CHAPTER IV

MEASUREMENT OF ELECTRON TEMPERATURE IN GLOW DISCHARGE IN TRANSVERSE MAGNETIC FIELD BY SPECTROSCOPIC METHOD.

4.1. Introduction

It is now well established that plasma parameters undergo a significant change when the plasma column is acted upon by a magnetic field. Whereas in an axial field at low gas pressure the electron temperature is reduced and the axial electron density increased, in a transverse field the electron temperature is raised and the electron density lowered when the positive column is pressed against the wall of the vessel (Beckman, 1948, Sen et al 1971, 1972, Kaneda, 1979). The nature of change is dependent upon the alignment of magnetic field with respect to the discharge current. In chapter III, we have measured parameters of magnetoplasma by Langmuir probe method in air, hydrogen, oxygen and nitrogen in both transverse and longitudinal magnetic fields. In a transverse magnetic field the radial electron density decreases and electron temperature increases in accordance with theory of Beckman (1948) and Sen et al (1972) specially for small values of B/p (B = magnetic field, p = gas pressure). Kaneda (1979) has reported that in a plasma a transverse magnetic field causes the wall losses and electron temperature to increase. Sen et al (1972) observed that the intensity of spectral lines increases with transverse magnetic field and after attaining a maximum value gradually decreases with the field. The field for maximum intensity depends upon the nature of the gas and wavelength of radiation. Assuming that the radial electron density decreases and electron temperature increases in a transverse magnetic field, the results have been quantitatively explained. To verify whether the assumption that T_{eB} rises with B is valid, $T_{eB} = f(B)$ is measured by spectroscopic method.

In chapter III, it has been observed that in transverse magnetic field, the probe characteristics became distorted owing to collision in the sheath. In this chapter measurement of electron temperature in glow discharge in hydrogen and helium plasma in transverse magnetic field by spectroscopic method has been described. Spectroscopic method has one advantage over the probe method that it does not disturb the plasma unduly. Electron temperature can be deduced from relative intensities of spectral lines. Electron temperature enters into the emission line intensities through excitation rate coefficients which are also dependent on electron number density. Both T_e and n_e are affected by a transverse magnetic field. When ratio of spectral intensities of two lines of some element is considered, dependence on n_e cancels out and the ratio depends explicitly on T_e. So dependence of T_e on transverse magnetic field could be determined by this method.

4.2. Method of measurement

It is well known that T_e cannot be determined from measured spectral intensities without some assumptions regarding the type of equilibrium that prevails inside the discharge tube. At the very beginning the question arises whether the LTE or SC (semi-corona) model is appropriate. To obtain LTE, the ~~xx~~ reverse of all fast processes must be maintained and exact balancing of total rates for complimentary processes must be allowed to take place. Since the discharge tube plasma considered here is optically thin to internal radiation (except perhaps for the resonance lines) collisional processes are usually more important in establishing LTE than radiative processes. Consequently collisional decay rates must exceed radiative decay rates. Thus, at a sufficiently high electron density, collisional LTE can be achieved. When electron densities are too low for the establishment of LTE it is still possible to obtain equilibrium whereby collisional excitation and ionisation is balanced by radiative decay and recombination respectively for all levels except the high lying ones. This model is known as semi-corona model. Some necessary, but not sufficient creteria for the establishment of LTE or SC inside a discharge tube have been discussed in chapter I. Utilising the criteria of Griem (1964), Hey (1978) and Wilson (1962) the value of electron number density n_e has been calculated and entered in Table 4.1 for hydrogen and helium gas.

TABLE 4.1

Limiting values of n_e for different types of equilibria.

Criteria	For LTE			For partial LTE	For SC	Estimated
Glow discharge	Griem (1964)	Hey (1976)	Wilson (1962)	Griem (1964)	Wilson (1962)	
Hydrogen P = 1 torr T_e = 1 eV.	$n_e \geqslant 10^{17}$	$n_e \geqslant 10^{18}$	$n_e \geqslant 2\times10^{17}$	$n_e \geqslant 1.7\times10^{14}$	$n_e \leqslant 6\times10^{12}$	10^{10} to 10^{11}
Helium P = 1 torr T_e = 5 eV.	$n_e \geqslant 2\times10^{18}$	$n_e \geqslant 10^{19}$	$n_e \geqslant 2\times10^{18}$		$n_e \leqslant 3\times10^{14}$	10^{10} to 10^{11}

It is evident from the table that the estimated electron density of glow discharge is well within the range of applicability of SC model, n_e has been estimated a bit higher than typical value in glow discharge for the constriction of the discharge tube in the central region from where the radiations are escaping.

In SC model, for a weakly ionised plasma,

$$n_1 n_e X_{1j}(T_e) = n_j^* \sum_{m<j} A_{jm} \tag{4.1}$$

X_{1j} (T_e) is the collisional excitation rate coefficient from ground state (designated by subscript 1) to j^{th} state. n_j^* is number density of excited neutral atoms in the j^{th} level and n_1 is the number density of ground state atoms.

Considering the plasma to be steady, homogeneous and optically thin the intensity of a transition $j \rightarrow i$

$$I_{ji} = \frac{h l \nu_{ji}}{4\pi} n_1 n_e X_{1j} \frac{A_{ji}}{\sum_{m<j} A_{jm}} \tag{4.2}$$

h is the Planck's constant, l is the length of the emitting plasma column along the line of sight. ν_{ji} and A_{ji} are the frequency and transition propability of the transition respectively. $\sum A_{jm}$ is the reciprocal of life time τ_j of the j^{th} state.

Considering two transitions ($j \longrightarrow i$) and ($l \longrightarrow i$) and taking the ratio of the spectral intensities,

$$\frac{I_{ji}}{I_{li}} = \frac{\lambda_{li}}{\lambda_{ji}} \frac{X_{1j}}{X_{1l}} \frac{A_{ji}}{A_{li}} \frac{\sum A_{lm}}{\sum A_{jm}} \tag{4.3}$$

since

$$X_{1j} = \int_{\vartheta_0}^{\infty} Q_{1j}(\vartheta)\, \vartheta f(\vartheta)\, d\vartheta \tag{4.4}$$

where $Q_{1j}(\vartheta)$ is the collisional excitation cross section of electrons from the ground level and ϑ_0 is the threshold value for the transition. Estimation for X_{1j} can be made if we have the knowledge of Q_{1j} and $f(\vartheta)$ which is the electronic velocity distribution function. After integration in (4.4) X_{1j} becomes a function of electron temperature and T_e may be determined from equation (4.3).

4.2.1. Excitation rate coefficient for hydrogen

For hydrogen we have considered M.J.Seaton's cross section for collisional excitation by electron. For a Maxwellian electron energy distribution, Allen (1963) has calculated

$$X_{1j} = 17.0 \times 10^{-4} \frac{f_{1j}}{\sqrt{T_e}\, E_j} 10^{-5040 E_j / T_e} P\left(\frac{E}{kT_e}\right) \tag{4.5}$$

where E_j is the excitation energy in eV of the j^{th} level and f_{1j} is the absorption oscillator strength which is characteristic to an optically allowed transition ($1 \rightarrow j$, 1 is the ground state). $P(E/K\,T_e)$ is g Gaunt factor which is slowly varying function of frequency and T_e. Allen has calculated that when $E/kT_e > 10$

$$P\left(\frac{E}{kT_e}\right) = 0.066\Big/\left(\frac{E}{kT_e}\right)^{1/2} \tag{4.6}$$

utilising equations (4.5) and (4.6) in (4.3)

$$I_{ji}/I_{li} = \frac{\lambda_{li}}{\lambda_{ji}}\,\frac{f_{1j}}{f_{1l}}\left(\frac{E_l}{E_j}\right)^{3/2}\frac{A_{ji}}{A_{li}}\,\frac{\sum A_{lm}}{\sum A_{jm}}\,10^{5040(E_l-E_j)/T_e} \tag{4.7}$$

or,

$$T_e = \frac{5040(E_l - E_j)}{\log\left[\frac{I_{ji}}{I_{li}}\,\frac{\lambda_{ji}}{\lambda_{li}}\,\frac{f_{1l}}{f_{1j}}\left(\frac{E_j}{E_l}\right)^{3/2}\frac{A_{li}}{A_{ji}}\,\frac{\sum A_{jm}}{\sum A_{lm}}\right]} \tag{4.8}$$

4.2.2. Excitation rate coefficient for helium

The transitions chosen for the helium are 4471.5 Å and 5876 Å with upper levels, $1s4d\,^3D$ and $1s3d\,^3D$. Both the radiations are triplet lines. The excitation to these levels by electron impacts of the ground state helium atoms are known as optically forbidden transitions since the upper levels have no optically allowed spontaneous emission to the ground level. Values of the cross section of the forbidden transitions are virtually unknown. Green (1966) has suggested one semi-empirical formula for cross section of helium.

$$Q_{1j}(E) = \frac{4R^2 \pi a_0^2 f_{1j}}{E_0^2} \left[1-(E_0/E)\right]^{1/2} (E_0/E)^3 \qquad (4.9)$$

where E and E_0 are the energy of electrons and threshold energy of the transition ($1 \rightarrow j$). R = 13.6 eV is the Rydberg energy and a_0 is the Bohr radius. For f_{1j} Green suggests using the oscillator strength of a 'companion' allowed transition. Benson and Kulander (1972) have shown that equation (4.9) when averaged over a Maxwellian velocity distribution for electrons, can be represented as

$$X_{1j} = \beta_{1j} T_e^{\gamma_{1j}} \exp\left(-\alpha_{1j} \frac{E_j}{kT_e}\right) \qquad (4.10)$$

where β , γ and α are constants. Choosing $f_{1 \rightarrow 4^3D}$ = 0.01 and $f_{1 \rightarrow 3^3D}$ = 0.1, Benson and Kulander calculated values for the constant, for 5875.6 Å line ($1 \rightarrow j$):

β_{1j} = 1.17 x 10^{-7}, γ_{1j} =-0.368 and

α_{1j} = 1.01

for 4471.6 Å line ($1 \rightarrow l$): β_{1l} = 1.06 x 10^{-8},

γ_{1l} = -0.362 and α_{1l} = 1.01

A calculation for excitation rate coefficients considering T_e = 25,000°K shows

X_{1j} = 5.6765 x 10^{-14} and X_{1l} = 4.0065 x 10^{-15}

~~xxxGxS~~
in C.G.S. unit, so that $X_{1j} / X_{1\ell} \approx 14$. Subtracting the constributions from exp. $(-\alpha_{ij} E_{ij} / kT_e)$ the ratio $X_{1j} / X_{1\ell} \approx 10.5$. Since after substracting contribution from exp. $(-\alpha_{ij} E_{ij} / kT_e)$, the remaining contribution comes mainly from the β_{ij} in equation (4.10), we can write

$$\frac{\beta_{1j}}{\beta_{1\ell}} = \frac{X_{1j\ \text{subtracted}}}{X_{1\ell\ \text{subtracted}}} = 10.5$$

whereas the calculated values are $\beta_{1j} / \beta_{1\ell} = 11 \cdot 7 / 1 \cdot 06 = 11$, so that the X_{1j} in equation (4.10) is sensitive to β_{1j} But the factor f_{1j} enters into β_{1j} and an arbitrary choice for a 'companion' allowed transition for f_{1j}, influences β_{1j} and hence X_{1j}. The experimentally measured values by St.John, Miller and Lin (1964) are

$X_{1j} = 1.13 \times 10^{-15}$ and $X_{1\ell} = 4.4 \times 10^{-16}$

so that $X_{1j} / X_{1\ell} = 2.95$, which differs from calculated values of Benson and Kulander with arbitrary choice of $f_{1j,1\ell}$. From experimentally obtained values, the ratio $\beta_{1j} / \beta_{1\ell}$ is obtained as

$$\beta_{1j} / \beta_{1\ell} \approx X_{1j} / X_{1\ell} = 2.95$$

The difference in experimental and calculated values may be traced back to the arbitrary choice in f values.

For two lines $j \rightarrow i$ and $\ell \rightarrow i$

$$\frac{I_{ji}}{I_{li}} = \frac{\lambda_{li}}{\lambda_{ji}} \frac{\beta_{1j}}{\beta_{1l}} \frac{A_{ji}}{A_{li}} \frac{\sum A_{lm}}{\sum A_{jm}} \exp\left[\frac{\alpha}{kT_e}(E_l - E_j)\right] \tag{4.11}$$

and

$$kT_e = \alpha(E_l - E_j) \Big/ \ln\left[\frac{I_{ji}}{I_{li}} \frac{\lambda_{ji}}{\lambda_{li}} \frac{\beta_{1l}}{\beta_{1j}} \frac{A_{li}}{A_{ji}} \frac{\sum A_{jm}}{\sum A_{lm}}\right] \tag{4.12}$$

since $\alpha_{1j} = \alpha_{1l} = \alpha$ and $\gamma_{1j} = \gamma_{1l}$. For β_{1j} / β_{1l} in equation (4.12) we have utilised the value 2.95 obtained experimentally by St.John et al (1964).

4.2.3. Measurements in magnetic field

When the magnetic field B is applied

$$(I_{ji})_B = \frac{hl\nu_{ji}}{4\pi} n_{1B} n_{eB} X_{1j}(T_{eB}) \left[\frac{A_{ji}}{\sum_{m<j} A_{jm}}\right]_B \tag{4.13}$$

Hence

$$\frac{(I_{ji})_B}{I_{ji}} = \frac{n_{1B}}{n_1} \frac{n_{eB}}{n_e} \frac{X_{1j}(T_{eB})}{X_{1j}(T_e)} \frac{\left[\frac{A_{ji}}{\sum A_{jm}}\right]_B}{\left[\frac{A_{ji}}{\sum A_{jm}}\right]} \tag{4.14}$$

A. Hydrogen.

Putting values of X_{1j} from eqn. (4.5) and

$$P(E/kT_{eB})/P(E/kT_e) = (T_{eB}/T_e)^{1/2} \quad (4.14)$$

in eqn. (4.14) we get

$$\frac{(I_{ji})_B}{I_{ji}} = \frac{n_{eB}}{n_e}\,\frac{n_{1B}}{n_1}\,\frac{[A_{ji}/\sum A_{jm}]_B}{A_{ji}/\sum A_{jm}}\,10^{-5040\,E_j\left(\frac{1}{T_{eB}}-\frac{1}{T_e}\right)} \quad (4.16)$$

Now

$$\left[\frac{A_{ji}}{\sum A_{jm}}\right]_B \Big/ \frac{A_{ji}}{\sum A_{jm}} = \frac{(g_i)_B}{g_i}\,\frac{\tau_{jB}}{\tau_j}\,\frac{g_j}{(g_j)_B} \quad (4.17)$$

where g is the statistical weight of the level. Considering two lines

$$\frac{(I_{li})_B}{I_{li}} \Big/ \frac{(I_{ji})_B}{I_{ji}} = \frac{g_l (g_j)_B}{(g_l)_B\, g_j}\,\frac{\tau_{lB}\,\tau_j}{\tau_l\,\tau_{jB}}\,10^{5040(E_l-E_j)\left(\frac{1}{T_e}-\frac{1}{T_{eB}}\right)} \quad (4.18)$$

As the degeneracy is with respect to magnetic quantum number, in a magnetic field there will be splitting of the levels (Zeeman effect). As such, in a magnetic field they must be regarded as actually simple that is no longer degenerate. These different states are ascribed the same a "priori probability" or the same statistical weight. That is, it is assumed that they will appear equally often under the same conditions. Hence if the lines belong to one and the same series and if our spectroscope is unable to resolve the Zeeman splitting we can assume,

$$\left.\begin{aligned} &\frac{g_\ell}{g_{\ell B}} \cdot \frac{g_{jB}}{g_j} = 1 \\ &\tau_\ell = \tau_{\ell B} \text{ and } \tau_j = \tau_{jB} \end{aligned}\right\} \tag{4.19}$$

Then from eqns. (4.18) and (4.19)

$$\frac{1}{T_e} - \frac{1}{T_{eB}} = \log\left[\frac{(I_{\ell i})_B}{I_{\ell i}} \Big/ \frac{(I_{ji})_B}{I_{ji}}\right] / 5040\,(E_\ell - E_j) \tag{4.20}$$

B. Helium

Proceeding in the same manner and considering equation (4.12) it can be shown that

$$\frac{1}{kT_e} - \frac{1}{kT_{eB}} = \frac{\ln\left[\frac{(I_{\ell i})_B}{I_{\ell i}} \Big/ \frac{(I_{ji})_B}{I_{ji}}\right]}{\alpha(E_\ell - E_j)} \tag{4.21}$$

So from equations (4.8), (4.12), (4.20) and (4.21) electron temperatures with and without magnetic field may be determined for hydrogen and helium plasma in SC model. In the above calculations it has been assumed that plasma be optically thin, homogeneous and has a Maxwellian distribution for electronic energy. The validity for optical thinness and

Maxwellian distribution has been already discussed in Chapter I. For homogeniety of plasma column along the line of sight, equation (4.3) suggests that plasma need not be homogeneous along the diameter of discharge column which is also the line of sight, for $\int_{-R}^{R} I_r \, dr$ will be equal for the two transitions. Moreover, when a transverse magnetic field is present, electrons and ions are deflected towards the wall of the discharge tube in the direction of Lorentzian force. Thus the cylindrical symmetry of the plasma column is destroyed. But the column will be deflected in the direction of line of sight due to Lorentz force either towards the collimator or away from the collimator depending upon the direction of discharge current. In our experiment the deflection was towards the collimator. The inhomogeneity developed in the plasma column along (the line of sight by a transverse magnetic field would be equal for both the lines, so that measurements are not affected by the inhomogeniety developed. While determining T_e either from eqn. (4.8) or (4.12) the relative spectral response of the photomultiplier is to be considered. But to determine T_{eB} from eqns. (4.19) and (4.21), the relative spectral response does not affect the measurements since it is in no way a function of a distant magnetic field and the ratio of the line intensities has been considered.

4.3. Experimental arrangement

The discharge tubes which acted as the source of radiation were fitted with aluminium electrodes. Pure hydrogen and helium gases under a pressure of 1 torr have been introduced into the discharge tube and the discharge was excited by a stabilised d.c. power supply (1000 volts, 20 mA). An accurately calibrated constant deviation spectrograph was used to measure the wave length of the incident radiation. The slit of the spectrometer was illuminated by condensing the light from the source on to the slit. The spectral line to be studied was focussed on the cathode of the photomultiplier tube (M 10FS 29V$_\lambda$). The sensitivity of a photomultiplier tube is not the same for all wavelengths as sensitivity depends on the wavelength of the incident radiation and on the quantum efficiency of the cathode material.

So the radiation intensity which is determined experimentally was corrected for relative spectral response of the photomultiplier. Details of the method have been given in Chapter II. The intensity was measured by a microammeter in the detector circuit and this intensity was the total intensity $\int_0^\infty I_\nu \, d\nu$. The transverse magnetkc field was varied between zero to 1000 G and the experimental set up has been shown in fig. 2.5. For hydrogen, the spectral lines chosen are 4861.33 Å ($n = 4 \rightarrow n = 2$) and 6562.73 Å ($n = 3$ to $n = 2$). For helium, the 4471.5 Å ($4^3D \rightarrow 2^3P_1$)

and 5875.6 Å ($3^3D_{123} \longrightarrow 2^3P_1$) emission lines were chosen. The spectral lines chosen have been shown in fig. 4.1. For two emission lines I_{λ_1} and I_{λ_2} with wavelengths λ_1 and λ_2 and energies of the upper levels of the transisions E_1 and E_2,

$$\frac{d(I_{\lambda_1}/I_{\lambda_2})}{I_{\lambda_1}/I_{\lambda_2}} = \left[\frac{E_2 - E_1}{kT_e}\right] \frac{d(kT_e)}{kT_e} \qquad (4.22)$$

where kT_e is the electron temperature in eV. If $(E_2 - E_1)/kT_e \gg 1$, then an x % change in T_e results in $\gg x$ % change in $I_{\lambda_1}/I_{\lambda_2}$. But if $(E_2-E_1)/kT_e < 1$, then an x % change in T_e gives less than x % change in $I_{\lambda_1}/I_{\lambda_2}$. So it is desirable to have $(E_2 - E_1)/kT_e \gg 1$ in a measurement. But in our experiment, for visible atomic transitions with sufficient response to the photomultiplier, the above criterion is not fulfilled. In that case it is preferable to use one ionic lines. But in a magnetic field the way an ionic line is affected is different to that of an atomic line. Generally the ionic lines are enhanced more in a magnetic field. For this reason and as no strong ionic line was observable in the experiment, we have chosen the transisions that have been mentioned.

Other details of the experimental set up have been described in Chapter II.

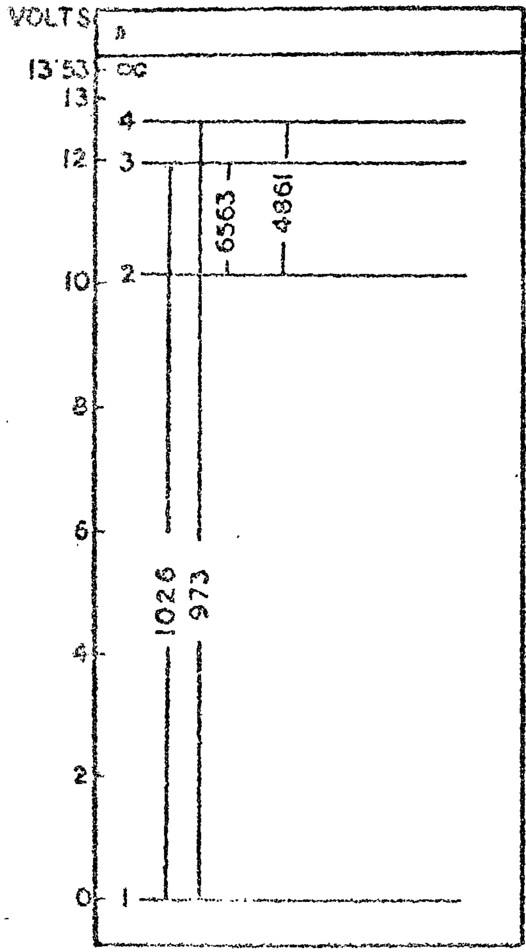

FIG. 4.1a ENERGY LEVELS OF THE LINES CHOSEN FOR HYDROGEN.

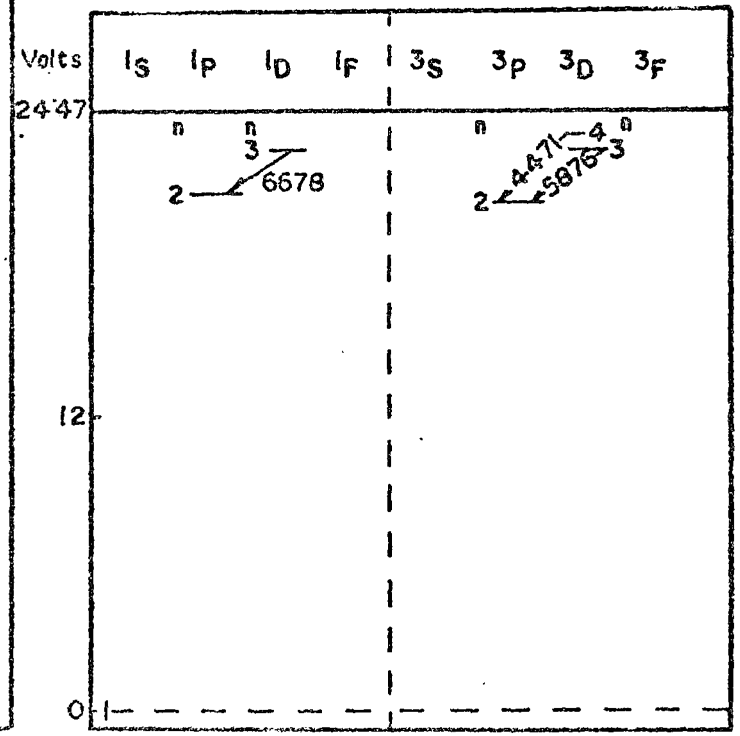

FIG. 4.1b. ENERGY LEVELS OF THE LINES CHOSEN FOR HELIUM.

4.4. Results and Calculations

A. Hydrogen

For hydrogen the values of the parameters appearing in eqn. (4.8) has been given in Table 4.2.

TABLE 4.2

Values of parameters for hydrogen.

λ_x (Å)	Transition	f_{1x}	$A_{xi} \times 10^{-6}$, Sec^{-1}	$1/\tau_x$ Sec^{-1} $\times 10^{-6}$	E_x V
4861.3	$n = 4$ to $n = 2$	0.0372	32.866	127.12	12.75
6562.7	$n = 3$ to $n = 2$	0.1034	93.413	260.61	12.00

A_{xi} and τ_x values have been taken from Griem (1964) and Corney (1977). f values have been calculating using formula of Bethe and Salpeter (1957), for $n \longrightarrow m$ transition,

$$f_{mn} = \frac{2^5}{3^{3/2}\,\pi}\left(\frac{1}{n^2 - m^2}\right)\frac{1}{n^5}\,\frac{1}{m^3} \tag{4.23}$$

Since our spectrograph cannot resolve the fine structure of the levels, equation (4.23) was used to determine f values.

For hydrogen,

$I_{6563} / I_{4861} = 1.2$ and T_e was calculated to be 1.017×10^4 °K.

Calculated values of electron temperature with and without a magnetic field have been entered in Table 4.3.

TABLE 4.3.

(Electron temperature calculated for hydrogen in SC model)

Magnetic field (G)	$B^2/p^2 \times 10^{-4}$ (G^2/torr^2)	$\ln\left[\frac{(I_{\ell i})_B}{I_{\ell i}} \Big/ \frac{(I_{ji})_B}{I_{ji}}\right]$	$T_e \times 10^{-4}$ °K	$\left(\frac{T_{eB}}{T_e}\right)^2 - 1$
0	0	0	1.017	0
250	6.25	0.005	1.034	0.0276
360	13	0.00847	1.046	0.05
440	19.4	0.01424	1.06	0.08
475	22.6	0.01623	1.067	0.094
525	27.6	0.01933	1.076	0.113
600	36	0.026195	1.098	0.158
700	49	0.03261	1.118	0.202
840	70.6	0.0361	1.130	0.228
950	90.3	0.03735	1.134	0.238

Though for plasma, we are considering, LTE cannot hold, we have nevertheless calculated the electron temperature and its variation in the magnetic field when the plasma is in LTE for comparison purpose. In LTE

$$\frac{I_{ji}}{I_{li}} = \frac{\lambda_{li}}{\lambda_{ji}} \frac{A_{ji}}{A_{li}} \frac{g_j}{g_l} \exp\left[(E_l - E_j)/kT_e\right] \tag{4.24}$$

Further

$$\left.\begin{aligned} A_{ji} &= 6.67 \times 10^{-1} \frac{g_i}{g_j} \frac{f_{ij}}{\lambda_{ji}^2} \\ \text{and} \quad A_{li} &= 6.67 \times 10^{-1} \frac{g_i}{g_l} \frac{f_{il}}{\lambda_{li}^2} \end{aligned}\right\} \tag{4.25}$$

Hence

$$kT_e = \frac{E_l - E_j}{\ln\left(\frac{I_{ji}}{I_{li}} \frac{\lambda_{ji}^3}{\lambda_{li}^3} \frac{f_{il}}{f_{ij}}\right)} \tag{4.26}$$

and

$$\frac{1}{T_e} - \frac{1}{T_{eB}} = \frac{k}{E_l - E_j} \ln\left[\frac{(I_{li})_B}{I_{li}} \Big/ \frac{(I_{ji})_B}{I_{ji}}\right] \tag{4.27}$$

Calculated values of electron temperature with and without magnetic field has been entered in Table 4.4.

TABLE 4.4

T_e of hydrogen with and without magnetic field in LTE.

Magnetic field B (Gauss)	$T_e \times 10^{-4}$ (°K)	$(T_{eB}/T_e)^2-1$
0	1.4614	0
250	1.4945	0.0458
360	1.5181	0.079
440	1.5569	0.1351
475	1.5743	0.1606
525	1.5977	0.1953
600	1.6562	0.2789
700	1.7065	0.3637
840	1.7292	0.4000
950	1.7504	0.4341

B. Helium.

For helium the values of parameters determing T_e have been given in Table 4.5.

TABLE 4.5

λ_x (Å)	$\frac{A_{xi}}{\sum A_{xm}}$	$\beta_{1j}/\beta_{1\ell}$	α_{1x}	f_{ix}	E_x (V)
4471.5	1	2.95	1.01	0.12	23.7355
5875.6	1		1.01	0.62	23.0731

For the levels considered $A_{xi} / \sum A_{xm}$ = 1 because there is no other optically allowed transition which is sufficiently strong, from the levels. The choice for β_{1j} / β_{1l} has been already discussed in section 4.2.2. α_{1x} values have been taken from Benson and Kulander (1972), those of f_{ix} for the transitions from Griem (1964). Energy values of levels have been taken from Moore (1971). Calculated values of electron temperature and its variation with transverse magnetic field have been shown in Table 4.6.

TABLE 4.6.

Electron temperature calculation for helium.

Magnetic field (G)	0	290	430	580
I_{5876} ($j \rightarrow i$) (arbitrary unit) μA reading.	160.0	162.0	163.5	166.0
I_{4471} ($l \rightarrow i$) (arbitrary unit) μA reading.	16.0	16.5	17.0	17.5
$\ln\left[\frac{(I_{li})_B}{I_{li}} \Big/ \frac{(I_{ji})_B}{I_{ji}}\right]$	0.0184	0.0393	0.0528	0.069
kT_e (eV) (SC model)	5.69	6.74	8.54	9.67
kT_e (eV) (LTE model)	5.1	5.95	7.32	8.61

For calculating T_e without magnetic field the ratio I_{ji} / I_{li} = 10 as evident from Table 4.6 was corrected for photomultiplier response factor (Fig. 2.7) and after correction I_{ji} / I_{li} was found to be 2.

4.5. Discussions and Conclusions

To verify whether equation

$$T_{eB} = T_e \left[1 + C_1 \frac{B^2}{p^2} \right]^{1/2} \tag{4.28}$$

as deduced by Sen et al (1972) is valid for the plasma we are considering, we have plotted $\left[(T_{eB}/T_e)^2 - 1 \right]$ with B^2/p^2 in Figs. (4.2) and (4.3) for hydrogen and helium. For hydrogen, C_1 was calculated as

C_1 (Sc model) = 0.426×10^{-6}

C_1 (LTE model) = 0.74×10^{-6}

For helium, C_1 value is 6.25×10^{-6} and more or less same for both the models. C_1 is actually the square of the mobility of electrons in a gas of pressure one torr at 0°C and explicitely depends on λ_{e1}, the mean free path of electrons in a gas of one torr pressure. λ_{e1} is a function of electron energy, thus depends on E/N. The experimentally obtained ratio of C_1 for helium and hydrogen corresponding to E/N values of the discharges is 14.7. A compa-

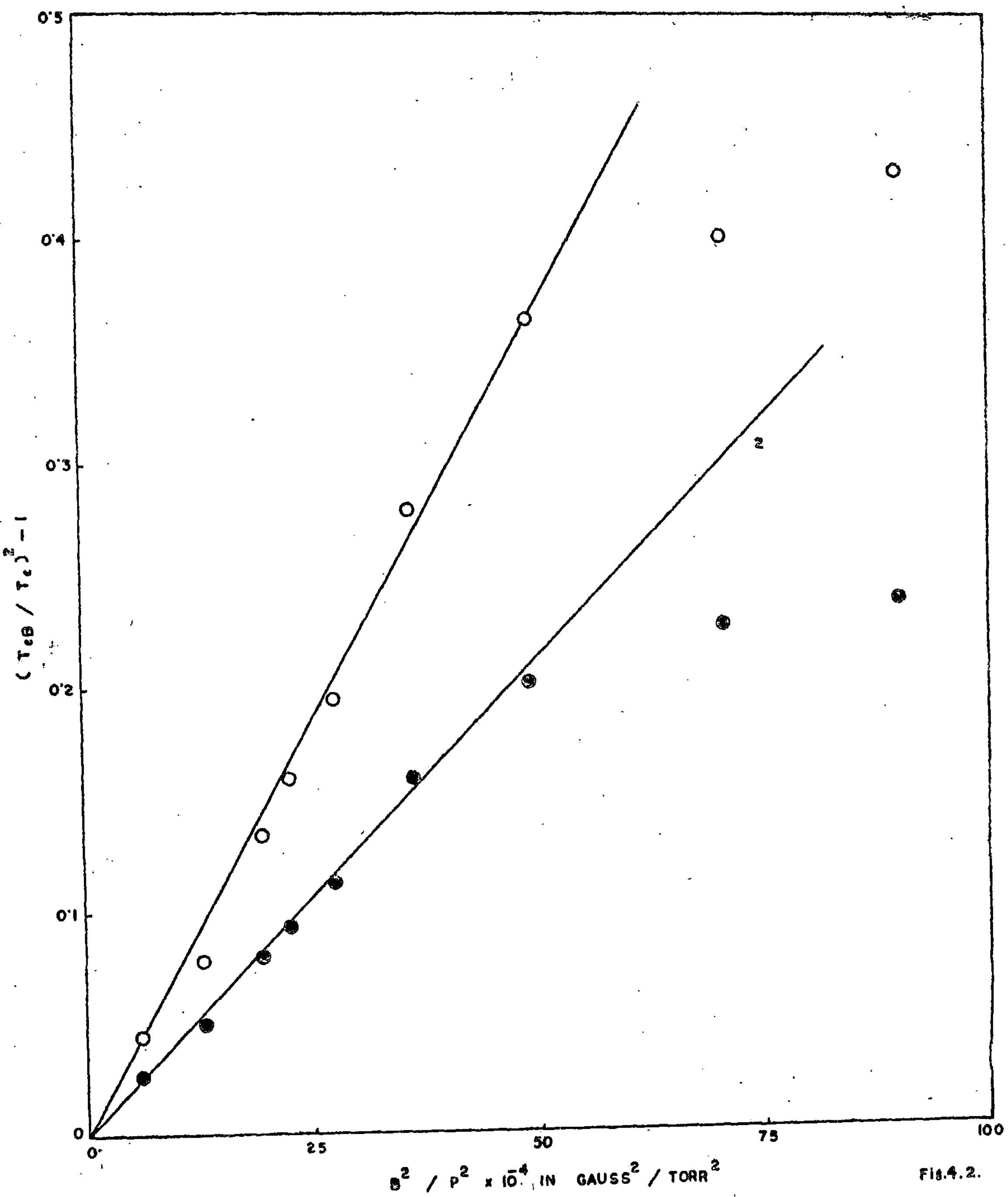

Fig. 4.2. Variation of $(T_{eB}/T_e)^2-1$ with B^2/p^2 for hydrogen in transverse magnetic field in SC (black circles) and LTE (white circles) models.

Fig. 4.3. Variation of $(T_{eB}/T_e)^2 - 1$ with B^2/p^2 for helium in transverse magnetic field in SC (white circles) and LTE (black circles) models.

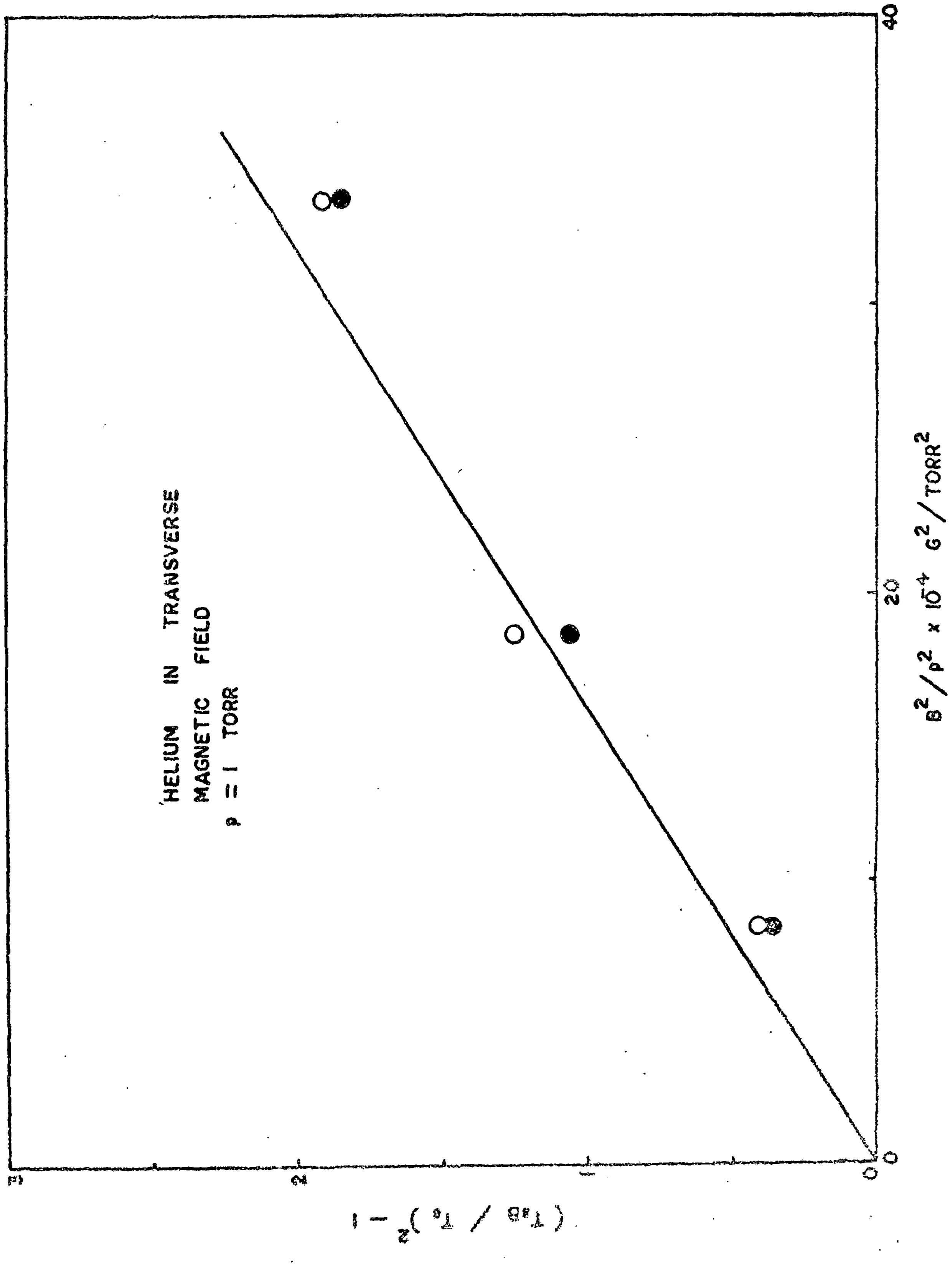

FIG. 4.3.

rison of this ratio with values obtained by other method is of interest. von-Engel (1965) has given the collision cross section Q_e (cm^{-1}) of electrons at a pressure of 1 torr at 0°C for electron energies 3×10^{-2} eV. (molecular energies). Different values (5 and 23) for Q_e in helium have been given. The ratio Q_e (helium)/ Q_e (hydrogen) calculated from A.H.von-Engel's table corresponding to 3×10^{-2} eV. energy, lies between 1.25 to 5.75 and the square of $\frac{Q_e \text{ (helium)}}{Q_e \text{ (hydrogen)}}$ is between 1.56 and 33. But for a true comparison, E/N values of discharges should be ~~made.~~ same

For hydrogen, C_1 values found by probe method in chapter III and by spectroscopic method differ. The reason is the different values of E/N for the discharges. If we consider that the temperatures determined by different methods for different discharges (with E/N differing) are correct then the temperatures would represent D/μ where D and μ are the coefficient of diffusion and mobility of electrons. Corresponding to these values of D/μ, if we find out the corresponding E/N for hydrogen, from the graph given by Huxley and Crompton (1974), the ratio of E/N for probe experiment to E/N for spectroscopic experiment become 15 - 20 and the ratio of C_1 value found by probe method and by spectroscopic method (SG model) is 625 whose square root is 25.

For hydrogen C_1 values obtained by SC and LTE models differ much but corresponding values for helium agrees very well. Actually LTE model is not a valid assumption under the conditions of discharges in our experiment, but we have calculated C_1 values by LTE method for comparison. Actually T_e values may differ depending upon the models chosen, but in determining C_1, the ratio of electron temperatures e.g. $(T_{eB}/T_e)^2 - 1$ is calculated which should not vary from model to model. So a comparison of C_1 value for hydrogen and helium in different models have been made. From the comparison it is evident that calculation in SC model for helium is more certain than that of hydrogen. For helium, we have used experimentally obtained cross section values by St.John et al (1964) for determining excitation rate coefficients. The measurements of St.John et al was carried out in a low pressure chamber so that effects of imprisonment radiation and collisional excitation transfer can be neglected and this type of experimental values is needed for a true SC model. In the actual plasma in our experiment, SC equilibrium assumption was justified. The influence of triplet metastable state 2^3S, on the population of higher triplet states have been neglected as no excitation from these metastable levels to the levels concerned has been taken into account. Phelp (1955) has shown that for helium below 10 torr pressure, 2^3S_1 level is quenched by diffusion to the walls. A constriction in the central region of the discharge tube where experiment was carried out enhances the diffusion loss. Moreover, for helium, since the radiation $3^3D_{123} \rightarrow 2^3P_2$ is very weak, we consider, $A_{ji}\tau_j = A_{li}\tau_l = 1$ i.e. the lines considered are the only transitions from the respective levels. Whereas for hydrogen values of $A_{ji}\tau_j / A_{li}\tau_l$ have been taken from Corney (1977) and the values were calculated theoretically from quantum mechanical basis which may have limited certainty.

It is thus evident that spectroscopic method can be adopted for measuring the electron temperature of a plasma in magnetic field as well. But a suitable choice of cross section data corresponding to the true model is to be found out.

The experimental result presented here show that equation (4.27) is valid for B/p as low as $\leqslant$ 700 G. torr^{-1}. The problem investigated here should be clearly distinguished from experimental studies by Aikawa (1976) in which anisotropy of the electron temperature and electron energy distribution function in a magnetised plasma has been studied. Here the average electron temperature and its variation in a magnetic field transverse to the direction of discharge current was measured assuming that the electron energy distribution is Maxwellian and ~~K~~ hence temperature dependent. It is evident that the majority of the electrons involved in the excitation and ionisation processes are those few with sufficiently high energy which occupy the tail of the distribution curve. Though the applied magnetic field will modify the energy distribution of these electrons only the distribution function of the majority of electrons remains unaltered and hence nearly Maxwellian and can be characterised by an electron temperature. The above measurement supports the theoretical deduction of Beckman (1948) and ~~x~~ Sen et al (1972).

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Measurement of electron temperature in glow discharge in transverse magnetic field by spectroscopic method

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Abstract. The electron temperature in the positive column of a glow discharge in hydrogen and helium as a function of the transverse magnetic field in the range of 0 to 1000 G has been obtained by measuring the intensities of two spectral lines having the same lower level. Since the electron number density in the column is of the order of 10^{10} cm^{-3} the semicorona model suitably modified has been used to calculate the plasma electron temperature in a magnetic field. For lower values of the field the variation of the electron temperature with the field can be represented by an expression deduced by Sen and Gupta following Beckman's theory. The electron energy distribution in a transverse field can thus be assumed to be nearly Maxwellian.

1. Introduction

It is now well established that plasma parameters undergo a significant change when the plasma column is acted upon by a magnetic field. Whereas in an axial field at low gas pressure the electron temperature is reduced and the electron density increased, in a transverse field the electron temperature is raised and the electron density lowered when the positive column is pressed against the wall of the vessel. (Beckman 1948, Sadhya *et al* 1979, Kaneda 1978.) The nature of the change is dependent upon the alignment of the magnetic field with respect to the direction of the discharge current. Sadhya *et al* (1979) have measured the axial electron density and electron temperature with Langmuir probes in air, hydrogen, oxygen and nitrogen in both the transverse and longitudinal magnetic fields. In a transverse magnetic field the radial electron density decreases and electron temperature increases in accordance with the theory of Beckman (1948) specially for small values of B/P; (B = magnetic field, P = gas pressure). To verify experimentally whether the nature of variation is the same when a glow discharge column is in a transverse magnetic field the electron temperature and its variation has been measured in this paper using a spectroscopic method as reported by Davies (1953). He observed that in a longitudinal magnetic field the electron temperature in a radio frequency discharge is raised. Ricketts (1970) has measured the electron temperature and density of argon ring discharge at 2 to 50 Torr in a longitudinal magnetic field. The temperature distribution showed a large decrease at the centre indicating a rapid cooling of the electrons which diffuse across the magnetic field. Kaneda (1978) has reported that in a plasma a transverse magnetic field causes the wall losses and electron temperature to increase.

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No spectroscopic measurement of the electron temperature of a plasma in a transverse magnetic field has been reported. Yet Sen and Gupta (1971) from the theoretical analysis of Beckman (1948) showed that in a transverse magnetic field B, the electron temperature T_{eB} should increase; for small B/P

$$T_{eB}=T_e[1+C_1(B^2/P^2)]^{1/2} \tag{1}$$

where $C_1=(eL/mv_r)^2$, L is the mean free path of the electrons in the gas at 1 Torr and v_r is their random velocity. C_1 is the square of the electron mobility at 1 Torr. Sen *et al* (1972) observed that the intensity of the spectral lines increases with transverse magnetic field and after attaining a maximum value gradually decreases with the field. The field for maximum intensity depends upon the nature of the gas and the wavelength of radiation. Assuming that the radial electron density decreases and the electron temperature increases in a transverse magnetic field the results have been quantitatively explained. To verify whether the assumption that T_{eB} rises with B is valid, $T_{eB}=f(B)$ is measured by a spectroscopic method.

2. Method of measurement

The standard method of spectroscopically measuring T_e in a plasma is to measure the half-width of a spectral line. To study the effect of magnetic field B on T_e, a low pressure plasma with a low input power is necessary in order to make Doppler broadening negligible. Hence the method of calculating T_e by measuring the relative intensity of spectral lines has been adopted here.

It is well known that T_e cannot be determined from measured spectral line intensities without assuming the type of equilibrium in the discharge plasma. The question arises whether LTE or SC (semicorona) model is appropriate. When electron densities are too low for the establishment of LTE it is still possible to obtain equilibrium whereby collisional excitation and ionisation is balanced by radiative decay and recombination respectively which is known as the semicorona model. For a weakly ionised plasma,

$$n_1 n_e X_{1j}(T_e)=n_j^* \sum_{m<j} A_{jm}. \tag{2}$$

$X_{1j}(T_e)$ is the collisional excitation rate coefficient from the ground state to jth state, and n_j^* is the number density of excited neutral atoms in the jth level. Considering the plasma to be steady, homogeneous and optically thin

$$I_{ji}=\frac{hl\nu_{ji}}{4\pi} n_1 n_e X_{1j} \frac{A_{ji}}{\sum\limits_{m<j} A_{jm}}. \tag{3}$$

Assuming the electron energy distribution function to be Maxwellian, Allen (1963) has calculated that

$$X_{1j}=17\times 10^{-4}(f_{1j}/\sqrt{T_e}E_j)P(E/kT_e)10^{-5040E_j/T_e}. \tag{4}$$

E_j is the excitation energy in electron volts and $P(E/kT_e)$ is the average gaunt factor which is a slowly varying function of frequency and T_e; when $E/kT_e>10$

$$P(E/kT_e)=0\cdot 066/(E/kT_e)^{1/2}. \tag{5}$$

Considering another transition $l \rightarrow i$ and taking the ratio of the spectral intensities we obtain from (3), (4) and (5)

$$\frac{I_{ji}}{I_{li}} = \frac{\lambda_{li} f_{1j}}{\lambda_{ji} f_{1l}} \left(\frac{E_l}{E_j}\right)^{3/2} \frac{A_{ji}}{A_{li}} \frac{\sum_{m<l} A_{lm}}{\sum_{m<j} A_{jm}} 10^{5040(E_l - E_j)/T_e} \tag{6}$$

or

$$T_e = 5040(E_l - E_j) \Bigg/ \lg \left[\frac{I_{ji}}{I_{li}} \frac{\lambda_{ji} f_{1l}}{\lambda_{li} f_{1j}} \left(\frac{E_j}{E_l}\right)^{3/2} \frac{A_{li}}{A_{ji}} \frac{\sum_{m<j} A_{jm}}{\sum_{m<l} A_{lm}} \right]. \tag{7}$$

This expression can be used for the measurement of electron temperature if the plasma is optically thin and excitations are by electron impact from the ground state and Wilson (1962) and de Vries and Mewe (1966) have shown that the approximate condition that has to be satisfied is

$$n_e \leqslant 10^{11} E_i^{3/2} (kT_e)^2 \text{ cm}^{-3}$$

where E_i and T_e are both in electron volts.

The electron density necessary for the semicorona model to hold is calculated from the above equation. For our present experiment the semicorona model is applicable.

2.1. Measurements in magnetic field

No measurement of electron temperature by the spectroscopic method in the transverse magnetic field has been reported so far. Sen *et al* (1972) observed that the intensity of a spectral line increases with the application of a transverse magnetic field. This effect has been utilised here to determine T_e by measuring the intensity of a line in the absence and in the presence of the field.

In the semicorona model we have shown that equation (3) holds. When the magnetic field is applied, from equations (3), (4) and (5) we obtain for the ratio of line intensities at B and at $B = 0$.

$$\frac{(I_{ji})_B}{I_{ji}} = \frac{n_{eB} n_{1B}}{n_e n_1} \frac{(A_{ji}/\Sigma A_{jm})_B}{(A_{ji}/\Sigma A_{jm})} 10^{-5040[(1/T_{eB}) - (1/T_e)]}. \tag{8}$$

Now

$$\left[\frac{A_{ji}}{\Sigma A_{jm}}\right]_B \Bigg/ \frac{A_{ji}}{\Sigma A_{jm}} = \frac{(g_i)_B}{g_i} \frac{\tau_{jB}}{\tau_j} \tag{9}$$

where τ_j is the mean life in the jth level. Considering another line having the intensity I_{li} (transition $l \rightarrow i$) and from equations (8) and (9)

$$\frac{(I_{ji})_B}{(I_{li})_B} \frac{I_{li}}{I_{ji}} = \frac{(g_l)_B}{(g_l)} \frac{g_j}{(g_j)_B} \frac{\tau_l}{\tau_{lB}} \frac{\tau_{jB}}{\tau_j} 10^{5040[(1/T_e) - (1/T_{eB})](E_l - E_j)}. \tag{10}$$

As the degeneracy is with respect to magnetic quantum number in a magnetic field there will be splitting of the levels and these states are ascribed the same *a priori* 'probability' or the same statistical weight. Hence if the lines belong to one and the same series and if our spectrograph does not resolve the Zeeman splitting, we can assume

$$\frac{(g_l)_B}{g_l} \frac{g_j}{(g_j)_B} = 1$$

and also by the same assumption $\tau_l=\tau_{lB}$ and $\tau_j=\tau_{jB}$ then

$$\frac{(I_{jt})_B}{I_{jt}}\frac{I_{lt}}{(I_{lt})_B}=10^{5040(E_l-E_j)[(1/T_e)-(1/T_{eB})]}$$

or

$$\frac{1}{T_e}-\frac{1}{T_{eB}}=\lg\left(\frac{(I_{jt})_B}{I_{jt}}\frac{I_{lt}}{(I_{lt})_B}\right)\Big/5040(E_l-E_j). \tag{11}$$

3. Experimental arrangement

The discharge tubes of 2·5 cm inner diameter and 15 cm length were fitted with aluminium electrodes and filled with pure hydrogen and helium at a pressure of 1 Torr. The discharge was excited by a stabilised DC power supply (1000 V, 10 mA). A constant deviation spectrograph was used. The slit was illuminated by condensing the light from the tubes on to the slit. The spectral line to be studied was focused on the cathode of the photomultiplier tube (M 10FS 29V_λ). Since the sensitivity of a photomultiplier tube depends on the wavelength and on the quantum efficiency of the cathode material, proper correction has been made to standardise the intensity ratios of the spectral lines after Griem (1964). The detector circuit of Sen *et al* (1972) has been used here. The transverse magnetic field is provided by an electromagnet. It is varied from 0 to 1000 G and calibrated by a flux meter.

4. Results and discussion

The electron temperature of both a hydrogen and a helium plasma have been measured in the absence and in the presence of a magnetic field. For hydrogen the spectral lines chosen were

$$\lambda_{lt}=4861\cdot33\ \text{Å} \qquad E_l=12\cdot75\ \text{V}$$

$$\lambda_{jt}=6562\cdot73\ \text{Å} \qquad E_j=12\cdot00\ \text{V}$$

$$\frac{I_{jt}}{I_{lt}}=1\cdot25.$$

f values are tabled in Griem (1964). Since our spectrograph cannot resolve the fine structure of the levels, the value of oscillator strength was found using the formula of Bethe and Salpeter (1957)

$$f_{mn}=\frac{2^5}{3^{3/2}\pi}\left(\frac{1}{n^2-m^2}\right)\frac{1}{n^5}\frac{1}{m^3}$$

for $n\rightarrow m$ transition. The values of A_{jt} and A_{jm} have been tabulated by Corney (1977) and Griem (1964). From equation (7) T_e in hydrogen was calculated $T_e=10\,200$ K.

For helium the spectral lines chosen were

$$\lambda_{lt}=4471\cdot5\ \text{Å} \qquad \lambda_{jt}=5875\cdot6\ \text{Å}$$

$$E_l=23\cdot7355\ \text{V} \qquad E_j=23\cdot0731\ \text{V}$$

For helium, excitations via metastable atoms have been neglected because the electron density is so low that the metastable levels are primarily quenched by diffusion to the walls and by two- and three-body collisions. Phelps (1955) has shown that helium 2^3S_1 level is quenched in this way. Since the transition $3^3D_{123} \rightarrow 2^3P_2$ is very weak $A_{ji}\tau_l/\tau_j A_{lm}$ may be taken to be unity. The electron temperature for $B=0$ in He is $T_e = 26\,730$ K.

T_e for H_2 and He in the presence of a magnetic field has been obtained from equation (11).

Sen *et al* (1972) showed that assuming equation (1) the increase in intensity and the occurrence of a maximum in the intensity curve for a particular value of the magnetic field can be explained. The experimental verification of the theoretical deduction can be made from the experimental results obtained here.

Figures 1 and 2 show the variation of $(T_{eB}{}^2/T_e{}^2)-1$ with B^2/P^2. The curves for H_2 and He and low B/P are found to be straight lines but rise less steeply for higher

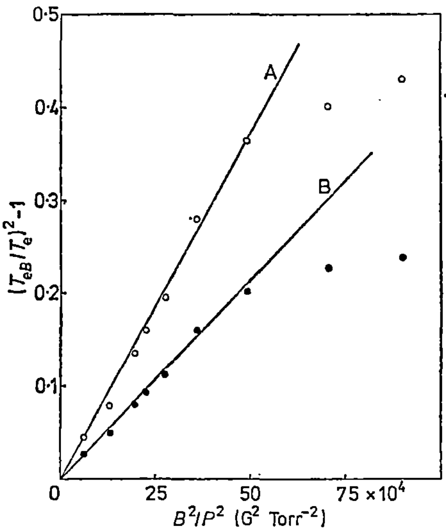

Figure 1. Variation of $(T_{eB}/T_e)^2-1$ with B^2/P^2 for hydrogen. A, from LTE model; B, from SC model.

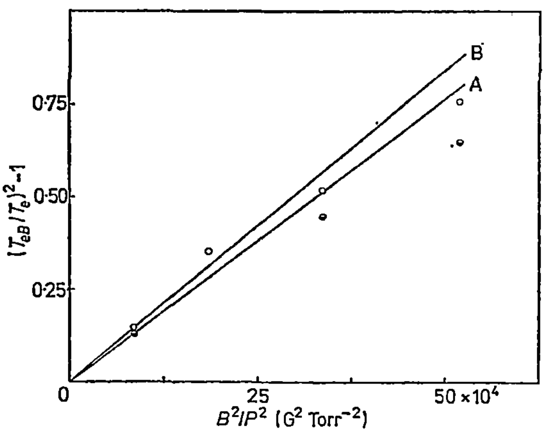

Figure 2. Variation of $(T_{eB}/T_e)^2-1$ with B^2/P^2 for helium. A, from LTE model; B, from SC model.

values of (B/P). From the slope of the curve the value of $C_1 = 0{\cdot}426 \times 10^{-6}$ for H_2 and $C_1 = 1{\cdot}68 \times 10^{-6}$ for He. These values agree quite well with the results obtained from radio frequency conductivity and microwave methods (Sen and Gupta 1970). By measuring the electron temperature by the probe method in a plasma in a transverse magnetic field it has been noted (Sadhya *et al* 1979) that T_e increases with B as given by expression (1).

The problem investigated here should be clearly distinguished from experimental studies by Aikawa (1976) in which the anisotropy of the electron temperature and electron distribution function in a transversely magnetised plasma has been studied. Here the average electron temperature and its variation in a magnetic field transverse to the direction of discharge current was measured assuming that the electron energy distribution is Maxwellian and hence temperature dependent. It is evident that the majority of the electrons involved in the excitation and ionisation processes are those few with sufficiently high energy which occupy the tail of the distribution curve. Though the applied magnetic field will modify the energy distribution of these electrons only, the distribution function of the majority of electrons remains unaltered and hence nearly Maxwellian and can be characterised by an electron temperature. The above measurements support the theoretical deductions of Beckman (1948).

It is thus evident that the spectroscopic method can be adopted for measuring the electron temperature of a plasma in magnetic field as well. The semicorona model is only valid for the determination of electron temperature in the positive column of a glow discharge if the electron density does not exceed 10^{10} cm^{-3}. It is also valid when the plasma is in a transverse magnetic field. Though in this case local thermodynamic equilibrium cannot hold we have nevertheless calculated the electron temperature and its variation in the magnetic field from

$$kT_e = (E_l - E_j) \Big/ \ln \left(\frac{I_{ji}}{I_{li}} \frac{\lambda_{ji}^3}{\lambda_{li}^3} \frac{f_{il}}{f_{ij}} \right) \tag{12}$$

and

$$\frac{1}{T_e} - \frac{1}{T_{eB}} = \frac{k}{(E_l - E_j)} \ln \left(\frac{(I_{li})_B}{I_{li}} \frac{I_{ji}}{(I_{ji})_B} \right).$$

The results have been plotted in figures 1 and 2 for comparison. It is seen that in a hydrogen plasma there is considerable difference between the results obtained from the two models, whereas in helium the results are almost identical. This is due to the fact that in helium the term $A_{li}\Sigma A_{jm}/A_{ji}\Sigma A_{lm}$ in equation (7) becomes unity and equation (7) becomes nearly identical with equation (12) whereas in hydrogen this ratio differs considerably from unity.

Sen *et al* (1972) assumed that equation (1) is valid over a wide range of B/P values and that the value of the magnetic field for which the intensity becomes a maximum can be obtained. However this proved to be not practicable because of numerous uncertain factors in the expression for B_{max}. The experimental results presented here show that equation (1) is valid for B/P as low as $\leqslant 700$ G Torr^{-1}. Although the abscissae in figures 1 and 2 are in similar units of variable B/P the experimental variable for H_2 and He was B since P and the tube diameter were kept constant (1 Torr, $2{\cdot}5$ cm).

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