CHAPTER - VII

Elastic - Plastic Analysis of Shallow Shells of Arbitrary Shape

In the previous chapters it has been observed that the "Constant Deflection Contour" method can be effectively applied to study the vibrations of elastic plates and shells and the analysis appears to be easier than the other existing methods.

To make further investigation, this method is applied to study the vibration of elastic plastic shallow shells in this chapter.

Though several studies have been made . . on elastic-plastic analysis of plates and shells [107, 129, 130], most of them deal with the linear analysis . This initiates the present investigator to make an attempt to apply the " the Constant Deflection Contour " method to study the non-linear vibration of elastic plastic shallow shells. Regarding the application of " Constant Deflection Contour " method, on elastic plastic analysis of plates, Mazumdar et - al [107] made some useful studies on this sphere.

Considering an elastic plastic shallow shell of thickness h , the equation of the middle surface of the shell is given by

$$z=\frac{x^2}{2R_1}+\frac{y^2}{2R_2}+\frac{xy}{R_{12}}$$

The shell is called shallow if $r=(x^2+y^2)^{1/2}$ is small compared to the least of the radii of curvature R_1, R_2, R_{12}

The basic equations to study the elastic plastic analysis of shell may be written as :

$$\frac{d^3w}{du^3}\oint(1-\Omega)R\,ds+\frac{d^2w}{du^2}\oint(1-\Omega)J\,ds+\frac{dw}{du}\oint(1-\Omega)G\,ds$$

$$+\frac{d^2w}{du^2}\oint D\left(\frac{\partial\Omega}{\partial x}\frac{\partial u}{\partial x}+\frac{\partial\Omega}{\partial y}\frac{\partial u}{\partial y}\right)t^{1/2}\,ds$$

$$+\frac{dw}{du}\oint\frac{D}{t^{1/2}}\left(\frac{\partial\Omega}{\partial x}K+\frac{\partial\Omega}{\partial y}L\right)ds+\iint\left[\rho h\,w_{,tt}+\frac{1}{R_1}F_{,yy}+\frac{1}{R_2}F_{,xx}\right.$$

$$\left.-\frac{2}{R_{12}}F_{,xy}-p\right]dx\,dy=0 \qquad [7.1]$$

$$\frac{d^3F}{du^3}\oint R\,ds+\frac{d^2F}{du^2}\oint J\,ds+\frac{dF}{du}\oint G\,ds-\frac{12D^2(1-\nu^2)(1-\Omega)}{h^2}\frac{dw}{du}\oint\left[\frac{u_{,x}^2}{R_2}+\frac{u_{,y}^2}{R_1}\right]t\,ds$$

$$=0 \qquad [7.2]$$

where $\Omega = 0$, when $e \leqslant 1$ and the region is elastic

$$\Omega = \lambda\left[1 - \frac{3}{2e} + \frac{1}{2e^3}\right] \text{ when } e \geqslant 1 \text{ and the region is plastic} \quad [7.3a]$$

$$\text{where } e = \frac{h}{\sqrt{3}\, e_s}\left(w_{,xx}^2 + w_{,yy}^2 + w_{,xx} w_{,yy} + w_{,xy}^2\right)^{1/2} \quad [7.3b]$$

$$t = u_{,x}^2 + u_{,y}^2$$

$$R = -D t^{3/2}$$

$$J = \frac{-D}{t^{1/2}}\left[3 u_{,xx} \cdot u_{,x}^2 + 3 u_{,yy} u_{,y}^2 + u_{,xx} u_{,y}^2 + u_{,yy} u_{,x}^2 + 4 u_{,xy} u_{,x} u_{,y}\right]$$

$$G = \frac{-D}{t^{3/2}}\Big[u_{,xxx} u_{,x}^3 + u_{,yyy} u_{,y}^3 + (2-\nu)\big(u_{,xxx} u_{,x} u_{,y}^2$$

$$+ u_{,yyy} u_{,y} u_{,x}^2 + u_{,xyy} u_{,x}^3 + u_{,xxy} u_{,y}^3 + (2\nu-1)\big(u_{,xyy} u_{,x} u_{,y}^2$$

$$+ u_{,xxy} u_{,x}^2 u_{,y}\big) - 2(1-\nu) u_{,xy}\big(u_{,x} u_{,y} u_{,xx} - u_{,y}^2 u_{,xy}$$

$$- u_{,x}^2 u_{,xy} + u_{,x} u_{,y} u_{,yy}\big) + (1-\nu)\big(u_{,xx} - u_{,yy}\big)$$

$$\big(u_{,xx} u_{,y}^2 - u_{,yy} u_{,x}^2\big) + \frac{2D(1-\nu)}{t^{5/2}}\Big[u_{,xy}\big(u_{,x}^2 - u_{,y}^2\big)$$

$$- u_{,x} u_{,y}\big(u_{,xx} - u_{,yy}\big)\Big]^2$$

$$K = u_{,x}\left(u_{,xx} + \nu u_{,yy}\right) + u_{,y}\left[(1-\nu) u_{,xy} - H/D\right]$$

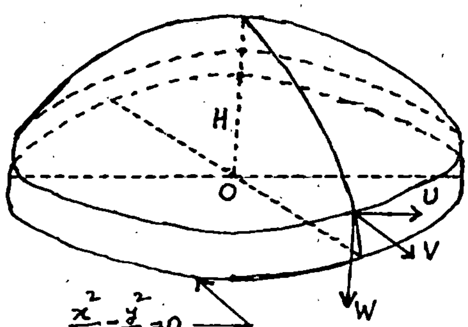
$$L = u_{,y}\left(u_{,yy} + \nu u_{,xx}\right) + u_{,x}\left[(1-\nu)u_{,xy} + \frac{H}{D}\right]$$

$$H = \frac{D(1-\nu)}{t}\left[u_{,xy}\left(u_{,x}^2 - u_{,y}^2\right) - u_{,x}u_{,y}\left(u_{,xx} - u_{,yy}\right)\right]$$

[7.4]

Equations (7.1) and (7.2) are the basic equations to study the vibration of elastic plastic shallow shell . One cannot proceed further unless the geometry of the shell is known.

A clamped dome of non-zero curvature upon an elliptic base is considered (Fig - 6).



The first approximation for the lines of constant deflection for this case due to symmetry consideration may be taken as

$$u = 1 - \frac{x^2}{a^2} - \frac{y^2}{b^2} \qquad [7.5]$$

R_{12} has been assumed to be zero in accordance with the shallow shell theory with the form of u given by equation (7.5) and carrying out the necessary but lengthy calculation the differential equations (7.1) and (7.2) takes the form

$$(1-\Omega)(1-u)^2\frac{d^3w}{du^3} - 2(1-\Omega)(1-u)\frac{d^2w}{du^2}$$

$$-\frac{d\Omega}{du}\left[(1-u)^2\frac{d^2w}{du^2} - 2\frac{P'}{P}(1-u)\frac{dw}{du}\right] - \frac{\Upsilon}{D}(1-u)\frac{dF}{du}$$

$$= \frac{-p(1-u)}{2DP} - \frac{\rho h}{2DP}\int_1^u w_{,tt}\,du \qquad [7.6]$$

$$(1-u)\frac{d^3F}{du^3} - 2\frac{d^2F}{du^2} + \Upsilon(1-\Omega)Eh\frac{dw}{du} = 0 \qquad [7.7]$$

where $P_1 = \left(\frac{1}{a^4} + \frac{1}{b^4} + \frac{2\nu}{a^2b^2}\right)$

$$\Upsilon = \frac{1}{P}\left[\frac{1}{R_1 b^2} + \frac{1}{R_2 a^2}\right] \qquad [7.8]$$

Using the following non-dimensional parameters

$$w^* = \frac{wh}{e_s a^2} \qquad F^* = \frac{F}{Ee_s a^2} \qquad p^* = \frac{pa^2h}{2De_s} \qquad [7.9]$$

Equation (7.6) and (7.7) takes the form

$$(1-\Omega)(1-u)^2\frac{d^3w^*}{du^3} - 2(1-\Omega)(1-u)\frac{d^2w^*}{du^2}$$

$$-\frac{d\Omega}{du}\left[(1-u)^2\frac{d^2w^*}{du^2} - 2\frac{P_1}{P}(1-u)\frac{dw^*}{du}\right]$$

$$-\frac{Eh\Upsilon}{D}(1-u)\frac{dF^*}{du} + \frac{\rho h}{2DP}\int_1^u w^*_{,tt}\,du = -\frac{p^*(1-u)}{Pa^4} \qquad [7.10]$$

$$(1-u)\frac{d^3F^*}{du^3} - 2\frac{d^2F^*}{du^2} + \Upsilon(1-\Omega)\frac{dw^*}{du} = 0 \qquad [7.11]$$

Considering the shell completely clamped along the boundary, the boundary conditions can be expressed in terms of the deflection function and its derivatives with respect to u.

$w'=0$ at $u=0$ and $F=0$ at $u=0$ and

$$\frac{dw}{du}=0 \text{ at } u=0 \qquad \frac{dF}{du}=0 \text{ at } u=0 \qquad [7.12]$$

With these conditions expressed in equation (7.12), equations (7.10) and (7.11) are to be solved for W* and F*.

Let $w=\sum_{i=2}^{\alpha} A_i u^i \psi(t)$ compatible with the boundary condition expressed by equation (7.12), A_i's are to be evaluated. Let a rough approximation be considered with the first term only.

$$w = Au^2\psi(t)$$

$$F = Au^2\phi(t) \qquad [7.13]$$

Where $\psi(t)$ and $\phi(t)$ are functions of time only. Since equation (7.13) does not represent the exact solution, Galerkin procedure is applied to minimize the error.

Substituting equation (7.13) in to equation (7.11) a relation between $\phi(t)$ and $\psi(t)$ is first established

$$\phi(t) = \frac{3}{8}\gamma(1-\Omega)\psi(t) \qquad [7.14]$$

Making use of equations (7.30) and (7. 3b) $\frac{d\Omega}{du}$ may be evaluated in the following way.

$$e = \frac{h}{\sqrt{3}\,e_s}\left(w_{,xx}^2 + w_{,yy}^2 + w_{,xx}w_{,yy} + w_{,xy}^2\right)^{1/2}$$

$$\text{i.e.,} \quad e = \frac{h}{\sqrt{3}\,e_s}\left[M\left(\frac{dw}{du}\right)^2 + N\frac{dw}{du}\frac{d^2w}{du^2} + t^2\left(\frac{d^2w}{du^2}\right)^2\right]^{1/2} \qquad [7.15]$$

$$\text{where } M = u_{,xx}^2 + u_{,yy}^2 + u_{,xx}u_{,yy} + u_{,xy}^2$$

$$N = 2u_{,xx}u_{,x}^2 + 2u_{,yy}u_{,y}^2 + u_{,xx}u_{,y}^2 + u_{,yy}u_{,x}^2 + 2u_{,xy}u_{,x}u_{,y}$$

$$t = u_{,x}^2 + u_{,y}^2 \qquad [7.16]$$

$$\Omega = \lambda\left[1 - \frac{3}{2e} + \frac{1}{2e^3}\right]$$

$$\frac{d\Omega}{du} = \frac{d\Omega}{de}\frac{de}{du} \qquad [7.17]$$

$$\frac{de}{du} = \frac{h^2}{6e_s^2 e}\left[\frac{d}{du}\left\{M\left(\frac{dw}{d}\right)^2 + N\frac{dw}{du}\frac{d^2w}{du^2} + t^2\left(\frac{d^2w}{du^2}\right)^2\right\}\right]$$

[7.18]

Substituting the values of M, N, t from equation (7.16), equation (7.18) takes the form

$$\frac{d\Omega}{du} = 8Q_2\left[\left(\frac{1}{a^2}+\frac{1}{b^2}\right)^2 + 2\left(\frac{\cos^2\theta}{a^4}+\frac{\sin^2\theta}{b^4}\right)\right]\frac{dw^*}{du}\frac{d^2w^*}{du^2}$$

$$-8(1-u)\left[\left(2\frac{\cos^2\theta}{a^4}+2\frac{\sin^2\theta}{b^4}+\frac{1}{a^2b^2}\right)\right.$$

$$\left.-4\left(\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}\right)^2\right]\left(\frac{d^2w^*}{du^2}\right)^2$$

$$-8(1-u)\left[2\left\{\frac{\cos^2\theta}{a^4}+\frac{\sin^2\theta}{b^4}\right\}+\frac{1}{a^2b^2}\right]\frac{dw^*}{du}\frac{d^3w^*}{du^3}$$

$$+32(1-u)^2\left[\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}\right]^2\frac{d^2w^*}{du^2}\frac{d^3w^*}{du^3} \qquad [7.19]$$

where $Q_2 = \dfrac{\lambda a^4}{4}\,\dfrac{e^2-1}{e^5}$

θ = eccentric angle

substituting equations (7. 13) and (7. 19) into equation (7.10) and applying Galerkin procedure to minimize the error one can arrive at the final equation

$$(1-\Omega)\left[\frac{1}{3}+\frac{3}{80}\left(\frac{2r}{h}\right)^2(1-\nu^2)\right]A\psi(t)$$

$$+\frac{32}{5}Q_2\left[\frac{1}{6}P_2+\frac{1}{12}P-\frac{1}{6}P_1-\frac{P_1P_2}{P}\right]A^3\psi^3(t)$$

$$+\frac{1}{36}\rho\frac{hA\psi_{,tt}(t)}{DP}=\frac{p^*}{12Pa^4} \qquad [7.20]$$

$$\text{where } P_2=\frac{1}{a^4}+\frac{1}{b^4}+\frac{1}{a^2b^2}$$

$$\text{and } Q_2=\frac{\lambda a^4}{4}\frac{e^2-1}{e^5}$$

and e should be replaced by the average value of e i.e.

$$\bar{e}^2=\frac{8}{9}A^2(3m^4+2m^2+3) \quad \text{where } m=\frac{a}{b}$$

Equation (7. 20) can easily be utilized to study the static and dynamic behaviours of an elastic-plastic shallow shell. No numerical results have been presented in the sense that one of the co-research workers is engaged in such studies on the basis of the equation(7.20).

It can be concluded that the application of " Constant Deflection Contour" method for elastic plastic bending analysis of plates and shell is quite straight forward and efficient. Although the method is illustrated to study the elastic plastic analysis of a shell upon an elliptic base, its application to other plate or shell geometries is quite simple. This method highly relies on the accuracy of the choice of the isodeflection contour lines u (x, y), however it is very difficult to find out the exact form of contour lines for a plate or shell of arbitrary shape. In the present study, the contour line function is assumed to be that for the corresponding fully elastic case. The present investigator wishes to continue further studies in this sphere in near future .