CHAPTER IV

A. RADIO FREQUENCY ELECTRIC FIELD BREAKDOWN OF GASES IN PRESENCE OF TRANSVERSE MAGNETIC FIELD (LOW INTENSITY).

B. RADIO FREQUENCY ELECTRIC FIELD BREAKDOWN OF GASES IN PRESENCE OF TRANSVERSE MAGNETIC FIELD (HIGH INTENSITY).

A. RADIO FREQUENCY ELECTRIC FIELD BREAKDOWN OF GASES IN PRESENCE OF TRANSVERSE MAGNETIC FIELD (LOW INTENSITY).

INTRODUCTION.

The condition of breakdown of a gas excited by high frequency electric field depends mainly upon the factors such as the pressure of the gas, the dimension of the discharge tube and frequency of excitation. The two dominant factors by which electrons are lost are diffusion and mobility and if the gas is an electron attaching one then by electron attachment also. It is found that when the gas pressure is high so that electronic mean free path is much smaller than the dimension of the vessel, electrons are lost mainly by diffusion whereas when the mean free path is comparable to the dimension of the vessel, the electron mobility is the chief loss process.

The experimental values of the breakdown voltage, using microwave fields, are found to be consistent with those calculated theoretically taking the above electron removal processes into consideration. To test the limitations of the diffusion theory when the gas is subjected to radio frequency electric field of a few MHz., some breakdown experiments have been performed in this laboratory when the pressure of the gas was maintained in the range of a few torr. The factors responsible for the loss of electrons namely diffusion and mobility and also the gain of energy of electrons from the electric field are affected by the presence of magnetic field. It is worthwhile to see whether the above breakdown conditions are valid in case where a magnetic field, transverse to the electric field, is applied.

The breakdown of a gas excited by a radio frequency field in presence of magnetic field either longitudinal or transverse has been studied previously. Mention may be made of the work of Townsend and Gill (1938) who carried out experiments in air for two frequencies, viz. 48 MHz. and 30 MHz. and in the range of pressure varying from a few m torr to 0.24 torr. Lax et al (1950) performed experiment on the breakdown voltage of a gas in a microwave field in presence of transverse magnetic field. The gas used was helium containing a small admixture of mercury vapour. Ferritte and Veronese (1955) performed experiments for frequencies ranging from 10 to 30 MHz. in air, the magnetic field ranging from 0 to 600 gauss. They used cylindrical electrodes and observed a lowring of breakdown potential in presence of a magnetic field. The breakdown of air and nitrogen excited by radio frequency field of frequency between 7 to 100 MHz. and within the pressure range of 10 to 300 m torr in presence of transverse magnetic field varying from 0 to 80 Gauss have been studied by Sen and Ghosh (1963). Sen and Bhattacharjee (1969) performed similar experiments on air, hydrogen, oxygen and carbondioxide using radio frequency field of frequency 17.6 MHz. and in transverse magnetic field varying from 0 to 1.8 K.Gauss.

In the present paper, the effect of transverse magnetic field on breakdown potential of gases, when excited by radio frequency electric field, in the pressure range of a few torr has been studied in case of air, hydrogen and oxygen. Attempts have been made to explain the results incorporating the possible effects of magnetic field on the different mechanisms as are postulated in the breakdown theory of gases.

EXPERIMENTAL ARRANGEMENT.

The method of measurement of breakdown voltage was the same as was used previously by Sen and Ghosh (1963). The gas was contained in a cylinder of length 0.7 cm. and radius 2.0 cm. and fitted with two external electrodes of radius 4.0 cm. each perpendicular to the length of the cavity. The radio frequency voltage was supplied from a Colpitt oscillator, the frequency of which being kept fixed at 6.2 MHz. The r.m.s. output voltage was measured by a V.T.V.M. * The pressure of the gas was measured with a mercury manometer. The discharge tube was placed between the pole pieces of an electromagnet in such a way that the lines of force being perpendicular to the discharge tube and the field was measured with a gaussmeter.

Keeping the magnetic field at a constant value, the pressure of the gas was varied and breakdown voltage determined for different values of gas pressure. The experiments were repeated and the results were found to be reproducible.

Hydrogen was prepared by the electrolysis of x warm saturated solution of barium hydroxide taken in a hard glass U-tube fitted with nickel electrodes. The gas was passed over hot platinum gauge to burn any oxygen in it. The gas was then dried by passing it through caustic potash and finally through pure redistilled phosphorous pentoxide.

Oxygen was prepared by heating potassium permanganate. The oxygen gas so formed was washed with water and dried by passing it through fused calcium chloride, caustic potash and finally through pure redistilled phosphorous pentoxide.

Air was first washed with water and then dried by passing it through fused calcium chloride, caustic potash and finally through pure redistilled phosphorous pentoxide.

RESULTS AND DISCUSSION.

The radio frequency electrical breakdown potentials of hydrogen, oxygen and air at different gas pressures were measured in presence of transverse magnetic field. The variation of breakdown voltage against pressure have been plotted in Figs. 4.1, 4.2 and 4.3 for hydrogen, oxygen and air respectively. The values of the magnetic field for which variation of breakdown voltage against pressure have been measured are 0, 350 and 500 Gauss.

It is found that in absence of the magnetic field, the breakdown potential attains a minimum for all the three gases studied. The pressure at which the breakdown voltage becomes minimum and minimum breakdown voltage are different for different gases. The general nature of the variation of breakdown voltage with pressure remains unaltered under the application of the magnetic field for all the gases studied. In presence of the magnetic field, however, and at pressures where the collision frequencies are either lower than or comparable to the electron cyclotron frequency, magnetic field greatly reduces the breakdown voltage; ~~by~~ but at

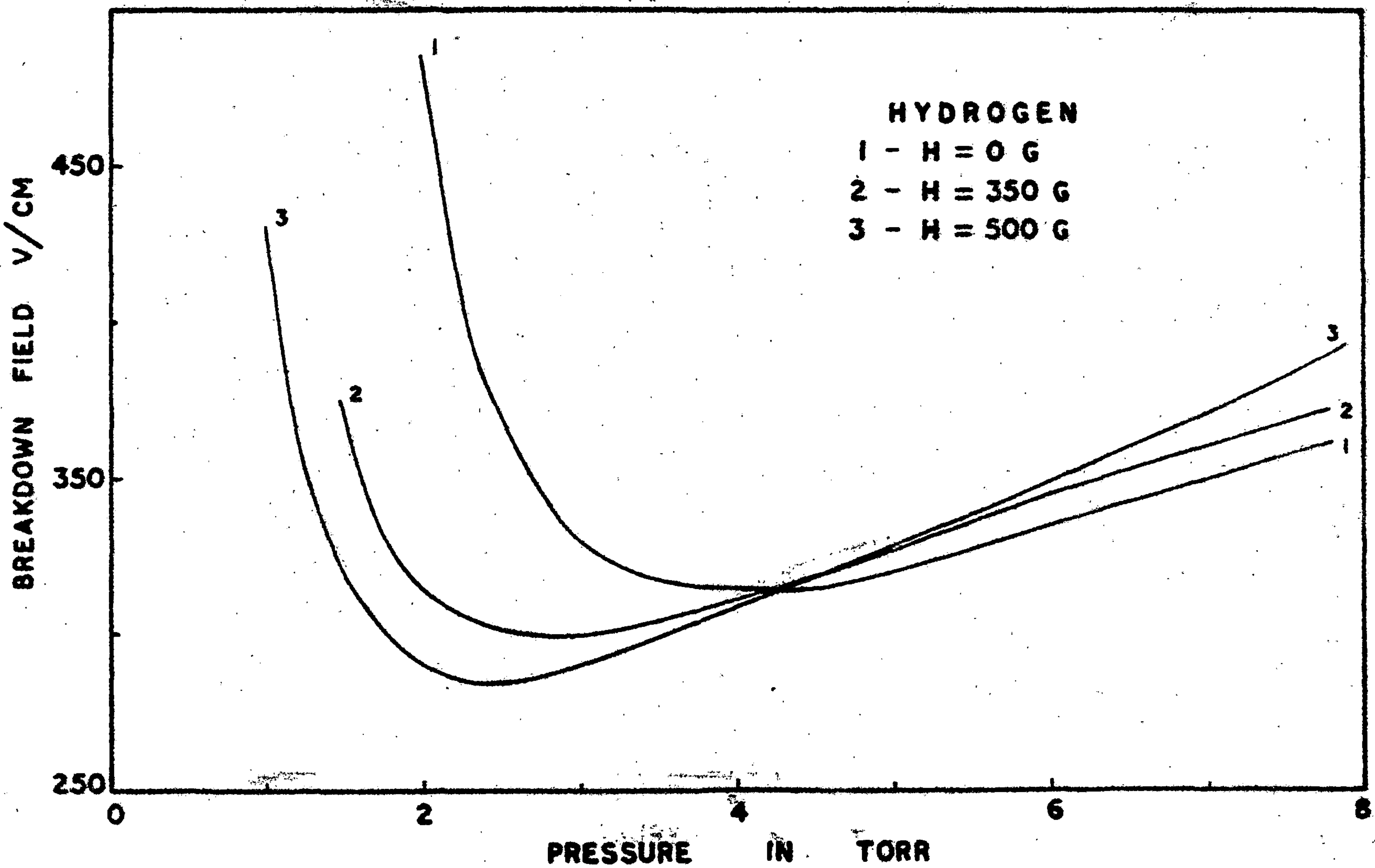
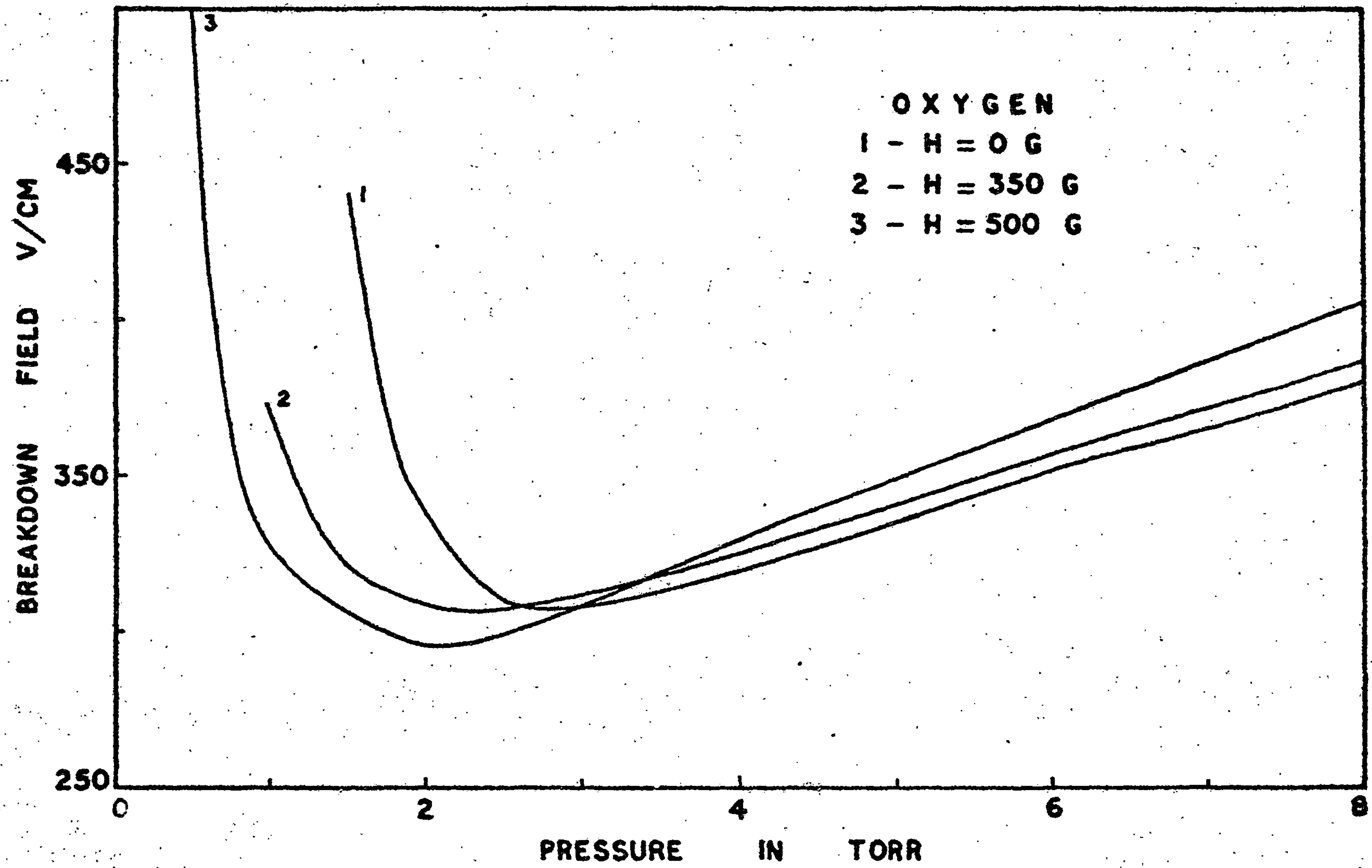
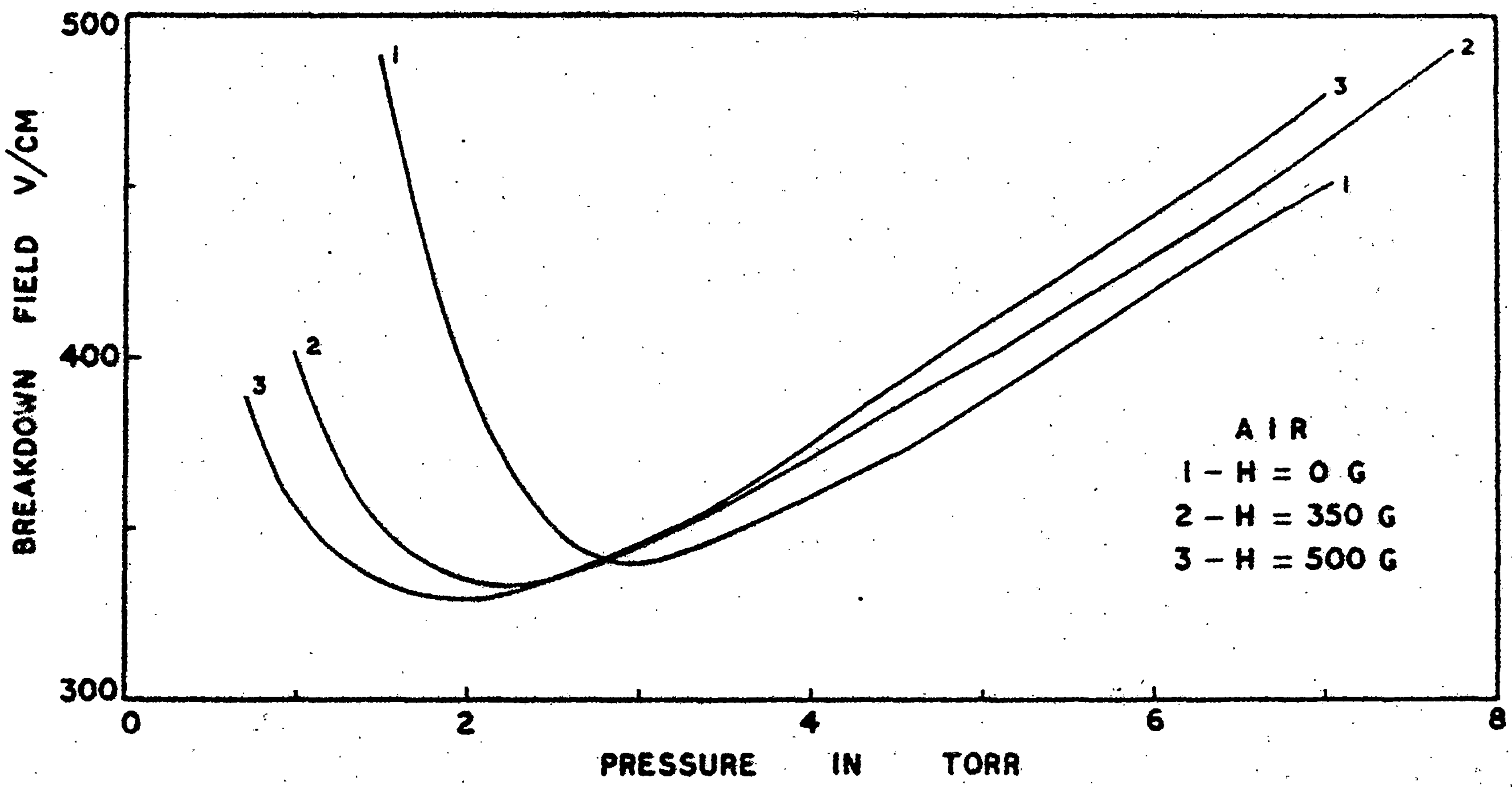

HYDROGEN
1 - H = 0 G
2 - H = 350 G
3 - H = 500 G
450
350
250
0
2
4
6
8
1
2
3
3
2
1
BREAKDOWN FIELD V/CM
PRESSURE IN TORR

FIG. 4.1.



OXYGEN
1 - H = 0 G
2 - H = 350 G
3 - H = 500 G
BREAKDOWN FIELD V/CM
450
350
250
0
2
4
6
8
PRESSURE IN TORR
1
2
3

FIG. 4.2.



500
400
300
BREAKDOWN FIELD V/CM
0
2
4
6
8
PRESSURE IN TORR
1
2
3
A I R
1 – H = 0 G
2 – H = 350 G
3 – H = 500 G

FIG. 4.3.

pressures where collision frequencies are much higher than the cyclotron frequency, increases of breakdown voltages are observed. Also the pressure at which the breakdown voltage becomes minimum shifts towards lower values of the pressure.

In order to ascertain which process is dominant in electron removal under the present experimental set up, the following points have been considered:-

i) According to Francis (1960), for the diffusion theory of breakdown to be valid, the dimension of the discharge tube must be small compared with the wavelength of the applied radio frequency voltage. As the wavelength is approximately 48.4 meter and the length of the discharge tube is 0.7 cm. and diameter 4.0 cm., this condition is satisfied.

ii) The maximum mean free path of the electrons in gases used here is less than 0.2 cm., at pressure of 1 torr. as is given by Townsend (1947), which is much smaller than the length and radius of the discharge tube. For higher pressure, the mean free path will be still smaller.

iii) The collision frequency $\nu_c = v_r/\lambda_e$ where v_r is the random velocity and λ_e the mean free path of electrons, is of the order of 10^9 collision/sec. and is much greater than frequency (6.2 MHz.) of the applied radio frequency field even at a pressure of 1 torr.

iv) The amplitude of electron oscillation when calculated from the equation (Francis, 1960)

$$x = \frac{q\,E_\circ}{m\,\omega}\,\frac{1}{\sqrt{\omega^2 + \nu_c^2}}$$

where $E_\circ$ is the peak value of the electric field and ω is the frequency of the applied field, is found to be less than 0.05 cm. at pressure of 1 torr. The amplitude of oscillation will be still smaller at higher pressures.

Under the above conditions it is thus apparent that the electrons make many oscillations of small amplitude because the motion is restricted by collisions and the cloud of electrons appear stationary, spreading out only by diffusion. Hence the loss due to drift can be neglected. New charged particles are formed due to ionizing collisions and the loss due to diffusion ~~in~~ only predominates.

The condition of breakdown of a gas is given by (Brown, 1956)

$$\frac{\nu_i}{D} = \frac{1}{\Lambda^2} \qquad \text{... (4.1)}$$

where ν_i = ionization frequency of electrons .

D = diffusion coefficient of electrons

Λ = characteristic diffusion length.

If μ be the coefficient of mobility of electrons in the gas and 'E' be the effective applied field, then the ionization frequency is given by (Allis, 1956)

$$\nu_i = \alpha \mu E \qquad \ldots (4.2)$$

where α is the Townsend's first ionization coefficient and is given by Townsend's empirical equation as

$$\frac{\alpha}{p} = A_o \exp\left(-\frac{B_o p}{E}\right) \qquad \ldots (4.3)$$

where A_o and B_o are constants, characteristic of the gas and 'p' is the pressure of the gas. Combining eqns. (4.1), (4.2) and (4.3) the breakdown condition becomes

$$A_o E p \ (\mu/D) \ \exp. \ (- B_o p/E) = \frac{1}{\Lambda^2} \qquad \ldots (4.4)$$

Allis (1956) has shown that when collision frequency is independent of energy of electrons, the ratio

$$\frac{D}{\mu} = \frac{2}{3} u_{avg}$$

where u_{avg} is the average energy of electrons. Writing u_{avg} in terms of electron temperature T_e

$$\frac{D}{\mu} = \frac{kT_e}{e}$$

where k = Boltzman's constant

e = electronic charge.

A mathematical expression involving T_e and E/p has been deduced by von Engel (1965) as

$$\frac{D}{\mu} = \frac{kT_e}{e} = 0.47 \cdot \frac{L}{\chi^{1/2}} \left(\frac{E}{p}\right) \qquad \ldots (4.5)$$

where 'L' is the mean free path of electron at a pressure of 1 torr and χ is fraction of energy lost by electron per collision. For elastic collisions χ is a constant, but it becomes a function of E/p when collisions are ~~x~~ inelastic (von Engel, 1965). From eqns. (4.4) and (4.5) the breakdown condition becomes

$$\frac{A_o p^2}{L} \cdot \frac{\chi^{1/2}}{0.47} \exp\left(-\frac{B_o p}{E}\right) = \frac{1}{\Lambda^2} \qquad \ldots (4.6)$$

For a cylindrical cavity of length 'd' and radius 'r' the characteristic diffusion length is given by

$$\frac{1}{\Lambda^2} = \left(\frac{\pi}{d}\right)^2 + \left(\frac{2.405}{r}\right)^2$$

Following Townsend (1947) it is assumed that $L \approx 1/A_o$.
If E_o exp. ($j\omega t$) be the applied field, the effective equivalent d.c. field 'E' is given by

$$E^2 = \frac{E_o^2}{2} \cdot \frac{\nu_c^2}{\nu_c^2 + \omega^2}$$

which is for the present experimental condition becomes $E^2 \approx E_0^2/2$ = square of r.m.s. value of applied high-frequency field as $\nu_c \gg \omega$. Eqn. (4.6) can then be written as

$$\frac{A_0^2 p^2 \chi^{1/2}}{0.47} \cdot \exp\left(-\frac{B_0 p}{E}\right) = \left(\frac{\pi}{d}\right)^2 + \left(\frac{2.405}{r}\right)^2 \qquad \ldots (4.7)$$

The values of A_0 and B_0 for hydrogen, oxygen and air are obtained by fitting the Townsend's empirical equation (4.3) to experimental values of α/p for different E/p values, given in standard literature for the range of E/p comparable with the present experimental range, which is the usual procedure. The calculated values are given in Table-4.1 with respective range of E/p in which they are valid

TABLE-4.1

Gas	A_0 Ion pair cm^{-1} $torr^{-1}$	B_0 Volt cm^{-1} $torr^{-1}$	E/p range volt cm^{-1} $torr^{-1}$	E_{min} volt cm^{-1}	p_{min} from eqn. (4.8) torr	p_{min} from observation torr.
Air	8.2	222	50–250	340	3.06	3.0±0.1
Hydrogen	5.5	143	50–250	315	4.00	4.0±0.1
Oxygen	7.4	200	25–120*	305	3.05	2.8±0.1

* No experimental results available for wider range.

Eqn. (4.7) may be compared with the results of the present observations provided the value of χ is known. Non-availability of any experimental values of χ and its variation with E/p makes the comparison difficult. However, if χ is assumed to be constant in the range of E/p values of the present observation as an approximation, then from eqn. (4.7) the pressure p_{min} at which the breakdown potential will attain minimum value E_{min} can be obtained as

$$p_{min} = \frac{2E_{min}}{B_o} \quad \ldots (4.8)$$

Using the observed values of E_{min}, the values of p_{min} are obtained from eqn. (4.8) and compared with the experimental values of p_{min} in Table-4.1. The discripency in values of p_{min} between theoretical ~~xkaxxka~~ calculation and experimental results may be due to the assumption that has been treated as constant in deducing eqn. (4.8). Considering the gain of energy by electrons suffering inelastic collisions with the atoms in presence of magnetic field transverse to high frequency electric field and steady d.c. magnetic field H, Allis (1956) has shown that the effect of high frequency electric field in transfering the energy to the electrons can well be represented by an effective field E_{eff} given by

$$E_{eff}^2 = \frac{E_o^2}{2} \cdot \frac{\nu_c^2\left(\nu_c^2 + \omega^2 + \omega_b^2\right)}{\left[\nu_c^2 + (\omega + \omega_b)^2\right]\left[\nu_c^2 + (\omega - \omega_b)^2\right]} \qquad \ldots (4.9)$$

where $\omega_b = \frac{qH}{m}$ is the electron cyclotron frequency. In the present experiment ν_c & ω_b are much greater than ω and hence eqn. (4.9) reduces to

$$E_{eff}^2 \approx \frac{E_o^2}{2} \cdot \frac{\nu_c^2}{\nu_c^2 + \omega_b^2} = E^2 \cdot \frac{\nu_c^2}{\nu_c^2 + \omega_b^2} \qquad \ldots (4.10)$$

where $E = E_o / 2^{1/2}$ is the r.m.s. value of the high-frequency field. The concept of eqn. (4.9) is useful when ν_c is independent of electron velocity because then this single function takes into account the effect of frequency and magnetic field on the energy of electron (Allis, 1956). For air, hydrogen and oxygen ν_c is nearly independent of energy of electrons and only function of gas pressure. The a.c. mobility measurements of oxygen by Nielson and Bradbury (1937) gave the value of $\nu_c = 3.5 \times 10^9 p$ for oxygen and by Nielson (1936) and Riemann (1944) yields for air $\nu_c = 4.3 \times 10^9$ p. From the experimental value of collision probability Brown (1959) obtained the value of collision frequency for hydrogen and is equal to 4.8×10^9p.

It was shown by Lax et al. (1950) that the effect of magnetic field is equivalent to expanding the cavity in the ratio $(\nu_c^2 + \omega_b^2)/\nu_c^2$ in all directions perpendicular to the magnetic field. As the breakdown measurements are made in a cylindrical cavity with the magnetic field transverse to the axis, the effective diffusion length Λ_e for the whole cavity is given by

$$\frac{1}{\Lambda_e^2} = \frac{\nu_c^2}{\nu_c^2+\omega_b^2}\left[\left(\frac{\pi}{d}\right)^2 + \frac{1}{2}\left(\frac{2\cdot405}{r}\right)^2\right] + \frac{1}{2}\left(\frac{2\cdot405}{r}\right)^2$$

$$= \frac{\nu_c^2}{\nu_c^2+\omega_b^2}\cdot\Lambda_o + \Lambda_r \qquad \ldots (4.11)$$

where $\Lambda_o = \left(\frac{\pi}{d}\right)^2 + \frac{1}{2}\left(\frac{2\cdot405}{r}\right)^2$ and $\Lambda_r = \frac{1}{2}\left(\frac{2\cdot405}{r}\right)^2$
Replacing Λ by Λ_e and E by E_{eff} in eqn. (4.6) from eqn. (4.10) and (4.11) respectively the breakdown condition in a transverse d.c. magnetic field becomes

$$\frac{A_o^2 p^2 \chi^{1/2}}{0\cdot47}\cdot \exp\left[-\frac{B_o p}{E_H}\left(\frac{\nu_c^2+\omega_b^2}{\nu_c^2}\right)^{1/2}\right] = \frac{\nu_c^2 \Lambda_o}{\nu_c^2+\omega_b^2} + \Lambda_r \qquad \ldots (4.12)$$

where E_H is the breakdown field in presence of the magnetic field. Eqn. (4.12) gives the relation between breakdown field E_H, pressure 'p' and the magnetic field 'H'. Comparing eqns. (4.7) and (4.12) for the same pressure 'p', the relation of the breakdown voltage with and without magnetic field is

$$\frac{E}{E_H} = \left(\frac{\nu_c^2}{\nu_c^2+\omega_b^2}\right)^{1/2}\left[1 - \left(\frac{E}{B_o p}\right)\log_e\left\{\frac{\frac{\nu_c^2}{\nu_c^2+\omega_b^2}\cdot\Lambda_o+\Lambda_r}{\Lambda_o+\Lambda_r}\right\}\right] \quad \ldots (4.13)$$

Taking the experimental observed values of 'E' at different pressures, the ratio is plotted using eqn. (4.13) for H = 500 G and compared with some of the representative points of the observed ratio (obtained from experimental breakdown potentials as given in Figs. 4.1, 4.2 & 4.3) for H = 0 and H = 500 G and are shown in Figs. 4.4, 4.5 & 4.6 for oxygen air and hydrogen respectively. The representative points agree fairly well with eqn. (4.13) for all the three gases.

From eqn. (4.12) the pressure p_{HM} at which the breakdown field will be minimum $(E_H)_{min}$ is given by

$$\frac{B_o p_{HM}}{2(E_H)_{min}} = \left(\frac{\nu_c^2+\omega_b^2}{\nu_c^2}\right)^{1/2} - \frac{\omega_b^2\Lambda_o\left(\frac{\nu_c^2}{\nu_c^2+\omega_b^2}\right)^{1/2}}{\nu_c^2\Lambda_o+(\nu_c^2+\omega_b^2)\Lambda_r} \quad \ldots (4.14)$$

No simple expression can be obtained as ν_c is a function of pressure. However, some simplification is possible if 'r' is assumed very large so that $\Lambda_r \approx 0$ when eqn. (4.14) reduces to

$$p_{HM}^2 = \left[2 \cdot \frac{(E_H)_{min}}{B_o}\right]^2 - \frac{\omega_b^2}{a^2} \quad \ldots (4.15)$$

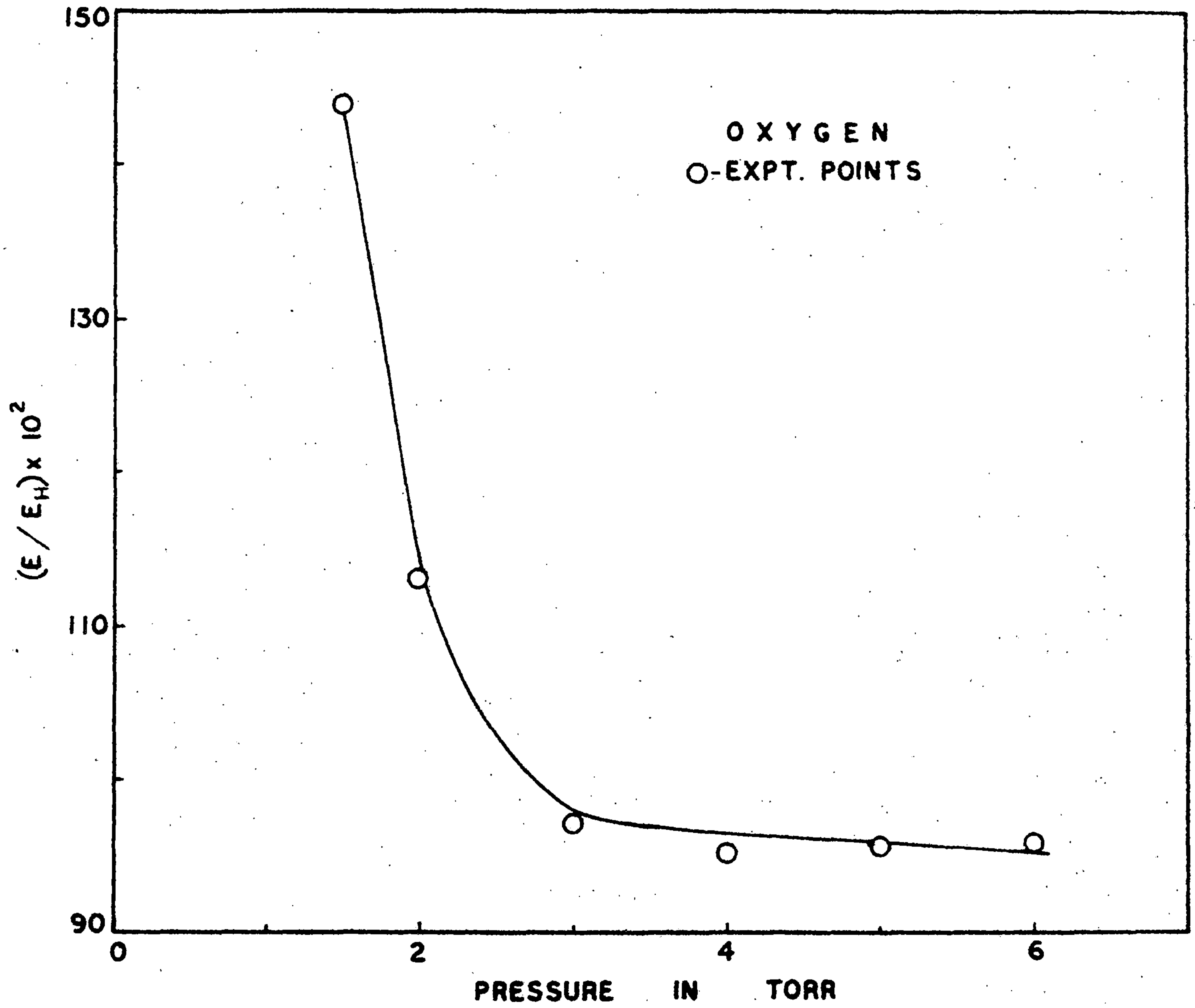

150
130
110
90
$(E/E_H) \times 10^2$
OXYGEN
O-EXPT. POINTS
0
2
4
6
PRESSURE IN TORR

FIG.4.4

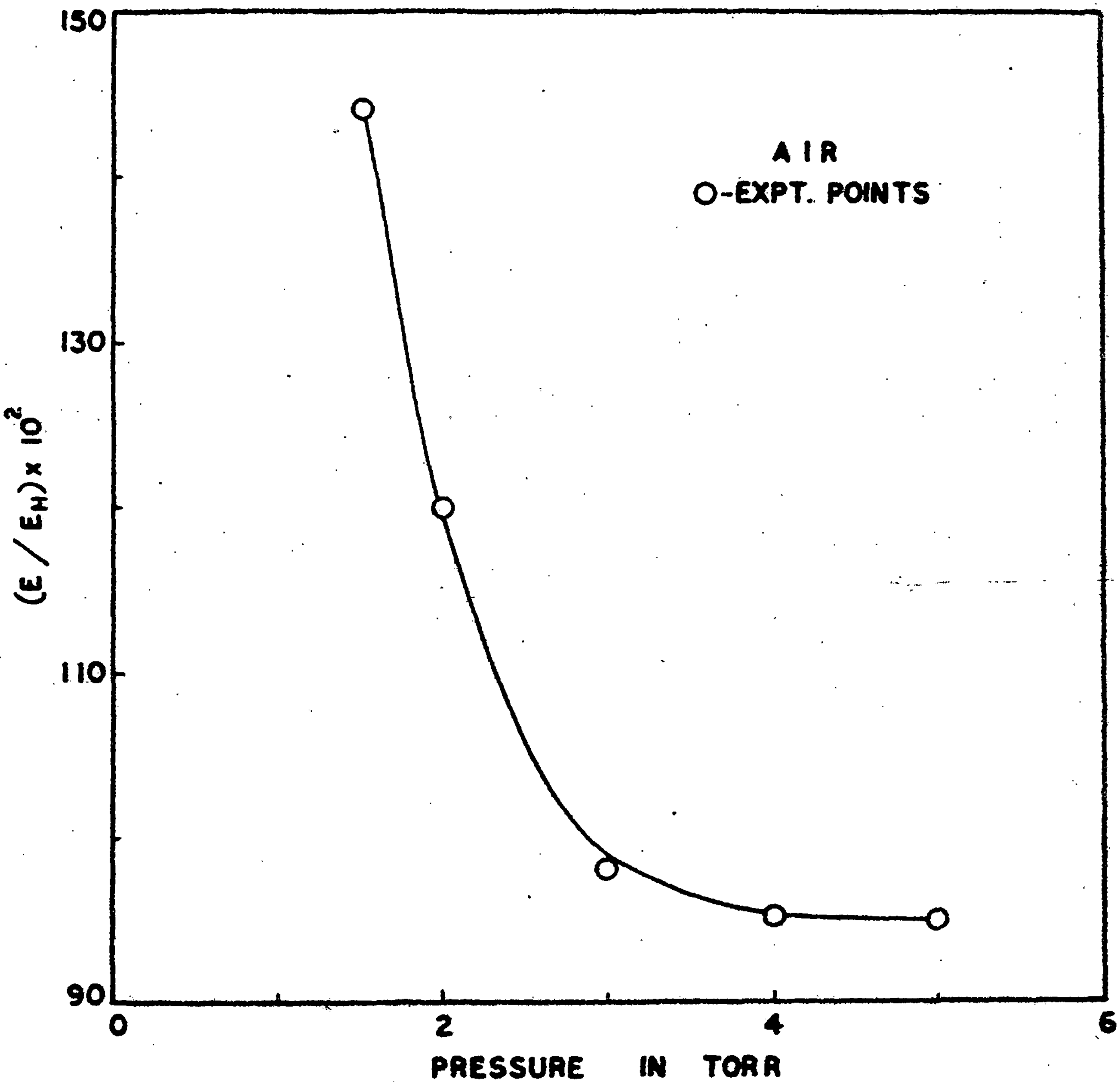

150
130
110
90
$(E/E_H) \times 10^2$
A I R
O-EXPT. POINTS
0
2
4
6
PRESSURE IN TORR

FIG. 4.5.

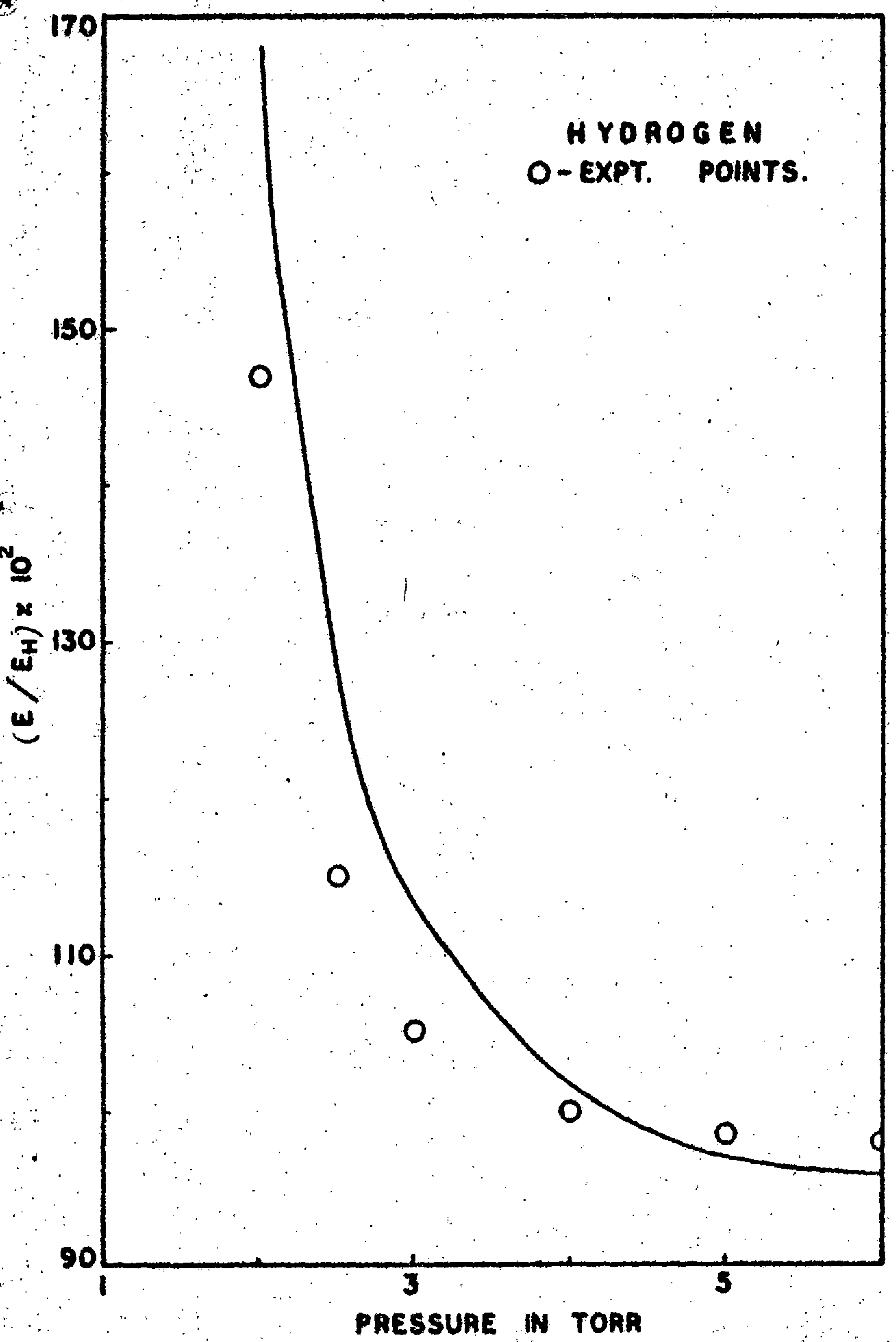

HYDROGEN
O - EXPT. POINTS.
170
150
130
110
90
(E / E_H) x 10^2
1
3
5
PRESSURE IN TORR

FIG. 4.6.

where "a" is a constant given by $\nu_c = a\ p$.

Eqn. (4.15) shows that with increase of magnetic field the pressure at which the breakdown potential is minimum shifts towards lower pressure which is also enhanced by the decrease of $(E_H)_{min}$ with increase of the magnetic field. Using experimental values of $(E_H)_{min}$ the values of p_{HM} are calculated from equation (4.15) and compared with the experimentally observed values in Table-4.2 for the fixed values of the magnetic field.

TABLE-4.2.

Gas	H Gauss.	$(E_H)_{min}$ V/cm.	p_{HM} from eqn.(4.15) torr.	p_{HM}^{*} from observation torr.
Air	350	325	2.5	2.3
	500	320	2.0	2.0
Oxygen	350	305	2.4	2.4
	500	295	1.4	2.1
Hydrogen	350	300	3.9	3.0
	500	285	3.5	2.4

* Deviation $\pm$ 0.1 torr.

CONCLUSION

It can thus be concluded that the very high frequency breakdown theory, when modified for radio frequency, can explain the present experimental results with fair amount of agreement. Present theory successfully accounted for the shift of pressure towards lower values corresponding to minimum breakdown potential when magnetic field is "on" taking the assumption which is nearly valid for the present experimental set up. Also the change of breakdown potential with magnetic field in the pressure range of this investigation has been explained using the present theory. When ν_c is comparable to or less than ω_b, the effect of reduction in electron loss by diffusion predominates over the effect of reduction in energy gain of the electrons from the applied field when magnetic field is "on" and as a result a decrease of breakdown potential is observed. But when ν_c is much higher than ω_b, the capability of the magnetic field in confining the electron within the discharge region decreases rapidly with increase of gas pressure and an increase of breakdown potential is observed when the magnetic field is "on".

The quantitative discrepency between theoretical and experimental observations may be attributed to the variation of χ with E/p for inelastic collisions which has not been considered in the present theory and also to the uncertainties in the values of different parameters used and some of the simplifying assumptions made in the theoretical deduction. However, it can be concluded that the present theory has explained the different results with fair amount of success.

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Electrical Breakdown of Gases in External Magnetic Field

The variations of breakdown potentials of dry air, oxygen and hydrogen using rf electric field (6·2 MHz) have been studied in the range of gas pressure 0·5-8 torr and also in the presence of uniform magnetic fields (350 G, 500 G). transverse to rf electric field. The theory of breakdown using high frequency electric field has been modified for crossed rf electric and steady magnetic fields to explain the results.

The diffusion theory of electrical breakdown of gases is found to be consistent with the experimental observations using the microwave electric field.[1] To test the limitations of the diffusion theory when the gas is subjected to radio frequency (rf) excitation of a few MHz, some experiments have been performed in this laboratory, the pressure of the gas being maintained in the range of a few torr when electron diffusion is recognized as the dominant electron removal process. It is worthwhile to see whether the above breakdown conditions are valid in the case where a magnetic field, transverse to electric field, is applied, because the factors responsible for the loss of electrons, namely diffusion and also the gain of energy of electrons from the electric field, are affected by the presence of magnetic field. The experimental arrangement was similar to that used previously by Sen and Bhattacharjee.[2] The cylindrical vessel (length 0·7 cm, radius 2 cm) containing the gas was fitted with two external electrodes (radius 4 cm each), which are connected to oscillator output of frequency 6·2 MHz.

Electrical breakdown potentials of dry air, oxygen and hydrogen at different gas pressures were measured with uniform magnetic field (values 0, 350 and 500 G) perpendicular to the axis of the cylinder and the results are plotted in Fig. 1. It is observed that the breakdown potential becomes a minimum for all the three gases studied at different pressures in the absence of magnetic field. Application of magnetic field leaves the general nature of variation of the breakdown potential with pressure unaltered. However, at pressure where $\nu_c \lesssim \omega_b$ (ν_c the collision frequency, ω_b = electron cyclotron frequency) application of magnetic field highly reduces the breakdown potential; but at pressures when $\nu_c \gg \omega_b$ the breakdown potential is raised. Also, the pressure at which the breakdown potential becomes minimum shifts towards lower values when the magnetic field is increased for the gases studied.

The breakdown condition for high frequency electric field is given by[1]

$$\nu_i / D = 1/\Lambda^2 \qquad \ldots(1)$$

where ν_i = ionisation frequency of electrons D = electron diffusion coefficient Λ = characteristic diffusion length.

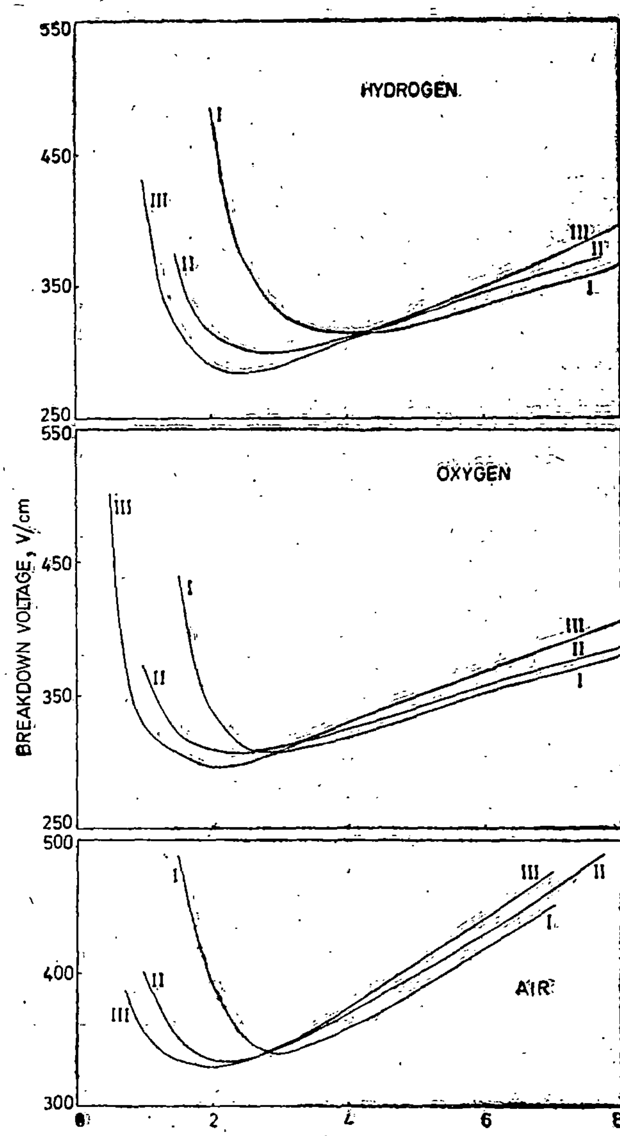

Fig. 1—Variation of breakdown voltage with pressure for different gases, hydrogen, oxygen and air [For curve I, H = 0 G; II, H = 350 G and III, H = 500 G]

Using Townsends' empirical equation for the first ionization coefficient, ν_i can be expressed as[3]

$$\nu_i = A_0 P \mu E \exp(-B_0 P/E) \quad \ldots(2)$$

where A_0 and B_0 are constants characteristic of the gas. E is the applied effective dc electric field, P is the pressure of the gas and μ is the mobility coefficient of electrons in the gas. When ν_c is independent of energy of the electrons, the ratio D/μ can be written as[3-5]

$$D/\mu = 0{\cdot}47\,(L/\chi^{1/2})\,(E/P) \quad \ldots(3)$$

where L=mean free path of electron at 1 torr=$1/A_0$ (Townsend[5]), χ = fraction of energy lost by electron per collision. If $E_P \exp(j\omega t)$ be the applied high frequency field, then it transfers the same power to the electron, as the effective equivalent dc field E given by

$$E^2 = E_P^2\,\nu_c^2/2\,(\nu_c^2 + \omega^2)$$

which for the present experimental condition $\nu_c \gg \omega$ can be written as $E \approx E_P/2^{1/2}$=rms value of applied high frequency field. The values of A_0 and B_0 for hydrogen, oxygen and air are obtained by fitting the Townsend's empirical equation to experimental values of first ionization coefficient given in standard literature for the range of E/P comparable with the present experimental range. The values are given in Table 1 with the respective range of E/P in which they are valid. Expressing Λ in terms of length d and radius r of the tube, the breakdown condition becomes

$$(A_0 P^2 \chi^{1/2}/0{\cdot}47) \exp(-B_0 P/E) = (\pi/d)^2 + (2{\cdot}405/r)^2 \quad \ldots(4)$$

Nonavailability of experimental values of χ and its variation with E/P makes the comparison between Eq. (4) and experimental results difficult. If χ is assumed constant in the range of E/P values of the present observation as an approximation, then from Eq. (4) the pressure P_M at which the breakdown potential will become minimum (E_M) will be

$$P_M = 2\,E_M/B_0 \quad \ldots(5)$$

Using the observed values of E_M, the values of P_M are obtained from the Eq. (5) (Table 1). The discrepancy in the values of P_M between theoretical calculation and experimental results may be due to the assumption that χ has been treated as constant in deducing Eq. (5).

In the presence of dc magnetic field transverse to rf electric field, the effective equivalent dc field is given by[3] $E\,(\nu_c^2/\nu_c^2 + \omega_b^2)^{1/2}$ and the characteristic diffusion length[1] by

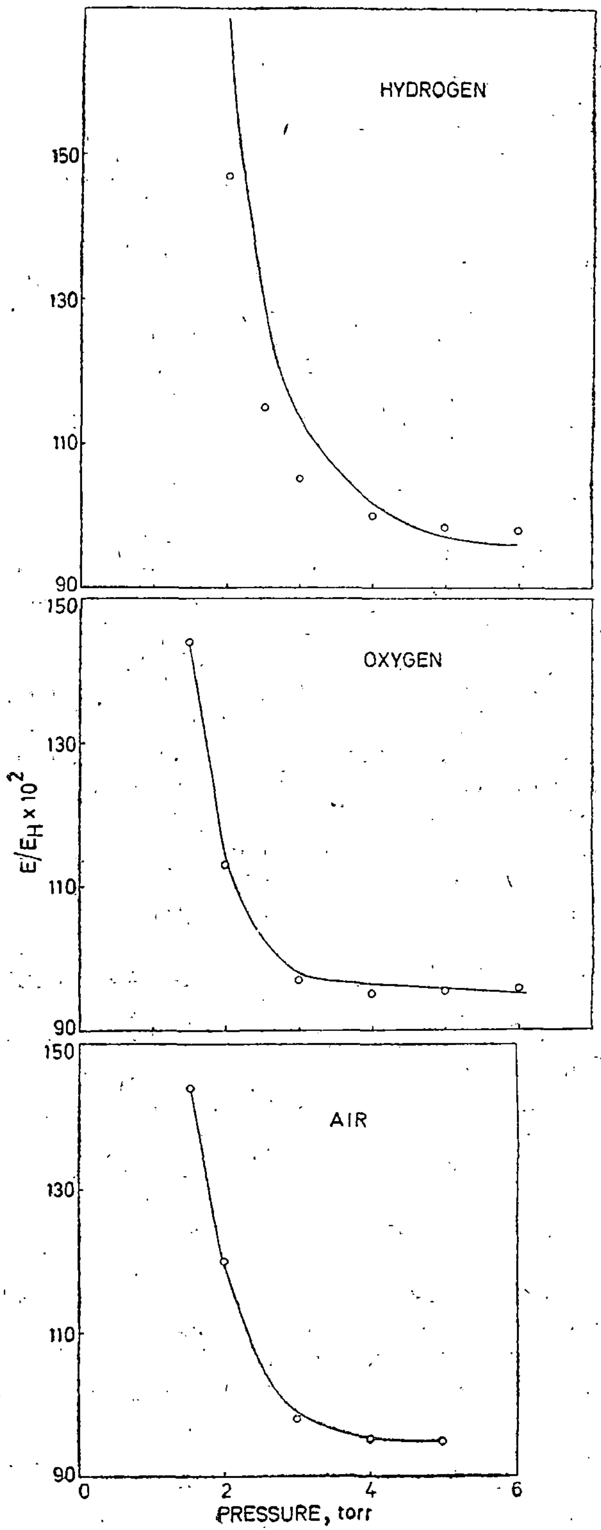

Fig. 2—Variation of E/E_H with pressure for hydrogen, oxygen and air [——,theo; O, exptl]

Table 1—Values of A_0, B_0, E/P, E_M and the Theoretical and Experimental Values of P_M for the Gases Studied

Gas	A_0 Ion pair/ cm torr	B_0 V/cm torr	E/P range V/cm torr	E_M V/cm	P_M from Eq. (5), torr	P_M from Expt, torr
Air	8·2	222	50-250	340	3·06	3±0·1
Hydrogen	5·5	143	50-250	315	4	4±0·1
Oxygen	7·4	200	25-120*	305	3·05	2·8±0·1

* No experimental result available for wider range.

$$[\nu_c^2/\nu_e^2 + \omega_c^2]\,[(\pi/d)^2 + \tfrac{1}{2}\,(2{\cdot}405/r)^2]$$

$$+ \tfrac{1}{2}\,(2{\cdot}405/r)^2 = 1/\Lambda_b^2$$

Combining Eqs. (1)-(3) and using the above two expressions, the breakdown condition in the presence of magnetic field can be obtained. The ratio of breakdown potentials without and with the magnetic field is given by

$$E/E_H = (\nu_c^2/\nu_e^2 + \omega_b^2)^{1/2}\,[1 - 2\,(E/B_0\,P)\,\ln \Lambda/\Lambda_b] \quad \ldots(6)$$

where E_H = breakdown potential in the presence of magnetic field.

Taking the experimentally observed values of E at different pressures, the ratio is plotted using Eq. (6) for $H = 500$ G and compared with some of the representative points of the observed ratio (obtained from Fig. 1) for $H = 0$ and 500 G and are shown in Fig. 2. The representative points agree fairly well with Eq. (6) for all the three gases (Fig. 2).

As ν_c is function of pressure, no simple expression in presence of magnetic field for pressure P_{HM} at which the breakdown potential will be minimum E_{HM} can be obtained. However if r is assumed very large then

Table 2—Values of H, E_{HM} and P_{HM} (Theo. & Exptl)

Gas	H Gauss	E_{HM} V/cm	P_{HM} from Eq. (7), torr	P_{HM} (obs.),* torr
Air	350	325	2·5	2·3
	500	320	2	2
Oxygen	350	305	2·4	2·4
	500	295	1·4	2·1
Hydrogen	350	300	3·9	3
	500	285	3·5	2·4

* Deviation ± 0·1 torr.

$$P_{HM}^2 = [2E_{HM}/B_0]^2 - \omega_b^2/a \quad \ldots(7)$$

where a is a constant given by $\nu_c = a\,P$

Eq. (7) shows that with increase of magnetic field the pressure at which breakdown potential is minimum shifts towards lower pressure which is also enhanced by the decrease of E_{HM} with increase of the magnetic field. Using experimental values of E_{HM}, the values of P_{HM} are calculated from Eq. (7) and compared with the experimentally observed values (Table 2) for fixed values of the magnetic field. It can thus be concluded that the very high frequency breakdown theory, when modified for rf breakdown and effects of magnetic field incorporated, can explain the present experimental results with a fair amount of agreement. The quantitative discrepancy between theoretical and experimental observations may be attributed to the variation of χ with E/P for inelastic collisions which has not been considered in the present theory and also to the uncertainties in the values of different parameters used and some of the simplifying assumptions made in the theoretical deduction.

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B. RADIO FREQUENCY ELECTRIC FIELD BREAKDOWN OF GASES IN PRESENCE OF TRANSVERSE MAGNETIC FIELD (HIGH INTENSITY).

INTRODUCTION.

The breakdown of a gas excited by a radio frequency voltage in the presence of a magnetic field, either longitudinal or transverse, has been studied previously. Mention may be made of the works of Lax et al (1950) and Ferritti and Veronesi (1955). Lax et al (1950) performed experiments on the breakdown potential of a gas in a microwave field in the presence of a transverse magnetic field and obtained breakdown curves for different values of the gas pressure. Ferritti and Veronesi (1955) performed experiments for frequencies ranging from 10 to 30 MHz. in air, with the magnetic field varying from 0 to 600 G. They used cylindrical electrodes and observed a lowering of the breakdown potential in the presence of a magnetic field. Brown (1956) has also discussed some of the microwave breakdown measurements in the presence of a magnetic field. A good account of work on the influence of the magnetic field on the primary ionization coefficient and breakdown potential using the equivalent pressure concept of Blevin and Haydon (1958) has been given by Grey Morgan (1965).

Sen and Ghosh (1963), working in the present laboratory, obtained the breakdown potential of some molecular gases using a radio frequency voltage of frequency 7-10 MHz. with the gas pressure g varying from 10-300 m. torr. in the presence of a transverse magnetic field varying from 0-80 G. Bhattacharjee and Das (1977) measured the variation of the breakdown potential of gases with pressure in the range of a few Torr. in a cylindrical cavity placed in a transverse magnetic field.

Since there has been little work reported on the variation of radio frequency breakdown potential with change in magnetic field over a wide range of magnetic field values for molecular gases : it is proposed to study the variation of radio frequency breakdown potential for molecular gases in a small cavity length in the presence of a continuously variable magnetic field. Following Grey Morgan (1965), attempts have been made to explain the results by modifying Townsend's first ionization coefficient using the equivalent pressure concept of Blevin and Haydon (1958). However, the use of the equivalent pressure concept at high values of H/p is not free from criticism and so an attempt has been made to explain the experimental results by analysing the motion of electrons in an $\vec{E} \times \vec{H}$ field configuration.

While studying the high frequency breakdown potential of gases in the presence of a magnetic field in this laboratory, we have noticed that at some specific range of gas pressure and intensity of magnetic field, the discharge once established can be extinguished either by increasing the electric field or by decreasing the magnetic field. A similar observation was made previously in a very low pressure discharge (Francis,1960) where the pressure of the gas was about 10 m. torr and the magnetic field was below 100 G. No explanation, either qualitative or quantitative, was given for these observations.

Grey Morgan (1965) has shown theoretically that, for a very low pressure discharge, the discharge should go off when the cyclotron diameter of an average electron equals the dimension of the vessel. He obtained a theoretical expression for the electric field intensities at the extinction in plane electrodes and coaxial cylindrical electrodes when the magnetic field is transverse to the electric field.

In the present set up, we have observed identical phenomena for gas pressures and transverse magnetic field values such that the electron mean free path and cyclotron diameter are much smaller than the dimensions of the discharge vessel. A detail study of the phenomena has been undertaken and we attempt to explain the results both qualitatively and quantitatively using the average motion of electrons in a transverse magnetic field.

EXPERIMENTAL ARRANGEMENT.

The method of measurement of breakdown voltage was the same as that used by Sen and Ghosh (1963). The discharge cavity was rectangular (length 0.6 cm., width 2.0 cm. and height 2.2 cm.) and was fitted with two external parallel plane electrodes perpendicular to the length, the distance between them being 1.0 cm. for all the gases studied. The radio frequency voltage was supplied from a tuned-plate-tuned-grid oscillator. The frequency used to excite the discharge was 8.9 MHz. and the output of the oscillator could be varied continuously from 0-500 V. The r.m.s. output voltage was measured with a vacuum-tube voltmeter. The pressure of the gas was measured by a Pirani gauge which had previously been calibrated by means of a Mc Leod gauge. * The uniform magnetic field was supplied by an electromagnet, the lines of force being parallel to the height of the vessel. The field strength was measured by a calibrated Hall probe. Keeping the pressure of the gas at a constant value, the magnetic field was varied and breakdown voltage determined for various values of the magnetic field strength. The experiments were repeated and the results were found to be reproducible.

PREPARATION OF GASES.

Hydrogen was prepared by the electrolysis, with nickel electrodes, of warm saturated barium hydroxide solution. The gas was passed over hot platinum gauge to burn any oxygen in it. The gas was then dried by passing it through caustic potash and finally through pure redistilled phosphorous pentoxide.

Oxygen was prepared by heating potassium permanganate. The oxygen gas so formed was washed with water and dried by passing it through fused calcium chloride, caustic potash and finally through pure redistilled phosphorous pentoxide.

Air was dried by passing it through fused calcium chloride, caustic potash and finally through pure redistilled phosphorous pentoxide.

RESULTS AND DISCUSSION.

The electric breakdown fields of gases like dry air, hydrogen and oxygen excited by a radio frequency field of fixed frequency (8.9 MHz.) have been measured in the presence of an external d-c magnetic field. Keeping the pressure of the gas and the magnetic field constant at certain values, the radio frequency electric field is gradually increased until a glow appears in the discharge vessel; the value of the field at which this occurs is taken as the breakdown field. The measurements are repeated for different steady values of the magnetic field and the variation of the breakdown field with magnetic field are recorded. Throughout the measurements, the pressure of the gas in the discharge vessel is kept constant. The same procedure is repeated for air, hydrogen and oxygen and the variation of breakdown field with magnetic field are plotted - curve A in Figs. 4.7, 4.8 and 4.9 for air, hydrogen and oxygen respectively.

For all the three gases studied, it is found that the nature of variation of the breakdown field 'E' with the magnetic field 'H' is similar to the variation of the breakdown field with gas pressure with or without magnetic field as observed by the earlier workers. As 'H' is increased from zero value, 'E' decreases at first, reaches a minimum E_m at a magnetic field value H_m and then increases with increase of 'H' values. The value of H_m at which the breakdown field becomes minimum is dependent on the nature of the gas and is tabulated in Table-4.3.

Observations made at high value of 'H', beyond ~~tx~~ the value at which 'E' is minimum, indicate the presence of another breakdown field at a much lower 'E' value ~~tx~~ than given by curve 'A' for the same magnetic field.

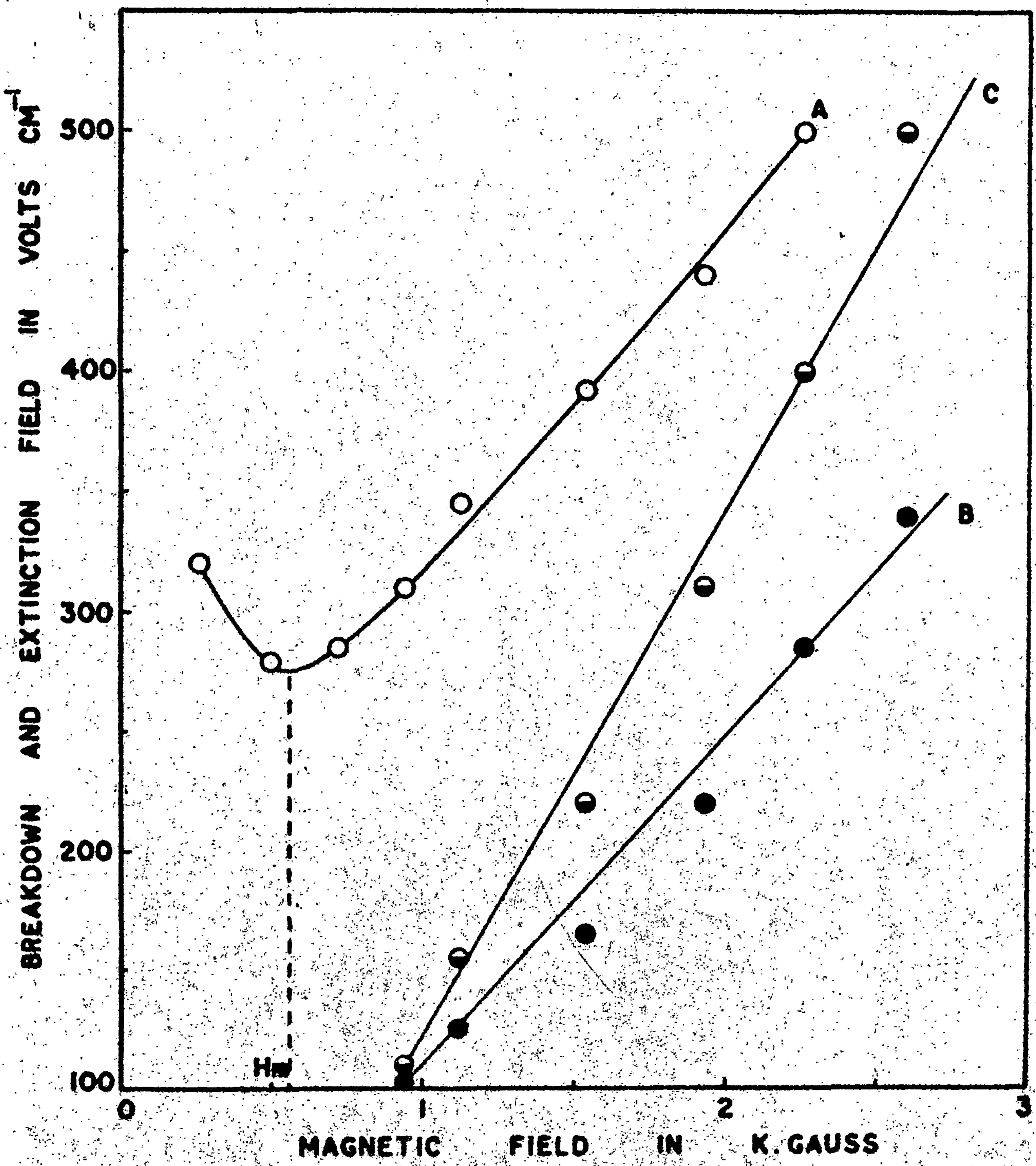
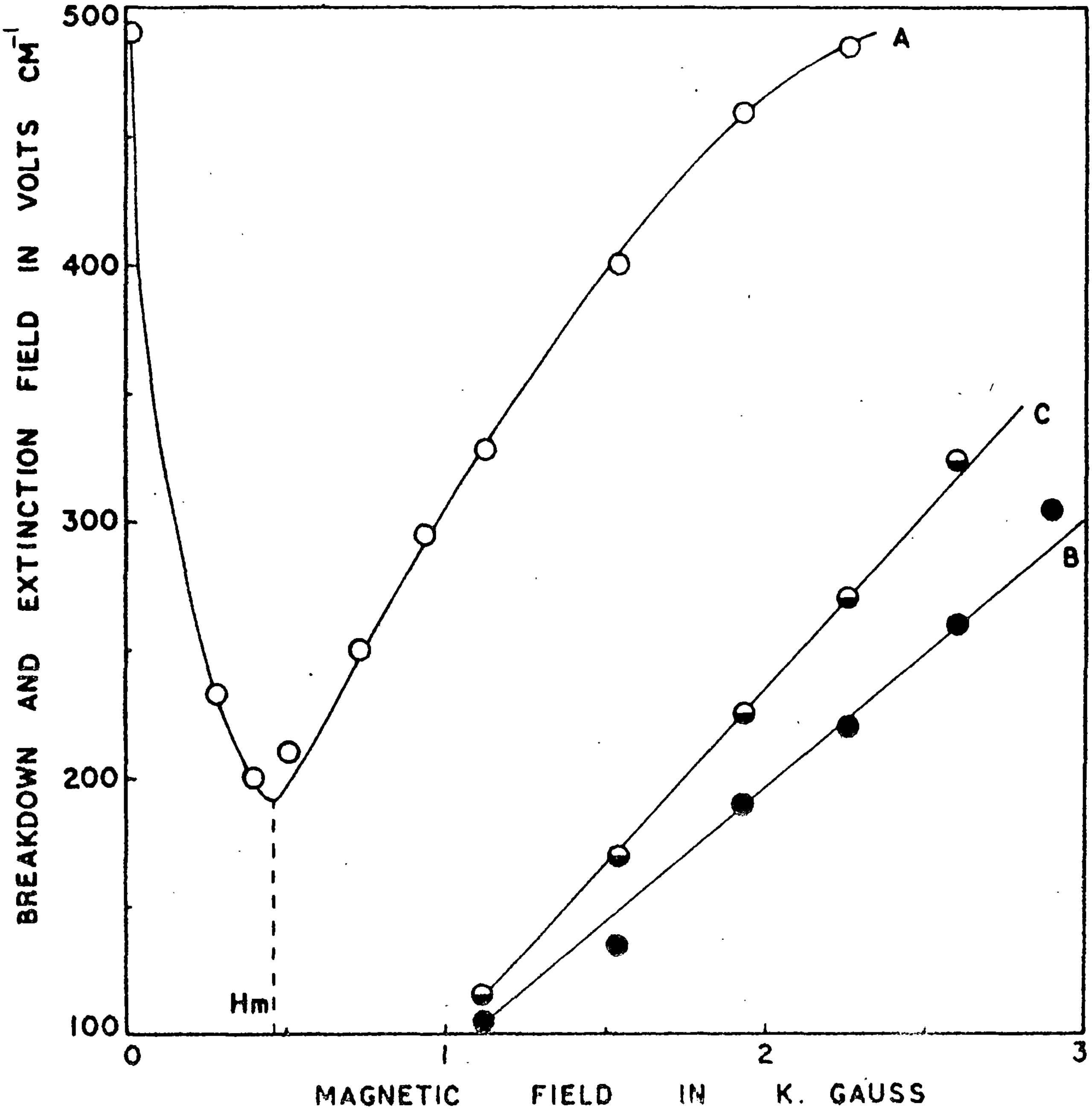

BREAKDOWN AND EXTINCTION FIELD IN VOLTS CM^{-1}
500
400
300
200
100
0
1
2
3
MAGNETIC FIELD IN K. GAUSS
A
B
C
Hm

FIG. 4.7.



BREAKDOWN AND EXTINCTION FIELD IN VOLTS CM^{-1}
500
400
300
200
100
A
B
C
Hm
0
1
2
3
MAGNETIC FIELD IN K. GAUSS

FIG. 4.8.

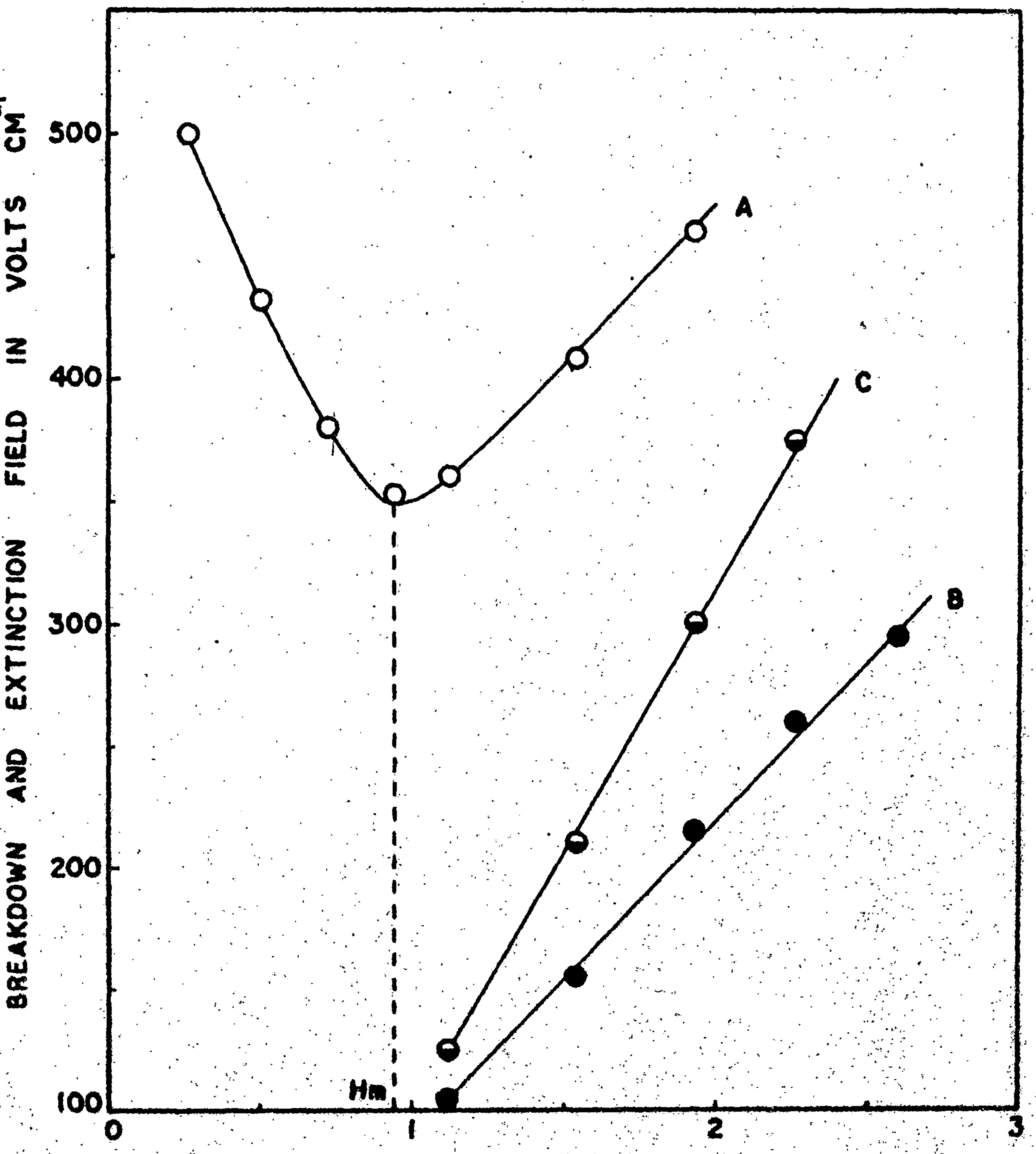

500
400
300
200
100
0
1
2
3
A
C
B
Hm
BREAKDOWN AND EXTINCTION FIELD IN VOLTS CM^{-1}
MAGNETIC FIELD IN K. GAUSS

FIG. 4.9.

A faint glow appears in the discharge vessel, indicating the establishment of a self-sustained discharge at a much lower potential than given by curve 'A' for the corresponding value of 'H'. This breakdown field is found to increase linearly as the magnetic field increases. The nature of variation of this breakdown field is same for all the three gases and is shown by curve 'B' in Figs. 4.7, 4.8 and 4.9. The slope of this linear variation is tabulated in Table-4.4.

Keeping the magnetic field constant if the electric field is raised to a value higher than the value given by curves 'B', it is found that the faint glow disappears, indicating the extinction of the discharge. This extinction field is also found to increase linearly as the magnetic field increases for all the three gases studied and are shown by curve 'C' in figures 4.7, 4.8 and 4.9. However, if the applied electric field is again lowered from a value between the curves 'A' and 'C' to the corresponding value given by curve 'B' at a constant 'H', the glow reappears. The slope of curves 'C' are tabulated in Table-4.4 for all the three gases.

Now if the value of 'E' is raised from the extinction field to the field given by curve 'A' for the respective 'H' values, the self sustained discharge is again established.

This discharge cannot be extinguished by further raising the field, limited in the present case by the maximum available potential from the high frequency source. Instead the glow becomes more intense as the applied field is increased further; this increase in intensity can even be observed with the naked eye.

The phenomena of two breakdown field with one extinction field in between at high values of 'H' are ~~fa~~ found to be absent at value of 'H' lower than the magnetic field at which the breakdown field given by curve 'A' is minimum. For such values of 'H', only one breakdown field, given by curves 'A', is observed for the three gases investigated.

If $E_0 \exp(j\omega t)$ be the applied field the effective equivalent d.c. field 'E' is given by

$$E^2 = \frac{E_0^2}{2} \cdot \frac{\nu_c^2}{\nu_c^2 + \omega^2} \qquad \ldots (4.16)$$

where E_0 is the peak value and ν_c is the electron neutral collision frequency. Since in the present investigation ν_c is much greater than ω, the frequency of the applied field, the r.m.s. value of the high frequency field will be treated as effective equivalent d.c. field.

Now the breakdown of a gas in a cavity occurs when the loss of electrons to the side walls, mainly by electron diffusion, is compensated by the production of electrons by ionization due to collisions between electrons and neutral atoms and the breakdown condition excited by radio frequency electric field can be written as (Brown, 1956)

$$\frac{\nu_i}{D} = \frac{1}{\Lambda^2} \qquad \text{... (4.17)}$$

where ν_i is the ionization frequency, 'D' is the isotropic electron diffusion coefficient and Λ is the characteristic diffusion length given by

$$\frac{1}{\Lambda^2} = \pi^2\left[L_x^{-2} + L_y^{-2} + L_z^{-2}\right]$$

where L_x L_y and L_z are the length, width and height respectively of the discharge vessel.

The ionization frequency can be written as (Allis, 1956)

$$\nu_i = \alpha\mu E \qquad \text{... (4.18)}$$

where μ is the mobility coefficient of electrons and is Townsend's first ionization coefficient given by (von-Engel, 1965)

$$\alpha = A_o p \exp\left(-\frac{B_o p}{E}\right)$$

where 'p' is the gas pressure in Torr and A_o and B_o are two constants whose values are given in standard texts with their respective ranges of validity for various E/p values. Hence from equation (4.18) the high frequency ionization coefficient can be written as

$$\nu_i = A_o p \mu E \exp\left(-\frac{B_o p}{E}\right) \qquad \ldots (4.19)$$

Blevin and Haydon (1958) have shown that the presence of a magnetic field causes an increase in pressure in the transverse direction of the magnetic field. When ν_c is independent of the energy of the electron, which is approximately true for the gases studied in the present investigation (Brown, 1956), the equivalent pressure can be written as (Grey Morgan, 1965)

$$p_e = p\left(1 + \frac{\omega_b^2}{\nu_c^2}\right)^{1/2} \qquad \ldots (4.20)$$

where 'p' is the original gas pressure and ω_b (= $\frac{eH}{m}$) is the electron cyclotron frequency. The ratio D/μ can be written as (Allis, 1956)

$$\frac{D}{\mu} = \frac{kT_e}{e} \qquad \ldots (4.21)$$

where 'e' is the electronic charge, T_e is the electron temperature and 'k' is the Boltzmann constant. A mathematical expression involving T_e and E/p has been deduced by von Engel (1965) and is given by

$$\frac{kT_e}{e} = \frac{L}{\chi^{1/2}} \cdot \left(\frac{E}{p}\right) = r \cdot \left(\frac{E}{p}\right) \qquad \ldots (4.22)$$

where $r = L/\chi^{1/2}$, 'L' is the mean free path of electrons at a pressure of 1 Torr. and χ is the average energy lost by an electron in one collision. Hence the breakdown condition (4.17) in presence of magnetic field becomes

$$A_o \cdot \frac{p_e^2}{r} \cdot \exp\left(-\frac{B_o p_e}{E}\right) = \frac{E}{\Lambda^2} \qquad \ldots (4.23)$$

Equation (4.23) gives the variation of the breakdown field with applied magnetic field.

The values of the constants A_o and B_o are obtained from von Engel (1965) and are tabulated in Table-4.3. The value of χ is found to vary with E/p (von Engel, 1965) for inelastic collisions between electrons and neutral atoms. However, no published work is found available, either theoretical or experimental, concerning the variation of χ with E/p for values of E/p where inelastic collisions predominate. For E/p values less than a few V cm^{-1} $Torr^{-1}$, when elastic collisions predominate, χ was found to be constant and is equal to $2mM^{-1}$ where 'm' and 'M' are the masses of electron and atom respectively.

Assuming that χ is a constant independent of E/p, 'r' is a constant. Then differentiating eqn. (4.23) with respect to H we have,

$$\frac{2A_0 p_e}{r} \cdot \frac{dp_e}{dH} - \frac{A_0 p_e^2}{r} \cdot \frac{B_0}{E} \cdot \frac{dp_e}{dH} + \frac{B_0 p_e}{E^2} \cdot \frac{dE}{dH} = 0 \qquad \ldots (4.24)$$

Since, $p_e = p\left(1+\frac{\omega_b^2}{\nu_c^2}\right)^{1/2} = \left(1+\frac{e^2H^2}{m^2\eta^2 p^2}\right)^{1/2}$

where $\nu_c = \eta p$, η being a constant we have,

$$\frac{dp_e}{dH} = \frac{e}{m\eta} \cdot \frac{1}{2p_e}$$

with this value of $\frac{dp_e}{dH}$ and $\frac{dE}{dH}$ = 0, the magnetic field at which the breakdown field becomes minimum, E_m is given by

$$H_m = \left(\frac{\eta m}{e}\right)\left[\left(\frac{2E_m}{B_0}\right)^2 - p^2\right]^{1/2} \qquad \ldots (4.25)$$

The values of η for the three gases studied have been obtained from Brown (1956) and are tabulated in Table-4.3. Taking the experimental values of E_m , the values of H_m are calculated and compared with the observed H_m in Table-4.3.

In presence of a magnetic field in the z-direction, all velocity components of the charged particles will remain unchanged and hence the coefficient of diffusion in that direction remains same. But the charged particles moving in the x- and y- directions are accelerated in the y- and x-directions respectively and describe arc of a circle until deflected by collision. Hence the charged particles proceed along x- and y-directions at a slower rate than in absence of magnetic field which corresponds to decrease in diffusion coefficient. Thus the diffusion of electrons is anisotropic in the presence of a magnetic field and the diffusion coefficients are given in cartesian co-ordinate system as (Allis, 1956)

$$D_x = D_y = \frac{\langle v^2 \rangle}{3} \cdot \frac{\nu_c}{\nu_c^2 + \omega_b^2}$$

and $$D_z = \frac{1}{3} \cdot \frac{\langle v^2 \rangle}{\nu_c} = D$$

Hence $$D_x = D_y = \frac{D \nu_c^2}{\nu_c^2 + \omega_b^2}$$

where D_x , D_y , and D_z are the diffusion coefficients for diffusion of electrons along the x-, y-, and z- directions respectively and $\langle v^2 \rangle$ is the average of the square of random velocity of the electrons.

With anisotropic diffusion terms, the breakdown condition (4.16) becomes (Allis, 1956)

$$\frac{\nu_i}{D} = \frac{\nu_c^2}{\nu_c^2 + \omega_b^2} \cdot \pi^2 \left(L_x^{-2} + L_y^{-2} \right) + \pi^2 L_z^{-2} \qquad \ldots (4.26)$$

In presence of a steady d.c. magnetic field transverse to the high frequency electric field, Allis (1956), considering the gain of energy by electrons suffering inelastic collisions with the atoms, have shown that the effect of high frequency electric field in transfering the energy to the electrons can well be represented by equivalent d.c. field given by

$$E_e^2 = \frac{E_o^2}{2} \cdot \frac{\nu_c^2(\nu_c^2 + \omega^2 + \omega_b^2)}{[\nu_c^2 + (\omega + \omega_b)^2][\nu_c^2 + (\omega - \omega_b)^2]}$$

since ω is much smaller than ν_c and ω_b of electrons even for a magnetic field strength of a few gauss we have

$$E_e = E \cdot \frac{\nu_c}{\sqrt{\nu_c^2 + \omega_b^2}} \qquad \ldots (4.27)$$

'E' being the r.m.s. value of the high frequency field. Using this equivalent field expression, Townsend's first ionization coefficient is modified in the presence of magnetic field as

$$\frac{\alpha}{p} = A_o \exp\left(-\frac{B_o p}{E_e}\right) \qquad \ldots (4.28)$$

Also

$$\nu_i = \alpha v_x \qquad \ldots (4.29)$$

where v_x is the electron drift velocity along the field direction and is given by

$$v_x = \frac{e}{m} \cdot \frac{\nu_c}{\nu_c^2 + \omega_b^2} \cdot E \qquad \ldots (4.30)$$

von Engel (1965) obtained the relation between random velocity and the drift velocity of the electrons along the applied field as

$$\frac{v_x}{v} \approx \left(\tfrac{1}{2}\chi\right)^{1/2} \qquad \ldots (4.31)$$

Hence

$$\frac{\nu_i}{D} = \frac{\alpha v_x}{D} = \frac{\alpha v_x \nu_c}{\frac{1}{3}\langle v^2 \rangle} = \frac{3}{2} \cdot \frac{\alpha \chi \nu_c}{v_x} \qquad \ldots (4.32)$$

Putting α, v_x and E_e from eqns. (4.28), (4.30) and (4.27) respectively in eqn. (4.32) we have,

$$\frac{\nu_i}{D} = \frac{3}{2} \cdot \frac{\chi A_o p}{\left(\frac{e}{m}\right) E} \cdot \left(\nu_c^2 + \omega_b^2\right) \exp\left[-\frac{B_o p}{E} \cdot \frac{\left(\nu_c^2 + \omega_b^2\right)^{1/2}}{\nu_c}\right] \qquad \ldots (4.33)$$

Hence the breakdown condition (4.26) can be written as

$$\frac{3}{2} \cdot \frac{\chi A_o p}{\left(\frac{e}{m}\right) E} \cdot \left(\nu_c^2 + \omega_b^2\right) \exp\left[-\frac{B_o p}{E}\left(1 + \frac{\omega_b^2}{\nu_c^2}\right)^{1/2}\right]$$

$$= \pi^2\left[\left(1 + \frac{\omega_b^2}{\nu_c^2}\right)^{-1}\left(L_x^{-2} + L_y^{-2}\right) + L_z^{-2}\right] \qquad \ldots (4.34)$$

Differentiating both side of eqn. (4.34) with respect of 'H' and assuming χ as a constant, independent of 'E', we have, with $\frac{dE}{dH}$ = 0 at the minimum breakdown voltage,

$$\frac{3}{2}\cdot\frac{\chi A_0 p}{\left(\frac{e}{m}\right)E}\cdot\left[2-\frac{B_0 p}{E}\left(1+\frac{\omega_b^2}{\nu_c^2}\right)^{1/2}\right]\exp\left[-\frac{B_0 p}{E}\left(1+\frac{\omega_b^2}{\nu_c^2}\right)^{1/2}\right]$$

$$=-\pi^2\left(1+\frac{\omega_b^2}{\nu_c^2}\right)^{-2}\cdot\frac{2}{\nu_c^2}\cdot\left(L_x^{-2}+L_y^{-2}\right)$$

which becomes using eqn. (4.34)

$$\frac{E}{\nu_c^2+\omega_b^2}\cdot\pi^2\left[\left(1+\frac{\omega_b^2}{\nu_c^2}\right)^{-1}\left(L_x^{-2}+L_y^{-2}\right)+L_z^{-2}\right]\left[\frac{2}{E}-\frac{B_0 p}{E^2}\left(1+\frac{\omega_b^2}{\nu_c^2}\right)^{1/2}\right]$$

$$=-\pi^2\left(1+\frac{\omega_b^2}{\nu_c^2}\right)\cdot\frac{2}{\nu_c^2}\left(L_x^{-2}+L_y^{-2}\right)$$

from which the condition at which the breakdown field becomes minimum can be obtained as

$$\left[\left(1+\frac{\omega_b^2}{\nu_c^2}\right)^{-1}\left(L_x^{-2}+L_y^{-2}\right)+L_z^{-2}\right]\left[\frac{B_0 p}{2E}\left(1+\frac{\omega_b^2}{\nu_c^2}\right)^{1/2}-1\right]$$

$$=\left(1+\frac{\omega_b^2}{\nu_c^2}\right)^{-1}\left[L_x^{-2}+L_y^{-2}\right] \qquad \ldots (4.35)$$

The solution of this equation for H_m cannot be performed in the usual way and hence a transcendental solution of the equation has been obtained with the experimental values of E_m . The values of H_m obtained for the three gases are tabulated in Table-4.3.

TABLE-4.3

Gas	A_0 cm^{-1} $torr^{-1}$	B_0 V cm^{-1} $torr^{-1}$	η x 10^{-9}	V_i Volts	p Torr	E_m Expt. V cm^{-1}	H_m from eqn. (4.25) K.Gauss	H_m from eqn. (4.35) K.Gauss	H_m Expt. K.Gauss.
Air	15	365	4.3	15.0	0.25	275	0.36	0.43	0.56
Oxygen	13	230	3.5	12.2	0.30	345	0.58	0.64	0.94
Hydrogen	5.4	139	5.93	15.4	0.50	190	0.88	1.10	0.46

When the applied electric field is much smaller than those indicated by eqns. (4.23) and (4.34) and if the magnetic field is such that $\omega_b^2 \nu_c^{-2} \gg 1$, then high increase of equivalent pressure will increase the number of electron-neutral collisions and presence of much lower electric field will decrease the magnitude of average velocity of electrons. Also in low applied electric field, a much smaller number of electrons will be produced by collisions with neutrals. Since the electron diffusion coefficients are

$$D_z = \frac{1}{3} \cdot \frac{\langle v^2 \rangle}{\nu_c}$$

$$D_x = D_y = D_z \left(1 + \frac{\omega_b^2}{\nu_c^2}\right)^{-1}$$

so from the above argument it can be assumed that there is practically no loss of electrons to the side walls due to diffusion. Most of the electrons will remain confined in the cavity and the centre of the cyclotron orbit of the electron, executing and elliptical motion at the frequency of the applied field, will receive energy from the applied field being in phase with the field. The collisions will interrupt this motion and cause excitation and/or ionization of the neutral atom. If the average energy received by the electron during one period of the applied field becomes just sufficient to ionize an atom, then after every period of the applied field, a fresh pair of electrons will start to receive energy from the field and multiply them causing the establishment of a self sustained discharge.

The average drift velocity components of the electron in $\vec{E} \times \vec{H}$ field configuration are (Uman, 1964)

$$v_x = \frac{e}{m} \cdot \frac{\nu_c}{\nu_c^2 + \omega_b^2} \cdot E, \quad v_y = -\frac{e}{m} \cdot \frac{\nu_c}{\nu_c^2 + \omega_b^2} \cdot E \quad \& \quad v_z = 0$$

where length, breadth and height are taken along x-, y-, and z- directiions respectively. Hence for the breakdown field given by curve 'B' we get,

$$eEv_xT = eV_i \qquad \ldots (4.36)$$

where $T = 2\pi/\omega$ and V_i = ionization potential, ω being the frequency of the applied electric field. Assuming $\omega_b^2 \nu_c^{-2} \gg 1$, the breakdown field for curve 'B' can be written as

$$E \approx \left[\frac{V_i \omega \nu_c}{2\pi\left(\frac{e}{m}\right)}\right]^{1/2} \cdot \frac{\omega_b}{\nu_c} \qquad \ldots (4.37)$$

The E vs. H curve from eqn. (4.37) will have slope given by

$$\left[\frac{V_i \omega (e/m)}{2\pi \nu_c}\right]^{1/2} \times 10^{-8}$$

when all the quantities are given in e.m.u. Since all the quantities are known, so the slope is calculated for all the three gases and compared with experimentally observed slope of curve 'B' in Table-4.4.

As the electric field intensity is increased in this active discharge so developed, the energy supplied per unit time to the plasma by the external source is $eE_i v_x$ and if we assume that almost all of this energy goes to ionization, then approximately the supply of energy creates per unit time 'n' number of new electrons given by

$$n = \frac{eE_i v_x}{eV_i}$$

But the motion along y- direction causes electrons to move away from the main plasma column. An average displacement of L_y /2 causes an electron to be lost permanently in the side wall and will fail to contribute anything to the discharge current. The number of electrons that will run-away from the main stream of discharge current per unit time, on the average, is given by

$$n' = \frac{2v_y}{L_y}$$

If the number of new electrons n are not lost to ~~th~~ the side walls, by the drift motion along y- direction as mentioned above, then the electron density will grow causing a density gradient. This will in turn allow electrons to be lost by diffusion when the maintainance of discharge will be a balance between diffusion loss and production of the electrons by collisions and the required voltage is given by curves A. So the discharge will go off as the acting applied field is much lower than that necessary for sustaining the discharge. Hence the condition for extinction can be written as

$$n \geqslant n', \quad \text{i.e.} \quad \frac{eE'v_x}{eV_i} \geqslant \frac{2v_y}{L_y}$$

or
$$E \geqslant \frac{2V_i\,\omega_b}{L_y\,\nu_c} \qquad \ldots (4.38)$$

Assuming $\omega_b^2\nu_c^{-2} \gg 1$. From eqn. (4.38) the slope of E vs.

H curve is

$$\frac{2V_i\,(e/m)}{L_y\,\nu_c}\times 10^{-8}$$

when all the quantities are measured in e.m.u. The slope is calculated for the three gases and compared with experimentally observed slope of curve C in Table-4.4.

TABLE-4.4

f = 8.9 MHz. L_y = 2.0 cm.

Gas	V_i Volts	$\eta \times 10^{-9}$	P Torr	E/H (curve B) V cm^{-1} $gauss^{-1}$ Theoretical from eqn. (4.37)	E/H (curve B) Experimental	E/H (Curve C) Theoretical from eqn. (4.38)	E/H (Curve C) Experimental.
Air	15.0	4.3	0.25	0.149	0.136	0.251	0.227
Oxygen	12.2	3.5	0.30	0.136	0.135	0.209	0.218
Hydrogen	15.4	5.93	0.50	0.092	0.107	0.0934	0.137

CONCLUSION

There is discrepancy in magnitude between experimental and the theoretical values of H_m obtained either from relation (4.25) or from (4.35) for all the three gases. However, agreement is better between experimental and theoretical values given by (4.35) for air and oxygen gas; but for hydrogen the relation (4.25) gives value of H_m which is in better agreement with experimental value.

The variation of the value χ with the energy of the electrons and hence with E/p, ignored in the present theoretical discussions, and also the uncertainty in the accepted values of A_o and B_o for the range of E/p values of the present experiment may be some of the reasons for the deviations between experimental and theoretical values of H_m. The range of values of E/p for the validity of the values taken for A_o and B_o in Table-4.3 are much smaller than the present experimental E/p values, specially for oxygen, which are not available and obtained by comparing Townsend's emperical relation

$$\frac{\alpha}{p} = A_o \exp\left(-\frac{B_o p}{E}\right)$$

with the available experimental variation of α/p with E/p for the maximum E/p value equal to 50 V/cm. torr. (Brown, 1959). However, existance of a minimum breakdown field in equations (4.23) and (4.34) indicates qualitative agreement of the theories with the experimental results. If the values of A_o and B_o for the present range of E/p values and the exact functional dependence of χ on E/p in presence of magnetic field be available, then both the equations (4.23) and (4.34) are expected to give better quantitative agreement with the experimentally observed variation of breakdown field with magnetic field (curves A).

There is good agreement in slope between curve B and eqn. (4.37) and breakdown curves C and equation (4.38) The small amount of discrepancy observed between theoretical and experimental values of the slopes may be due to some symplifying assumptions that have been made to obtain the equations (4.37) and (4.38) and also uncertainties in the energy independent values of ν_c . Equations (4.37) and (4.38) indicate that even when gas pressure is such that neither mean free path nor the cyclotron diameter of electron is comparable to the dimensions of the discharge cavity then also the discharge once started can be extinguished either by increasing the electric field keeping magnetic field constant or increasing the magnetic field keeping electric field constant provided the condition $\omega_b^2 \nu_c^{-2} \gg 1$ is satisfied. The success of the equation (4.37) and (4.38) in explaining quantitatively the experimental observations of curves B and C justifies the proposed mechanism operative in the discharge for small intensity of electric field when $\omega_b^2 \nu_c^{-2} \gg 1$.

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Radiofrequency electric field breakdown of gases in the presence of a transverse magnetic field

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Abstract. The variation of radiofrequency (8.9 MHz) electric field breakdown of gases like air, hydrogen and oxygen is obtained, for gas pressures of 0.25, 0.5 and 0.3 Torr respectively, in the presence of a steady external uniform magnetic field varying from zero to 3 kG, the magnetic field being transverse to the electric field direction. For each gas a minimum value of the breakdown field is found at a certain value of the magnetic field, both values being different for each of the three gases. In addition, in a strong magnetic field when the electron cyclotron frequency is much higher than the electron–neutral collision frequency, both being much higher than the frequency of the applied field, a second breakdown field of much lower magnitude was found. The discharge once established at this lower field can be extinguished by increasing the electric field. The second breakdown field and the extinction field are both found to be linearly increasing functions of the applied strong magnetic field.

The theory of high-frequency electric field breakdown is modified by taking the 'equivalent pressure concept' of Belvin and Haydon in the presence of a magnetic field and an expression for the magnetic field at which the breakdown field becomes minimum is obtained and compared with experimental observations. An attempt is also made to explain the experimental observations by analysing the motion of electrons in an $\boldsymbol{E} \times \boldsymbol{H}$ field configuration without using the equivalent pressure concept.

By considering the average motion of the electrons in very strong magnetic field, transverse to an electric field of small magnitude, linear relations are obtained between the second breakdown field and the corresponding magnetic field and also between the extinction field and the magnetic field. The theoretical slopes of these linear equations compare well with the experimental observations.

1. Introduction

The breakdown of a gas excited by a radiofrequency voltage in the presence of a magnetic field, either longitudinal or transverse, has been studied previously. Mention may be made of the works of Lax *et al* (1950) and Ferritti and Veronesi (1955). Lax *et al* (1950) performed experiments on the breakdown potential of a gas in a microwave field in the presence of a transverse magnetic field and obtained breakdown curves for different values of the gas pressure. Ferritti and Veronesi (1955) performed experiments for frequencies ranging from 10 to 30 MHz in air, with the magnetic field varying from 0 to 600 G. They used cylindrical electrodes and observed a lowering of the breakdown potential in the presence of a magnetic field. Brown (1956) has also discussed some of the microwave breakdown measurements in the presence of a magnetic field. A good account of work on the influence of the magnetic field on the primary ionisation coefficient and breakdown potential using the equivalent pressure concept of Blevin and Haydon (1958) has been given by Grey Morgan (1965).

Sen and Ghosh (1963), working in the present laboratory, obtained the breakdown potential of some molecular gases using a RF voltage of frequency 7–10 MHz with the gas pressure varying from 10–300 mTorr in the presence of a transverse magnetic field varying from 0–80 G. Bhattacharjee and Das (1977) measured the variation of the breakdown potential of gases with pressure in the range of a few Torr in a cylindrical cavity placed in a transverse magnetic field.

Since there has been little work reported on the variation of RF breakdown potential with change in magnetic field over a wide range of magnetic field values, it is proposed to study the variation of RF breakdown potential for molecular gases in a small cavity length in the presence of a continuously variable magnetic field. Following Grey Morgan (1965), attempts have been made to explain the results by modifying Townsend's first ionisation coefficient using the equivalent pressure concept of Blevin and Haydon (1958). However, the use of the equivalent pressure concept at high values of H/P is not free from criticism and so an attempt has been made to explain the experimental results by analysing the motion of electrons in an $\boldsymbol{E} \times \boldsymbol{H}$ field configuration.

While studying the high-frequency breakdown potential of gases in the presence of a magnetic field in this laboratory, we have noticed that at some specific range of gas pressure and intensity of magnetic field, the discharge once established can be extinguished either by increasing the electric field or by decreasing the magnetic field. A similar observation was made previously in a very low-pressure discharge (Francis 1960) where the pressure of the gas was about 10^{-5} Torr and the magnetic field was below 100 G. No explanation, either qualitative or quantitative, was given for these observations.

Grey Morgan (1965) has shown theoretically that, for a very low-pressure discharge, the discharge should go off when the cyclotron diameter of an average electron equals the dimension of the vessel. He obtained a theoretical expression for the electric field intensities at the extinction in plane electrodes and coaxial cylindrical electrodes when the magnetic field is transverse to the electric field.

In the present set-up, we have observed identical phenomena for gas pressures and transverse magnetic field values such that the electron mean free path and cyclotron diameter are much smaller than the dimensions of the discharge vessel. A detailed study of the phenomena has been undertaken and we attempt to explain the results both qualitatively and quantitatively using the average motion of electrons in a transverse magnetic field.

2. Experimental arrangement

The method of measurement of breakdown voltage was the same as that used by Sen and Ghosh (1963). The discharge cavity was rectangular (length 0.6 cm, width 2.0 cm and height 2.2 cm) and was fitted with two external parallel plane electrodes perpendicular to the length, the distance between them being 1.0 cm for all the gases studied. The radiofrequency voltage was supplied from a tuned-plate–tuned-grid oscillator. The frequency used to excite the discharge was 8.9 MHz and the output of the oscillator could be varied continuously from 0–500 V. The RMS output voltage was measured with a vacuum-tube voltmeter. The pressure of the gas was measured by a Pirani gauge which had previously been calibrated by means of a McLeod gauge. The uniform magnetic field was supplied by an electromagnet, the lines of force being parallel to the height of the vessel. The field strength was measured by a calibrated Hall probe. Keeping the pressure of the gas at a constant value, the magnetic field was varied and breakdown voltage determined for various values of the magnetic field strength. The experiments were repeated and the results were found to be reproducible.

3. Preparation of gases

Hydrogen was prepared by the electrolysis, with nickel electrodes, of warm saturated barium hydroxide solution. The gas was passed over hot platinum gauze to burn any oxygen in it. The gas was then dried by passing it through caustic potash and finally through pure redistilled phosphorus pentoxide.

Oxygen was prepared by heating potassium permanganate. The oxygen gas so formed was washed with water and dried by passing it through fused calcium chloride, caustic potash and finally through pure redistilled phosphorus pentoxide.

Air was dried by passing it through fused calcium chloride, caustic potash and finally through pure redistilled phosphorus pentoxide.

4. Results and discussion

The electric breakdown fields (BF) of gases like dry air, hydrogen and oxygen, excited by a radiofrequency field of fixed frequency, have been measured in the presence of an external DC magnetic field. Keeping the magnetic field constant at a certain value, the RF electric field is gradually raised until a glow appears in the discharge vessel; the value of field at which this occurs is taken as the breakdown field. These measurements were repeated for different steady values of the magnetic field and the variation of the breakdown field with magnetic field recorded. Throughout the measurements, the pressure of gas in the vessel is kept constant. The same procedure is repeated for the three gases kept at three different pressures and the results are plotted as curve A in figures 1, 2 and 3 for air, hydrogen and oxygen, respectively.

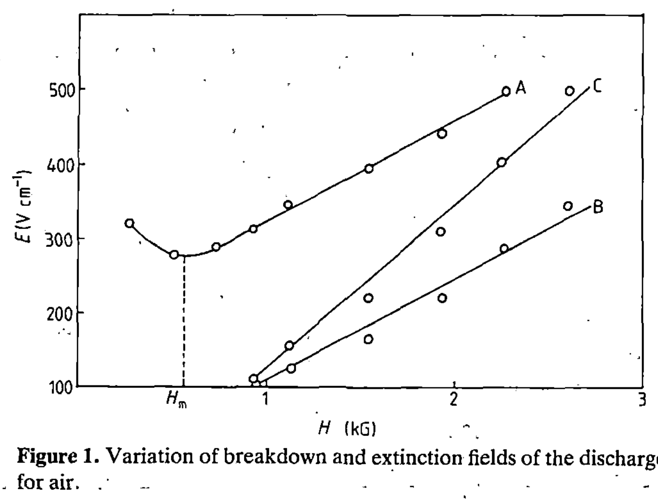

Figure 1. Variation of breakdown and extinction fields of the discharge with magnetic field for air.

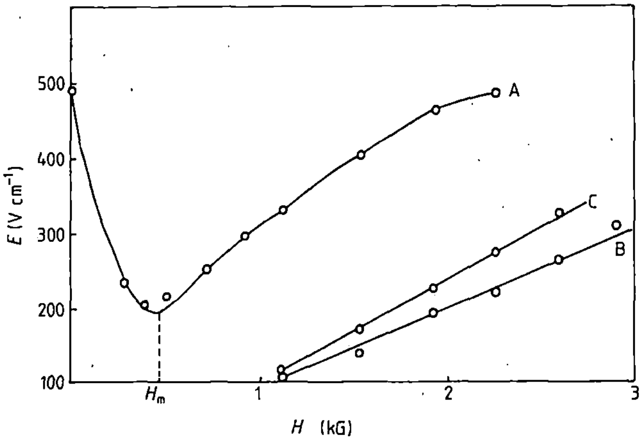

Figure 2. Variation of breakdown and extinction fields of the discharge with magnetic field for hydrogen.

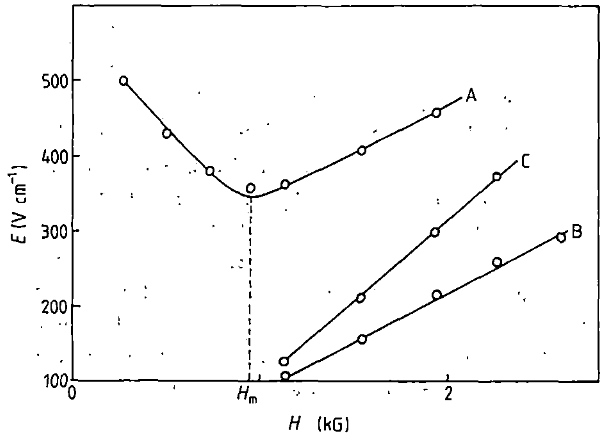

Figure 3. Variation of breakdown and extinction fields of the discharge with magnetic field for oxygen.

Table 1.

Gas	A (cm^{-1} Torr^{-1})	B (V cm^{-1} Torr^{-1})	$\eta \times 10^{-9}$	V_i (V)	p (Torr)	E_m (exp) (V cm^{-1})	H_m (equation (8)) (kG)	H_m (equation (17)) (kG)	H_m (exp) (kG)
Air	15	365	4.3	15	0.25	275	0.36	0.43	0.56
Oxygen	13	230	3.5	12.2	0.30	345	0.58	0.64	0.94
Hydrogen	5.4	139	5.93	15.4	0.50	190	0.88	1.1	0.46

For all three gases studied, it is found that the nature of the variation of the breakdown field E with the magnetic field H is similar to the variation of the BF with gas pressure observed by earlier workers. As H increases from zero, E decreases, reaches a minimum E_m at a magnetic field value H_m and then increases as H increases further. The value of H_m at which the BF is minimum depends on the nature of the gas and is tabulated in table 1.

Observations made at high values of H, beyond the value at which E is minimum, indicated the presence of another BF at a much lower E value than given by curves A for the same H value. A faint glow appears in the discharge vessel, indicating the establishment of a self-sustained discharge at a much lower potential than given by curves A for the corresponding value of H. This breakdown field is found to increase linearly as H increases for all three gases (curves B, figure 1, 2 and 3). The slope of the linear variation is tabulated in table 2.

Table 2.

				E/H ($\mathrm{V\,cm^{-1}\,G^{-1}}$)			
Gas	V_i (V)	$\eta \times 10^{-9}$	p (Torr)	Theoretical (equation (19))	Experimental	Theoretical (equation (20))	Experimental
Air	15	4.3	0.25	0.149	0.136	0.251	0.227
Oxygen	12.2	3.5	0.30	0.136	0.135	0.209	0.218
Hydrogen	15.4	5.93	0.50	0.092	0.107	0.0934	0.137

When the electric field is raised to a higher value than the value given by curves B, it is found that the faint flow disappears, indicating the extinction of the discharge. This extinction field is also found to increase linearly as H increases for all three gases (curves C, figure 1, 2 and 3). However, if the E value is again lowered to the corresponding value given by curve B at constant H, the glow reappears. The slopes of curves C are tabulated in table 2 for all three gases.

Now if the value of E is raised from the extinction field to the field given by curves A for the respective H values, the self-sustained discharge is again established. This discharge cannot be extinguished by further raising the potential, limited in the present case by the maximum available potential from the high-frequency source. Instead the glow becomes intense as E is increased further; this can also be observed with the naked eye.

The phenomena of two BF with one extinction field in between at high values of H are found to be absent at values of H lower than the value of H at which the BF given by curves A is minimum. For such values of H, only one BF, given by curves A, is observed for the three gases investigated.

In the present investigation the frequency of the applied field ω is much lower than the electron–neutral collision frequency ν_c and also the cyclotron frequency ω_b of electrons even for a magnetic field strength of a few gauss. So, the RMS value of the high-frequency field will be treated as effective equivalent DC field (Brown 1956).

The gas in the cavity will break down when the loss of electrons to the side walls, mainly by electron diffusion, is compensated by the production of electrons by ionisation due to collisions between electrons and neutral atoms and the breakdown condition excited by a HF electric field can be written as (Brown 1956)

$$\nu_i D^{-1} = \Lambda^{-2} \tag{1}$$

where ν_i is the ionisation frequency, D is the isotropic electron diffusion coefficient and Λ is the characteristic diffusion length given by

$$\frac{1}{\Lambda^2} = \pi^2\left(\frac{1}{L_x^2} + \frac{1}{L_y^2} + \frac{1}{L_z^2}\right)$$

where L_x, L_y and L_z are the length, width and height respectively of the discharge vessel.

Now (Allis 1956)

$$\nu_i = \alpha\mu E \tag{2}$$

where μ is the mobility coefficient of electrons and α is Townsend's first ionisation coefficient given by (von Engel 1965)

$$\alpha = Ap\exp((-Bp/E)$$

where p is the gas pressure in torr and A and B are two constants whose values are given in standard texts with their respective ranges of validity for various E/p values. Hence from equation (2) the high-frequency ionisation coefficient can be written as

$$\nu_i = Ap\mu E\exp(-Bp/E). \tag{3}$$

The effect of the magnetic field can be introduced using the equivalent pressure concept of Blevin and Haydon (1958). When ν_c is independent of the energy of the electron, which is approximately true for the gases studied in the present investigation (Brown 1956), the equivalent pressure can be written as (Grey Morgan 1965)

$$p_e = p(1 + \omega_b^2\nu_c^{-2})^{1/2} \tag{4}$$

where p is the original gas pressure. The ratio D/μ can be written as (Allis 1956)

$$D/\mu = kT_e/e \tag{5}$$

where T_e is the electron temperature and e is the electronic charge. A mathematical expression involving T_e and E/p has been deduced by von Engel (1965) and is given by

$$kT_e/e = L\chi^{-1/2}(E/p) = r(E/p) \tag{6}$$

where $r = L\chi^{-1/2}$; L is the mean free path of the electron at a pressure of 1 Torr and χ is the average energy lost by an electron in one collision. Hence the breakdown condition (1), in the presence of magnetic field, becomes

$$r^{-1}Ap_eE\exp(-Bp_e/E) = \Lambda^{-2}E/p_e. \tag{7}$$

Equation (7) gives the variation of the breakdown potential with applied magnetic field.

The values of the constants A and B are obtained from von Engel (1965) and are tabulated in table 1. The value of χ was found to vary with E/p (von Engel 1965) for inelastic collisions between electrons and neutral atoms. However, we were unable to find any published work, either theoretical or experimental, concerning the variation of χ with E/p for values of E/p where inelastic collisions predominate. For E/p values less than a few V cm^{-1} $Torr^{-1}$, when elastic collisions predominate, χ was found to be constant and equal to $2\,mM^{-1}$ where m and M are the masses of the electron and atom respectively.

Assuming that χ is a constant independent of E/p will lead, from equation (7), to a value of the magnetic field H_m at which the breakdown field E becomes minimum, E_m, of

$$H_m = \left(\frac{\eta m}{e}\right)\left[\left(\frac{2E_m}{B}\right)^2 - p^2\right]^{1/2} \tag{8}$$

where $\nu_c = \eta p$, η being a constant. The values of η for the three gases studied have been obtained from Brown (1956) and are tabulated in table 1. Taking the experimental values of E_m, the values of H_m are calculated and compared with the observed H_m in table 1.

The diffusion of electrons is anisotropic in the presence of a magnetic field and the diffusion coefficients are given in the Cartesian coordinate system as (Allis 1956)

$$D_x = D_y = \frac{\langle v^2\rangle}{3}\frac{\nu_c}{\nu_c^2 + \omega_b^2}$$

and

$$D_z = \tfrac{1}{3}\langle v^2\rangle / \nu_c = D.$$

Hence

$$D_x = D_y = D\nu_c^2/(\nu_c^2 + \omega_b^2).$$

Here D_x, D_y and D_z are the diffusion coefficients for diffusion of electrons along the x, y and z directions respectively and $\langle v^2\rangle$ is the square of the average random velocity of the electrons.

With anisotropic diffusion terms, the breakdown condition (1) becomes (Allis, 1956)

$$\frac{\nu_i}{D} = \frac{\nu_c^2}{\nu_c^2 + \omega_b^2}\pi^2(L_x^{-2} + L_y^{-2}) + \pi^2 L_z^{-2} \tag{9}$$

for anisotropic diffusion loss of electrons.

Since $\omega \ll \nu_c$ and $\omega \ll \omega_b$, the effect of a high-frequency electric field in the presence of a magnetic field can be equated from an equivalent DC field given by (Brown 1956)

$$E_e = E[\nu_c^2/(\nu_c^2 + \omega_b^2)^{1/2}] \tag{10}$$

where E is the RMS value of the high-frequency field. Using this equivalent field expression, Townsend's first ionisation coefficient is modified in the presence of a magnetic field as

$$\alpha/p = A\exp(-Bp/E_e). \tag{11}$$

Also

$$\nu_i = \alpha v_x \tag{12}$$

where v_x is the electron drift velocity along the electric field:

$$(e/m)[\nu_c/(\nu_c^2 + \omega_b^2)]E.$$

Von Engel (1956) obtained the relation between random velocity and the drift velocity of electrons along the applied electric field as

$$v_{\text{drift}}/v_{\text{random}} \approx (\tfrac{1}{2}\chi)^{1/2}. \tag{13}$$

Hence
$$\nu_i/D = \alpha v_x/D = \alpha v_x \nu_c/\tfrac{1}{3}\langle v^2\rangle = 3\alpha\chi\nu_c/2v_x. \tag{14}$$

Putting (10) and (11) in (14)
$$\frac{\nu_i}{D} = \frac{3}{2}\frac{\chi A p}{(e/m)E}(\nu_c^2 + \omega_b^2)\exp\left(-\frac{Bp}{E}\frac{(\nu_c^2+\omega_b^2)^{1/2}}{\nu_c}\right). \tag{15}$$

Hence the breakdown condition (9) can be written as
$$\frac{3}{2}\frac{\chi A p}{(e/m)E}(\nu_c^2 + \omega_b^2)\exp\left(-\frac{Bp}{E}\frac{(\nu_c^2+\omega_b^2)^{1/2}}{\nu_c}\right)$$
$$= \frac{\nu_c^2}{\nu_c^2+\omega_b^2}\pi^2(L_x^{-2} + L_y^{-2}) + \pi^2 L_z^{-2}. \tag{16}$$

As before, assuming χ as a constant, the condition at which the BF becomes minimum can be written as
$$\left(\frac{L_x^{-2}+L_y^{-2}}{1+(\omega_b^2/\nu_c^2)} + L_z^{-2}\right)\left[+\frac{Bp}{2E}\left(1+\frac{\omega_b^2}{\nu_c^2}\right)^{1/2} - 1\right] = \frac{L_x^{-2}+L_y^{-2}}{1+(\omega_b^2/\nu_c^2)}. \tag{17}$$

A graphical solution of this equation taking the experimental values of E_m yields the values H_m. The values of H_m for the three gases are tabulated in table 1.

When the applied electric field is much smaller than indicated by equation (7) and (16) and if the magnetic field is such that $\omega_b^2\nu_c^{-2} \gg 1$, then a great increase in the equivalent pressure will increase the number of electron–neutral collisions and the presence of a much lower electric field will decrease the magnitude of the average electron velocity. Also, in a low applied field E, a much smaller number of electrons will be produced by collisions with neutrals. Since the electron diffusion coefficients are (Allis 1956)
$$D_z = \tfrac{1}{3}\langle v^2\rangle/\nu_c$$
and
$$D_x = D_y = D_z[1 + (\omega_b^2/\nu_c^2)]^{-1}$$
from the above argument it can be assumed that there is practically no loss of electrons to the side walls due to diffusion. Most of the electrons will remain confined in the cavity and the centre of the cyclotron orbit of the electron, executing an elliptical motion at the frequency of the applied field, will receive energy from the applied field which is in phase with the field. The collisions will interrupt this motion and cause excitation and/or ionisation of the neutral atom. If the average energy received by the electron during one period of the applied field becomes just sufficient to ionise an atom, then after every period of the applied field, a fresh pair of electrons will start to receive energy from the field and multiply them, causing the establishment of a self-sustained discharge.

The average drift velocity components of the electron in an $\boldsymbol{E} \times \boldsymbol{H}$ field configuration are (Uman 1964)
$$v_x = \frac{e}{m}\frac{\nu_c}{\nu_c^2+\omega_b^2}E \qquad v_y = -\frac{e}{m}\frac{\omega_b}{\nu_c^2+\omega_b^2}E \qquad v_z = 0$$
where length, width and height are taken along the x, y and z directions respectively. Hence for the BF given by curves B in the figures we get
$$eEv_xT = eV_i \tag{18}$$

where $T = 2\pi/\omega$ and V_i is the ionisation potential, ω being the frequency of the applied electric field. The BF for curves B can be written as

$$E \approx \left(\frac{V_i \omega \nu_c}{2\pi(e/m)}\right)^{1/2} \frac{\omega_b}{\nu_c} \tag{19}$$

assuming $\omega_b^2 \nu_c^{-2} \gg 1$.

The E against H curve from equation (19) will have a slope given by $[V_i\omega(e/m)/2\pi\nu_c]^{1/2} \times 10^{-8}$ when all the quantities are given in emu. Since all the quantities are known, the slope can be calculated for all three gases and compared with the slopes of curves B observed experimentally and given in table 2.

As the electric field intensity in this active discharge so developed, is increased, the energy supplied per unit time to the plasma by the external source is eEv_x and if we assume that almost all of this energy goes to ionisation, then the supply of energy creates n new electrons per unit time where n is given by

$$n = Ev_x/V_i$$

However, the motion along the y direction causes the electrons to move away from the main plasma column. An average displacement of $\frac{1}{2}L_y$ causes an electron to be lost permanently in the side wall, and thus this electron will fail to contribute to the discharge current. The number of electrons that will run away from the main stream of the discharge current per unit time is, on average,

$$n' = 2v_y/L_y.$$

If the number of new electrons, n, is not lost to the side walls by the drift motion along the y direction as mentioned above, then the electron density will grow, causing a density gradient. This will in turn allow electrons to be lost by diffusion and the maintainance of the discharge will be a balance between diffusion loss and production of electrons by collisions; the required voltage is given by curves A. The discharge will therefore be extinguished if the acting applied field is much lower than that necessary to sustain it. Hence the condition for extinction can be written as

$$n \geqslant n' \qquad Ev_x/V_i \geqslant 2v_y/L_y$$

or

$$E \geqslant 2V_i\omega_b/L_y\nu_c \tag{20}$$

assuming $\omega_b^2\nu_c^{-2} \gg 1$. From equation (20) the slope of the E against H curve is $[2V_i(e/m)/L_y\nu_c] \times 10^{-8}$ when all the quantities are measured in emu. The slope is calculated for the three gases and compared with the slopes of curves C observed experimentally in table 2.

5. Conclusion

There is discrepancy in magnitude between the experimental and theoretical values of H_m obtained either from relation (8) or from (17) for all three gases. However, agreement is better between the experimental and theoretical values given by (17) for air and oxygen; for hydrogen the relation (8) gives a value of H_m which is in better agreement with the experimental value.

The variation of the value of χ with the electron energy and hence with E/p, ignored in the present theoretical discussions, and also the uncertainty in the accepted values of

A and B for the range of E/p values of the present experiment may partially account for the deviations between the experimental and theoretical values of H_m. The ranges of values of E/p for the validity of the values taken for A and B in table 1 are much smaller than the present experimental E/p values, especially for oxygen, which are not available and obtained by comparing Townsend's empirical relation

$$\alpha/p = A \exp(-Bp/E)$$

with the available experimental variation of α/p with E/p for the maximum E/p value (equal to 50 V cm^{-1} $Torr^{-1}$) (Brown 1959). However, the existence of a minimum BF in equations (7) and (16) indicates qualitative agreement between the theories and the experimental result. If the values of A and B for the present range of E/p values and the exact functional dependence of χ on E/p in the presence of a magnetic field become available, then both equations (7) and (16) are expected to give better quantitative agreement with the experimentally observed variation of BF with magnetic field (curves A).

There is good agreement in slope between curves B and equation (19) and between curves C and equation (20). The small discrepancy observed between the theoretical and experimental values of the slopes may be due to some simplifying assumptions that have been made to obtain equations (19) and (20) and also uncertainties in the energy-independent values of ν_c. Equations (19) and (20) indicate that even when gas pressure is such that neither mean free path nor the cyclotron diameter of the electron are comparable with the dimensions of the discharge cavity then the discharge once established can be extinguished either by increasing the electric field and keeping the magnetic field constant or increasing the magnetic field and keeping the electric field constant provided the condition $\omega_b^2\nu_c^{-2} \gg 1$ is satisfied. The success of equations (19) and (20) in explaining quantitatively the experimental observations of curves B and C justifies the operation of the mechanism proposed in discharges for small electric field intensities when $\omega_b^2\nu_c^{-2} \gg 1$.

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