Chapter 4

Compact object with charge in Pseudo-spheroidal geometry

4.1 Introduction

In relativistic astrophysics, exact solutions of Einstein-Maxwell equations for a compact object described by a sphere containing matter become important because of various physical applications. To obtain a physically realistic stellar model the relativistic solution for the interior content of the charged fluid sphere is matched with the exterior Reissner-Nordström metric at the boundary. In the presence of electromagnetic field, the gravitational collapse may be counter balanced by the electrostatic repulsion of Columbian force along with the pressure gradient. The gravitational collapse of a spherically symmetric matter distribution with charge and nuclear density may avoid point singularity. Stellar models of compact objects considering a solid sphere with electromagnetic fields [66, 67] are recently studied. There are some compact objects with maximum mass and radius both less than that of neutron star, with higher compactification factor (ratio of mass to radius) have been reported from observations. The physics inside the superdense compact object are not well understood yet. The equation of state of matter of compact objects namely neutron stars, strange stars are also not known clearly. As the EoS is not known an alternative approach suitable for description of compact objects in relativity is possible because of Einstein field equations. The Vaidya-Tikekar approach will be applied here to obtan useful stellar solution for a compact star with Einstein-Maxwell field equations. The physical properties of static star with electromagnetic field is studied in different contexts [20, 66, 68] considering Vaidya-Tikekar space time metric [45]. The physical plausibility of Vaidya-Tikekar approach is discussed in Ref. [60]. Tikekar and Thomas [30], analysed compact stars with 3-pseudo spheroidal geometry for the 3-space of the interior space-time and obtained a class of relativistic solutions suitable for model building. In this chapter a class of relativistic stellar models will be presented using Tikekar-Thomas ansatz which prescribes 3-pseudo spheroidal geometry for the 3-space of the interior space-time in the presence of electromagnetic field. Consequently, physical properties of a charged relativistic star assuming its 3-geometry described by pseudo-spheroidal geometry will be studied. As the EOS of a neutron star is not known so we adopt a different technique [48] here to determine the EOS for a given geometry as was considered in previous chapter. A relation for pressure and density in the Einstein gravity are established and its physical properties are explored.

4.2 Einstein-Maxwell Equations

We consider a spherically symmetric static star represented by the line element

$$ds^2 = -e^{2\nu(r)}dt^2 + e^{2\mu(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{4.1}$$

where from eq.(1.2) $A(r) = e^{\nu(r)}$ and $B(r) = e^{\mu(r)}$ are the two unknown metric functions. We also consider electrically charged star with most general form of energy-momentum tensor given by,

$$T_{ij} = \text{diag}\ (-(\rho + E^2),\ p - E^2,\ p + E^2,\ p + E^2), \tag{4.2}$$

where ρ and p represent the energy-density and pressure respectively and E represents intensity of the electric field.

The components of the Einstein field equation are:

$$\rho + E^2 = \frac{(1 - e^{-2\mu})}{r^2} + \frac{2\mu' e^{-2\mu}}{r}, \tag{4.3}$$

$$p - E^2 = \frac{2\nu' e^{-2\mu}}{r} - \frac{(1 - e^{-2\mu})}{r^2}, \tag{4.4}$$

$$p + E^2 = e^{-2\mu}\left[\nu'' + \nu'^2 - \nu'\mu' + \frac{\nu'}{r} - \frac{\mu'}{r}\right], \tag{4.5}$$

where $()'$ represents derivative w.r.t. r. From eqs.(4.4) and (4.5) we obtain

$$\nu'' + \nu'^2 - \nu'\mu' - \frac{\nu'}{r} - \frac{\mu'}{r} - \frac{(1 - e^{-2\mu})}{r^2} - 2E^2 e^{2\mu} = 0. \tag{4.6}$$

We use the ansatz [69] to solve the above differential eq.(4.6),

$$e^{2\mu} = \frac{1 + \lambda\frac{r^2}{R^2}}{1 + \frac{r^2}{R^2}}, \tag{4.7}$$

where λ is the spheroidicity parameter and R is a geometrical parameter related with the configuration of a stellar model. The ansatz given by eq.(4.7) corresponds to pseudo-spheroidal geometry. Using eq. (4.7) in eq. (4.6) , one obtains a second order differential equation in x, given by

$$(1-\lambda+\lambda x^2)\Psi_{xx} - \lambda x\Psi_x + \lambda(-1)\Psi - \frac{2E^2R^2(1-\lambda+\lambda x^2)^2}{(x^2-1)}\Psi = 0 \tag{4.8}$$

where $\Psi = e^{\nu(r)}$, with $x^2 = 1 + \frac{r^2}{R^2}$, and ψ_x represents derivative of ψ w.r.t x.

Now for simplicity we choose β^2, such that it is related to the electric field intensity as

$$E^2 = \frac{\beta^2(x^2-1)}{R^2(1-\lambda+\lambda x^2)^2} = \frac{\beta^2 r^2}{R^4(1+\frac{\lambda r^2}{R^2})^2}. \tag{4.9}$$

The electric field intensity given by eq. (4.9) shows that the regularity at the center of the compact object is ensured [46]. The choice of E generates a model for a charged star which is physically realistic in pseudo-spheroidal geometry. We now use a further transformation from x variable to z variable as $z = \sqrt{\frac{\lambda}{\lambda-1}}\,x$, consequently the eq. (4.8) becomes

$$(1-z^2)\Psi_{zz} + z\Psi_z + [(1-\lambda) + 2\beta^2/\lambda]\Psi = 0. \tag{4.10}$$

Differentiating the above equation once again with respect to z, we get

$$(1-z^2)\Psi_{zzz} - z\Psi_{zz} + \Omega^2\Psi_z = 0 \tag{4.11}$$

where $\Omega^2 = (2-\lambda+2\beta^2/\lambda)$ is a constant. Now positivity of ω puts an upper bound on the spheroidicity parameter determined by β which is $\lambda < (1+\sqrt{1+2\beta^2})$. In pseudo spheroidal space-time positivity of central density puts further constraint on λ ($\lambda > 1$). Eq. (4.11) is a third order differential equation in z, which admits a general solution given by:

$$\Psi = \Psi_1 \left[\Omega\sqrt{z^2-1}\, cos(\Omega\eta) - z\, sin(\Omega\eta)\right] + \Psi_2\left[\Omega\sqrt{z^2-1}\, sin(\Omega\eta) - z\, cos(\Omega\eta)\right]. \tag{4.12}$$

where $\lambda > 1+\sqrt{1+2\beta^2}$, $\Omega = \sqrt{\lambda - 2 - 2\beta^2/\lambda}$ and $\eta = \cos^{-1} z$ for any β.

For $\beta = 0$ one recovers the solution obtained by Tikekar and Jotania [61]. The two unknown constants Ψ_1 and Ψ_2 are to be determined from the boundary conditions. The total charge in the compact star of radius r is defined as follows:

$$q(r) = 4\pi \int_0^r \sigma r^2 e^{\mu} dr = r^2 E^2(r) \tag{4.13}$$

where σ is the proper charge density.

4.3 Physical applications

The general relativistic solutions obtained above will be used in the section to study compact objects with charge. The physical parameters namely, energy density (ρ), pressure (p) and charge density (σ) can be expressed as a function of r which are given by,

$$\rho = \frac{(\lambda-1)}{R^2(1+\frac{\lambda r^2}{R^2})}\left[1 + \frac{2}{(1+\frac{\lambda r^2}{R^2})} - \frac{\beta^2 r^2}{R^2(\lambda-1)(1+\frac{\lambda r^2}{R^2})}\right], \tag{4.14}$$

$$p = -\frac{(\lambda-1)}{R^2(1+\frac{\lambda r^2}{R^2})}\left[1 - \frac{2R^2(1+\frac{r^2}{R^2})}{r(\lambda-1)}\frac{\Psi_r}{\Psi} - \frac{\beta^2 r^2}{R^2(\lambda-1)(1+\frac{\lambda r^2}{R^2})}\right], \tag{4.15}$$

$$\sigma = \frac{2\beta(1+\frac{r^2}{R^2})(3+\frac{\lambda r^2}{R^2})}{R^2(1+\frac{\lambda r^2}{R^2})^{\frac{5}{2}}}. \tag{4.16}$$

The above equations will be used to study the physical property of compact objects. The central density of a star is independent of β *i.e.*, on electric field which is evident from eq. (4.14). It however depends on the geometric parameter R and the spheroidicity parameter λ [69]. The central density of a compact star is a constant which is given by,

$$\rho_0 = \frac{3(\lambda - 1)}{R^2}. \tag{4.17}$$

which is independent of charge. The positivity of central density ensures that $\lambda > 1$ and in pseudo spheroidal geometry $z > 0$. The expressions for energy density and pressure of an uncharged star corresponds to the case when $\beta = 0$. Inside the charged sphere, ρ, p and σ are well-behaved , bounded, finite and regular even at the center. The boundary of the star is determined from $p(b) = 0$. The mass of a charged star contained within the radius b is given by

$$M(b) = \frac{(\lambda-1)b^3}{2R^2(1+\lambda\frac{b^2}{R^2})} - \frac{\beta^2}{4\lambda^2}\Big[\frac{b(3+2\lambda\frac{b^2}{R^2})}{1+\lambda\frac{b^2}{R^2}} - \frac{3R\,tan^{-1}(\frac{b}{R}\sqrt{\lambda})}{\sqrt{\lambda}}\Big]. \tag{4.18}$$

The compactness factor u (defined as the ratio of mass to radius) is given by

$$u = \frac{M(b)}{b} = \frac{(\lambda-1)y^2}{2(1+\lambda y^2)} - \frac{\beta^2}{4\lambda^2}\Big[\frac{(3+2\lambda y^2)}{1+\lambda y^2} - \frac{3tan^{-1}(y\sqrt{\lambda})}{y\sqrt{\lambda}}\Big] \tag{4.19}$$

where $y = \frac{b}{R}$. To construct a physically viable stellar model for compact charged star, the following conditions are imposed:

- At the boundary of the charged star ($r = b$), the first fundamentals of the interior solutions is matched with that of the exterior Reissner-Nordström metric given by,

$$ds^2 = -\left(1 - \frac{2M}{r} + \frac{q^2}{r^2}\right) dt^2 + \left(1 - \frac{2M}{r} + \frac{q^2}{r^2}\right)^{-1} dr^2 + r^2(d\theta^2 + Sin^2\theta d\phi^2). \tag{4.20}$$

Thus the metric potential must be matched at the boundary as follows:

$$e^{2\nu(r=b)} = e^{-2\mu(r=b)} = \left(1 - \frac{2M}{b} + \frac{q^2}{b^2}\right). \tag{4.21}$$

where M and q denote the mass and charge of the compact star respectively, as measured by an observer at infinity.

- At the boundary $(r = b)$ of the star the pressure $p = 0$, which yields,

$$\frac{\Psi_r(b)}{\Psi(b)} = \frac{b(\lambda-1)}{2R^2(1+\frac{b^2}{R^2})}\left[1 - \frac{\beta^2 b^2}{R^2(\lambda-1)(1+\frac{\lambda b^2}{R^2})}\right] \tag{4.22}$$

- The condition that pressure $p \geq 0$ leads to

$$\frac{\Psi_r}{\Psi} \geq \frac{r(\lambda-1)}{2R^2(1+\frac{r^2}{R^2})}\left[1 - \frac{\beta^2 r^2}{R^2(\lambda-1)(1+\frac{\lambda r^2}{R^2})}\right]. \tag{4.23}$$

Differentiating eqs. (4.14) and (4.15) respectively once, we obtain squared speed of sound which is given by,

$$\frac{\partial p}{\partial \rho} = \frac{\frac{R^2}{r}(1+\frac{\lambda r^2}{R^2})\left[\frac{R^2}{r}(1+\frac{\lambda r^2}{R^2})(1+\frac{r^2}{R^2})(\frac{\Psi_r}{\Psi})^2 + (\lambda-1)(\frac{\Psi_r}{\Psi})\right] - \frac{\beta^2}{R^2}(3R^2+\lambda r^2)}{\left[\lambda(\lambda-1)(5+\frac{\lambda r^2}{R^2}) + \frac{\beta^2}{R^2}(R^2-\lambda r^2)\right]}. \tag{4.24}$$

The speed of sound should be less than the speed of light inside the star to maintain the causality condition for which $\frac{\partial p}{\partial \rho} < 1$. Thus the causality condition leads to the following inequality equations

$$\left(-\frac{\sqrt{\lambda-1}}{2\sqrt{\lambda\left(1+\frac{r^2}{R^2}\right)}}-D\right)\leq\frac{R^2(1+\frac{\lambda r^2}{R^2})\sqrt{1+\frac{r^2}{R^2}}}{r\sqrt{\lambda(\lambda-1)}}\frac{\Psi_r}{\Psi}\leq\left(-\frac{\sqrt{\lambda-1}}{2\sqrt{\lambda\left(1+\frac{r^2}{R^2}\right)}}+D\right) \quad (4.25)$$

where $D=\sqrt{\frac{\lambda-1}{4\lambda\left(1+\frac{r^2}{R^2}\right)}+5+\frac{\lambda r^2}{R^2}+\frac{4\beta^2}{\lambda(\lambda-1)}}$.

The above inequalities given by eqs.(4.23) and (4.25) lead to the following bounds on the spheroidicity parameter λ:

$$(i)\ \lambda<\frac{7-\sqrt{49-17(16\beta^2-3)}}{17} \quad (4.26)$$

$$(ii)\ \lambda>\frac{1}{2}\left[3+f_1+\sqrt{\frac{1}{3}(34+8\beta^2-\frac{f_2}{f_3}-f_3)+\frac{24+8\beta^2}{f_1}}\right] \quad (4.27)$$

where $f_1=\sqrt{\frac{1}{3}(17+4\beta^2+\frac{f-2}{f_3}+f_3)}$, $f_2=25+16\beta^2+16\beta^4$ and $f_3=(125+120\beta^2+312\beta^4+64\beta^6+24\sqrt{3}a^2\sqrt{25+28\beta^2+47\beta^4+16\beta^6})^{\frac{1}{3}}$. In an uncharged star *i.e.*, ($\beta=0$) one obtains (i) $\lambda<-\frac{3}{17}$ and (ii) $\lambda>5$ respectively as was obtained in Ref. [70]. The maximum compactness factor upto which a stellar model is permitted corresponds to $u=0.4167$ for $\beta=0$ with $\lambda=6$. An interesting case is noted here in a charged star for $\beta=0.65$, where the spheroidicity parameter satisfies a lower bound ($\lambda>5.16208$) for the same compactness factor $u=0.4167$.

4.3.1 Numerical results

The expressions for density and pressure of a compact star obtained in the previous section are functions of λ, R and β. It is not simple to obtain a known form of pressure in terms of density as these are highly non-linear functions of the above parameters. Consequently a numerical technique will be adopted here to study the physical properties of compact objects. The radial variation of different physical parameters such as p, and $\frac{\partial p}{\partial \rho}$ are plotted for different λ, β and R. We also tabulate radial variation of energy density with various λ and β. To construct a stellar model, first we determine R using eq.(4.21) in the uncharged limit for a given configuration of compact object namely, the spheroidicity parameter (λ), mass (M) and size (b). The effect of charge, on the different physical parameters are studied by considering the two unknown constants Ψ_1 and Ψ_2 of eq.(4.12). The arbitrary constants namely, Ψ_1 and Ψ_2 can be determined from eqs.(4.21) and (4.22) for a given mass (M), radius (b), spheroidicity parameter (λ) and charge parameter (β). Therefore, the radial variation of the physical parameters ρ, p, and $\frac{\partial p}{\partial \rho}$ are determined knowing Ψ_1, Ψ_2 and R for different β. One can determine the compactness factor $u = \frac{M}{b}$ using eq.(4.19) for a given spheroidicity parameter λ.

Case I: An X-ray pulsar, namely Her X-1 [15] characterized by Mass $M = 0.88M_{\odot}$, where $M_{\odot}$ = the solar mass, size of the star $b = 7.7$ km. we obtain stellar model having compactness factor $u = M/b = 0.1686$ for a spheroidicity parameter $\lambda = 6$. The known values of the parameters are then used to determine the geometrical parameter R using eqs. (4.7) and (4.21) at $r = b$. We obtain $R = 22.882$ km. for a star without charge, *i.e.*, $\beta = 0$. For a star with $\beta = 0.3$ and $\beta = 0.6$, we get a viable compact stellar model for $R = 22.9047$ km. and $R = 22.9741$ km. respectively. In fig. (4.1) the radial variation of pressure p are plotted for $\beta = 0$, 0.3 and 0.6 respectively.

It has been observed from the fig.(4.1) that the radial pressure p decreases with an increase of electric field intensity which is prominent at the center. In Table-4.1, radial variation of energy density (ρ) inside and on the surface of the star are tabulated for two sets of parameters. The energy density (ρ) decreases with an increase in charge

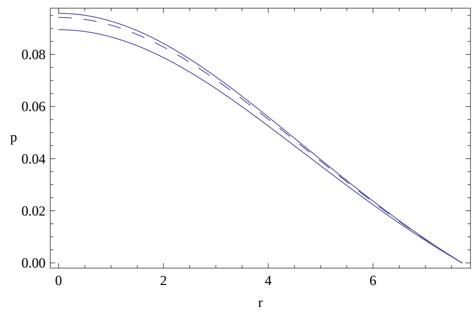

Figure 4.1: Radial variation of pressure(p) in Gev/fm^3 inside the star HER X-1. (Lines from top to bottom are for $\beta = 0, 0.3, 0.6$ respectively).

r in km.	Energy density(ρ)					
	$\lambda = 6$			$\lambda = 15$		
	$\beta = 0$	$\beta = 0.3$	$\beta = 0.6$	$\beta = 0$	$\beta = 0.3$	$\beta = 0.6$
$R \rightarrow$	22.8815	22.9047	22.9741	39.6572	39.6620	39.6764
0	0.85949	0.85775	0.85258	0.80117	0.80098	0.80040
2.0	0.79781	0.79627	0.79171	0.75283	0.75265	0.75212
4.0	0.65129	0.65019	0.64692	0.63374	0.63361	0.63321
b	0.37374	0.37324	0.37178	0.38853	0.38846	0.38826

Table 4.1: Energy density (ρ) in Gev/fm^3 in the interior and on the surface of X-ray pulser HER X-1.

inside the compact star for a given spheroidicity parameter. It is also noted here that for a large spheroidicity parameter the radial variation of energy density is same for all the cases.

In fig. (4.4) the variation of $\frac{\partial p}{\partial \rho}$ at the center and at the surface for different charge parameter β are plotted. The causality condition ($\frac{\partial p}{\partial \rho} < 1$) holds good inside the star even in the presence of electric field.

Case IIa: A millisecond pulsar namely, SAX J 1808.4-3658 which is characterised by mass $M = 1.435 M_{\odot}$, and size of the star $b = 7.07$ km. is considered which has compactness factor $u = M/b = 0.2994$ with spheroidicity parameter $\lambda = 6$. In this

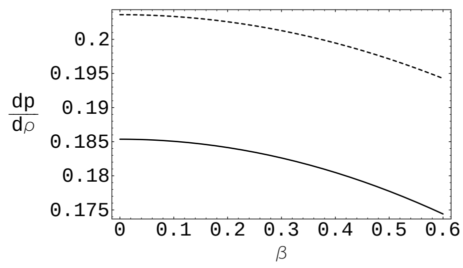

Figure 4.2: Variation of $\frac{\partial p}{\partial \rho}$ with charge parameter (β) at the center (Solid line) and at the surface (Dotted line) of the star HER X-1.

r in Km.	Energy density(ρ)					
	$\lambda = 6$			$\lambda = 15$		
	$\beta = 0$	$\beta = 0.3$	$\beta = 0.6$	$\beta = 0$	$\beta = 0.3$	$\beta = 0.6$
0	3.83003	3.80101	3.71712	3.00751	3.00519	2.99833
2.0	2.82074	2.80469	2.75792	2.41104	2.40951	2.40500
4.0	1.47593	1.47145	1.45822	1.44784	1.44726	1.44554
b	0.56168	0.56081	0.55821	0.61701	0.61687	0.61644

Table 4.2: Energy density (ρ) in Gev/fm^3 in the interior and on the surface of the compact star SAX-J for Mass $M = 1.435M_{\odot}$, and Radius $b = 7.07$ km. for $\lambda = 6$, and for $\lambda = 15$.

case eq. (4.7) and the boundary condition at $r = b$, leads to geometrical parameter $R = 10.839$ km. In fig. (4.5), the radial variation of pressure (p) is plotted for different β. It is noted that the pressure inside the charged compact star is lower than that for a compact star without electromagnetic field. It is maximum at the center and gradually decreases as one moves towards the surface due to the presence of electromagnetic field.

Variation of $\frac{\partial p}{\partial \rho}$ with β at the center and at the surface of star SAX J-1 is shown in fig. (4.8). The causality condition ($\frac{\partial p}{\partial \rho} < 1$) obeyed throughout the interior of the star.

Case IIb: For SAX J1 which is characterised by mass $M = 1.323M_{\odot}$, and

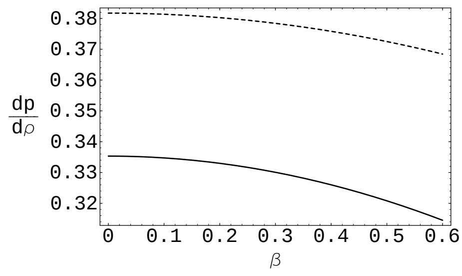

Figure 4.3: Variation of $\frac{\partial p}{\partial \rho}$ with anisotropy parameter β at the center (Solid line) and at the surface (Dotted line) of the star SAXJ-1.

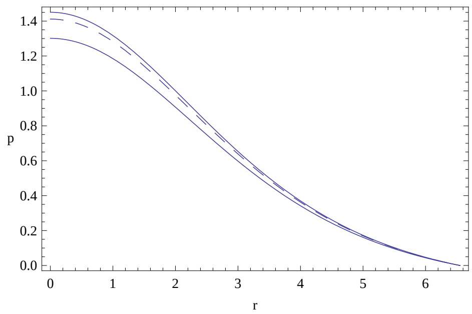

Figure 4.4: Variation of pressure p in Gev/fm^3 inside the star SAX J with the radial distance r in km.,lines from top to bottom are for $\beta = 0, 0.3, 0.6$ respectively.

size of the star $b = 6.550$ km. is considered here. It has compactness factor $u = M/b = 0.2994$ which can be accommodated with spheroidicity parameter $\lambda = 6$. The geometrical parameter R is determined from the eqs. (4.6) and (4.21) using the boundary condition at $r = b$, which is $R = 10.129$ km. The compactness factor is found slightly smaller than Case-IIa. The radial variation of energy density (ρ) is tabulated in Table (4.3).

The variation of $\frac{\partial p}{\partial \rho}$ with β at the center and surface of star SAX J-1 is shown in fig. (4.12). The square speed of sound i.e., ($\frac{\partial p}{\partial \rho} < 1$) is maintained inside the star.

This upper limit on the reduced size of a star is obtained from eq. (4.27) which

r in $km.$	Energy density(ρ)					
	$\lambda = 6$			$\lambda = 15$		
	$\beta = 0$	$\beta = 0.3$	$\beta = 0.6$	$\beta = 0$	$\beta = 0.3$	$\beta = 0.6$
$R\ (km.) \rightarrow$	10.1288	10.1668	10.2789	19.0915	19.0987	19.1203
0	4.38628	4.35355	4.25911	3.45693	3.45432	3.44652
2.0	3.10543	3.08867	3.03986	2.68859	2.68697	2.68212
4.0	1.53570	1.53149	1.51905	1.53270	1.53213	1.53044
b	0.65413	0.65311	0.65008	0.71797	0.71781	0.62298

Table 4.3: Energy density (ρ) in Gev/fm^3 in the interior and on the surface of the compact star SAX-J for Mass $M = 1.323M_\odot$, Radius $b = 6.55$ km. and compactness $u = 0.2979$ for $\lambda = 6$ and for $\lambda = 15$

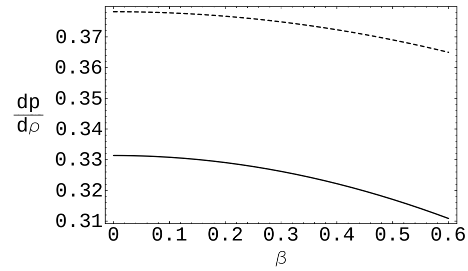

Figure 4.5: Variation of $\frac{\partial p}{\partial \rho}$ with anisotropy parameter β at the center (Solid line) and at the surface (Dotted line) of the star SAXJ-1.

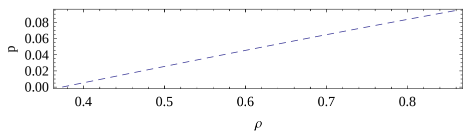

Figure 4.6: Plot of pressure with density of HER X1 for $\beta = 0.3$.

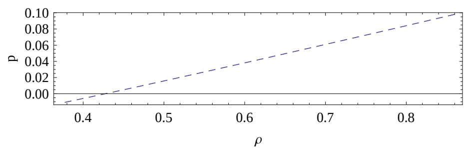

Figure 4.7: Plot of pressure with density of HER X1 for $\beta = 0$.

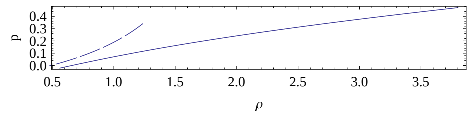

Figure 4.8: Plot of pressure with density of SAX J, solid line for $m = 7.07M_\odot$ and dashed line for $m = 6.55M_\odot$ for $\beta = 0.3$.

is given by

$$\frac{b}{R} = \sqrt{\frac{(12\lambda^2 - 12\lambda + 2\lambda\beta^2 + 6\beta^2) + X_0}{\lambda^4 - 6\lambda^3 + 5\lambda^2 - 2\lambda^2\beta^2 + 2\lambda\beta^2 + \beta^4}} \tag{4.28}$$

where $X_0 = (-12\lambda^2 + 12\lambda - 2\lambda\beta^2 - 6\beta^2)^2 + (\lambda^4 - 6\lambda^3 + 5\lambda^2 - 2\lambda^2\beta^2 + 2\lambda\beta^2 + \beta^4)(17\lambda^2 - 14\lambda + 16\beta^2 - 3)$. For uncharged star, we obtain a limiting value that was obtained in *Ref.*[70]. Consequently, we note that in the presence of charge the limiting values of the reduced radius is increased for same spheroidicity parameter which are tabulated in Table-4.5. In this chapter, the radius of a star for a given set of parameters in

$\frac{b}{R}$				
λ	$\beta = 0$	$\beta = 0.2$	$\beta = 0.4$	$\beta = 0.6$
6	2.1517	2.1663	2.2120	2.2935
7	1.5047	1.5090	1.5222	1.5449
8	1.2194	1.2214	1.2276	1.2380
9	1.0503	1.0515	1.0550	1.0609
10	0.9356	0.9363	0.9386	0.9423
11	0.8513	0.8518	0.8534	0.8560
15	0.6542	0.6543	0.6548	0.6557
50	0.3042	0.3042	0.3043	0.3043
100	0.2089	0.2088	0.2088	0.2088

Table 4.4: The variation of $\frac{b}{R}$ with β for a specific λ is shown.

the presence and in the absence of electromagnetic field are also determined. By considering mass of a star from observational data we determined the size of a star. For a given mass of a star considered here we present stellar models with two different

Star	Mass(m)	λ	Star Size(b) in Km.		
			$\beta = 0$	$\beta = 0.2$	$\beta = 0.4$
HER X-1	$0.88 M_{\odot}$	6	7.7000	7.7134	7.7543
		15	7.7000	7.7030	7.7121
SAX J SS1	$1.435 M_{\odot}$	6	7.0700	7.0748	7.0892
		15	7.0700	7.0710	7.0740
SAX J SS2	$1.323 M_{\odot}$	6	6.5500	6.5544	6.5678
		15	6.5500	6.5510	6.5537

Table 4.5: Size of different stars for spheroidicity parameter $\lambda = 6$ and $\lambda = 15$.

spheroidicity parameter $\lambda = 6$ and $\lambda = 15$ respectively and then determined the size of a star in the presence and absence of electromagnetic field (shown in Table-4.6). It is evident that in the presence of electromagnetic field the radial size of a star (b) is more than that in the absence of electromagnetic field.

Given Star	Mass(m)	β	Equation of State
HER X-1	$0.88M_\odot$	0	$p = 0.196\rho - 0.072$
		0	$p = -0.023\rho^2 + 0.226\rho - 0.081$
		0.6	$p = 0.186\rho - 0.068$
		0.6	$p = -0.023\rho^2 + 0.216\rho - 0.077$
SAX J	$1.435M_\odot$	0	$p = 0.396\rho - 0.208$
		0	$p = -0.017\rho^2 + 0.471\rho - 0.268$
SS1		0.6	$p = 0.375\rho - 0.194$
		0.6	$p = -0.018\rho^2 + 0.452\rho - 0.255$
SAX J	$1.323M_\odot$	0	$p = 0.394\rho - 0.242$
		0	$p = -0.014\rho^2 + 0.466\rho - 0.308$
SS2		0.6	$p = 0.372\rho - 0.223$
		0.6	$p = -0.012\rho^2 + 0.431\rho - 0.273$

Table 4.6: Equation of states for different stellar models.

4.3.2 Equation of State

Equation of state obtained from curve fitting for the models considered here are given in tabular form below in Table- 4.6.

A theoretical expression for EoS of matter inside a compact object from statistical analysis is given in Ref [17]. As the pressure and density cannot be expressed in closed form we adopt numerical technique to draw energy density and pressure curve for a given stellar configuration. In figs. (4.6) - (4.8) we plot pressure versus energy density. Then a numerical fitting of the curve leads to a a relation between pressure (p) and energy density (ρ), which are given in Table-4.6. It is evident that the models can be fitted with linear, quadratic even with higher order polynomial function in ρ. We present here two EoS in each case with charge and without charge. The EOS obtained here have similar form to the EoS considered by Maharaj and Mafa Takisa [64] to obtain stellar models. It is found for SAX J and SAX J-1 with masses $M = 1.435M_\odot$

and $M = 1.323M_{\odot}$ respectively, the gradient of the linear relation between pressure and density decreases as β, the charge parameter increases. In the first case we obtain strange star EoS of the form

$$p = \frac{1}{3}(\rho - 4B)\,. \tag{4.29}$$

For SAX J the EoS with $\beta = 1.1$ admits bag parameter $B = 0.12$ and for SAX J 1, with $\beta = 1.02$ admits bag parameter $B = 0.14$. Thus as the charge parameter increases the Bag parameter decreases.

4.4 Discussion

In this chapter, a class of relativistic solution for static compact star in the presence of electromagnetic field in pseudo spheroidal space-time geometry is obtained. The interior solutions of Einstein-Maxwell field equation for a compact object with electric field is matched with Reissner-Nordström metric at the boundary. The interior geometry of a compact star here contains five unknown parameters , mass (M), radius (b), spheroidicity parameter (λ), charge (β) and geometrical parameter (R). The solutions given by eq.(4.12) obtained from the Einstein field equation contains two unknown constants namely, Ψ_1 and Ψ_2. The constants are however can be determined from the two boundary conditions: (i) Pressure at the boundary of the star vanishes and (ii) By matching the first fundamentals of metrics at the boundary. The radial variation of density, pressure and charge configuration are then determined.

In fig. (4.1) the radial variation of pressure for a given star namely HER-X1 is plotted which has a maximum at the centre and decreases as one approaches the boundary. The observed mass and radius of HER X1 are taken to study the physical properties of the star with pseudo-spheroidal geometry. It is found that for a given spheroidicity parameter, the pressure inside the star is less compared to that of of an uncharged star. If one increases charge (q), the pressure inside the star is found to decrease. In the case of energy density, for a given star with different R parameter the central density is found to decrease with charge. At the surface, energy density (ρ) depends on R but independent of charge. The causality condition is also obeyed throughout the star from the centre to the boundary (see fig.(4.4)).

The physical properties of SAX J1 808.4-3658 for two different cases with $u = 0.2994$ is investigated. We note that $q(r) = r^2E^2 \geq \frac{4.00391\times10^{-6}r^4}{(1+0.051067r^2)^2}$ when $\beta = 0.2351$ with $\lambda = 6$. However the radial variation of pressure and density are found same as was obtained in the case of HER X-1. It is also evident that energy density is maximum at the center which decreases radially outward from the center. The energy density ρ of a charged compact star decreases compared to that of an uncharged compact object [70] except at the center. In the case of higher spheroidicity parameter (λ) the effect of charge in deciding physical properties of a star is negligible. The variation of $\frac{dp}{d\rho}$ with radial distance (r) for Case I, Case IIa and Case IIb are shown in figs. (4.2) and (4.4), respectively which shows that the causality is always maintained inside the charged compact star for the stellar models. The variation of pressure with density is plotted in figs. (4.6)-(4.8), it is evident that EoS for a star without charge is linear. However, for a compact charged star, the EoS is nonlinear. A deviation of EoS from linearity is found for the stellar models considered here which is shown in Table-4.6 for different configurations. The EoS obtained here have the same form to that recently considered for different stellar models by Maharaj and Mafa Takisa

[64, 71]. An interesting observation is made for SAX J with mass $M = 1.435M_{\odot}$, the gradient of the linear relation between pressure and density decreases as β, the charge parameter increases. For $\beta = 1.1$, the fitted curve obeys an EOS of the form of strange stars.