CHAPTER - 1

BIBLIOGRAPHY

With the progress of modern civilization, the applications of elastic and plastic properties of solids in the field of structural Engineering are gaining momentum day by day. Most of the modern structurs are subjected to severe vibrations and hence it becomes a must to design structural components to withstand high dynamic stresses. Dynamic characteristics of structural system are essential for design and control.

Since Robert Hook (1635-1716) gave a simple relation between stress and strain of elastic substance many research workers become inclined to investigate the elastic behaviour of matter. Since then the linear analysis of plates and shells has long attracted the attention of several investigartors resulting in a wealth of papers published by several authors. An extensive study on this subject has been presented by Gontkevich[1] and later by Leissa [2].

A study on the dynamic response of structures reveals the fact that much has been investigated so far as linear analysis is concerned as determination of the natural frequency plays important role in designing a structure prone of vibration. Some of the works previously made may include the problem of symmetrical bending of circular and rectangular plates of variable thickness investigated by H. Holzer[3], R.G. Olson[4] and H.D. Conway[5].

The linear frequencies of in-plane vibrations of polar orthotropic annular plates with linearly varying thickness have been analyzed by Ganesan and Soamidas[6]. A semi-analytical method of analysis has been used where the radial and tangential displacements are expanded in the circumferential direction as Fourier series and the radial behaviour is solved using finite element method and the frequencies have been studied with respect to various boundary conditions, aspect ratio, thickness ratios, ratio of moduli and two fiber directions. The vibration and stability analysis of polar orthotropic circular plates using the finite element method is discussed by Gerard and C-Pardoen[7].

Free torsional vibrations of conical and cylindrical shells of thickness varying as a power of distance have been studied by Soni, Jain and Prasad[8]. The numerical values of the frequency parameter for the first three modes of vibration are computed for shells of linearly and parabolically varying thickness for different ratio of treminal radii.

Laura et.al. [84] analyzed the vibration and stability of a circular plate elastically restrained against rotation. Forced vibration of a circular plate elastically restrained against rotation has been discussed by Laura el.al.[85].

The resonant response of simply supported thin and thick orthotropic cylindrical shell is determined by Warburton and Soni [9] by using model analysis.

The transverse vibration of free elliptical plates with rectangular orthotropy is analyzed by T. Naritra[10], while the natural frequencies of rectangular and polygonal plates have been obtained by R.B. Bhat [11,12]. Dickinson and Blasio[13] analyzed the use of orthogonal polynomials to study the flexural vibration and buckling of isotropic and orthotropic rectangular plates. Sing and Chakraborty [14,15] studied the transvese vibrations of circular and elliptic plates of variable thickness. Flexural vibration of skew plates was investigated by Singh and Chakraborty [16].

Unfortunately, the linear classical theory is no longer applicable in cases of practical interest and this leads to the non linear analysis of such problem. The analysis of non linear vibration of plates and shells with their symmetrical and unsymmetrical bending character has drawn the attention of many research workers because of their applications in engineering design. It is , however, difficult to solve the vibration problems of plates and shells of practical interest due to their highly non-linear behaviour. A number of research workers tried to solve the necessary differential equation by linearzing those through proper approximation. The results thus obtained do not agree with the experimental results for complicated plate geometry which are actually used in practice. Approximate solutions of such problems can be obtained from Karman [20] field equations. These equations involve the deflection and membrane stress functions as two dependent variables coupled together. Many workers have used Karman equations to solve the vibrational problems of elastic plates, among which Chu and Hermann [21] and Yamaki [22] need special mention.

Several techniques have been used to solve the equations; for example Levy [23] subtitutes a double Fourier series in the equation for rectangular plates. Chi-Teh-Wang [24] wrote the equation for rectangular plates in a finite difference form and solved them by the method of successive approximations. S.Way, [117] solved the circular plate equations by substituting a power series solution into the energy expression determining the coefficient by setting the first variation of the strain energy equal to the first variation of potential energy due to the external loading for any variation of each coefficient. Many investigators used Von-Karman equations to analyze the non-linear vibrational problems of plates of various shape. Rectangular plates were analyzed by Smith, Malme and Gogos [25] Yamaki [22]; Eisely [26]; Murthy and Shebourne [27]; Bayles, Lowery and Boyd [28]; Cawford and Atluri [29]. Circular plates are treated by Crawford and Atluri[30]; Fornsworth and Evan-Iwanowski [31]; Sridhar, Mook, Nayfeh [32].Ring sector plates were treated by Chisyaki and Takashi [33] and elliptic plates were treated by Lobitz, Nayfeh and Mook [34]. Wu and Vinsion [86] investigated the influence of large amplitudes, transverse shear deformation and rotatory inertia on lateral vibrations of transversely isotropic plates.

Vendhan and Das [35] investigated the non-linear vibration of plates by the application of Raylegh Ritz and Galerkin methods to the Von-Karman equation, expressed in terms of the three displacement variables, governing the non-linear dynamic behaviour of thin elastic plate. It was seen that the Rayleigh-Ritz approximation are consistently better than the Galerkin approximation, which however, tend to be equally good after a few terms. It was also noted that the above two aproximations are idential for a linear problems and the difference between them is solely due to non-linearity, even though they tend to ultimately converge to a common value. J. Ramchandran [134] studied the free vibration of rectangular plates carrying concentrated mass .

The large amplititude free flexural vibrations of thin elastic anisotropic skew plates were studied by Prathap and Vardhan [36]. They used Von-Karman field equations in which the governing non-linear dynamic equations are derived in terms of the stress-function and the lateral displacement. Clamped boundary conditions are chosen and in-plane edge conditions considered are either immovable or movable. Solutions are obtained by Galerkin method. The relationship between amplitude and period of frequency was shown to exhibit a hardening type non-linearity, irrespective of the boundary conditions, skew angle, angle of fiber oreientation or aspect ratio.

Non-linear transverse vibrations of elastic orthotropic shells were investigated by Nowinski [37] using Von Karman-Tsien equations, generalized to dynamic and orthotropic case. A sharp decrease of the period of non linear vibrations with an increase in amplitude was corroborated the mode pattern influencing the period more than the degree of anisotropy.

B.R.El. Zaouk and C.L.Dym [38] studied the effect of curvature, meterial orthotropy and internal pressure upon the non-linear vibtarions of shallow shells.

Nath , Mahrenholtz and Varma [39] investigated the non linear response of a doubly curved shallow shell on an elastic foundation. They studied the large dynamic response of a doubly curved shallow spherical shell of rectangular platform, supported on two parameter elastic subgarde and subjected to uniformly distributed step and sinusoidal loading.

Hu-Nan-Chu [40] investigated the influence of large amplitude of fexural vibrations of a thin circular cylindrical shell. Axial body force terms which may be of practical importance are included. Non linear periods are obtained for the free vibration case. The numerical results are compared with a previous study on flat plates. Nonlinear effects are found to be considerably less manifest in cylinders than in corresponding flat plates.

Nonlinear equations of motion for a transversely isotropic plate having initial geometric imprefection are derived by Lin and Chen [41]. The effects of both transverse shear deformation and rotatory intertia are included. Equation of motion for a simply supported imperfect plate is obtained by performing the Galerkin procedure and solved by Runge-Kutta method. It is found that the vibration frequencies are very much dependent on the order of initial amplitude and imperfection.

A number of investigators analyzed the non linear oscillation of anisotropic plates using Von-Karman equations. Yu [42]; Yu and Lai [43] studied the non linear vibrations of sandwich plate. Yu [44] investigaed the nonlinear vibration of layered plates and shells. Hassert and Nowinski [45], Sathyamoorthy and Pandalai [46], Ramachandran [47] treated rectangular plates with special rectangular orthotrophy. Nowinski [48] analyzed orthotropic circular plates, Sathyamoorthy and Pandalai [49] analyzed rectilinear orthotropic skew plates, while Venkateswara Rao, Kanaka Raju and Raju [50] used a finite element method, to orthotropic circular plates. Bert [52] investigated the nonlinear oscillations of an arbitary laminated rectangular plate.

Banerjee, Mazumdar and Chanda [54] investigated the non-linear vibrations of elastic plates and shell applying the Karman field equations, extended to the dynamic case, these equations involved the deflection and membrane stress functions as two dependent variables. As consequence, the solutions for almost all problems require considerable computation. But they found that to study the non linear dynamic behaviour of plates and shells, Karman equations pose difficulties in obtaining the required solution. In such cases other methods may be employed. Berger equation may provide acceptable results when the relative amplitudes assumes value less that 2.0.

Due to very complicated nature of the basic equations governing the motion of a structure exhibiting large deflection it has always been a difficult task for investigator to obtain even an approximate solution. Attempts have also been made to find ways to ease such problems. Berger [55] proposed an alternative method which enabled one to replace the coupled Karman equations by simpler set of uncoupled and quasilinear equations. Bergers assumption was based on the idea that the second strain invariant in the middle plane of the plate can be neglected wihtout including any appreciable error in the solution. However, he did not put forward any physical justification for this assumption.

Following this idea J. Nowinski [57], S.N.Sinha [58] studied the large deflection analysis of plates. Later this technique was extended to the dynamic case by Nash and Mooder [59]. Since then this method has been followed by different authors [60-65] for the analysis and dynamic behaviour of plates exhibiting large deflections.

Nash and Mooder [59] extended the Berger method to a dynamic case. M.M.Banerjee with his co-workers published a large number of papers [65-67] based on Berger's hypothesis. Most of their works are related to the problem concerning the variation of thickness of plates and shells. Neglecting in-plane intertia Nash and Mooder [59] showed that the use of such equation for simply-supported plates yeilds results which are in excellent agreement with those obtained from Karman equations. S.Das and B.Banerjee [68] investigated the damped oscillations of moderatly thick plates of arbitary shapes. They used the concept of "Lines of Equal Deflection".

M.M. Banerjee and S. Chanda [69] investigated the large deflections of thin plats of arbitrary shape placed on elastic foundation and subjected to both uniform and concentrated load at the centre as well. They followed Berger's method in conjunction with the method of "Constant Deflection Contour Lines". Nonlinear free vibrations and thermal buckling of a elastic rectangular plate at elevated temperature has been analyzed by P.Biswas [87]. The analysis was based on Berger approximation. S.Datta [70] analyzed the large deflection of clamped circular plate on elastic foundation under non-uniform but symmetrical loads, following Berger's approximate method. Here the deflections are obtained in the form of an infinite series involving Bessel function. S.Dutta [71] again investigated the large amplitude free vibrations of irregular plates placed on elastic foundation by introducing conformal mapping technique and Galerkin method.

Berger's technique was extensively used till Nowinski and Ohanbe [72] examined Berger equation critically and initiated the criticism on the free hand application of these euqations. They observed that the method may lead to grave inaccuracies and even become meaningless if the edge of the plate is free to move in in-plane directions. Lee,Blotter and Yen [73] found that the errors introduced by applying the Berger's hypothesis to a clamped circular plate, depend on Poisson's ratio and the ratio of the radius to the thickness of the plate. Moreover they found that the error is minimized when the Poisson's ratio increases. Huang and Al-Khattat [74] showed that for radially restrained circular plates, solutions base don Berger's hypothesis are accurate at low amplitude but the accuracy decreases as the amplitude increases. Moreover they found that Berger hypothesis is entirely unsuitable for plates with moveable edges. Banerjee [75] while dealing with the large amplitude vibrations of nonuniform rectangular plates, observed that the value of the relative time period (nonlinear and linear) differ from what has been calculated by Bouer [76] by approximately 2% for unit relative ampitude. In an attempt to explore some limitation on the use of Berger equations for large amplitude vibrations of thin elastic plates Banerjee and Sarker [77] further observed that the acceptability of Berger's hypothesis may be restricted to the cases of clamped square and circular plates with immovable edges, and to some extent to simply supported circular or rectangular plates having smaller aspect ratio. They suggested that Berger's method may be restricted to circular and rectangular plates and to some extent to skew plates with smaller skew angles for clamped immovable edge conditions. Banerjee and Das [78] aimed at finding a few points in support of Berger equation without rejecting them totally. They suggested to be cautious against the freehand application of this method. Mention may also be made regarding the relative exactness of Berger's technique as studied by Vendhan [79], by Prathap and Vardan [80] and by Prathap [81].

Sinharay et.al. [82] proposed some modification of Berger's approximation by expressing the total potential energy expression due to bending and streching of the middle surface of a plate or shell in a different way. They preferred to replace e_2 by a new expression without rejecting it totally. The idea is novel one but the limit of its accuracy is yet to be established. For, like Berger's hypothesis, it lacks in providing with a rigorous physical justification. A simple application of the hypothesis proposed by Sinharay has been made by Banerjee et.al. [83] to test its validity. The method was aplied to the problem of finding the temperature effect on the dynamic response of shallow spherical shells. The findings are not very encouraging. Rather, one of the vital equations in Ref. [82] appeared to be fallacious denying the claim of the accuracy of the new approach, at least on the basis of the very problem treated in Ref. [83]. Moreover, Banerjee et.al. [83] observed that assumption of a certain parameter less than unity appeared to be impractical when the radius of the base circle of the spherical shell is large enough compard to the thickness of the shell whereas the authors of Ref. [82] have assumed values of the parameter less than unity. Taking into consideration the different options expressed by authors working on this method it may be stated that Berger method, simpliest of all the existing ones for the analysis of vibrating structures, cannot be discarded altogether. Its applicability may be restricted to the cases of clamped square and circular plates with immovable edge. There is every scope that the deficiency in Berger's method can be overcome and it will then be applied to all possible cases of structures with various boundary conditions. Further studies which deal with membranes, shells of different shapes, flat plates, spinning disks, spinning membranes have been citied in Ref. [88-105].

Mazumdar [19,137-139] put forward a new method to solve the problems of elastic plates of arbitrary shape. The method as it was termed is "Constant Deflection Contour" method. Mazumdar with his co-workers published a series of papers [19, 137–139] on linear vibration of plates and shells utilizing this technique. The outstanding feature of this method is that it is entirely independent of the shape of the plate. Using this method, Jones, Mazumdar and Fu-Pen-Chiang [106] investigated the vibrations of plates under various boundary and load conditions. As illustrations, the case of circular plate clamped on one part of its boundary and simply - supported on the remainder and the case of clamped elliptical plate under elliptical line loading, have been discussed. A simple method for the analysis of the elastic-plastic bending of plates of arbitrary shape was developed by Jain and Mazumdar [107]. The procedure was based upon the concept of "Constant Deflection Contour" method. Again Mazumdar and Bucco [108] analyzed the transverse vibrations of shells of visco-elastic material under arbitray time - dependent load. Banerjee [109] developed an idea of extending the " Constant Deflection Contour" method to the non-linear analysis of plates vibrating at large amplitudes.

The present thesis is based on this idea and it will be followed in all problems considered in this thesis. The details of the method needs a separate chapter to explain the procedure of deriving the basic equations as well as the method of finding their solutions. [See chapter III]