CHAPTER VI

MEASUREMENT OF HIGH FREQUENCY DISCHARGE CURRENT IN A GAS DISCHARGE EXCITED BY A RADIO FREQUENCY ELECTRIC FIELD.

INTRODUCTION

The current flowing through an ionised medium by the application of small alternating electric field was considered in many problems involving microwave and also in the analysis of the ionosphere. For low frequency of the applied electric field and high gas pressure, the current flowing through the ionised medium has the same phase as the applied field and may be obtained from the mobility formula due to Langevin (1905). For high frequencies and low gas pressure, on the other hand, the current is in phase quadrature with the applied field. The intermediate case between the two extremes was considered by Margenau (1946) who obtained the current density as having two components: one in phase with the applied alternating field and the other in quadrature with it.

Sodha (1960) obtained expressions for the complex electrical conductivity when a high power radio frequency wave is incident on the plasma. According to the same author, the velocity distribution of electrons in a plasma for constant mean free path λ_e and energy loss factor χ is Druyevesteyn. Everhart and Brown (1949) measured the complex admittance in the microwave region for helium filled in the cavity of a magnetron.

When a gas discharge excited by radio frequency electric field is maintained between two plane parallel electrodes, as is conventionally done in laboratory, then in addition to the two components of gas discharge current there will be current flowing due to capacitative effect of the electrodes. Francis and von Engel (1953) measured the growth of discharge current during the onset of discharges at low pressures such that the electron mean free path is always greater than the size of the vessel. An experimental technique has been employed to measure the current actually flowing through the discharge. The large capacitative current flowing across the external electrodes is balanced out by a bridge method, the bridge becoming unbalanced when current flows through the gas. The growth with time of this unbalanced component which is proportional to the discharge current is recorded in oscillograms for helium and hydrogen at different pressures. Applying similar bridge technique with modifications, Clark, Earl and New (1970) measured the complex impedance of a radio frequency discharge in hydrogen. Penfold and Warder, Jr. (1967) designed and tested a circuit to measure current in high voltage radio frequency plasma discharge with better accuracy compared to the measurement by conventional r.f. meters and probes.

In the present paper, a new technique has been reported to isolate and measure the current flowing through the active r.f. discharge which has the same phase as the applied field and also the current which is in phase quadrature with the applied field. From these measurements the resistive impedance and ~~a~~ hence the real part of the electrical conductivity of the discharge and its variation with the applied electric field is obtained for discharges with three gas pressures. Attempts to explain all the experimental observations from theoretical stand have been made.

EXPERIMENTAL TECHNIQUE.

(a) Circuit Design:-

A plasma produced in a cavity by a radio frequency electric field is viewed as a resistive impedance in parallel with a capacitative ~~imdxp~~ impedance. Some authors have measured the total r.f. discharge current by using balanced a.c. bridge to eliminate vacuum displacement current (Francis & von Engel, 1953). The process of balancing the bridge and use of different external inductive and capacitative impedances require lot of adjustments and screening throughout the measurements and at different ranges of applied voltages.

In the present experimental technique developed, the balancing of bridge and introduction of additional capacitative and inductive impedances have been avoided. The basic circuit diagram to measure the current in phase with the applied field and the current having some phase difference with the applied field, which simultaneously flow through a R-C combination in parallel is shown in Fig. 6.1 where two high speed operational amplifiers (A_1 & A_2) have been used and two resistances R_1 & R_2 in series are connected across the same r.f. source.

To simulate the electrical equivalent of a fully developed r.f. plasma column between two electrodes, a resistance 'R' and a capacitance 'C' are put in parallel combination as shown in the dotted region. The resistance 'r' must be much smaller than 'R' and impedance of 'C' at the applied **freqmency**. The value of $r = 100\,\Omega$ is taken. The maximum value of the variable resistance $R_1 = 1\ K\Omega$ must be much smaller than the fixed resistance $R_2 = 1\ M\Omega$.

A_1 is a high speed operational amplifier using IC chip No. μA 715 (Slew rate 100 v/μs; Band width 65 MHz). It is used as unit gain voltage amplifier with phase reversal at the output by 180°. A_2 is also a high speed operational amplifier using IC chip No. μA 715. It is used as unit gain summing amplifier. Both the IC's are initially tested to perfection for their respective operations by

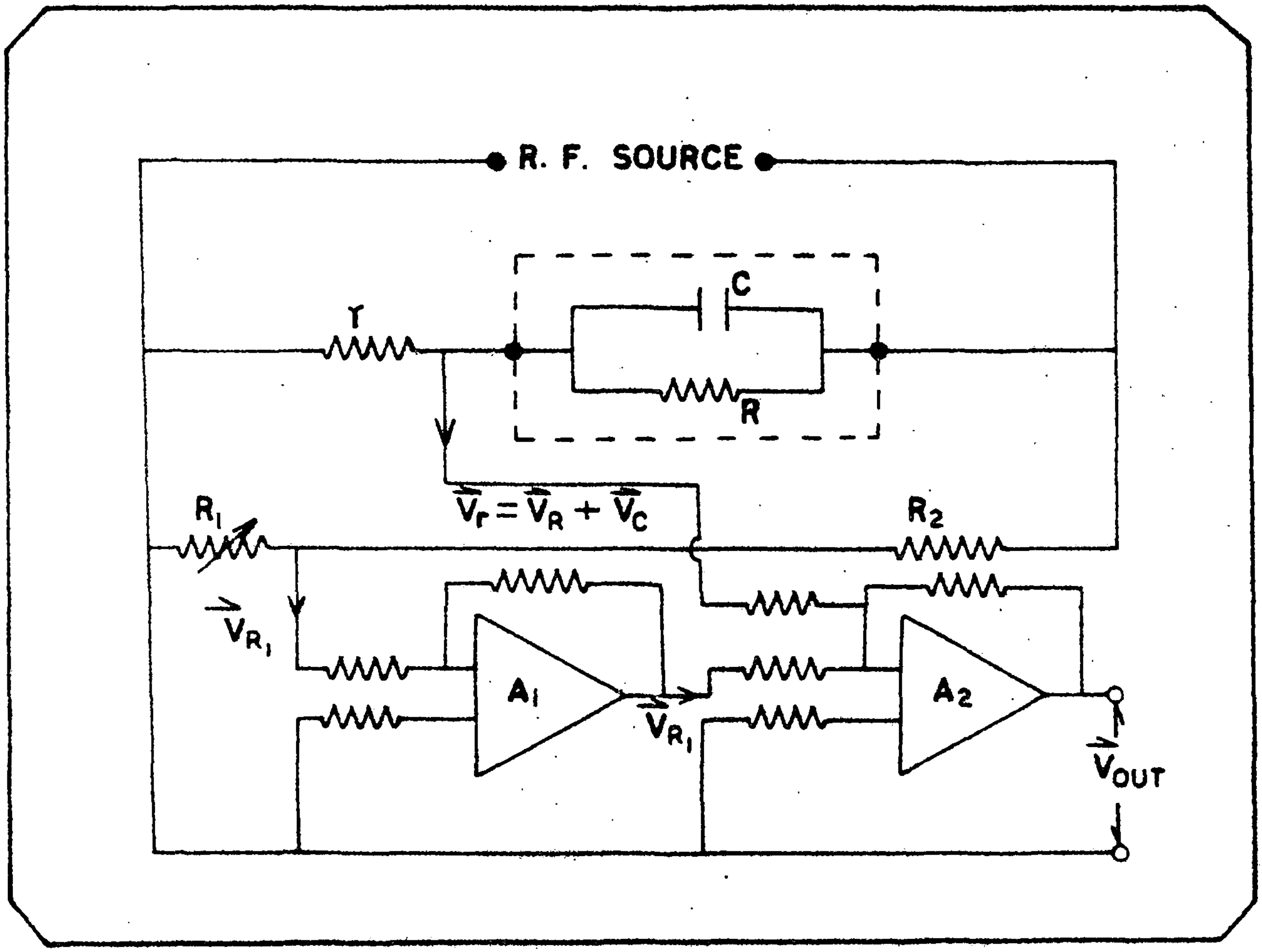


FIG. 6.1. SCHEMATIC DIAGRAM OF EXPERIMENTAL ARRANGEMENTS.

proper choice of external circuit components. The operation of A_1 and A_2 are tested in the following way: Signal from a r.f. voltage source is applied in the input of A_1 and its output is fed to one beam of a double bean C.R.O. (Type Dumond 766 H/F). In the other beam of the C.R.O. the same source potential is applied directly. The source voltage was in the range of less than one volt r.m.s. Observing simultaneously two beam traces on the same screen, the external circuit elements of A_1 are adjusted to obtain the output of A_1 as exactly inverted image of source potential trace. The output from A_1 is then fed to input of A_2 which also receives simultaneously the signal potential from the source at its input. The output of A_2 is then measured by a precision solid state a.c. voltmeter capable of measuring accurately 10^{-7} volt. The external circuit elements of A_2 are adjusted till the output of A_2 becomes zero in the solid state a.c. voltmeter. The whole process of testing the functioning of the different sections of the circuit as stated above is repeated by varying source signal voltage from 1 mV. to 1 volt (r.m.s.). For this purpose a low power r.f. source with operating frequency at 1.45 MHz. (measured by absorption wavemeter) was used. This repeated procedure of adjustments ensured the operation of A_1 as unit gain inversion voltage amplifier and A_2 as unit gain summation voltage amplifier.

(b) Theory of Measurement:

Let the r.f. signal applied from the source be given by

$$\vec{V} = \vec{V}_o e^{j\omega t} \qquad (6.1)$$

The current flowing through the capacitor of capacitance 'C' is $\vec{I}_c = \frac{\vec{V}}{X_c}$ (6.2)

where X_c is the impedance of the condenser and given by

$$X_c = \frac{1}{j\omega C}$$

Hence, $\vec{I}_c = j\omega C\,\vec{V} = j\,\omega\,C\,\vec{V}_o\, e^{j\omega t}$ (6.3)

The current flowing through 'R' is

$$\vec{I}_R = \frac{\vec{V}}{R} = \frac{\vec{V}_o e^{j\omega t}}{R} \qquad (6.4)$$

The potential drop across 'r' due to flow of current I_c and I_R is

$$\vec{V}_r = \vec{I}_c r + \vec{I}_R r = j\omega C r \vec{V}_o e^{j\omega t} + \frac{r}{R}\vec{V}_o e^{j\omega t} \qquad (6.5)$$

The potential drop across R_1 due to current I_{R_1} flowing from the source through R_1 is

$$\vec{V}_{R_1} = R_1 \vec{I}_{R_1} e^{j\omega t} = \vec{V}_{R_{10}} e^{j\omega t} \qquad (6.6)$$

The potential $\vec{V}_{R_1}$ is applied to the input of A_1 and in inverted potential $-\vec{V}_{R_1}$ and $\vec{V}_r$. are simultaneously applied to the input of A_2 and the output of A_2 is obtained as

$$\vec{V}_{out} = j\omega C r \vec{V}_o e^{j\omega t} + \frac{r}{R}\vec{V}_o e^{j\omega t} - \vec{V}_{R_{10}} e^{j\omega t}$$

$$= j\vec{V}_{co} e^{j\omega t} + \vec{V}_{R_o} e^{j\omega t} - \vec{V}_{R_{10}} e^{j\omega t}$$

$$= \left[\left(\vec{V}_{R_o} - \vec{V}_{R_{10}}\right) + j\vec{V}_{co} \right] e^{j\omega t}$$

$$= \left(\vec{V}_{out}\right)_o e^{j\omega t + \phi} \qquad (6.7)$$

where $\left(\vec{V}_{out}\right)_o \cos\phi = \vec{V}_{R_o} - \vec{V}_{R_{10}}$ and $\left(\vec{V}_{out}\right)_o \sin\phi = \vec{V}_{co}$

so that $\tan\phi = \dfrac{|\vec{V}_{co}|}{|\vec{V}_{R_o} - \vec{V}_{R_{10}}|}$

and the amplitude of the output voltage of A_2 is

$$\left(V_{out}\right)_o = \left[V_{co}^2 + \left(V_{R_o} - V_{R_{10}}\right)^2 \right]^{1/2} \qquad (6.8)$$

As R_1 is varied, $V_{R_{10}}$ will vary. Since $R_1 \ll R_2$ so variation of R_1 will not create any significant change in the current distribution from the r.f. source to discharge tube and to $(R_2 + R_1)$ combination respectively. Hence by changing magnitude of $V_{R_{10}}$, the output voltage of A_2 $(V_{out})_o$ is made minimum when

$$V_{R_o} = V_{R_{10}} \qquad (6.9)$$

their phase being same. Then the phase angle ϕ will be equal to $\pi/2$ and the out-put voltage

$$(V_{out})_o = V_{co} \tag{6.10}$$

when the condition (6.9) is satisfied.

Keeping the r.f. source potential constant at a certain value, R_1 is varied and V_{R_1} and V_{out} is measured simultaneously by the precision solid state a.c. voltmeter mentioned earlier. The plot of V_{out} against V_{R_1} will have a minimum value of V_{out}. At this minimum point V_{R_1} will give V_R and the corresponding V_{out} will give V_c. Hence both the resistive and capacitative currents are being isolated and measured by dividing V_R and V_{out} by the value of 'r' respectively.

(c) Varification of Working Principle:

To verify experimentally the working principle of the circuit we have introduced, in place of R-C combination in Fig. 6.1, a cylindrical glass tube fitted with two internal circular plane metal electrodes parallel to each other and separated by a distance of 5.0 cm. The tube is filled with distilled water. Keeping the r.f. voltage from the source fixed at some value ($\sim$ 100 volt r.m.s.) R_1 is varied and V_{R_1} and V_{out} are measured by the solid state a.c. voltmeter. It was observed that V_{out} always shows minimum as V_{R_1} is varied. Identical measurement was repeated for different source voltage to test the reproducibility of the working of the designed circuit. Measurement at every source voltage yielded respective minimum value of V_{out} corresponding to some value of V_{R_1} i.e. different values of current components. These observations confirmed the principle of the measurement. After ensuring the reproducibility of the working of the measuring circuit it is used to isolate and measure the resistive and capacitative part of the electric currents flowing through a plasma column of radio frequency discharge due to discharge maintainance potential.

(d) Measurements with Plasma:

A cylindrical glass vessel fitted with two internal aluminium electrodes at the two end-faces formed the discharge cavity. The parallel plane circular electrodes are of diameter 3.0 cm. and separated by a distance of 4.8 cm. (Fig. 6.2). The vessel with electrodes is cleaned and properly baked. Nitrogen from commercial grade cylinder is allowed to pass through different trap for cleaning and drying of the gas which is then passed to the discharge vessel via needle valve. Rotary exhaust pump coupled with needle valve is connected to discharge vessel for evacuation of nitrogen gas from the cavity. The gas pressure is measured by a pre-calibrated pirani gauge. The needle valve is adjusted to keep the gas pressure of the vessel constant at desired value. The discharge tube is connected in place of R-C combination of Fig. 6.1. Keeping the gas pressure of the vessel constant, the r.f. voltage is increased until a self sustained discharge develops. The r.f. potential is

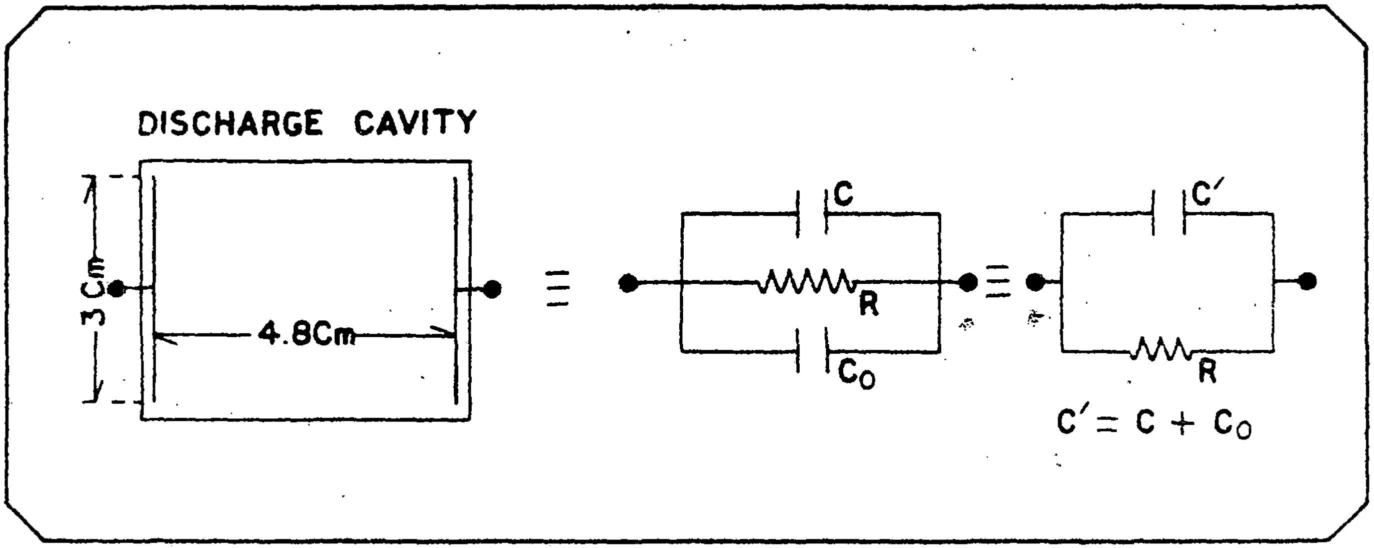


FIG. 6.2. DISCHARGE CAVITY AND ITS ELECTRICAL EQUIVALENT

then lowerdd to a value slightly above the maintainance potential. Keeping this potential constant at a certain value, the resistance R_1 is varied and potential drop across R_1 i.e. V_{R_1} and the output of A_2 i.e. V_{out} are recorded by solid state a.c. voltmeter and plotted in Figs. 6.(3,4,5) where increasing the r.f. potential in steps of 20 volts, the same procedure is repeated at every voltage step to obtain different curves of the respective figures. Fig. 6.3, Fig. 6.4 & Fig. 6.5 represent the above measurements at gas pressures of 0.2 torr, 0.15 torr and 0.1 torr respectively. Limitation of r.f. power from the source restricted our observations to the present gas pressure range studied. Too high or too low gas pressure in the present cavity requires much higher r.f. voltage to initiate the discharge in the cavity. Works in future will be undertaken to study these gas pressure regions.

The electrical equivalent of the cavity described earlier and filled with plasma excited by r.f. voltage, is shown in Fig. 6.2. Here R and C represent the resistive and capacitative impedances respectively of the plasma. C_0 represent the capacitative impedance due to electrodes for which strong vacuum displacement current flows (Francis and von Engel, 1953).

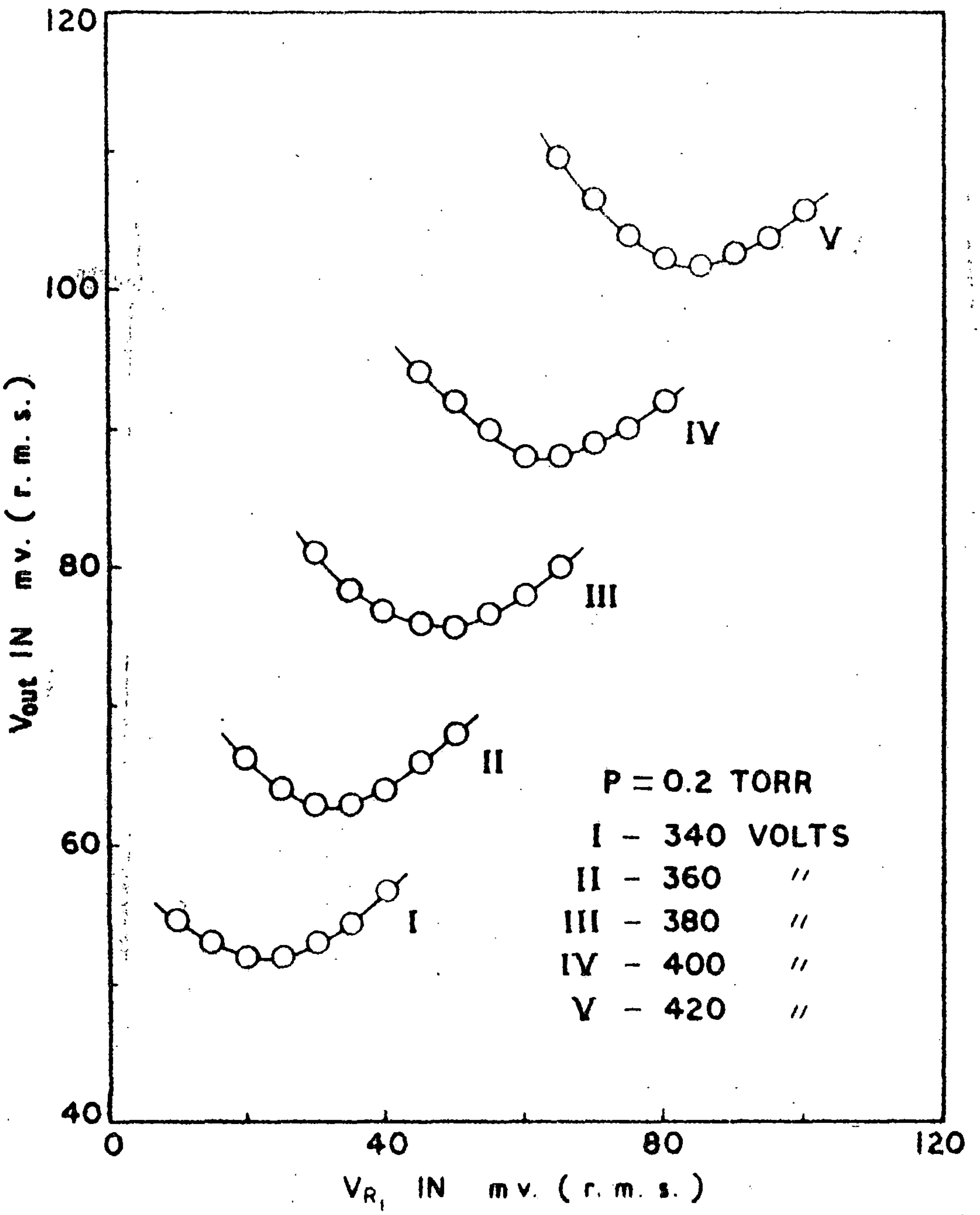
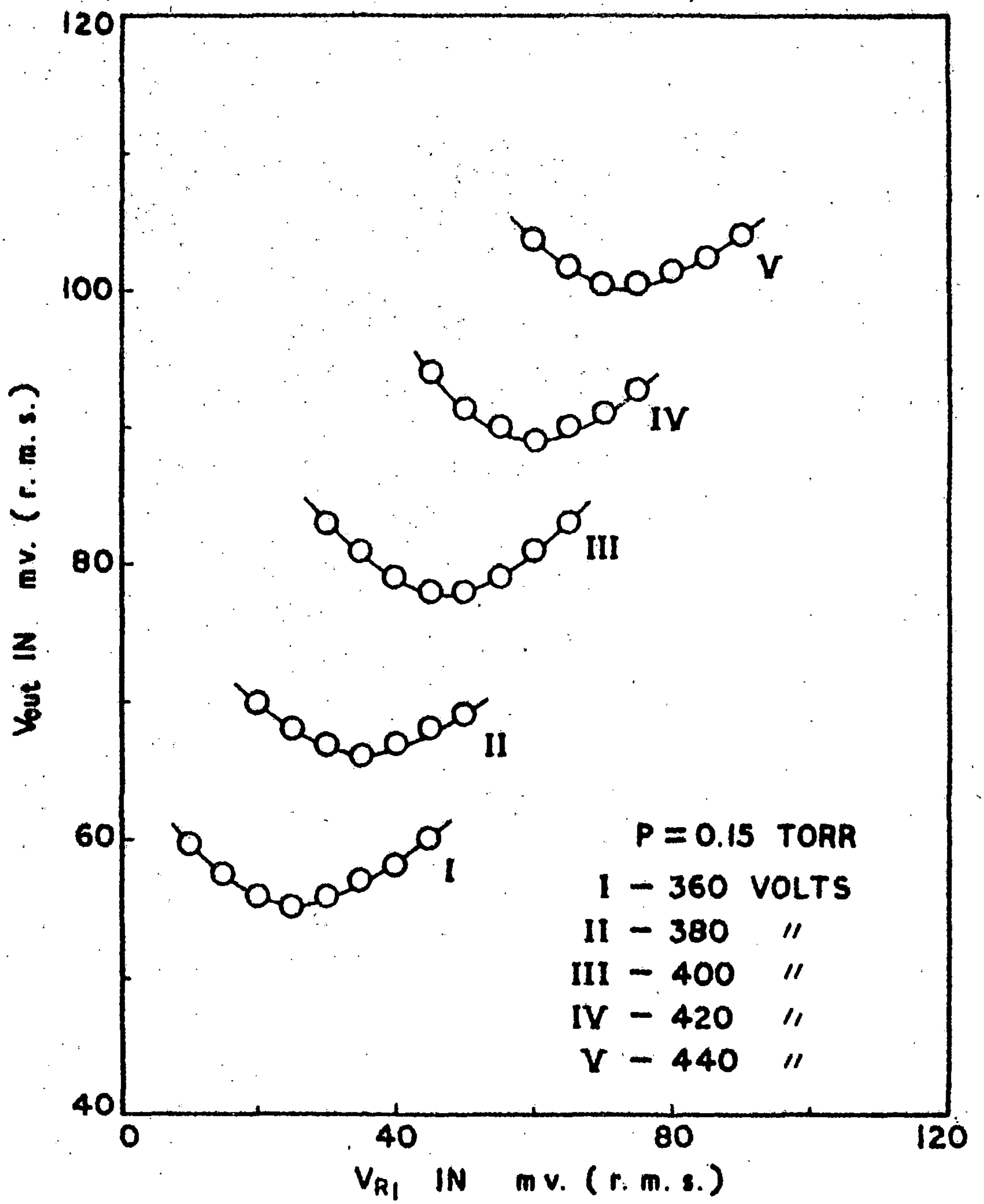

120
100
80
60
40
0
40
80
120
V_{out} IN mv. (r. m. s.)
V_{R_1} IN mv. (r. m. s.)
V
IV
III
II
I
P = 0.2 TORR
I - 340 VOLTS
II - 360 "
III - 380 "
IV - 400 "
V - 420 "

FIG. 6.3.



120
100
80
60
40
0
40
80
120
V_{out} IN mv. (r. m. s.)
V_{R1} IN mv. (r. m. s.)
I
II
III
IV
V
P = 0.15 TORR
I – 360 VOLTS
II – 380 "
III – 400 "
IV – 420 "
V – 440 "

FIG. 6.4.

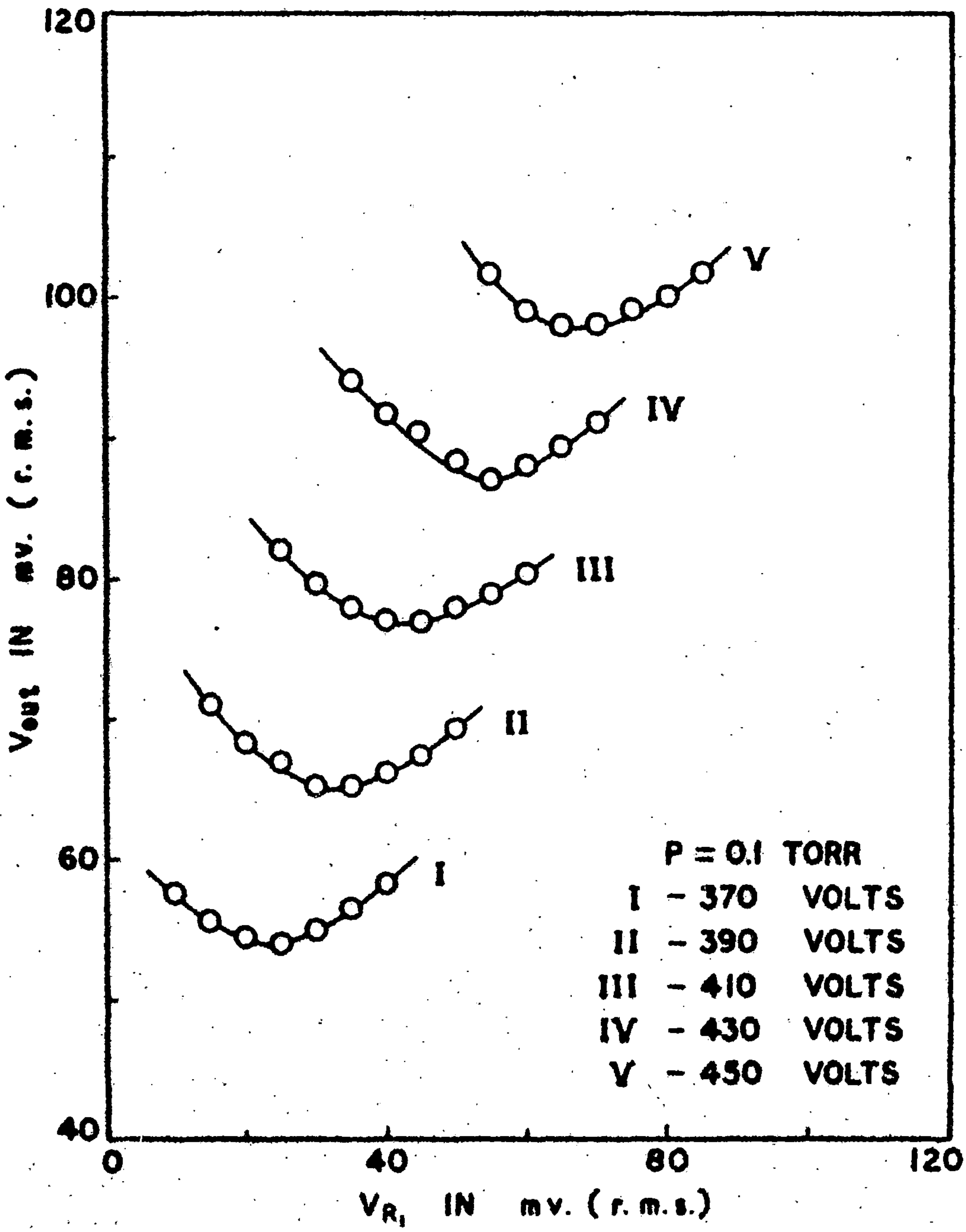

120
100
80
60
40
0
40
80
120
V_{out} IN mv. (r.m.s.)
V_{R_1} IN mv. (r.m.s.)
V
IV
III
II
I
P = 0.1 TORR
I - 370 VOLTS
II - 390 VOLTS
III - 410 VOLTS
IV - 430 VOLTS
V - 450 VOLTS

FIG. 6.5.

The current flowing through the cavity is given by

$$\vec{I} = \frac{\vec{V}}{R} + i\omega C' \vec{V} \qquad \text{...(6.11)}$$

Here $$\vec{I}_R = \frac{\vec{V}}{R} \qquad \text{... (6.12)}$$

and

$$\vec{I}_{C'} = i\omega C' \vec{V} \qquad \text{... (6.13)}$$

But $$C' = C + C_0 \qquad \text{...(6.14)}$$

Unless accurate value of C of the cavity is known, the individual values of

$$\vec{I}_{C_0} = i\omega C_0 \vec{V} \qquad \text{... (6.15)}$$

and

$$\vec{I}_C = i\omega C \vec{V} \qquad \text{... (6.16)}$$

can not be calculated. Works are in progress in this lab. to isolate $\vec{I}_{C_0}$ and $\vec{I}_C$ and the results will be communicated soon for publication.

From the curve of V_{out} vs. V_{R_1} at the minimum value of V_{out}, the corresponding V_{R_1} when divided by the value of 'r' yields the value of I_ℓ . The minimum value of V_{out} when divided by the value of 'r' yield the value of $I_{c'}$. Hence knowing 'V', the applied r.f. potential both R, the resistance of the plasma and $X_{c'}$, the effective capacitative impedance can be calculated for different applied r.f. voltage. As R and V are known, the variation of current (I_R) flowing through the plasma due to real part of conductivity for the variation of V is obtained and plotted in Fig. 6.6 against 'V' for three gas pressures.

If it is assumed that the plasma between the electrodes is uniform with no space gradient of the charged particles i.e. electrons, then the real part of the plasma conductivity can be related to plasma resistance by

$$R = \frac{1}{\sigma_r} \cdot \frac{d}{A} \qquad (6.17)$$

where 'd' is the separation of the electrodes and 'A' is the area of the electrodes. Knowing 'd' and 'A' and from the measured value of 'R', σ_r is calculated for discharges of three gas pressures and plotted against applied r.f. voltage in Fig. 6.7.

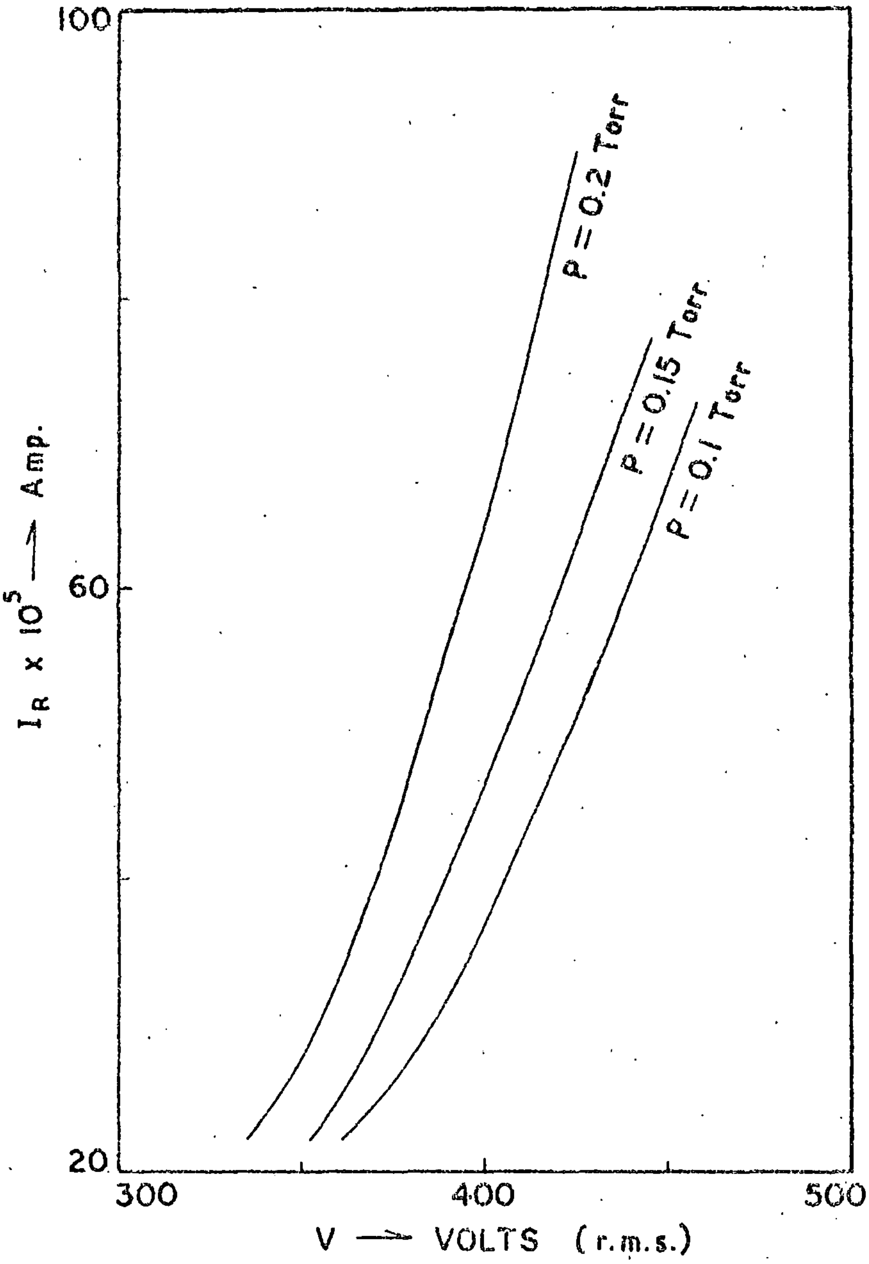
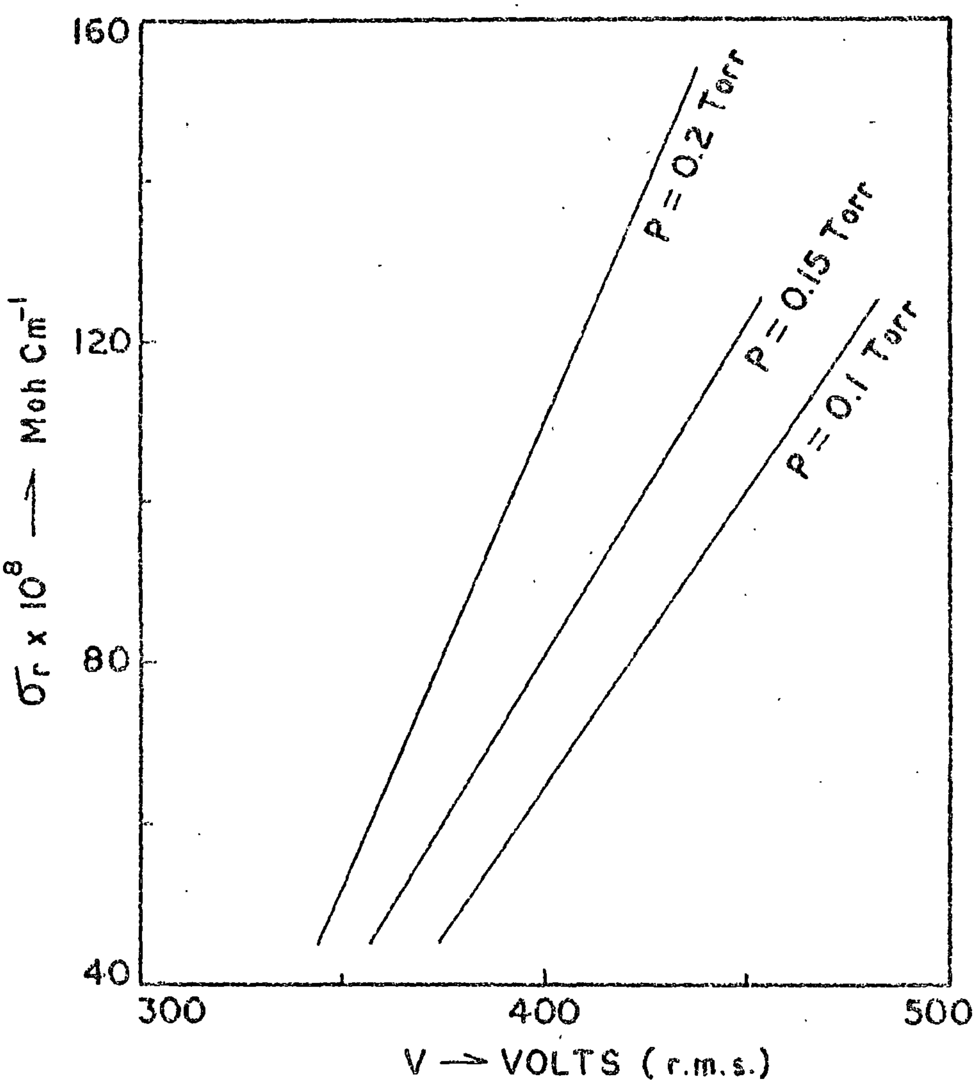

100
60
20
300
400
500
$I_R \times 10^5 \longrightarrow$ Amp.
V ⟶ VOLTS (r.m.s.)
P = 0.2 Torr
P = 0.15 Torr
P = 0.1 Torr

FIG. 6.6.



160
120
80
40
300
400
500
$\sigma_r \times 10^8 \longrightarrow$ Moh Cm^{-1}
$V \longrightarrow$ VOLTS (r.m.s.)
P = 0.2 Torr
P = 0.15 Torr
P = 0.1 Torr

FIG. 6.7.

RESULTS AND DISCUSSION

A general solution of the differential equation giving the velocity distribution function for the electrons in a plasma is extremely difficult with gradients. The simplest procedure is to assume a uniform plasma with no space gradients and to neglect all source terms. In strong electric field when gas temperature is much less compared to electron temperature and if it is assumed that the energy loss factor χ and the mean free path $\lambda_e = v/\nu_c$ of the electrons are constants, then the velocity distribution of electrons follows the Druyvesteyn distribution given by

$$f_0 = A_0 \exp(-\gamma v^4) \tag{6.18}$$

where A_0 and γ are constants and 'v' is the velocity of electron.

To evaluate the value of γ for Druyvesteyn distribution we proceed as follows:

The general form of electron velocity distribution is $f_0 = A_0 \exp(-W)$

$$\text{where } W = \int_0^v \frac{mv\,dv}{kT_g + \dfrac{2e^2E^2}{3m\chi(\nu_c^2+\omega^2)}} \tag{6.19}$$

and T_g = gas temperature,

E = r.m.s. value of the applied high frequency field,

ω = frequency of the applied field,

ν_c = collision frequency of the electrons with neutral atoms.

For Druysteyn distribution kT_g term is much smaller compared to electric field term. For the case $\nu_c \gg \omega$ as in the present case, we get putting

$$W = \int_0^v mv \left[\frac{2e^2E^2}{3m\nu_c^2\chi}\right]^{-1} dv$$

$$= \int_0^v mv \cdot \frac{3}{2} \cdot \frac{v^2 \chi}{\lambda_e^2 \left(\frac{e}{m}E\right)^2} dv$$

or
$$W = \frac{3}{8} \cdot \frac{\chi}{\left(\frac{e}{m} \cdot E\lambda_e\right)^2} \cdot v^4 \tag{6.20}$$

Hence
$$\gamma = \frac{3}{8} \cdot \frac{\chi}{\left(\frac{e}{m} \cdot E\lambda_e\right)^2} \tag{6.21}$$

The total number density of electrons is

$$n = 4\pi \int_0^\infty f_0 v^2 dv = 4\pi A_0 \int_0^\infty e^{-\gamma v^4} v^2 dv \tag{6.22}$$

Putting $\gamma v^4 = x^4$ we get

$$n = 4\pi A_0 \int_0^\infty \gamma^{-3/4} e^{-x^4} x^2 dx$$

$$= 4\pi A_0 \gamma^{-3/4} \left(\tfrac{1}{4}\right) \Gamma\left(\tfrac{3}{4}\right)$$

$$= \pi A_0 \gamma^{-3/4} \Gamma\left(\tfrac{3}{4}\right)$$ where Γ is gamma function.

Putting the value of γ

$$n = \left(\frac{8}{3}\right)^{3/4} \pi A_0 \Gamma\left(\frac{3}{4}\right) \frac{\left(\frac{e}{m} E\lambda_e\right)^{3/2}}{\chi^{3/4}} \tag{6.23}$$

When collision frequency is proportional to velocity i.e. constant mean free path, the energy loss factor χ is independent of velocity and when the frequency of the applied field is low enough to satisfy the relation

$$(\sqrt{3\chi})\,\lambda_e m \omega^2 \ll 2eE$$

then the real and imaginary part of the conductivity of a plasma in presence of external strong electric field i.e. when velocity distribution is Druyvesteyn is given by (Shkarofsky et al. 1966)

$$\sigma_r = \frac{\sqrt{2\pi}\, n\, e^{3/2} \chi^{1/4}}{3^{3/4}\,\Gamma(\frac{3}{4})\left(\frac{mE}{\lambda_e}\right)^{1/2}} \qquad (6.24)$$

$$\sigma_i = -j \cdot \frac{2\Gamma(\frac{5}{4})}{\sqrt{3}\,\Gamma(\frac{3}{4})} \cdot \frac{\lambda_e \omega e n \chi^{1/2}}{E} \qquad (6.25)$$

Putting the value of 'n' from (6.23) to (6.24) we get

$$\sigma_r = \beta E \lambda_e^2 \chi^{1/4} \qquad (6.26)$$

where $\beta = \left(\frac{8}{9}\right)^{3/4} \cdot \pi^{3/2} \cdot \frac{e^3}{m} \cdot A_0 \sqrt{2}$ and σ_r is the real part of the electrical conductivity.

As $E = \frac{V}{d}$, so eqn. (6.26) shows that σ_r is a linear function of applied potential. The variation of σ_r with the applied potential as obtained experimentally in Fig. 6.7 agrees qualitatively with the relation (6.26), variation for three gas pressures being all linear.

As plasma is uniform i.e. there is no gradient of electron density in space, so the resistance of the plasma column of cross sectional area 'A' and length 'd' is, from eqn. (6.17)

$$R = \frac{d}{A} \cdot \frac{1}{\beta E \lambda_e^2 \chi^{1/4}} = \frac{d^2}{A \beta V \lambda_e^2 \chi^{1/4}} \qquad (6.27)$$

Since the values of χ , λ_e , etc. are unknown, so R cannot be calculated from eqn. (6.27).

The current flowing through the plasma due to real part of the conductivity is

$$I_R = \frac{V}{R} = \frac{A \beta \lambda_e^2 \chi^{1/4}}{d^2} \times V^2 \qquad (6.28)$$

Eqn. (6.28) shows that the current through plasma resistance does not follow Ohm's law. The experimental observations in Fig. 6.6 also indicates sharp rise of current with increase of voltage rather than a smooth linear rise. Since the value of the coefficient of V^2 in eqn. (6.28) i.e. $A \beta \lambda_e^2 \chi^{1/4}/d^2$ is not known, so I_R can not be calculated. ~~However, if we take arbitrarily~~

CONCLUSION

The qualitative agreement between eqn. (6.26) and experimental results of Fig. 6.7 shows clearly that in an active discharge, the real part of the plasma conductivity varies linearly with the applied r.f. voltage unlike ordinary solid conductor where, conductivity value remains constant with change of applied voltage. Also for studying the bulk properties such as electrical conductivity, as done in this present experiment, the assumption of spatially uniform plasma is fairly good.

The success of the present experimental tēchnique, in isolating and measuring the discharge current due to resistive part of the plasma and its variation with the applied r.f. potential and gas pressure, may be concluded from the good qualitative ~~as well as quantitative~~ agreement with experimental and theoretical values of the measured currents and resistances. Hence the same experimental technique may be used to isolate and measure different components of discharge current in a wide variety of high frequency discharges without going for balancing the capacitance effect of the electrodes pairs for each cavity.

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