Chater III

A Genaral Discussion

In the linear theories of motion of bodies, deflections are assumed to be small in comparison with the plate thickness. But in most practical cases this basic assumption is nolonger valid, instead the deflections have the order of the magnitude of the plate thickness. Hence, derivation of governinrg differential equations considering large deflections needs special attention in such analysis. In general Karman type filed equations are employed for almost all types of structures.

The paucity of literature concerning non linear (large amplitude) vibration analysis, probably, due to the fact that the two basic Von Karman field equations extended to the dynamic case, involve the deflection and stress functions in a coupled form. Moreover, these equations are of fourth order, posing analytical problems and necessitating a numerical approach. Several methods are available to investigate such problems and thereby elucidate non linear response for some simpler cases. For one type of method the analytical difficulties are overcome by using modern high speed computers and finite elements or finite differences. Yet, classical approach is still preferable, even for some approximate solution, wher-ever possible .

The basic equations for free flexural vibrations of rectangular elastic plates have been explicitily discussed by G. Hermann (21) These equations are Karman type equations extended to the dynamic case. Chu and Herman studied the influence of large amplitude on free flexural vibration of rectangular plates with hinged immovable edges, by applying Herman's theory. Approximate solution for the nonlinear response of rectangular plate to sinusoidal acoustic pressure were obtained by Kirchman and Greenspon (17) by using the static load-deflection relations previously obtained . Eringen (18) studied vibrations of strings and bars exhibiting large vibrations . During the last thirty years, several problems of practical interest have been investigated by different authors using different approaches. Laminated isotropic and orthotropic plates or sandwiched plates have also gained importance during this period.

Due to the very complicated nature of the basic equations governing the motion of a structure exhibiting large deflections , it has always been a difficult task for an investigator to obtain an approximate solution. Attempts have also been made to find ways to ease such problems .Berger [55] proposed an alternative method which enable one to replace the coupled Karman equations by a simpler set of uncoupled and quasi-linear equations. Following this novel idea authors [56,57,58] studied the large deflection analysis of plates. This technique was extended to the dynamic case by different authors for the analysis of static and dynamic behaviour of plates exhibiting large deflections. Berger's assumption was based on the idea that the second strain invariant in the middle plane of the plate can be neglected without inducing any appreciable error in the solution. However , he didnot give any physical justification for this assumption. This technique was extensively used till Nowinski[72] examined Berger's equations critically and initiated the criticism on the freehand application of these equations . However, this method may be applied to some specific problems with some limitations [78]

Sinharay and Banerjee [82] tried to improve Berger's approach to the solutions of large amplitude vibrations of orthogonal plates with some modification of Berger's hypothesis. But like Berger's one this method lacks in providing with the rigorous physical justification.

Most of the investigations employed Galerkin approximation as a tool for the nonlinear analysis of vibration of plates and shells but it has been observed by Vendhan [53] that the first order Galerkin approximation may involve substantial error in the case of plates with unrestricted boundary conditions. Moreover Bayeles et. al. [140] have pointed out inadequacy of a first order Galerkin approximation to the solution of inplane equilibrium or of the compatibility equation.

The Rayleigh -Ritz method has now become a versatile method for obtaining the approximate solutions of vibrational problems in solid mechanics though the Galerkin method has a wider applicability the one term Rayleigh Ritz approximation yields better results compared to the former one. But it remains wide open to justify the validity of a method merely on the basis of a one term approximation.Yet we have no alternative for the increase of just one term more in the approximation series which will multiply the mathematical and computational labour considerably.Vendhan and Das [35] have made a nice comparative study between the Rayleigh Ritz method and Galerkin method, Investigating on the nonlinear vibrations of triangular and rectangular plates, the authors of Ref[141] have discussed the nature of Rayleigh-Ritz approximation based on the variational for mulation and the Galerkin approximation based on minimization of the error function. Presenting the numerical results for convergence of the relative time periods of the nonlinear and linear vibrations for triangular and rectangular plates of orthotropic materials for various aspect ratios, they have shown that the Rayleigh Ritz approximation is consistently better than the Galerkin approximation but they become equally good after a few terms. It has further been observed that the results obtained from both the methods approach the true value from the opposite sides of it. In fact this is what is expected from the Rayleigh Ritz method for the approximation is associated with a potential energy formutation corresponding to a stiffer structure . On the other hand Galerkin approximation which turns out to be an upper bound to the true value for a given lateral displacement corresponding to a more flexible structure. The bounding property of the Galerkin approximation may be the characteristic of the constrained in palne edge condition and the single mode expression for the deflection function. However, the observation that the modified nonlinear equations gives a less stiff model than the Rayleigh Ritz method may be considered to be of general significance.

The expression for defining the transverse deflections and in plane displacements are often assumed as polynomials in space co-ordinates, or., in terms of trigonometric series. The proper choice of the coordinate functions is very imortant for obtaining good accuracy. For example. Vendhan's [142] investigation into the non linear vibrations of thin plates including inplane inertia effects reveals that sometimes polynomial expressions for the displacement functions may be found to be good enough for obtaining good accuracy . Leissa [111] making on extensive survey on the free vibrations of rectangular plate has shown that the percent difference of the 36- terms solution for eigen-frequency from the single term Rayleigh Ritz solution with respect to the first term solution is negative indicating that the one term solution is more exact than the 36- term solution.

Besides the methods discussed above, several methods based on computer application are being used to analyze the vibration problems. Finite Elements method is now regarded as one of the most powerful method for problems on structural and continuum mechanics[143]

Using Karman type field equations for solution of plate problems having uncommon boundaries is a difficult task and a more complicated one when geometrical non-linearities are involved.

Recently a new idea has been put forward by Banerjee [109] to study the dynamic response of structures of arbitar.y shapes based on "Constant Deflection Contour" method. The method has been previously developed by Mazumdar [137–139]. Further application of this method has been made by Majumder and others [106-108, 115, 116]. However the application of this method has been restricted to linear analysis only.

The present thesis aims at extending this method to the nonlinear analysis of plates vibrating at large amplitudes. It begins with a review of the basic ideas developed by Banerjee [109]. The analysis carried out in this thesis may readily be applied to other geometrical structures and as a by- product, the static deflection is also obtainable. A combination of the ' constant Deflection Contour' method and the Galerkin procedure is employed. The numerical results obtained, are in excellent agreement with those from previous studies. Application of the present analysis to structures with complicated geometry is in progress.

Some preliminary Remarks about the constant Deflection contour Method :

The fundamental concept of the constant deflection contour method may be best explained by considering transverse vibrations of plate, referred to a system of orthogonal coordinates oxyz. for which oz is the transverse direction (positively downward). The horizontal plane oxy coincides with the middle plane of the plate. Such a plate is either statically deflected, vibrating freely or forced to vibrate, all due to normal static or dynamic loads. When the plate vibrates in a normal mode, then at any instant t_o, the intersections between the deflected surface and the parallels z = constant will yield contours which after projection onto z = o surface are a set of level curves, u(x,y) = constant, called the " Lines of Equal Deflection", which are iso-amplitude contours. The boundary of the plate, irrespective of any combinations of support, is also a simple closed curve C_o belonging to the family of lines of Equal Deflections C_u. As defined by Mazumdar [138], the family of non-intersecting curves may be denoted by C_u for $O<u<u^*$, so that C_o $(u = O)$ is the boundary and C_u conicides with the point (s) at which the maximum $u = u^*$ is obtained

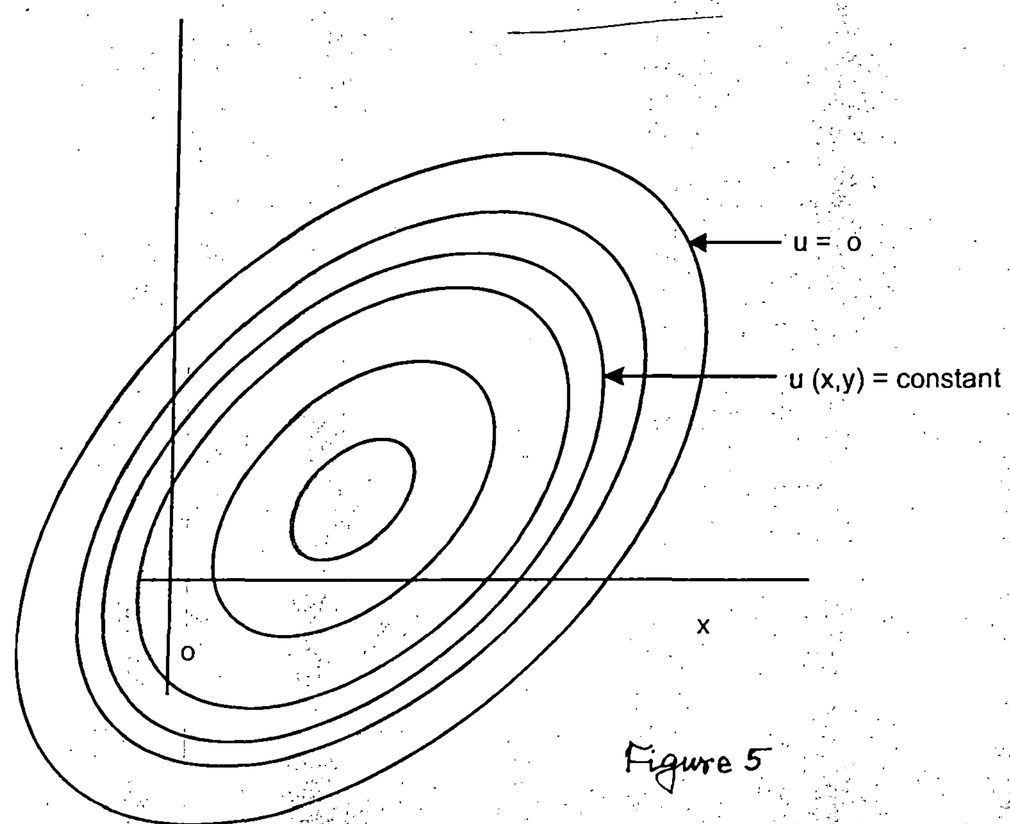

Figure 5

Governing / Basic Equations :

A different apporach will be utilised in establishing the governing equations to that used by Mazumdar [137]. We consider a thin elastic plate which vibrates with a moderately large amplitude in the transverse direction, under the action of an uniform load p. The usual procedure is to consider Karman type field equations extended to the dynamic case

$$D\nabla^4 w = \mathcal{L}(F, w) + p + \rho h w_{,tt} \qquad [3\cdot1]$$

$$\nabla^4 F = -\frac{Eh}{2}\mathcal{L}(w, w) \qquad [3\cdot2]$$

in which the flexural rigidity D and two dimensional Laplacian operator ∇^2 are defined by

$$D = \frac{Eh^3}{12(1-\nu^2)}, \quad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \qquad [3\cdot3]$$

with h the thickness of plate, E Young's modulus, p the uniform normal load. ρ the material density, w the deflection function and F the Airy stress function. In addition a suffix is taken as an indication of partical differentiqtion with respect to the implied variable and the operator $\mathcal{L}$ is defined by

$$\mathcal{L}(I, J) = I_{,xx} J_{,yy} + J_{,xx} I_{,yy} - 2I_{,xy} J_{,xy} \qquad [3\cdot4]$$

As an alternative to the derivation of the subsequent equations directly from basic equations. as done by Mazumdar [137-139] we will establish them directly from the above equations by introducing the concept of constant deflection contour lines.

It should be noted here that the use of the stress function is equivalent to disregard of intertia terms in the equations of inplane motions of the particles of the plate. This assumption is legitimate when the oscillations primarily take place in the transverse direction, perpendicular to the middle plane of the plate. We choose the deflection function and the stress function in the separable form

$$w(x, y, t) = W(x, y)\, h\, \psi(t)$$
$$F(x, y, t) = F'(x, y)\, h\, \psi^2(t) \qquad [3\cdot5]$$

where ψ (t) is an unknown function of time to be determined. Let us make the following transformations

$$\frac{\partial W}{\partial x} = W_{,x} = u_{,x}\frac{dW}{du}$$

$$W_{,xx} = u_{,x}^2\frac{d^2W}{du^2} + u_{,xx}\frac{dW}{du}$$

$$W_{,y} = u_{,y}\frac{dW}{du}$$

$$W_{,yy} = u_{,y}^2\frac{d^2W}{du^2} + u_{,yy}\frac{dW}{du}$$

$$W_{,xy} = u_{,xy}\frac{dW}{du} + u_{,x}u_{,y}\frac{d^2W}{du^2} \quad ; \text{ etc } \left(u_{,x} = \frac{\partial u}{\partial x}\right) \qquad [3.6]$$

with transformations examplified by those shown in equations (3.5) and (3.6), equations (3.1) and (3.2) are transformed to

$$D\left[A_1\frac{d^4W}{du^4} + A_2\frac{d^3W}{du^3} + A_3\frac{d^2W}{du^2} + A_4\frac{dW}{du}\right]\psi(t)$$

$$= \left[\left\{A_5\frac{dW}{du}\frac{dF'}{du}\right\} + \left\{A_6\frac{d}{du}\left(\frac{dW}{du}\frac{dF'}{du}\right)\right\}\right]\psi^3(t) + \left[p - \rho h W\psi_{,tt}\right] \qquad [3.7]$$

$$\left[A_1\frac{d^4F'}{du^4} + A_2\frac{d^3F'}{du^3} + A_3\frac{d^2F'}{du^2} + A_4\frac{dF'}{du}\right]$$

$$= -\frac{Eh}{2}\left[A_5\left(\frac{dW}{du}\right)^2 + A_6\frac{d}{du}\left(\frac{dW}{du}\right)^2\right] \qquad [3.8]$$

where

$$A_1 = \left(u_{,x}^2 + u_{,y}^2\right)^2$$

$$A_2 = 6\left(u_{,x}^2 u_{,xx} + u_{,y}^2 u_{,yy}\right) + 8u_{,x}u_{,y}u_{,xy} + 2\left(u_{,x}^2 u_{,yy} + u_{,y}^2 u_{,xx}\right)$$

$$A_3 = 3\left(u_{,xx}^2 + u_{,yy}^2\right) + 4u_{,xy} + 4\left(u_{,x}u_{,xxx} + u_{,y}u_{,yyy}\right)$$
$$+ 2u_{,xx}u_{,yy} + 4\left(u_{,x}u_{,xyy} + u_{,y}u_{,xxy}\right)$$

$$A_4 = u_{,xxxx} + u_{,yyyy} + 2u_{,xxyy}$$

$$A_5 = 2\left(u_{,xx}u_{,yy} - u_{,xy}^2\right)$$

$$A_6 = u_{,xx}u_{,y}^2 + u_{,yy}u_{,x}^2 - 2u_{,x}u_{,y}u_{,xy}$$

Since equations (3.7) and (3.8) are valid for all points of the whole domain, it is clear that

$$\iint_{\Omega_u} D\left[A_1\frac{d^4W}{du^4} + A_2\frac{d^3W}{du^3} + A_3\frac{d^2W}{du^2} + A_4\frac{dW}{du}\right]\psi(t)\,d\Omega$$
$$= \iint_{\Omega_u}\left[A_5\frac{dW}{du}\frac{dF'}{du} + A_6\frac{d}{du}\left\{\frac{dW}{du}\frac{dF'}{du}\right\}\right]\psi^3(t)\,d\Omega$$
$$+ \iint_{\Omega_u}\left[p - \rho h W\psi_{,tt}\right]d\Omega \qquad [3.9a]$$

$$\iint_{\Omega_u}\left[A_1\frac{d^4F'}{du^4} + A_2\frac{d^3F'}{du^3} + A_3\frac{d^2F'}{du^2} + A_4\frac{dF'}{du}\right]d\Omega$$
$$+ \frac{Eh}{2}\iint_{\Omega}\left[A_5\left(\frac{dW}{du}\right)^2 + A_6\frac{d}{du}\left(\frac{dW}{du}\right)^2\right]d\Omega = 0 \qquad [3.9b]$$

Where the region of integration is taken over the region enclosed by some contour C_u. To integrate equations (3 .8) and (3.9) previous authors have usually employed Green's theorem. However we pursue a different approach and change the variables using the general formula

$$\iint_{\Omega_u} f\left(u, u_{,x}, u_{,xx}, u_{,y}, u_{,yy}, u_{,xy} \text{---------------} \frac{dW}{du}, \frac{d^2W}{du^2}, \text{--------} \frac{d^nW}{du^n}\right) d\Omega$$

$$= -\int_{u^*}^{u} \phi_1(u) \left\{ \oint_{C_u} \phi_2(x,y) \frac{ds}{\sqrt{A_1}} \right\} du \qquad [3\cdot10]$$

Which is a generalisation of a formula adopted in Ref[115]. Often it has been encountered that the contour integral in (3.10) turns out to be dependent of u and hence care should be taken to evaluate first the contour integral to avoid any confusion that may arise from equation (25) of Ref. [180].

On evaluation of integrals (3.9a) and (3.9b), they may be further reduced to the forms

$$\Lambda_1\left[W, \psi(t), \psi_{,tt}(t)\right] \equiv 0$$

or,
$$D\left[f_{11}\frac{d^3W}{du^3} + f_{12}\frac{d^2W}{du^2} + f_{13}\frac{dW}{du} + f_{14}W\right]\psi(t) + f_{15}\left(\frac{dW}{du}\frac{d\psi}{du}\right)\psi^3(t)$$

$$+ f_{16}\, p + \rho h\, \psi_{,tt}(t) \int_{u^*}^{u} W\, du_0 = 0 \qquad [3\cdot11]$$

$$\Lambda_2\left[F'(t), \frac{dW}{du}\right] \equiv 0$$

$$\left[g_1\frac{d^3F'}{du^3} + g_2\frac{d^2F'}{du^2} + g_3\frac{dF'}{du} + g_4F'\right]$$

$$+ \frac{Eh}{2} g_5 \left(\frac{dW}{du}\right)^2 = 0 \qquad [3\cdot12]$$

To avoid the integral appearing in equation (3.11) Mazumdar and others [108] have taken the derivative of it making the equation of order four again, viz

$$D\left[f_{21}\frac{d^4W}{du^4} + f_{22}\frac{d^3W}{du^3} + f_{23}\frac{d^2W}{du^2} + f_{24}\frac{dW}{du}\right]\psi(t)$$

$$+\left[f_{25}\frac{dW}{du}\frac{dF'}{du} + f'_{25}\frac{d}{du}\left(\frac{dW}{du}\frac{dF'}{du}\right)\right]\psi^3(t) + f_{26}\, p + \rho h\, \psi_{,tt}(t) W = 0 \qquad [3\cdot13]$$

Equations (3.11) and (3.12) or equations (3.12) and (3.13) form the basic equations governing the vibrations of any structure. These equations have been derived without specifying the geometry of the structure and they may therefore be specialized to deal with any type of geometry. Moreover (3.11 - 3.13) from a system of ordinary differential equations which may be solved for a variety of structures and subjected to several forms of boundary conditions.

Method of Solution :

The method of solution may be considered in a two fold way.

Considering (3.11) and (3.12) as the basic equations with appropriate boundary conditions, it starts with finding the exact or approximate solution for ψ (t) from equation (3.12). However the exact solution for ψ(t) may only be feasible for linear analysis like what has been followed by Mazumder[137]For non-linear analysis one may has to seek for approximate solution for which the form of deflection function must be first assumed compatible with the boundary condition, next to solve for ψ (t) from equation (3.12) in conjunction with a Galerkin procedure. With this expression of ψ (t), equation (3.11)or equation (3.13) will again yield an ordinary time differential equation in combination with the Galerkin procedure. Mathematically, the above steps may be explained in the following way.

Let u(x,y) = u be the representative of one of the family of the iso-deflection curves. Then for any prescribed boundary conditions the deflection function w (u,t) can be assumed to take the form

$$w = \sum_{i=1}^{n} A_i W(u)\psi(t) \qquad [3{\cdot}14]$$

$$F = F'(u)\psi^2(t) \qquad [3{\cdot}15]$$

Equation (3.12) in combination with (3.14) and (3.15) will yield

F = F (u) .... (3.16)

Substituting this value of F with u was in (3.14) equation (3.11) will yield the error function

$$\epsilon = \Lambda\left[u, \psi(t), \psi_{,tt}(t)\right] \qquad [3{\cdot}17]$$

rather an approximation. The associated error function may be minimised using Galerkin method. The appropriate orthogonality condition applied to equation (3.17) will yield the following " Time Differential Equation"

$$Z \psi_{,tt}(t) + C_1 \psi(t) + C_2 \psi^3(t) = C_3 p \qquad [3 \cdot 18]$$

Equation (3.18) is a Dufling type equation and its solution is well known. Equation (3.18) will enable one to find the frequency response for various case in respect of linear and non-linear response and free or forced vibrations. Aditionally, it may also be used to determine static deflection to a uniform load. This equation and the method just detailed may be applied to any structure provided the contour - lines are known. Before we proceed to give any specific illustration, we must have the expressions for the boundary conditions to be satisfied by the deflection function as well as by the stress function.

Boundary conditions and their transformations for u- variables :

The boundary conditions imposed on structures play an important role both in obtaining the exact or approximate solution and also on the class of possible response. We now therefore look at several commonly used boundary conditions and their implications in respect of the concept of Constant Deflection Contour method. In two dimensional problems the following boundary conditions are usually imposed.

3·1 Supporting Conditions and their Transformations :

Case - I Clamped edges

a) For rectangular plates

$$w = \frac{dw}{dx}\left(= w_{,x}\right) = 0 \, , \text{ on the edge perpendicular to } x\text{-direction}$$

$$w = \frac{dw}{dy}\left(= w_{,y}\right) = 0 \, , \text{ on the edge perpendicular to } y\text{-direction}$$

or, $$w = \frac{dw}{dn}\left(= w_{,n}\right) = 0 \, , \text{ when } n \text{ is the outward unit-normal to the edge} \qquad [3 \cdot 1 \cdot 1]$$

The appropriate form of these conditions in terms of the variable u are given by

$$w = \frac{dw}{du} = 0, \text{ on the boundary } (\text{i.e., } u = 0) \qquad [3.1.2]$$

Case - II Simply-Supported edges

In the usual notation the conditions for a simply supported edge may be expressed in the form

$$w_{,xx} + \nu w_{,yy} = 0, \text{ on the edge normal to } x\text{-direction}$$
$$w_{,yy} + \nu w_{,xx} = 0, \text{ on the edge normal to } y\text{-direction} \qquad [3.1.3]$$

which when transformed in terms of u variable become

$$\frac{d^2w}{du^2}\left(u_{,x}^2 + \nu u_{,y}^2\right) + \frac{dw}{du}\left(u_{,xx} + \nu u_{,yy}\right) = 0$$
$$\frac{d^2w}{du^2}\left(u_{,y}^2 + \nu u_{,x}^2\right) + \frac{dw}{du}\left(u_{,yy} + \nu u_{,xx}\right) = 0 \qquad [3.1.4]$$

on the edge normal to x and y directions respectively.

Case - III Free edges :

For free edge conditions, on an edge normal to the x - direction the usual boundary conditions are

$$w_{,xxx} + (2-\nu)\, w_{,xyy} = 0$$
$$w_{,xx} + \nu w_{,yy} = 0 \qquad [3.1.5]$$

Together with two similar conditions for the edge normal to the y - direction, if it is also free obtained by interchanging x and y.

The above boundary conditions may be recast in terms of u to yield

$$\left(u_{,y}^{2}+\nu u_{,x}^{2}\right)\frac{d^{2}w}{du^{2}}+\left(u_{,xx}+\nu u_{,yy}\right)\frac{dw}{du}=0$$

$$\left[u_{,x}^{3}+(2-\nu)u_{,x}u_{,y}^{2}\right]\frac{d^{3}w}{du^{3}}+\left[3u_{,x}u_{,xx}+(2-\nu)\left(2u_{,x}u_{,xy}\right.\right. \qquad [3.1.6]$$

$$\left.\left.+u_{,x}u_{,yy}\right)\right]\frac{d^{2}w}{du^{2}}+\left[u_{,xxx}+(2-\nu)u_{,xyy}\right]\frac{dw}{du}=0$$

for free edge normal to x-direction.

and a similar expressions for a free edge normal to y-direction changing x to y and y to x

3.2 - Stress conditions and their transformation to u - variables :

a) Stress free edge (normal to x - direction)

$F_{yy}=F_{xy}=0$ .... (3.2.1)

which on transformation to u variable turns

$$u_{,y}^{2}\frac{d^{2}F}{du^{2}}+u_{,yy}\frac{dF}{du}=0 \qquad [3.2.2]$$

$$u_{,x}u_{,y}\frac{d^{2}F}{du^{2}}+u_{,xy}\frac{dF}{du}=0$$

Two similar expression if needed for a free edge normal to y-direction are obtained by interchanging x and y .

b) Immovable edge (normal to x - direction)

$$F_{,xy} = 0$$

$$u_{,x} \, u_{,y} \frac{d^2F}{du^2} + u_{,xy} \frac{dF}{du} = 0$$

$$U = \int_0^x \left[\frac{1}{E}\left(F_{,yy} - \nu F_{,xx}\right) - \frac{1}{2} w_{,x}^2 \right] dx = 0$$

$$U = \int_0^x \left[\frac{1}{E} \left\{ \left(u_{,y}^2 - \nu u_{,x}^2\right) \frac{d^2F}{du^2} + \left(u_{,yy} - \nu u_{,xx}\right) \frac{dF}{du} \right\} - \frac{1}{2} u_{,x}^2 \left(\frac{dw}{du}\right)^2 \right] dx = 0 \qquad [3.2.3]$$

Similarly for an edge normal to y - direction may be directly put with variable u as

$$u_x u_{,y} \frac{d^2F}{du^2} + u_{,xy} \frac{dF}{du} = 0$$

$$V = \int_0^y \left[\frac{1}{E} \left\{ \left(u_{,x}^2 - \nu u_{,y}^2\right) \frac{d^2F}{du^2} + \left(u_{,xx} - \nu u_{,yy}\right) \frac{dF}{du} \right\} - \frac{1}{2} \left(\frac{dw}{du}\right)^2 u_{,y}^2 \right] dy = 0 \qquad [3.2.4]$$

c) Movable edge :

The stress condition for edge normal to x -direction may be directly written here as

$$u_{,x} u_{,y} \frac{d^2F}{du^2} + u_{,xy} \frac{dF}{du} = 0$$

$$U = \text{Constant}$$

and for the edge normal to y direction

$$u_{,x} u_{,y} \frac{d^2F}{du^2} + u_{,xy} \frac{dF}{du} = 0$$

$$V = \text{Constant} \qquad [3.2.5]$$

3·3 Since in most practical cases polar co ordiantes having circular symmetry are of importance. We rewrite here the transformed boundary conditions for deflection and stress functions below in polar co ordinates (r, θ)

Case - I : Clamped

$$w = w_{,r} = 0 \Longrightarrow w\Big|_{u=0} = \sqrt{1-u}\,\frac{dw}{du}\Big|_{u=0} = 0 \qquad [3.3.1]$$

Case - II simply supported

$$w = w_{,rr} + \frac{\nu}{r} w_{,r} = 0 \Longrightarrow w\Big/_{u=0} = \left\{2(1-u)\frac{d^2w}{du^2} - (1+\nu)\frac{dw}{du}\right\}\Big/_{u=0} = 0$$

[3.3.2]

For stress conditions, in brief we may state

a) Stress free edge

$$\sqrt{1-u}\;\frac{dF}{du}\Big/_{u=0} = 0 \qquad [3.3.3]$$

b) Immovable edge

$$\left\{2(1-u)\frac{d^2F}{du^2} - (1-\nu)\frac{dF}{du}\right\}\Big/_{u=0} = 0 \qquad [3.3.4]$$

Perhaps it would be proper to mention here that though the above boundary conditions are in fact the transformed expressions from cartesian or polar co ordinates to u - variables in the light of the isodeflection contour method; sometimes in general "it is impossible in the simply supported case to find the exact functions u and w such that they satisfy mechanical boundary conditions" as observed by Mazumdar et.al [106] . In such cases some conditions should be imposed on demand without violating the normal conditions [106]. This will be further discussed in the foregoing illustrations whenever such cases arise.