Chapter 7

Conclusion

In this chapter, we are going to discuss various applications of counterfeit coins problem and contributions made in this thesis to solve the problem along with the future scopes to deal with in this area of research. Section 7.1 describes various application perspectives that can be inferred from the presented work. In this thesis, we have devised a number of algorithms to solve single and two-counterfeit coins problem along with the evidences of the versatility of the combinatorial reasoning that has been summarized in Section 7.2. In Section 7.3, future scopes of the domain have been discussed.

7.1 Application Perspectives

Several fields of applications are hereby envisaged to gear up with help of algorithms developed to obtain the counterfeit coin solutions. Being a complex search problem in combinatory, counterfeit coins problem has all-round applications in real life problems.

7.1.1 Hidden Graph Learning and Counterfeit Coins

The problems conferred in the thesis accomplish various distinctive areas of research, the most important of which is graph learning [69]. In case of a graph learning problem, a hidden graph is known to belong to a given family of labeled graphs on some predefined vertex set. With reference to the information, the objective is to identify the graph under consideration by edge-detecting queries. Each query tells whether a subset of the vertices induces an edge of the graph. This problem is motivated by applications in DNA physical mapping [70]. Applications of this model extend to bioinformatics, where learning a hidden matching [70] is of extreme importance in DNA sequencing.

In resemblance, if the set of coins is assumed to be the vertices of a graph and there lies an edge between two vertices (say, v_i and v_j), if and only if the coins representing v_i and v_j are equal in weight and each edge weight represents that weight. Hence, in this context, all the true coins in the search space always form a complete graph whereas the false coins are disconnected from the true coins forming either isolated vertices or connected among themselves depending on their weights, i.e. those are isolated vertices if their weights are different, otherwise, there may be edges among the false coins if all or some of them are of equal weight. Thus, this problem now reduces to graph construction problem or finding hidden graph problem as we have no information of the weights of the coins initially. As all the coins are identical in their appearance and we do not have their weights, we may assume that all the *n* nodes form a complete graph as shown in Figure 7.1(a), i.e. we assume all the coins are of equal weight or true coins. At this point, the graph reconstruction problem is to find out the false edges and accordingly remove those to obtain the preferred graph. In Figures 7.1(b) and 7.1(c), we have depicted the obtained graphs in case of two counterfeit coins problem. In case of Figure 7.1 (b), the counterfeit coins are mutually different in weight, whereas Figure 7.1(c) cites the scenario of equally weighed two false coins.

As reduction of the number of weighings to recognize false coins is a prime challenge, proper subsets of coins must be selected for subsequent weighing that would lead to the conclusion efficiently. Moreover, in case of graph learning, less the number of queries less is the cost of construction of the graph. Thus, query minimization is a key challenge here that is basically analogous to the comparison minimization of the counterfeit coins problem.

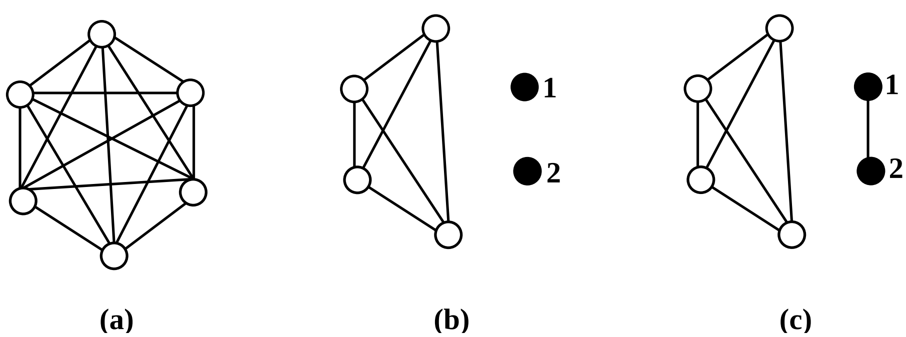

Figure 7.1: (a) 6 coins or nodes with identical appearance form a complete graph**. (b)** Nodes 1 and 2 are false nodes with mutually different weight other than the standard (or correct) weight. **(c)** Nodes 1 and 2 are false having mutually equal weight.

Learning a hidden general graph can be viewed as a variant of group testing that is a well-known combinatorial search problem [17]. Given a set of items, each of which is either positive or negative, a group test on a subset of items determines whether it contains any positive item. The key task of conventional group testing problem is to distinguish positive items by group tests. An extension of group testing is the complex model, where a set of complexes, each of which is a subset of fundamental items, is given and the property (positive or negative) of each complex is not yet determined; the corresponding query to identify positive complexes answers whether a subset of basic items contains at least one positive complex.

In a different background [17], identifying which pairs of chemicals react in a solution is modeled by counterfeit detection queries. Here, the coins, i.e. the vertices correspond to the chemicals, edges assign chemical reactions, and a set of chemicals 'reacts' if and only if, it induces an edge.

7.1.2 Electrical Circuit Analysis and Counterfeit Coins

As counterfeit coins problems possess analogy with graph learning, its application appears in many different contexts. Suppose, for a given circuit containing a set of chips on a board, we must test the resistance between two chips with an ammeter. In as few measurements as possible, our objective is to learn whether the circuit is connected entirely, or whether we need to provide power to the components individually [17]. This can be seen as a counterfeit coins problem, in which the chips are assumed to be the coins and the ammeter measurements are queries like weighing with a scale, which tell whether a set of coins are of same weight, i.e. they are connected. If we are provided an efficient enough ammeter to tell not only whether two chips are connected, but also how far apart they lie in the underlying circuit, we obtain the stronger 'shortest path' queries.

7.1.3 Consumer Products

The word *counterfeit* most frequently describes forgeries of currency or documents, but can also describe software, pharmaceuticals, etc. Counterfeit antique coins are generally made to a very high standard so that they often fool collectors. This is not easy, and many coins still stand out. It is not only limited to coins but potentially used global trade market, archaeological departments and many others. The raising issue of counterfeits violates intellectual property right and causing damage to both producer and consumer. To identify the counterfeit goods like pirated electronic gadgets, counterfeit batteries used in a digital camera, pharmaceuticals, valuable ornaments solution of counterfeit problem are used.

Counterfeiting of original versions of manufactured products is often observed to be a serious case of fraudulent activities in different ages of industrialization. Proposed algorithms, upon suitable modifications, can be useful to detect the originality of consumer product [65].

Moreover, Machine Learning aspects along with regression statistical techniques are the most promising attribute that can be added on top of counterfeit algorithms to materialize the product detection facilities more vibrant and stable [67]. A case study can be assimilated at this point such as follows. Suppose *X* is a famous women bag producer company. *Y* is another fraud company that produces bags which are very similar to the *X*. A customer goes to market with a smart hand-held device and places the infra-red emitter side of the device on the surface of the bag. The microscopic image as seen by the image detector is instantaneously checked with the pre-loaded counterfeit enabled application. Upon completion of a certain pattern matching formulation on the received image of bag surface is immediately cross-checked with the counterfeit values. Thus, the customer gets assurance about the originality or the manufactured product.

7.1.4 Drug Check

Medicine in form of drugs is indispensable part of human life to get rid of illness [66]. If medicine is going to be distributed in form of swindle format, then it must be dangerous for the patient who is taking it. Counterfeit algorithms are there to solve this type of problem with very effective way. Usually what happens is that the chemical behaviour of some organic compounds such as methanol, ethanol etc. make a practitioner puzzle to detect which substance it is in current form. Thus, medicines that are based on such chemicals are easy to be wrongly picked up than other forms of chemicals.

We may visualize a case study where this type of discrepancy can be sorted out. Suppose *A* is a drug dispenser who dispenses drug to the required unit of medicine design. *A* gets two sets of samples of look-alike chemical products. *A* is in confusion state to which to select for what design chain. Fortunately, *A* is equipped with the advanced Ramanspectroscopy machine that runs on the counterfeit engine. *A* targets the machine to the sample, the photo-metric values are then easily gets sorted with intervention from counterfeit-engine and the correct set of chemical is momentarily processed with accurate estimation.

7.1.5 Bill Detection

Nowadays, the printer-scanner technology has become too much advanced that can be easily used to recreate some forgery on the original bills. Such problem can be solved with the valuable interventions from the proposed counterfeit algorithms. A scenario may be opted to better understand the case as follows. Suppose an officer performs his duty toward checking of originality of the bills that are submitted to the office for appropriate procurement. This issue can be solved by using counterfeit techniques in prescribed manner. A deep learning model can be developed that inherits the convolution neural networks by comprising rectified as well as max-pooling linear units into the system. The learning network should be fed with the samples from different printer-scanner models. The extraction should be done based a prescribed set of features. The features will then be utilized to accurately predict as well as detection the faulty bills, thus reducing the opportunity of manual monetary loss.

7.1.6 Quantum Query Detection

Quantum computing can be seen as the next big thing in computing paradigm after the Boolean-conceptualization [64]. Researchers are in trouble to answer the question of quantum query complexity for finding k faulty coins, which is responsible to identify the actual input x. This problem can be seamlessly formatted with proper orientation from the balanced-oracle and inner product oracle algorithms. Developed counterfeit algorithms are well framed to realize the effectiveness of the quantum query complexity detection. But, it is known fact that the balanced-oracle algorithm can be simulated to the query by the inner-product algorithm by $O(k^{1/4})$. To do so, an appropriate stochastic lower bound under the random partition assumption can be validated against the $O(k^{1/4})$ complexity. Pan-wise coin distribution could be done in randomized trails.

As counterfeit coins problem initially belongs to combinatorial group testing problem, beside the aforementioned fields, it can be mapped into utilization in medical field like finding of any odd spike in MRI scan or in technical field to find any set of damaged pixels in a digital image.

7.2 Contributions

This section presents the key contributions made in this research work. Several new algorithms are designed, developed and simulated in this course of action. In this work, we have focused on single and two counterfeit coins problem. First, we have considered the eight coins problem, one that is well-known in the literature. Only one out of eight coins is a false coin, which is either heavier or lighter than a true coin. The usual objective of this problem is to find the false coin using a minimum number of comparisons. We have developed two new solutions for the eight coins problem, and they are as good as or better than the existing classical solution. Moreover, we have generalized it for all values of n, where n is an even number. We categorize the search space in two ways, ones which are powers of two, and the other is the rest of the even numbers. After developing the algorithm for solving the Counterfeit Coin Problem where the number of coins is even, we widen our algorithm for handling the situation of an odd number of coins problem. Thus, the algorithm is generalized.

Next, we have considered the two counterfeit coins problem. The common objective of this problem is to find two false coins among a set of identical coins using a minimum number of comparisons. As there are several variations for two counterfeit coins depending on the mutual relation of the counterfeit coins, we have first discussed all the variations in detail and devised algorithms for solving the problem considering all the cases individually. Here we have followed decision tree method. The nobility of these two algorithms is that the worst case running time of the algorithms are $O(\log n)$, except a typical one, i.e. when one false coin is heavier and the other is lighter and the weight difference between the heaver and a true coin is same as the difference of the lighter one and a true coin, the worst case run time becomes linear with respect to the cardinality of the search space. We have considered all the possible cases for any number of coins, i.e. the search space can be a set of coins of no restricted cardinality. Thus, we have generalized the algorithms to make it applicable for all the variations of two counterfeit coins problem. We established a clear picture of two counterfeit coins problem without any ambiguity.

7.3 Future Direction

This research is all about the design and development of several novel counterfeit algorithms to minimize the risk of fraudulent cases in various scene-specific aspects. This research has covered all variants of single and double counterfeit coins problem. As we have observed that the number of variants increases exponentially with the increase in number of false coins; hence the degree of fraudulence becomes very high. For an example, when the number of false coin is three, there can be twenty six different scenarios to take care of. We have investigated all the cases for three counterfeit coins in detail [26] and developed an algorithm to solve a few of those. Still there are several critical scenarios to be resolved.

Moreover, in some cases where volumes or density of any liquid is attackable for forgery, the scenario becomes much more difficult. In that case, we need to incorporate improved graph learning mechanisms to deal with the fact. Thus, the generalized counterfeit identification problem becomes a crucial domain of research. However, the study can be further engraved into interdisciplinary domain of information communication technology some of which have been cited as follows.

Internet of Counterfeit Things [66]: Internet of Things is the most recent bud of smart computing hierarchy as seen by the Gartner periodicals. Unlimited number of "*things*" are envisaged to get co-existed in near future all over the world. Node-oriented fraudulences can be minimized with proper incorporation with the developed counterfeit algorithms.

Big Counterfeit Data Analytics [68]: When there is lots of different data coming into internetwork in various speed and form factors, big data analytics becomes equally crucial. However, the requirement of analytics on the "big counterfeit data" can be taken as a next step of meaningfulness schematic for bringing data-independence in current ear of computing.

Deep Counterfeit Learning [67]: Machine Learning is key to artificially organize any relevant perspectives. Existing machine learning techniques should be intervened with the new counterfeit strategies so that a novel "counterfeit learning" theme could be visualized. Deep neural networking may act as the key enabler of assimilation of such notion in near future.

We strongly believe that this list will grow in the future, as more researchers involve themselves towards applying the counterfeit coins problem as a tool to solve the problems in their relevant research areas.