0.985
6	-0.037	-0.025	2.9548	0.814	14	-0.021	-0.016	4.5598	0.991
7	0.022	0.023	3.0295	0.882	15	-0.069	-0.066	5.3262	0.989
8	0.037	0.037	3.2341	0.919	16	-0.009	-0.000	5.3396	0.994

9.6.2 Breusch-Godfrey LM Test for Serial Correlation

The results of Breusch-Godfrey Lagrange Multiplier test for serial correlation have been presented through Table-9.11. As reported by F-statistic and $T\times R^2$ value and their corresponding probabilities of B-G LM test of Table-9.11, the null hypothesis of no autocorrelation is strongly rejected. The B-G LM test implies that residuals of equation (9.14) are serially correlated. Due to the presence of serial correlation, the estimated VAR model of equation (9.14) is not considered as the consistent model for representing the relationship between M_2 money supply and price level. However, the residuals of VAR model of equation (9.14) are free from serial correlation problem based on Correlogram of Squared of Residuals.

Table-9.11: Breusch-Godfrey Serial Correlation LM Test of Equation (9.14)

F-statistic	14.82834	Prob. F(1,132)	0.0002
$T \times R^2$	14.34072	Prob. Chi-Square(1)	0.0002

Now, we test serial correlation of residuals of VAR model of equation (9.15) using B-G LM test in order to confirm whether there is any need of remodeling of VAR between M_2 money supply and price level. The results of Breusch-Godfrey Lagrange Multiplier test for serial correlation based on equation (9.15) have been presented through Table-9.12.

Table-9.12: Breusch-Godfrey Serial Correlation LM Test of Equation (9.15)

F-statistic	0.823678	Prob. F(2,130)	0.4411
$T \times R^2$	1.764388	Prob. Chi-Square(2)	0.4139

The value of F-statistic and $T \times R^2$ based on the corresponding probability suggest that there is no evidence of rejecting the null hypothesis of serial correlation (Table-9.12). This clearly indicates that the residuals of VAR model of equation (9.15) are not serially correlated representing the robustness of the model, no need of remodeling of VAR model of relationship between M_2 money supply and price level.

9.6.3 Residuals Heteroscedasticity Test

The null hypothesis of White's (1980) is H_0: there is homoscedasticity in the residuals. The null hypothesis is not rejected if the F-statistic and χ^2 –statistic are not significant. No rejection of null hypothesis confirms that residuals of estimated regression do not suffer from heteroscedasticity problem and estimated regression is claimed to be consistent. Table-9.13 presents the Residual Heteroscedasticity test of estimated VAR model of equation (9.14).

From the Table-9.13, it is observed that F-statistic and $T \times R^2$ value are not significant as reported by the probability values of the fourth column. There is no evidence of rejecting the null hypothesis that residuals are homoscedastic. It means, the residuals of estimated VAR model of equation (9.14) do not suffer heteroscedastic problem. Hence, it is claimed that the estimated VAR model of equation (9.14) representing the relationship between broad money supply and price level is consistent model.

Table-9.13: White Heteroscedasticity Test of Residuals of Equation (9.14)

Test Summary	Value	Degree of Freedom	Probability
F-statistic	0.503655	Prob. F(44,97)	0.9938
$T \times R^2$	26.40832	Prob. Chi-square(44)	0.9835

9.7 Stability Test of Estimated VAR Models
($dLnCPI_t$ Regressed on Lagged $dLnM_{2t}$)

To examine the stability of the estimated regression equation (9.14), we apply some test such as Inverse Roots of Characteristic Polynomials; Ramsey's RESET test and Coefficient Variance Decomposition (coefficient diagnostic) test.

9.7.1 Inverse Roots of Characteristic Polynomials

The Inverse Roots of Characteristic Polynomials of Estimated VAR model of equation (9.14) is presented through Table-9.14. In the table, the modulus value of all roots does not exceed one, which means no root lies outside the unit circle. Thus, the estimated VAR model of equation (9.14) satisfies the stability condition, i.e. the estimated VAR model is stable.

Table-9.14: Inverse Roots of Characteristic Polynomials of Equation (9.14)

Root	Modulus
0.0064 - 0.9104i	0.9104
0.0064+ 0.9104i	0.9104
-0.8777	0.8777
0.7347 - 0.0254i	0.7352
0.7347 + 0.0254i	0.7352
0.0362 - 0.6981i	0.6990
0.0362 + 0.6981i	0.6990
-0.5769	0.5769
0.0064 - 0.9104i	0.9104
0.0064 + 0.9104i	0.9104

9.7.2 Ramsey's RESET Test

Table-9.15 demonstrates the results from Ramsey's RESET test of estimated VAR model of equation of equation (9.14).

In the upper part of the Table-9.15, F-statistic, t-statistic and likelihood ratio are not significant as reported by the corresponding probability values. The null hypothesis 'correct specification is linear' is not rejected even if the variable $Fitted^2$ term is included in to the regression equation (9.14). It means the estimated VAR model of equation (9.14) does not avoid the property of linearity. Likewise, in lower part of Table-9.15, H_0: $\gamma = 0$, is not rejected as reported by the t-statistic. Hence, the Ramsey's RESET test implies that the estimated regression equation (9.14) is stable model containing the properties of linearity and it is not misspecified model.

Table-9.15: Ramsey's RESET Test of VAR Equation (9.14)

Test-statistic	Value	Degree of Freedom	Probability
t-statistic	0.5376	132	0.5918
F-statistic	0.2890	(1, 132)	0.5918
Likelihood ratio	0.3105	1	0.5773

Unrestricted Test Equation

Variable	Coefficient	Std. Error	t-Statistic	Prob.
$Constant$	γ_1 = 0.0130	0.0054	2.3950	0.0180
$dLnM_{2t-1}$	α_1 = 0.0952	0.0625	1.5240	0.1299
$dLnM_{2t-2}$	α_2 = 0.1605	0.0870	1.8433	0.0675
$dLnM_{2t-3}$	α_3 = - 0.0450	0.0622	-0.7223	0.4713
$dLnM_{2t-4}$	α_4 = -0.1404	0.0602	-2.3300	0.0213
$dLnCPI_{t-1}$	β_1 = -0.0379	0.0721	-0.5254	0.6001
$dLnCPI_{t-2}$	β_2 = -0.0466	0.0713	-0.6541	0.5142
$dLnCPI_{t-3}$	β_3 = -0.1314	0.0887	-1.4809	0.1410
$dLnCPI_{t-4}$	β_4 = 0.5790	0.1547	3.7421	0.0003
$Fitted^2$	ρ =-2.7708	5.1540	-0.5376	0.5918

9.7.3 Coefficient Variance Decomposition

The results from Coefficient Variance Decomposition based on VAR model of equation (9.14) are presented through Annex-V.

The top line of the Annex-V shows the Eigen values, sorted from largest to smallest, with the condition numbers below. Note that the final condition number is always equal to 1. None of the ten Eigen values have condition numbers smaller than 0.001, with the smallest condition number being: 0.0350, which would indicate no colinearity amongst the regressors.

The second section of the table displays the decomposition proportions. The proportions associated with the smallest condition number are located in the first column. None these ten values are larger than 0.5. This indicates that there is no colinearity between those ten regressors.

Since the regressors are not collinear, the estimated coefficients of VAR model of equation (9.14) are claimed to be reliable and stable that bear consistent positive relationship between M_2 money supply and price level in the economy of Nepal during the study period.

9.8 Conclusion of Chapter Nine

Chapter Nine has the following conclusions.

- The VAR model, with $dLnCPI_t$ as dependent variable and lagged $dLnCPI_t$ and $dLnM_{1t}$ as independent variables, implies that a ten percent increase in change of M_1 money supply of two periods back causes the change in price level of current time to increase by 1.23%.
- The change in price in current time is caused by the change in price itself of four periods back. A ten percent increase in change of price of four periods back causes the change in price of current time to increase by 3.5%.
- The VAR model, with $dLnM_{1t}$ as dependent variable and lagged $dLnCPI_t$ and $dLnM_{1t}$ as independent variables, implies that a ten percent increase in change of price level of preceding period causes the change in M_1 money supply of current time to increase by approximately 3%.
- The change in M_1 money supply is not only affected by the change in price level but also by the change in M_1 money supply itself. A ten percent rise in change of M_1 money supply of four periods back causes the change in M_1 money supply of current time to increase by 4.2%.
- The estimated VAR model, with $dLnCPI_t$ as dependent variable and lagged $dLnCPI_t$ and $dLnM_{1t}$ as independent variables, is reasonably fitted. It is because it does not suffer autocorrelation and heteroscedasticity problems. This model satisfies the property of linearity and all the parameters are stable and no colinearity problem emerges so that this model is claimed to be

suitable for forecasting of inflation with the help of price and M_1 money supply in the economy of Nepal.

- The VAR model, with $dLnCPI_t$ as dependent variable and lagged $dLnCPI_t$ and $dLnM_{2t}$ as independent variables, implies that the change in M_2 money supply of two periods back causes the change in price level of current time. A ten percent rise in change of M_2 money supply of two periods back causes the change in current price level to increase by 1.26%.
- A ten percent increase in change of price itself of four periods back causes the change in M_2 money supply of current time to increase by approximately 5%.
- The estimated VAR model, with $dLnCPI_t$ as dependent variable and lagged $dLnCPI_t$ and $dLnM_{2t}$ as independent variables, is claimed to be suitable model for forecasting of inflation with the help of price level and M_2 money supply since it does not suffer autocorrelation and heteroscedasticity problems. It is the stable model containing the property of linearity and all parameters are also stable, and no colinearity problem is detected.
- The VAR model, with $dLnM_{2t}$ as dependent variable and lagged $dLnCPI_t$ and $dLnM_{2t}$ as independent variables, implies that the change in M_2 money supply of current time is caused by the change in M_2 money supply itself of four periods back. However, the change in M_2 money supply of current time is not affected by the change in price level.