CONCLUDING CHAPTER :

The main objective of the present thesis is to investigate the feasibility of the application of " Constant Deflection Contour " method in extending it to the non-linear analysis, so far as the static and dynamic behaviour of plates and shells under various geometrical as well as boundary conditions. It has already been accepted that the brilliant works have been made by Mazumdar. J, of Department of Applied Mathematics, University of Adelaide, Australia and some of his co-research workers in the field of static and dynamic response of structures. However their works were mainly restricted to linear cases only. The novelty of this method was, perhaps, missed the attention of the research workers on an international basis during the seventies of the last century . An attempt to extend this method to a quasi-linear problem was made by S. Das and B. Banerjee [68] . Unfortunately the essence of the " Constant Deflection Contour " method was lost in dealing with the illustrative example. It appears that , perhaps, the authors in reference [68] have not deduced the required equations following the method offered by Mazumdar [19, 137 - 139]. The present investigator has deduced the same equations which are not in exact agreement with those obtained by [68] .

In 1997 Banerjee [109], probably first initiated the extension and application of ' Constant Deflection Contour ' method to non-liner analysis of structures vibrating at large amplitude. Mention may be made the work of Chanda [131] dealing with some linear and quasi - linear problems using the ' Constant Deflection Contour ' method. The present investigator started her work under the guidance of M. M. Banerjee to study the application of 'Constant Deflection Contour' method to strictly non-linear problems in 1997. Later on Banerjee and Rogerson [122] put forward a general study on the application of ' Constant Deflection Contour " method . Hence it may humbly be stated that the present work may be considered as the first concrete attempt to extend the " Constant Deflection Contour " method to problems associated with large vibrations.

In the present thesis some problems dealing with structures having a little complicated boundary have been considered to pave way for considering problems with more complicated and complex boundaries. In the fourth Chapter an elliptic plate clamped along the edges has been considered to start with. And the results have been compared with known available results as far as possible. The use of " Constant Deflection Contour " method has been found to be effective for such problems.

In all problems considered from Chap - IV onwards the basic governing differential equations have been deduced primarily on the basis of Karman field equations extended to a dynamic case. In some problems such equations have been used straight forward and in some cases where external applied forces or geometrical non-linearity or inhomogeneity has been considered, the basic equations have been deduced on the basis of the present theory. The results so deduced have been found satisfactory enough and support the applicability of this method. For example the problem dealing with large vibration of elliptic plates on elastic foundation have been deduced. The comparison of results so remarkably well with available results obtained using by a different approaches. Differences if there be any arise out of a different approach, method of solution and the approximating functions (limited number of terms out of a polynomial expressions). Of course the variation of results is very insignificant .

In problem-3, Chapter IV, an attempt has been made to justify the use of Karmań field equations over other simplified or modified equations. While considering the problem of static and dynamic behaviour of elliptic plates under damping condition, the problem has been rechecked using " Constant Deflection Contour ", method and the same set of equation as used in [68]. The present investigator states with much hesitance that the results cited in ref [68] are not those obtained from the expressions deduced by the author. Moreover the use of "Constant Deflection Contour" method is not at per with the basic idea of "Constant Deflection Contour" method; the detailed criticism has been given at the end of problem-3, Chap -IV.

In chapter VI, problem 3 and 4concern with the effect of rotatory inertia, damping and varying flexural rigidity. In all cases the present analysis appears to be in conformity with the proposed theory establishing the objective and applicability of "Constant Deflection Contour" method. Moreover two problems referred to above, establish the accuracy of Karmań equations over the other, as stated in the concluding remark made at the end of problem - 4, Chapter IV.

During the process of investigation, for a simplified approach equations (3.11) and (3.12) were utilized, later on Banerjee and Rogerson [122] proposed the use of fourth order equations (3.12) and (3.13) and they have been used throughout the whole Chapter-V and VI. Some typical examples have been dealt with in Chapter V and some more complicated problems have been considered besides the illustrative examples put forward by Banerjee and Rogerson [1.22]. These illustrations not only support the proposed theory but also establish the accuracy of numerical results for mixed boundary value problems. The concluding remark made at the end of Chapter-V includes that even in some cases the present approach provides results more accurate than those obtained by other author with different approaches. Some authors have utilized some simpler form of Karmań equations at the cost of accuracy and even inviting some absurities. For example equation (5.4.20) and equation (4.3.28) are compared with the relevant equation of reference [22], for static case, $\nu=0.3$, m=1 (wherever admissible)

For static case,

$$\frac{pa^4}{Eh^4} = 5\cdot8608\left(\frac{W_0}{h}\right) + 2\cdot76\left(\frac{W_0}{h}\right)^3 \quad \text{from Karman Equations}$$

$$= 5\cdot8608\left(\frac{W_0}{h}\right) + 3\cdot516\left(\frac{W_0}{h}\right)^3 \quad \text{from Berger's Equation}$$

[Putting $KE/G_c = 0$ in Equation (4.3.28) in problem 3, chapter IV]

$$= 5\cdot848\left(\frac{W_0}{h}\right) + 2\cdot754\left(\frac{W_0}{h}\right)^3 \quad \text{[Yamaki, Ref(22)]}$$

For dynamic case,

$$\frac{T^*}{T} = \left[1 + 0\cdot3531\left(A_0/h\right)^2\right]^{-1/2} \quad \text{from Karman Equation}$$

$$= \left[1 + 0\cdot4499\left(A_0/h\right)^2\right]^{-1/2} \quad \text{from Berger Equation}$$

{Putting $KE/G_c = 0$, $\mu = 0$ in Equation (4.3.21), problem 3 chapter IV}

$$= \left[1 + 0\cdot3531\left(A_0/h\right)^2\right]^{-1/2} \quad \{\text{Yamaki, Ref (22)}\}$$

Clearly the variation in coefficients of the non-linear term signifies the validity of Banerjee's approach. The same question had already been raised by Nowinski [72] and later on by Banerjee et al [77].

The latest problem utilizing Berger equations has been treated by Mondal and Biswas [128] years later cautioned by Banerjee et al [77]. Hence the present investigation also support the criticism first initiated by Nowinski [72] and later on by others. In Chapter IV problem - 3 Berger's equations have been utilized not to encourage to avail of the mathematical simplicity at the cost of accuracy.

In Chapter VI the extension of " Constant Deflection Contour " method to shell structures, has been made to break the monotony of plate structures.

An attempt has also been made to extend the present analysis to elastic plastic shell structures. The Governing differential equations have been deduced and they provide the primary tools for investigation of elastic plastic structures. The present thesis does not consider any numerical results in the sense that one of the co-research worker is engaged in such studies.

Concluding Remark

The following remarks may be made considering all aspects and investigations made during the research period.

1. For establishing the governing differential equations a new approach has been made different from what had been proposed by previous users of " Constant Deflection Contour " method.

2. The previous investigations based on " Constant Deflection Contour " method concern only with linearised problems whereas the present study is based on a non-linear approach.

3. Starting from structures having regular and common boundaries, gradually more and more complicated structures and mixed boundary value problems have been included in the present thesis.

4. Numerical results presented for various illustrative examples have been compared with all results available to present investigator. Failure in this respect, if there be any, may be due to the non-availability of required information and lack of information sources.

5. A comparative study has always been made in dealing with a specific problem with regard to different approaches made to investigate the problem.

6. The present investigator also humbly suggests that for simplicity the very essence of non-linear analysis should not be lost and basic equations like Berger or modified Berger equations should be avoided in the interest of future study.

7. True research activities never stop. The present investigation has to be cut off so as to finish present project within the specified period. However lot of works still remain to be done in different sphere not considered in the present study. The present investigator wishes to continue further studies in the related topic in near future.

8. Some of the new spheres which the present investigator thinks should attract the co-research workers and contemporary researchers interested in this field are :

i> Dynamic and static response of structures like triangular, annular and polygonal shaped plate structures.

ii> Shell structures, other than spherical and cylindrical shells.

iii> Extension of the " Constant Deflection Contour " method to elastic plastic behaviour of structures.

In conclusion, the following remarks may be added with reference to different illustrations on " Constant Deflection Contour " method :

i> The method is based on solid mathematical foundation.

ii> Unlike Karman equations it ultimately reduces to a problem of solving two ordinary differential equations.

iii> The method may be also applied to structures of arbitrary shape.

iv) The only disadvantage of this method is that the equation of the lines of equal deflection should be known.