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APPLICATION OF ARIMA METHODOLOGY

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APPLICATION OF ARIMA METHODOLOGY

In chapter III application of growth types of model to both types of energy consumption data are shown. They are simply the time dependent models. Although on the basis of selected criteria it is found that some form of growth types of models outfit the energy consumption data, but univariate ARIMA types of models have some advantages over growth or regression models. Thus, in this chapter, section 4.1 gives an outlook of ARIMA model. Basic concepts, advantages of univariate ARIMA models are given in section 4.2. Characteristics of a good ARIMA model are mentioned in section 4.3. Section 4.4 deals with the steps in building up an ARIMA model. In section 4.5 criteria used in diagnostic checks are discussed. In section 4.6 criteria adopted for testing the validation of models are discussed and in section 4.7 back shift notation of ARIMA model is given which is used in the study. In section 4.8 application and result of ARIMA model to gas consumption data (untransformed and transformed) are discussed. In section 4.9 application and result of ARIMA model to the electricity consumption data are given. Finally, discussion of the preference of variables is given in section 4.10 and in section 4.11 a summary of the findings in this chapter is reported in brief. The numerical results obtained for whole analysis is presented in Appendix in last part of the chapter.

4.1. INTRODUCTION

ARIMA models are flexible and widely used in time series analysis. The stochastic process for which the exponentially weighted moving average forecast is optimal and a member of the class of non-stationary processes is the ARIMA process. The abbreviation ARIMA(p,d,q) stands for Autoregressive-Integrated-Moving-Average with three parameters p, the order of autoregression, d, the degree of differencing and q, the order of moving average (details of the parameters are given in next section). The ARIMA model, commonly known as Box-Jenkins' model, due to Box and Jenkins'(1970) worked for forecasting of a large variety of time series data. The underlying assumption is that the time series to be forecasted has been generated by a stochastic process. In other words it is assumed that each value in time-series is drawn from a probability distribution. Again, in some types of statistical analysis the various observations within a single data series are assumed to be statistically independent, which is a standard assumption about the error term in traditional regression analysis. But in univariate Box-Jenkins ARIMA analysis it is assumed that the time-sequenced observations in a data series (e,g, c_1.... c_{t-1}, c_t, c_{t+1} ....) are statistically dependent.

However, it is necessary to discuss the basic concepts of ARIMA (p,d,q) models.

4.1.1 Three basic concepts of ARIMA(p,d,q) methodology

In this section three basic concepts of ARIMA(p,d,q) models are discussed.

ARIMA models are the combination of three types of processes such as autoregression

(AR) process, differencing to strip of the integration (I) and moving average (MA) process. All three are based on the simple concept of random disturbances or shocks. Between two observations in a series, a disturbance occurs which somehow affects the level of the series. These disturbances can be mathematically described by ARIMA models. Each of the three types of processes has its own characteristic way of responding to a random disturbance. The most general ARIMA model involves all the three processes. Although they are related, each aspect of the model can be examined separately.

Autoregression

The first of the three processes included in an ARIMA model is autoregression. In an autoregressive process each value in a series is linear function of the preceding value(s). Thus in a first order autoregressive process only the single preceding value is used as a function of current value, in a second order autoregressive process two preceding values are used, and so on. These processes are commonly denoted by the notation AR(n), where the number in parenthesis stands for the order. Hence, AR(1), the first order autoregressive process, is defined as

$$c_t = \beta c_{t-1} + \epsilon_t \qquad \text{..................} \quad (4.1).$$

The coefficient ß is estimated from the observed series and indicates how strongly each value depends on the preceding value. So it is clear that AR(n) model is the same as ARIMA (n,0,0) model.

In an AR(1) process the current value is a function of the preceding value, which is a function of one preceding to it, and so on. Thus, each shock or disturbance to the system has a diminishing effect on all subsequent time periods. When the coefficient ß is greater than -1 and less than +1, as is usually the case, the influence of earlier observations dies out exponentially. For a pure AR process of order p, the partial autocorrelation function (pacf) up to lag p will be AR coefficient, while beyond that all are expected to be zero. An AR(p) model has exponentially declining values of autocorrelation function (acf) and have precisely p spikes in first p values of pacf. So the AR order of a series can be found from the plot of acf and pacf. However, in this study the criteria suggested by Mallows (1973), Schwarz (1978), Hannan and Quinn (1979) and Hurvich and Tsai (1989) are also employed to determine the proper order of autoregression which are discussed in section 4.5.

Differencing

A time series often reflects the cumulative effect of some process. The process is responsible for change in the observed level of the series but is not responsible for the level itself. Inventory level, for example, is not determined by the receipts and sales in a single period. Those activities cause changes in inventory levels. The levels themselves are the cumulative sums of the changes in each period. A series that measures the cumulative effect of something is called 'Integrated'. In the long run, the average level of an integrated series might not change. But in short run values can wander quite far from the average level purely by chance. When a series wanders, the difference from one to the next is often small. Thus, the differences of even a wandering series often remain fairly constant. This steadiness, or 'stationarity' of the differences is highly desirable from a statistical point of view.

The standard notation for integrated models that need to be differenced once is I(1), or ARIMA(0,1.0). Occasionally one will need to look at the differences of the differences, such models are denoted by I(2) or ARIMA(0,2,0), and so on. Thus one way of looking at I(1) process is that it has perfect memory of previous value. Except for random fluctuations each value equals the previous value. This type of I(1) process is often called a random walk because each value is one step away from previous value. For a properly selected degree of differencing of a series which makes the series stationary, the acf plots of differenced series will exhibit no significant autocorrelation. To determine the true degree of differencing in order to make the series stationary, the criteria suggested by Koreisha and Pukkila (1993) are adopted, details of which is given in section 4.5.

Moving Average

Last type of process in ARIMA models, which is the most difficult to visualize, is the moving average. In a moving average process, each value is determined by the average of the current disturbance and one or more previous disturbances. The order of the moving average process specifies how many previous disturbances are averaged into the value. Thus, the equation for a first-order moving average process is

$$c_t = \epsilon_t - \beta\, \epsilon_{t-1} \qquad \text{.....................(4.2)}$$

In the standard notation, MA(q) or ARIMA(0,0,q) processes use q previous disturbances along with the current value.

The difference between the autoregressive process and a moving average process is subtle but important. Each value in a moving average series is a weighted average of the most recent random disturbances. Where as each value in an autoregression is a weighted average of the recent values of the series. Since these values in turn are weighted averages of the previous ones, the effect of a given disturbance in an autoregressive process dwindles as time passes. In a moving average process, a disturbance affects the system for infinite number of periods (order of moving average) and then abruptly ceases to affect it. In a MA(q) process, the pacf will die away toward zero with no clear 'cut off' and acf will exhibit a clear 'cut off' at qth point. MA(q) process has precisely q spikes in first q values of pacf. Hence, in this study the order of moving average is determined by observing the nature of acf and pacf plots.

4.1.2 Other concepts related to ARIMA models

In this section some other concepts which are pertinent to an ARIMA model are discussed.

***Stationarity*:**

The univariate Box-Jenkins' (UBJ) ARIMA method is applicable to a stationary data series. Hence, stationarity is a condition which must be met by time series to which a UBJ-ARIMA model is to be fitted. Pure moving average (MA) series are stationary, while autoregressive (AR) and autoregressive-moving-average (ARMA) series might not be so. So before dealing with the problem of fitting ARIMA model to a data, it is necessary to test the stationarity condition of the series. A stationary time-series have mean, variance and autocorrelation that are constant throughout the time.

The mean of stationary series indicates the overall level of the series. If the time-series is stationary then the mean of any major subset of the series does not differ significantly than any other subset.

Similarly, in case of the variance, if the data series is stationary, then the variance of any major subset of the series will differ from the variance of any other major subset only by chance.

Again, if the estimated acf of the series drops off to zero quite rapidly, i,e, the estimated acf quickly becomes insignificantly different from zero, then the series is said to be stationary, otherwise the series is non-stationary.

However, any non-stationary series can be transformed into stationary series by proper degree of differencing, by transfer function model or taking logarithmic value of the series before differencing.

Autocorrelation function (acf)

The role of acf is very important at the identification stage of UBJ method. It measures the direction and strength of statistical relationship between ordered pairs of observations in a single data series. An estimated autocorrelation coefficient (r_k, of order k) is not fundamentally different from any sample correlation coefficient. The standard formula of calculating autocorrelation coefficient is

$$r_k = \frac{\sum_{t=1}^{N-k}(c_t-\bar{c})(c_{t+k}-\bar{c})}{\sum_{t=1}^{N}(c_t - \bar{c})^2} \qquad \text{.....................(4.3)}$$

This is some dimensionless number that can take on values only between -1 and +1. A value of -1 means perfect negative correlation, +1 means perfect positive correlation and zero means c_t's are not correlated at all in the available data.

Looking at the pattern in an estimated acf is a key element at the testing phase of stationarity and at the identification stage of the UBJ method. Box-Jenkins' (1976; p.33) suggested that maximum number of useful autocorrelation was roughly N/4, where N is the number of observations.

The acf is a useful identification and diagnostic checking tool for selecting ARIMA model. In addition to the values obtained by using the Mallows criterion and Hannan and Quinn Criterion, the exponentially declining nature of acf plots are also observed for taking final decision about the order of autoregression. The number of significant spikes in acf plots also helps in taking decision about the degree of moving average. Again, for taking decision about the degree of differencing, the degree obtained by the criterion of Koreisha and Pukkila is compared with the number of significant autocorrelations of the acf plots of differenced series.

Partial autocorrelation function (pacf)

The estimated partial autocorrelation function (pacf) is broadly similar to an estimated acf. An estimated pacf is also a graphical representation of the statistical relationship between sets of ordered pairs (c_t , c_{t+k}) drawn from a single time series. The estimated pacf is used as a guide, along with the estimated acf in choosing one or more ARIMA models that might fit the available data.

The idea of partial autocorrelation analysis is that we want to measure how c_t and c_{t+k} are related, but with the effects of the intervening c's accounted for. For example, we want to show the relationship between ordered pairs (c_t, c_{t+2}) taking into account the effect of c_{t+1} on c_{t+2}. Next, we want the relationship between the pairs (c_t, c_{t+3}), but with the effects of both c_{t+1} and c_{t+2} on c_{t+3} accounted for and so on, each time adjusting the impact of any c's that fallen between the ordered pairs in question.

The most accurate way of calculating partial autocorrelation coefficient is to estimate a series of least squares regression coefficient. An estimated regression coefficient is interpreted as a measure of the relationship between the 'dependent' variable and the 'independent' variable in question, with effects of other variables in the equation taken into account. That is exactly the definition of a partial autocorrelation coefficient, denoted by ϕ_{kk}, measures the relationship between c_t and c_{t+k} while the effects of the y's falling between these ordered pairs are accounted for.

Like acf, the pacf is also an important identification and checking tool for an ARIMA model. The values obtained by using the criteria of Mallows, and Hannan and Quinn, are compared, with the number of significant spikes in the pacf plots for taking decision about the order of autoregression. Exponentially declining nature of pacf plots also helps in taking decision about the degree of moving average.

Various attempts to compare the Box-Jenkins ARIMA methods with different types of traditional forecasting techniques have been made. Reid (1971) compared the Box-Jenkins' method to a variety of forecasting techniques based on exponential smoothing methods. On analysis of 113 time series of annual, quarterly and monthly data of economic variables in the UK, Reid concluded that Box-Jenkins' approach produced minimal forecasting errors. Granger and Newbold (1986) suggested the use of Box-Jenkins' approach in forecasting time series data because it produced superior results, and it can be employed with confidence for relatively short period time series. Geurts and Ibrahim (1975) compared the Box-Jenkins' model to first and second order exponential smoothing models and showed that this method produced equally good results. Ferrer et. al.(1997) compared the forecasting performances of univariate

ARIMA models with unobserved component models using monthly time series data of automobile sales in Spain. He showed that the forecasting performances of ARIMA models were fairly better.

Moreover, the UBJ model has three specific advantages over many other traditional single-series methods.

First, the concepts associated with UBJ models are derived from solid foundation of classical probability theory and mathematical statistics.

Second, ARIMA models are a family of models, not just a single model. Box and Jenkins developed a strategy that guide the analyst in choosing one or more appropriate model(s) out of a larger family of models. Some works on the selection of parameters of an appropriate ARIMA model have also been done by some econometricians (e.g. Mallows (1973) and Hannan and Quinn (1979) developed criteria of selection of the order of autoregression. Koreisha and Pukkila (1993) developed the method of choosing appropriate degree of differencing for ARIMA models).

Third, it can be shown that an appropriate ARIMA model produces optimal univariate forecasts. "There seems to be general agreement among knowledgeable professionals that properly built UBJ models can handle a wide variety of situations and provide more accurate short-term forecasts than any other standard single series technique" (Pankratz, 1986; pp.19).

Again Hall (1994, p.17) stated that Box-Jenkins modelling strategy for pure time series forecasting had received considerable attention over recent years. This procedure may be seen as one of the early attempts to confront the problem of non-stationary data. Although many researchers now view the Box-Jenkins approach as having been superseded by contegration and multivariate non-stationary analysis, it should still be regarded as useful tool in the overall as armory of econometric techniques with a particularly important role in univariate modelling.

4.2. CHARACTERISTICS OF A GOOD ARIMA MODEL

A good ARIMA model has the following seven properties:

i) It is parsimonious i.e. uses the smallest number of coefficients needed to explain the available data.

ii) It is stationary.

iii) It is invertible

iv) It has estimated coefficients of high quality.

v) It has uncorrelated residuals

vi) It fits the available data enough to satisfy the analyst *i.e.* a) RMSE is acceptable b) MAPE is also acceptable.

vii) It forecasts the future satisfactorily.

4.3. STEPS IN THE BOX-JENKINS ITERATION APPROACH TO MODEL BUILDING

Box-Jenkins (1976, p.19) proposed a practical three stages' procedure for finding a good model. The three-stage UBJ procedure is as follows:

Step 1: Identification

At the identification stage two graphical devices are used to measure the correlation between the observations within a single data series. These devices are called estimated autocorrelation function (acf) and an estimated partial autocorrelation function (pacf). These estimations are helpful in giving an idea for the patterns of available data. The estimated acf and pacf are used as a guide to choosing one or more ARIMA models that seem appropriate. The basic idea is that every ARIMA model has a theoretical acf and a pacf associated with it. At the identification stage estimated acf and pacf calculated from the available data are compared with various theoretical acf's and pacf's. Then a model is tentatively chosen whose theoretical acf and pacf closely resemble the estimated acf and pacf of the data series.

Whichever model is chosen in identification stage, it is considered only tentatively, it is only a candidate for final model. To choose a final model next two stages are followed and perhaps it may require to return to the identification stage if the tentatively considered model proved inadequate.

Step 2: *Estimation*

At this stage precise estimates of the coefficients of the model chosen at the identification stage are obtained. This stage may provide some warning signals about the adequacy of the chosen model. The coefficients are estimated using nonlinear least square estimation (For details see Pankratz 1986; pp.192-199). If the estimated coefficients do not satisfy certain inequality conditions *viz.* stationarity and invertibility (details are available in Pankratz 1986; pp.130-136), the model is rejected.

Step 3: Diagnostic checking

The final step in the ARIMA modelling is the diagnostic checking of the model. Box and Jenkins suggest some diagnostic checks to help to determine whether an estimated model is statistically adequate. A model that fails these diagnostic tests is rejected. Furthermore, the results at this stage may also indicate how a model could be improved which may lead the researcher back to the identification stage. Then cycles of identification, estimation and diagnostic checking are to be repeated until a good final model is obtained.

This three-step iterative nature of UBJ modelling is important. Although the applications of these three stage procedures do not guarantee that one would finally arrive at the best possible ARIMA model, but it stacks the cards in favor of the researcher. Thus, in this step the following checks are essential.

1. The acf and the pacf of the error series should not be significantly different from zero, one or two high-order correlations may exceed the 95% confidence limit.
2. The residuals should be without a pattern. A common test for this is the Box-Ljung Q statistic, also, called modified Box-Pierce Statistic. This statistic should not be significant.

4.4 CRITERIA USED FOR DETERMINING THE ORDER OF AUTOREGRESSION AND THE DEGREE OF DIFFERENCING

The following criteria are used for determining the order of autoregression and the proper degree of differencing of ARIMA model.

Mallows Criterion: Details of this criterion is given in section 2.3.1.1 of chapter II. p is to be selected so that C_p is minimum.

Hannan and Quinn Criterion: Hannan and Quinn (1979) developed a criterion, namely, $\phi(p)$ criterion, defined as

$$\phi(p) = \ln \hat{\sigma}_p^2 + N^{-1} \, 2pc \ln \ln (N), \; c>1$$

c is a constant to be determined by the researcher, usually, c is taken to be 2. They also show that as compared to other procedures, this is a strongly consistent procedure and underestimate the order of autoregression to a lesser degree.

Koreisha and Pukkila Criterion: Koreisha and Pukkila (1993) introduced a new approach of determining the degree of differencing necessary to make a series stationary, which is one of the prerequisites for fitting ARIMA models. They propose to select the structure ARIMA (p,d,q) which minimises

$$\delta(p,d,q) = N \ln \hat{\sigma}^2_{p,d,q} + (p+q)\, g(N)$$

where, g(N) is a prescribed non-negative criterion which is usually set equal to $N^{1/2}$. They also show that this δ(p,d,q) criterion performs extremely well.

In addition to the criteria used for determining the order of autoregression and the degree of differencing, the following diagnostic checks are used in the study for selecting the ARIMA model.

The Corrected Akaike Information Criterion **(AIC_c)**: AIC is one of the leading statistics which helps in taking decision about the order of an autoregressive model. AIC takes into account both how well the model fits the observed series, and the number of parameters to be used in the fit. AIC, due to Akaike (1969), is defined as

$$AIC = N(\ln \hat{\sigma}^2 + 1) + 2(p+1)$$

where, the parameters bear usual meanings. Akaike also mentioned that the minimum AIC criterion produced a selected model which was hopefully closer to the best possible choice.

But, sometimes this AIC criterion does not provide efficient order of model selection, while asymptotic efficiency is more desirable criterion. Shibata (1976) showed that AIC criterion was not consistent too. Thus, Hurvich and Tsai (1989) provide a criterion of AIC correction for bias. The correction is of particular use when the sample size is small or when the number of fitted parameters is a moderate to a large fraction of sample size. They defined the criteria as

$$AIC_c = N \ln \hat{\sigma}^2_p + N \frac{1 + \frac{p}{N}}{1 - \frac{p+2}{N}}$$

i.e.

$$AIC_c = AIC + \frac{2(p+1)(p+2)}{N-p-2}$$

i.e. AIC_c is the sum of AIC and an additional non-stochastic penalty term $2(p+1)(p+2)/(N-p-2)$, where, the parameters bear usual meanings. They show that their corrected AIC_c is asymptotically efficient if the true model is finite dimensional. Shibata (1981) also shows that if the approximating model was linear, then AIC_c is asymptotically efficient. This was also found to provide a better criterion for selecting order of autoregression than any other method. The model which adequately describes the series has the minimum AIC_c.

***Schwartz Information Criterion* (SIC)**: Schwarz (1978) suggested a criterion SIC which helps in deciding the order of autoregression. Initially, he develops this criterion for taking decision about the regressors subset. Later, Engel *et. al.*(1992) use this criterion as a tool for determining the order of autoregression and they define this criterion as

$$SIC = \hat{\sigma}_p \left(1 - \frac{p}{N}\right)^{\frac{1}{2}} N^{\frac{p}{2N}}$$

where, the parameters have usual meaning. In this study this definition of SIC is used in whole analysis. Schwarz showed that this criterion was better than AIC. Hurvich and Tsai (1989) use this criterion for selecting the order of autoregression in their study. The model with minimum SIC is assumed to describe the data series adequately.

Box-Ljung statistic[1] : A test statistic which tests the null hypothesis that a set of sample autocorrelation is associated with a random series. The Ljung-Box (1978) statistic is used to test the adequacy of a fitted model. If a model fits well, the residuals should not be correlated and the autocorrelation should be small. In this case the hypothesis

$$Ho: \rho_1(a) = \rho_2(a) = \ldots\ldots\ldots = \rho_k(a) = 0$$

is tested with the Box-Ljung statistic $Q^* = N(N+1)\sum_{k=1}^{k}(N-k)^{-1}\, r_k^2(a)$

where, N is the number of observations used to estimate the model. This statistic Q^* approximately follows chi-square distribution with (k-m) DF, where m is the number of parameters estimated in the ARIMA model. If Q^* is large (significantly different from zero), it says that the residual autocorrelations as a set are significantly different from

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[1] Some analysts and computer programmers use a statistic suggested by Box and Pierce (1970) defined as

$Q = N\sum_{k=1}^{K} r_k^2(a)$. The Ljung-Box statistic is preferred to the Box-Pierce statistic since its sampling distribution more nearly approximates the chi-square distribution when the sample size is moderate.

zero and random shocks of the estimated model are probably autocorrelated. So, one should then consider reformulating the model. The values of this statistic Q^* which are shown in right-hand sides of acf plots in figures 4.1.1, 4.2.1, 4.3.1, 4.4.1, indicate that the autocorrelations are insignificant.

4.5. CRITERIA USED FOR TESTING THE VALIDITY OF MODEL

The values of the above-mentioned criteria AIC_c, SIC, Cp, $\phi(p)$ and $\delta(p,d,q)$ are compared for correct determination of the order of autocorrelation and the degree of differencing, these criteria are computed for estimation period only. For the selection of an ARIMA model which adequately describes the data series, the values against following criteria are also compared for three periods, viz. estimation period, validation (or prediction) period and total period.

Absolute mean error*(AME), *Root mean square error* (RMSE), *Mean absolute percent error (MAPE) which are defined in section 3.3 of chapter 3. An adequately good model has minimum AME, RMSE and MAPE.

4.6. NOTATION AND INTERPRETATION OF ARIMA MODEL

The ordinary algebraic form of two common ARIMA processes, the AR(1) and the MA(1) are respectively,

$$c_t = C + \phi_1 c_{t-1} + a_t \quad \text{.................(4.4)}$$

$$c_t = C - \theta_1 a_{t-1} + a_t \quad \text{.................(4.5)}$$

where, C is an intercept term, ϕ is coefficient of an autoregression term, θ is the coefficient of moving average terms and a is white noise.

Similarly, AR(2) process, MA(2) process and ARMA(1,1) process are respectively,

and

$$c_t = C + \phi_1 c_{t-1} + \phi_2 c_{t-2} + a_t \quad \text{.................(4.6)}$$

$$c_t = C - \theta_1 a_{t-1} - \theta_2 a_{t-2} + a_t \quad \text{.................(4.7)}$$

$$c_t = C + \phi_1 c_{t-1} - \theta_1 a_{t-1} + a_t \quad \text{.................(4.8)}$$

The equation (4.8) is ARMA(1,1) process because the AR order is one and the MA order is one.

The ARIMA equations are the generalization of (4.8). Let the AR order of a process be designated p, the MA order of the process is q and the number of times a realization must be differenced to achieve a stationary mean be d. Although ARIMA(p,d,q) is enough to express an ARIMA model with the parameters p,d,q, let us have a look at the equation of ARIMA(p,d,q) model in a bit different form, which is commonly used and known as back shift notation of ARIMA model. Here the back shift operator B is used to denote the time back *i.e.* if c_t is multiplied by B, c_{t-1} is obtained, *i.e.* $Bc_t = c_{t-1}$, $B^2c_t = c_{t-2}$ and so on. So, $(1-B)c_t = c_t - c_{t-1}$, $(1-B)^2c_t = c_t - 2c_{t-1} + c_{t-2}$ and so on. $(1-B)^2c_t$ is a compact and convenient way of writing the second difference of c_t.

Thus a non-seasonal ARIMA(p,d,q) process in back shift notation has the general form as

$$(1 - \phi_1 B - \phi_2 B^2 .... - \phi_p B^p)(1 - B)^d c\text{-}_t$$

$$= (1 - \theta_1 B - \theta_2 B^2 .... - \theta_q B^q)\ a_t \qquad ....(4.9)$$

where, $c\text{-}_t$ is the dth differenced series of c_t measured from mean of the dth differenced series. In case any constant term C occurs in the estimated equation, it is to be placed in right-hand sides as an additional term. While, if d=0, then C is related to the mean of the series by the relation $C = \mu(1 - \Sigma\phi_i)$.

A more compact form that often appears in time-series literature is as follows.

Let, $\Delta^d = (1 - B)^d$,

$$\phi(B) = (1 - \phi_1 B - \phi_2 B^2 - .... - \phi_p B^p) \qquad ... .(4.10)$$

and $\theta(B) = (1 - \theta_1 B - \theta_2 B^2 .... - \theta_q B^q) \qquad ....(4.11)$

then we get the general compact form of ARIMA(p,d,q) process as

$$\phi(B)\ \Delta^d\ c\text{-}_t = \theta(B)\ a_t \qquad ....(4.12)$$

However, in this study the form (4.9) will be used.

4.7. SELECTION OF ARIMA MODEL FOR GAS CONSUMPTION

In order to identifying the tentative ARIMA model to the gas consumption data, the steps described by Box and Jenkins (given in section 4.4) are followed. For this purpose the data of the sample period 1970-71 to 1988-89 are used in fitting stage and the remaining data *i.e.* the period 1989-90 to 1992-93 is used for testing the validity of the models. Thus as in the case of growth model, at first the models are fitted using the data for estimation period, then, using the estimated models the consumption figures for the validation period is computed for testing the validity.

4.7.1 *For untransformed series*

The steps followed for selection of appropriate ARIMA model for gas consumption are as below.

Identification stage

The first and the most subjective step in ARIMA modelling is the identification of the process underlying the series. One must determine the value of the integers p,d,q in ARIMA (p,d,q) processes generating the series. It is mentionable here that Paul and Haque (1995) verified the performance of ARIMA model to gas consumption data. Out of three preselected models they showed that ARIMA(1,1,1) model performed better. In this study performance of ARIMA model is restudied using some latest tools in determining the degree of differencing and the order of autocorrelation. The following stepwise methodology is followed here.

A. Tests for stationarity nature of data

The original series of gas consumption data are plotted in a graph in order to observe whether the historical series are stationary in nature, because, stationarity is one of the prerequisites of modelling ARIMA. The plot reveals that the data for gas consumption rise steadily over time *i.e.* the series exhibits a clear upward trend, beside some exceptions, (which is also clear from consumption data too) also indicates their non-stationary nature. Non-stationarity of original data series is clearly evident from same figures.

For testing homogeneity of the average and the dispersion, the whole series is divided into four quarters. The first three quarters containing six observations each and the last quarter containing last five observations. Then, the test of equality of means of four quarters is performed using F test statistic and the equality of variances are performed using Bertlett's (approximate chi-square) test statistic. The statistics show that the means and variances of four quarters change considerably significantly over time, which also indicate the non-stationary nature of the series.

Moreover, the ACF plot of the original series (Figure 4.1.1) starts out with large positive values which die out slowly at increasing lags. This pattern also confirms that the series is not stationary.

Hence an attempt is made to remove the non-stationarity of the data series by differencing. Thus, for determining the proper degree of differencing the following methodology is adopted.

B. Determination of the degree of differencing

For this purpose first and second difference of the original series are considered and their acf plots are observed. Thus for second difference it is seen that acf plots show no significant autocorrelation at any lag. This is also evident from the Box-Ljung statistics, presented in the right-hand side of acf figures (Figure 4.1.2), that all the differenced values fall within the 95% confidence limits in the acf plots. Moreover, these values are reduced into white noise indicating their randomness. So the second differencing transformed all the original series into stationary one. So the value of the second parameter 'd' of the ARIMA model is tentatively chosen as 2.

Again, to be more confirmed about the value of 'd', the ARIMA models with d = 1 and 2 are compared with the criterion suggested by Koreisha and Pukkila (1993). These values are reported in table 2.1, from which it is evident that the value of the criterion is minimum for d=2. So d=2 is selected for final model.

C. Determination of the order of autoregression

Once the appropriate degree of differencing has been determined, the next step of identification process is to assess the appropriate ARMA specification of the stationary series. This is done by examining both acf and pacf, relying on the fact that AR and

MA processes have more quite different theoretical properties. It is already mentioned in section 4.1.1 that for a pure autoregressive process of lag p, the pacf up to lag p will be autoregressive coefficient, while beyond that all of them are expected to be zero. So, in general there will be a 'cut off' in lag p in the pacf. The acf on the other hand, will decline asymptotically toward zero and not exhibit any discrete 'cut off' point. That means, AR(p) models have exponentially declining values of acf and have precisely p spikes in the first p values of pacf. Thus, it is observed from the figure 4.1.1 that the acf's and the pacf's for untransformed gas consumption data are different in respect to number of spikes. So the order of autoregression and the order of moving average will be different. The acf plot of original data shows exponentially declining values and the pacf has one significant spike, which suggest that the maximum order of autoregression 'p', the first parameter of an ARIMA model, is 1. However, ARIMA models with p = 0,1,2 are run by computer using SPSSPC+ software in order to determine the appropriate value of 'p'. But it is well known that one problem that regression analysis with time series data very often faces, is the problem of autocorrelation. Unfortunately, with fewer time series data it is not easily identified. Again Gujarati (1978; pp.238) also stated 'in a sample of fewer than 15 observations, it becomes very difficult to draw any conclusion about auto-correlations by examining only the estimated residuals'. Hence as mentioned earlier in section 4.5 for determining the proper order of autoregression, the values of C_p Criterion and $\phi(p)$ criterion are computed considering the values of p for all p= 0,1,2. Three of the models with smaller values of these criteria are reported in table 4.1.1. Then comparing these values it is found that the appropriate value of 'p' is 1. However, the appropriate ARIMA model is selected comparing all the criteria listed for diagnostic checking (section 4.5 and 4.6) and values are reported in table 4.1.1.

D. Determination of the order of moving average

It is mentioned in the previous section that appropriate ARMA specification of the stationary series is done by examining both the acf and the pacf, relying on the fact that AR and MA processes have more quite different theoretical properties. As already mentioned in section 4.1.1 (def. of MA) that in case of MA process, the MA process of order 'q' will exhibit reverse property to that of AR process of order 'p'. The pacf will die away slowly toward zero with no clear 'cut off', while the acf will exhibit a discrete 'cut off' at the qth point. That means, MA(q) models have precisely q spikes in the first q values of the acf and exponentially declining values of pacf. Thus from the acf and pacf plots (figure 4.1.1), it is observed that the acf has two significant spikes in first two values and the pacf shows sharply declining values. Hence, the maximum value of 'q' is tentatively chosen as 2. The appropriate value of 'q' is also determined comparing the values against criteria in table 4.1.1.

E. Estimation stage

Ten ARIMA models with tentatively selected different values of p,d,q described above, are estimated by computer using SPSSPC+ software. The maximum likelihood method of estimation is used here. The tentatively selected models are ARIMA(1,1,1),

ARIMA(0,1,2), ARIMA(1,2,0), ARIMA(1,2,1), ARIMA(0,2,1), ARIMA(2,1,1), ARIMA(2,1,0), ARIMA(2,0,2), ARIMA(1,0,1) and ARIMA(1,2,2). The most appropriate model is selected in the diagnostic check stage given in the following subsections.

F. Diagnostic checks and final model selection

The adequacies of the tentatively selected models are compared using the criteria discussed in sections 4.5 and 4.6. The values of the criteria for three more well performed models are reported in table 4.1.1. Box-Ljung statistics are not reported here due to the fact that they are used for testing the randomness of the series. The Box-Ljung statistics reported in figure 4.1.1 show that there is no significant autocorrelation, *i.e.* the residuals are autocorrelated. Thus, table 4.1.1 reveals that ARIMA model with p=1, d=2 and q=1 possesses minimum values of all the selected criteria AIC_c, SIC, C_p, $\phi(p)$ and $\delta(p,d,q)$ indicating that ARIMA model with above mentioned values of parameters, *i.e.* ARIMA(1,2,1) model, adequately describes the data series. Again, AME, MAPE and RMSE of the same model are the smallest in all the three periods *i.e.* estimation period, validation period and total period. Hence, it can be concluded that ARIMA(1,2,1) model is the best fitted ARIMA model for untransformed gas consumption data. Moreover, this model satisfies all characteristics of a good ARIMA model given in section 4.3. So finally ARIMA (1,2,1) model is selected for forecasting the gas consumption and used for comparison of prediction performance with selected other two types of models *i.e.* growth type of models and

ANN models.

The selected model

The structure of ARIMA(1,2,1) model selected for gas consumption is as follows:

$$\underset{(-4.8132)}{(1 + 0.05934^{*}B)}(1-B)^2\, \tilde{c}_t = 8.3858 + \underset{(7.3605)}{(1-1.1392^{*}B)}a_t$$

(values in the parentheses are corresponding t- values and
* statistical significance $P \leq 0.01$)

where, $\tilde{c}_t$ is the second differenced value of gas consumption after taking the deviation from the mean of the second differenced series.

Finally the gas consumption figures are forecasted up to 2000-2001 using the selected model and reported in table 4.1.3

4.7.2. *For log-transformed series*

It is mentioned in last chapter (section 3.2) that attempt is also made to examine the fitness of ARIMA modelling to a log-transformed data series of gas consumption. Kennedy (1985; p.209-210) states that Box-Jenkins model does not perform as well as expected. One suggestion he offers for the improvement, is the use of transfer function model. He also states that differencing is not the only transformation employed to generate stationarity, logarithms of observations before differencing, may sometimes improve the result. In this study the log-transformation is made with such an object.

The same type of analysis from sections 4.8.1.1 to 4.8.1.3 as in the case of untransformed series is performed for log-transformed series. The nature of acf and pacf plots of log-transformed series are exactly same as in the case of untransformed series. Thus, in this case too ten tentative ARIMA models are run. The models considered here are ARIMA(1,1,1), ARIMA(1,1,2), ARIMA(1,2,1), ARIMA(1,2,2), ARIMA(1,2,0), ARIMA(0,1,1), ARIMA(2,1,2), ARIMA(1,1,0), ARIMA(2,2,1) and ARIMA(2,1,1). The values of the criteria for three more well performed models are reported in table 2.1.2. Box-Ljung statistics are not reported here due to the fact that they are used for testing the randomness of series. The Box-Ljung statistics reported in figure 4.1.2 show that there is no significant autocorrelation.

It is evident from table 4.1.2 that ARIMA model with p=1, d=2 and q=1 possesses minimum values of all the selected criteria AIC_c, SIC, C_p, $\phi(p)$ and $\delta(p,d,q)$ indicating that ARIMA model with above mentioned values of parameters, *i.e.* ARIMA(1,2,1) model, adequately describes the data series. Again, AME and MAPE of the same model are the smallest in all the three periods *i.e.* estimation period, validation period and total period. Only RMSE in estimation period is minimum in case of ARIMA(2,1,2) model, while for other two periods it is minimum in case of ARIMA(1,2,1) model. Since the purpose of the study is to select a forecasting model, it is justified to consider ARIMA(1,2,1) model as the best fitted ARIMA model for log-transformed gas consumption data. This model also satisfies the characteristics of a good model given in section 4.3. So ARIMA (1,2,1) model is finally selected for forecasting the gas consumption for log-transformed series and used for comparison of prediction performance with selected other two types of models *i.e.* growth type of models and ANN models.

The estimated model

The estimated ARIMA(1,2,1) model for log-transformed series of gas consumption is as follows:

$$(1 + 0.03061^{*}B)(1-B)^2 \log \tilde{c}_t = -0.003478 + (1-0.9817^{*}B)a_t$$
$$(-11.5091) \qquad\qquad\qquad\qquad (3.6095)$$

(Values in the parentheses are corresponding t- values and
*statistical significance $P \leq 0.01$)

where $\log \tilde{c}_t$ is the logarithmic value of second differenced values of gas consumption after taking the deviation from the mean of the second differenced series.

Forecasting the original data from an ARIMA model of log-transformed series is not straightforward as that of untransformed series. Computing simply the antilog of the logarithmic forecasts to get the forecasts of original series creates a problem. The method of forecasting the original series from a log transformed series is discussed here in brief (for details see Pankratz 1986; pp.257). Let a log-transformed series be denoted by c_t', where c_t is the original series. Then it can be shown that the forecast of $\hat{c}'_{t+1}$ depends on both the forecast and the forecast-error variance of $\hat{c}'_{t+1}$ in this way

$$\hat{c}_{t+1} = \exp\left[\, \hat{c}'_{t+1} + \frac{1}{2}\hat{\sigma}^2\, [\hat{a}'_{t+1}]\right] \qquad \text{..................(4.13)}$$

Thus, one should not simply find the anti-log of $\hat{c}'_{t+1}$ to find $\hat{c}_{t+1}$. Instead, one must take into account the variance of the logarithmic forecast as shown in equation 4.13.

Finally, the gas consumption figures are forecasted up to 2000-2001 using the selected model and reported in table 4.1.3. This job is also performed by computer using SPSSPC+ software.

4.8. SELECTION OF ARIMA MODEL FOR ELECTRICITY CONSUMPTION

The selection procedure of ARIMA model for electricity consumption data is also the same as that for gas consumption data. The steps of choosing an ARIMA model described by Box and Jenkins (given in section 4.4 of this study) is also followed for electricity consumption data. In this case too, at first models are fitted to the data for estimation period, then, using the fitted model consumption figures for validation period is computed for testing the validity of models.

4.8.1. *For untransformed data*

As in the case of gas consumption in this case too in order to identifying the tentative ARIMA model to the electricity consumption data, the steps discussed in sections 4.8.1.1 to 4.8.1.3 are followed. For this purpose the data of the estimation period 1976-77 to 1990-91 is used in fitting stage and the data of the remaining period, *i.e.* 1991-92 to 1992-93 is used for testing the validity of models. In this case too, a number of ARIMA models with varied tentatively chosen values of parameters are run by computer. The appropriate model is selected using the values, reported in table

4.2.1, of criteria of diagnostic tools of three better performed models.

It is very clear in table 4.2.1 that as far as the values of the selected criteria are concerned, the ARIMA(1,2,1) model outperforms other two models uniquely. So ARIMA(1,2,1) model is finally selected as the best ARIMA model for untransformed electricity consumption data.

The estimated model

The structure of ARIMA(1,2,1) model for untransformed electricity consumption is as follows:

$$\underset{(-3.44301)}{(1 + 0.6424^{*}B)}\,(1-B)^2 \log \tilde{c}_t = 22.8211 + \underset{(4.8789)}{(1 - 0.9296^{*}B)}a_t$$

(values in the parenthesis are corresponding t- values and *statistical significance $P \leq 0.01$)

where, $\tilde{c}_t$ is the second differenced value of electricity consumption after taking the deviation from the mean of the second differenced series.

Finally, the consumption figures are forecasted up to 2000-2001 and reported in table 4.2.3.

4.8.2. *For log-transformed series*

As in the case of untransformed series of electricity consumption the values of the three better performed ARIMA models, from ten tentatively chosen models, are reported in table 4.2.2.

It is clear from table 4.2.2, that ARIMA(2,1,2) supersedes other two models only by AIC_c criterion and ARIMA(1,2,2) model supersedes other two models only by AME in validation period. ARIMA(1,1,1) model supersedes other two models by all other criteria selected for diagnostic checking. So, ARIMA(1,1,1) model is finally selected as the best ARIMA model for log-transformed series of electricity consumption.

The estimated model

The estimated ARIMA(1,1,1) model for log-transformed electricity consumption is as follows:

$$(1 + 1.0143^{*}B)(1-B) \log \tilde{c}_t = 0.1172 + (1 - 0.9991^{*}B)a_t$$
$$\qquad (-5.3111) \qquad\qquad\qquad\qquad (9.4875)$$

(values in the parentheses are corresponding t- values and *statistical significance $P \leq 0.01$)

where, $\log \tilde{c}_t$ are the logarithmic values of the first differenced series of electricity consumption after taking the deviation from the mean of the first differenced series. Then, the electricity consumption figures up to 2000-2001 are computed as in the case of log-transformed gas consumption following Pankartz (1986; pp.257) and reported in table 4.2.3.

4.9. PREFERENCE OF VARIABLE (UNTRANSFORMED OR TRANSFORMED)

The statistics r^2 and T defined in chapter III, section 3.2 are also used for the selected ARIMA model in order to take decision regarding the choice of the variables, c_t or $\log c_t$.

4.9.1. *For gas consumption*

The values of the statistics r^2 and T computed for untransformed series of gas consumption models are 0.998 and 235.8098 respectively. Those for log-transformed series are 0.9674 and 253.9723 respectively. The result shows that untransformed gas consumption series is preferable for ARIMA modelling.

4.9.2. *For electricity consumption*

Again for untransformed electricity consumption series $r^2 = 0.995$, T = 115.6385 and for log-transformed series $r^2 = 0.9772$, T = 107.7655, which implies that for ARIMA model, no variable is preferable to other. From this it can be assumed that both data series perform well. However, in chapter VI both transformed and untransformed series of both type of data will be considered for comparison of their predictive performance.

4.10. SUMMARY

The findings obtained in the study can be summarized as below.

i) Gas consumption data (untransformed and transformed) and electricity consumption data (untransformed and transformed) are non-stationary. In order to fit ARIMA models to these data they are made stationary by differencing. The degrees of differencing are determined by observing the nature of acf and pacf plots of differenced series and employing the criterion suggested by Koreisha and Pukkila (1993).

ii) The order of autoregression is determined by employing Mallows (1973) criterion, Schwarz (1978) criterion, Hannan and Quinn (1973) criterion and Hurvich and Tsai (1989) criterion. In this case too, the nature of acf and pacf plots of original series (transformed and untransformed) are observed.

iii) Ten types of tentatively selected ARIMA models (obviously, on the basis of the criteria mentioned above) with varied values of p,d,q are estimated and their validity is tested using AME, MAPE and RMSE in all the estimation, validation and total periods. Thus as far as the criteria selected for diagnostic checks are concerned, it is found that ARIMA(1,2,1) model outperforms the other models for untransformed gas consumption series.

iv) The same type of model ARIMA(1,2,1) is also found to adequately fit the log-transformed gas consumption series.

v) ARIMA(1,2,1) model is selected as the best ARIMA model for untransformed electricity consumption data series.

vi) In case of log-transformed electricity consumption, ARIMA(1,1,1) model is found to outperform other type of ARIMA model.

vii) Gas and electricity consumption figures (for transformed and untransformed) are forecasted using the selected models and reported in the study.

viii) The untransformed gas consumption data is preferable for ARIMA modelling, which is inconclusive in case of electricity consumption data.

APPENDIX V

Table 4.1.1. Values of diagnostic criteria for selecting ARIMA models for untransformed gas consumption data

Criteria	Period	Values of diagnostic criteria for		
		ARIMA(1,2,1)	ARIMA(1,2,2)	ARIMA(1,1,1)
AIC_c	Estimation	361.3482*	361.8367	364.6605
SIC	Estimation	1789.0283*	1812.1754	1939.1727
C_p	Estimation	2.0000*	2.4427	5.3229
$\phi(p)$	Estimation	17.9957*	18.0214	18.1568
$\delta(p,d,q)$	Estimation	346.3159*	351.1633	349.3783
AME	Estimation	5310.8522*	5423.9920	5372.7165
	Validation	4831.4556*	4875.8441	5791.2373
	Total	5219.5367*	5319.8242	5452.4348
MAPE	Estimation	18.2428*	19.9508	19.8679
	Validation	2.7156*	2.8132	3.2231
	Total	15.2853*	16.6865	16.6974
RMSE	Estimation	7216.7655*	7310.1381	7822.4355
	Validation	10837.3823*	10940.3072	13647.8631
	Total	6858.0550*	6878.5620	7623.0992

Note: The value of the criterion for a model with starlet shows that the model is better than other two models with respect to that criterion.

Table 4.1.2. Values of diagnostic criteria for selecting ARIMA models for log- transformed gas consumption data

Criteria	Period	Values of diagnostic criteria for		
		ARIMA(1,2,1)	ARIMA(1,1,2)	ARIMA(2,1,2)
AIC_c	Estimation	363.2452*	363.7490	365.6225
SIC	Estimation	1880.6086*	1905.7089	2006.9974
C_p	Estimation	2.0000*	2.5106	3.5330
$\phi(p)$	Estimation	18.0955*	18.1220	18.2979
$\delta(p,d,q)$	Estimation	348.2130*	353.0757	352.3753
AME	Estimation	4790.4799*	4801.8310	4868.5719
	Validation	9262.3819*	10987.3611	11547.2359
	Total	5642.2707*	5980.0272	6140.6983
MAPE	Estimation	17.8713*	18.1132	18.3412
	Validation	5.2699*	5.3564	5.5376
	Total	15.4711*	15.6833	15.9024
RMSE	Estimation	7586.1916	7687.4440	7492.3973*
	Validation	21413.7997*	28685.2191	35103.4221
	Total	8380.7155*	10209.0768	11457.6523

Note: The value of the criterion for a model with starlet shows that the model is better than other two models with respect to that criterion.

Table 4.1.3. Observed and predicted values obtained by ARIMA models for gas consumption

Year	Observed gas consumption* in 10^6 cft	For untransformed series by ARIMA(1,2,1) Model	For log-transformed series by ARIMA(1,2,1) Model
ESTIMATION PERIOD			
1970-71	9400	NA	NA
1971-72	12300	NA	NA
1972-73	8800	16038.579	16038.794
1973-74	20000	9155.615	9285.213
1974-75	28313	27024.067	19969.662
1975-76	19754	32702.410	35915.052
1976-77	28871	25850.470	27143.724
1977-78	32029	38697.308	31844.297
1978-79	34131	37628.886	38298.717
1979-80	38243	43911.966	40503.169
1980-81	45032	47081.248	44187.711
1981-82	49494	55099.464	51116.869
1982-83	63717	59241.068	56943.474
1983-84	70133	74557.596	70216.024
1984-85	80257	79276.437	80708.641
1985-86	90958	93240.524	90813.696
1986-87	101138	102637.039	102771.184
1987-88	118955	115056.905	114300.774
1988-89	146309	132415.986	131872.147
VALIDATION PERIOD			
1989-90	159071	157613.821	160242.519
1990-91	164191	170291.636	180031.412
1991-92	178668	182446.664	187030.235
1992-93	194100	202089.329	205776.161
FORECAST PERIOD			
1993-94		219892.328	220039.568
1994-95		239060.597	227783.437
1995-96		259036.193	250533.905
1996-97		279852.223	274846.173
1997-98		301506.723	300388.265
1998-99		323999.810	327191.550
1999-2000		347331.476	355139.806
2000-2001		371501.721	384139.872

* Source: Different issues of Bangladesh Statistical Yearbook, Bangladesh Bureau of Statistics, Government of Bangladesh, Dhaka and Annual Report 1994, Bangladesh Oil, Gas and Mineral Resources Corporation, Dhaka.

Table 4.2.1. Values of diagnostic criteria for selecting ARIMA models for untransformed electricity consumption

Criteria	Period	Values of diagnostic criteria for		
		ARIMA(1,2,3)	ARIMA(2,1,2)	ARIMA(1,1,2)
AIC_c	Estimation	176.8141*	181.3092	183.3070
SIC	Estimation	52.6350*	60.1852	65.3535
C_p	Estimation	2.0000*	5.2809	9.5832
$\phi(p)$	Estimation	10.7199*	11.0731	11.1528
$\delta(p,d,q)$	Estimation	172.3061*	173.6193	174.5339
AME	Estimation	143.3811*	177.1340	174.5339
	Validation	252.4369*	334.7481	445.7787
	Total	157.9219*	198.1492	210.6998
MAPE	Estimation	5.4436*	8.1937	7.8792
	Validation	4.7569*	5.6166	7.5595
	Total	5.3520*	7.8501	7.8365
RMSE	Estimation	186.2591*	194.5933	231.2658
	Validation	254.8609*	453.3534	577.0276
	Total	199.3390*	243.9912	310.6128

Note: The value of the criterion for a model with starlet shows that the model is better than other two models with respect to that criterion.

Table 4.2.2. Values of diagnostic criteria for selecting ARIMA models for log- transformed electricity consumption

Criteria	Period	Values of diagnostic criteria for		
		ARIMA(3,1,1)	ARIMA(2,1,2)	ARIMA(1,2,2)
AIC_c	Estimation	185.9117	185.1988*	186.6802
SIC	Estimation	67.6275*	68.5168	73.1308
C_p	Estimation	6.0000*	7.2139	12.1269
$\phi(p)$	Estimation	11.3915*	11.4125	11.5239
$\delta(p,d,q)$	Estimation	174.4089*	177.5089	178.3773
AME	Estimation	137.1802*	144.1952	179.5949
	Validation	305.8173	316.0629	198.2964*
	Total	158.2598*	165.6787	181.9325
MAPE	Estimation	5.1118*	6.2159	6.3068
	Validation	5.7933*	5.8627	5.2830
	Total	5.1970*	6.1717	6.1788
RMSE	Estimation	199.7811*	221.5321	258.7874
	Validation	368.6351*	381.3719	405.1132
	Total	220.6352*	236.7121	279.3564

Note: The value of the criterion for a model with starlet shows that the model is better than other two models with respect to that criterion.

Table 4.2.3. Observed and predicted values obtained by ARIMA models for electricity consumption

Year	Observed electricity consumption* in MKWH	For untransformed series by ARIMA(1,2,1) Model	For log-transformed series by ARIMA(1,1,1) Model
ESTIMATION PERIOD			
1976-77	932	NA	NA
1977-78	1012	NA	1047.901
1978-79	1205	1114.821	1145.430
1979-80	1381	1373.971	1354.319
1980-81	1406	1559.044	1503.272
1981-82	1594	1635.859	1615.970
1982-83	2028	1848.467	1860.358
1983-84	2399	2198.399	2198.631
1984-85	2704	2551.908	2563.489
1985-86	2841	2928.642	3017.150
1986-87	3307	3186.894	3204.461
1987-88	3485	3676.457	3850.871
1988-89	3772	3883.397	3780.339
1989-90	4694	4370.912	4549.311
1990-91	4705	4910.927	5078.040
VALIDATION PERIOD			
1991-92	4870	5157.504	5233.871
1992-93	6021	5803.630	6268.758
FORECAST PERIOD			
1993-94		6189.0633	6793.011
1994-95		6779.4423	7641.572
1995-96		7275.6488	8685.324
1996-97		7869.8312	9720.219
1997-98		8438.5567	10906.868
1998-99		9061.1163	12299.690
1999-2000		9686.5749	13816.457
2000-2001		10347.6524	15526.640

* Source: Different issues of Bangladesh Statistical Yearbook, Bangladesh Bureau of Statistics, Government of Bangladesh, Dhaka and Annual Report 1993-94, Bangladesh Power Development Board, Dhaka.

```
Autocorrelations:                      GAS CONSUMPTION

     Auto- Stand.
Lag  Corr.   Err. -1  -.75  -.5 -.25  0  .25  .5  .75  1  Box-Ljung  Prob.
                   |----|----|----|----|----|----|----|----|
  1   .844   .203               .      *******.*********      17.218   .000
  2   .672   .198               .      *******.*****          28.688   .000
  3   .495   .193               .      *******.               35.266   .000
  4   .361   .188               .      *******.               38.959   .000
  5   .253   .182                .     ***** .                40.884   .000
  6   .138   .176                .     ***   .                41.499   .000
  7   .036   .170                .     *     .                41.544   .000
  8  -.053   .164                .   *       .                41.651   .000
  9  -.141   .158                . ***      .                 42.448   .000
 10  -.205   .151                . ****     .                 44.291   .000
 11  -.259   .144                .*****     .                 47.534   .000
 12  -.303   .137               *.****     .                  52.471   .000
 13  -.331   .129              **.****     .                  59.066   .000
 14  -.350   .120              **.****     .                  67.496   .000
 15  -.358   .111             ***.***     .                   77.838   .000
 16  -.344   .102             ***.***     .                   89.263   .000

Plot Symbols:       Autocorrelations *     Two Standard Error Limits .
```

Figure 4.1.1. ACF and PACF of gas consumption (untransformed original series)

```
Partial Autocorrelations:                    GAS CONSUMPTION

  Pr-Aut- Stand.
Lag  Corr.   Err. -1  -.75  -.5 -.25  0  .25  .5  .75  1
                   |----|----|----|----|----|----|----|----|
  1   .844   .218            .       ********.********
  2  -.144   .218            .    ***       .
  3  -.115   .218            .     **       .
  4   .034   .218            .       *      .
  5  -.020   .218            .       *      .
  6  -.130   .218            .    ***       .
  7  -.045   .218            .      *       .
  8  -.045   .218            .      *       .
  9  -.107   .218            .     **       .
 10  -.027   .218            .      *       .
 11  -.059   .218            .      *       .
 12  -.077   .218            .     **       .
 13  -.034   .218            .      *       .
 14  -.057   .218            .      *       .
 15  -.064   .218            .      *       .
 16   .007   .218            .       *      .

Plot Symbols:       Autocorrelations *     Two Standard Error Limits .
```

```
Autocorrelations:                                    GAS CONSUMPTION

Transformations:  difference (2)

      Auto-  Stand.
Lag   Corr.   Err. -1  -.75  -.5 -.25  0   .25  .5   .75   1   Box-Ljung  Prob.
                    |----|-----|---|---|----|----|----|-----|

  1   -.381   .212             ********|    .                       3.226   .072
  2   -.220   .206             .   ****|    .                       4.358   .113
  3    .270   .200             .       |*****  .                    6.174   .103
  4   -.149   .194             .    ***|    .                       6.761   .149
  5   -.111   .187              .    **|   .                        7.115   .212
  6    .294   .181              .      |******.                     9.766   .135
  7   -.251   .173              . *****|   .                       11.863   .105
  8    .076   .166              .      |**  .                      12.069   .148
  9    .081   .158               .     |**  .                      12.330   .195
 10   -.208   .150               . ****|   .                       14.239   .162
 11    .127   .142               .     |***  .                     15.039   .181
 12    .081   .132                .    |**  .                      15.409   .220
 13   -.270   .123                *****|   .                       20.239   .089
 14    .132   .112                 .   |***.                       21.633   .086
 15    .175   .100                 .   |****                       24.704   .054
 16   -.168   .087                  ***|  .                        28.442   .028
```

Figure 4.1.2 ACF and PACF of differenced series of gas consumption (untransformed)

```
Partial Autocorrelations:                            GAS CONSUMPTION

Transformations:  difference (2)

   Pr-Aut- Stand.
Lag  Corr.   Err. -1  -.75  -.5 -.25  0   .25  .5   .75   1
                   |----|-----|---|---|----|----|----|-----|

  1  -.381   .229            .********|    .
  2  -.427   .229            *********|    .
  3  -.018   .229            .       *|    .
  4  -.147   .229            .     ***|    .
  5  -.200   .229            .    ****|    .
  6   .110   .229            .        |**  .
  7  -.156   .229            .     ***|    .
  8   .056   .229            .        |*   .
  9  -.038   .229            .       *|    .
 10  -.148   .229            .     ***|    .
 11  -.011   .229            .        |    .
 12  -.029   .229            .       *|    .
 13  -.178   .229            .    ****|    .
 14  -.168   .229            .     ***|    .
 15   .073   .229            .        |*   .
 16   .065   .229            .        |*   .
```

```
Autocorrelations:

     Auto-  Stand.
Lag  Corr.   Err. -1  -.75  -.5 -.25   0   .25  .5   .75   1   Box-Ljung  Prob.
                   +----+-----+---+----+----+----+-----+----+
  1  -.124   .212              .   **|     .                        .339   .560
  2  -.354   .206              .*******|     .                       3.272   .195
  3   .159   .200              .       |***  .                       3.901   .272
  4  -.066   .194              .      *|     .                       4.017   .404
  5  -.064   .187               .     *|    .                        4.132   .531
  6   .178   .181               .      |****.                        5.108   .530
  7  -.146   .173               .   ***|    .                        5.820   .561
  8  -.045   .166               .     *|    .                        5.894   .659
  9   .057   .158                .     |*  .                         6.022   .738
 10  -.170   .150                .  ***|   .                         7.308   .696
 11   .068   .142                .     |*  .                         7.536   .754
 12   .021   .132                 .    *  .                          7.560   .818
 13  -.222   .123                 .****|  .                         10.828   .625
 14   .118   .112                  .   |** .                        11.938   .611
 15   .307   .100                  .   |***.                        21.364   .126
 16  -.180   .087                  *.**|  .                         25.665   .059

Plot Symbols:      Autocorrelations *      Two Standard Error Limits .
```

Figure 4.1.3. ACF and PACF of error series of gas consumption (untransformed) for ARIMA(1,2,1) model

```
Partial Autocorrelations:

    Pr-Aut- Stand.
Lag  Corr.   Err. -1  -.75  -.5 -.25   0   .25  .5   .75   1
                   +----+-----+---+----+----+----+-----+----+
  1  -.124   .229              .   **|      .
  2  -.375   .229              .*******|      .
  3   .062   .229              .       |*     .
  4  -.194   .229              .   ****|      .
  5  -.019   .229              .      *       .
  6   .080   .229              .       |**    .
  7  -.142   .229              .    ***|      .
  8   .020   .229              .      *       .
  9  -.105   .229              .     **|      .
 10  -.173   .229              .    ***|      .
 11  -.002   .229              .      *       .
 12  -.169   .229              .    ***|      .
 13  -.210   .229              .   ****|      .
 14  -.055   .229              .     *|       .
 15   .203   .229              .       |****  .
 16  -.042   .229              .     *|       .

Plot Symbols:      Autocorrelations *      Two Standard Error Limits .
```

```
Autocorrelations:

     Auto- Stand.
Lag  Corr.   Err. -1  -.75  -.5 -.25   0   .25  .5   .75   1   Box-Ljung  Prob.
                   |----|----|----|----|----|----|----|----|
  1   .826   .203              .       |*******.*********          16.471   .000
  2   .686   .198              .       |*******.******             28.428   .000
  3   .512   .193              .       |*******.                   35.463   .000
  4   .387   .188              .       |********                   39.711   .000
  5   .304   .182               .      |******.                    42.505   .000
  6   .178   .176               .      |****  .                    43.521   .000
  7   .076   .170               .      |**    .                    43.722   .000
  8  -.017   .164               .      *      .                    43.732   .000
  9  -.111   .158                .   **|     .                     44.225   .000
 10  -.193   .151                . ****|     .                     45.857   .000
 11  -.258   .144                .*****|     .                     49.079   .000
 12  -.317   .137                *.****|    .                      54.455   .000
 13  -.351   .129               **.****|    .                      61.876   .000
 14  -.376   .120              ***.****|    .                      71.652   .000
 15  -.392   .111             ****.*** |   .                       84.041   .000
 16  -.376   .102             ****.*** |   .                       97.673   .000

Plot Symbols:      Autocorrelations *     Two Standard Error Limits .
```

Figure 4.2.1. ACF and PACF of gas consumption (transformed original series)

```
Partial Autocorrelations:

   Pr-Aut- Stand.
Lag  Corr.   Err. -1  -.75  -.5 -.25   0   .25  .5   .75   1
                   |----|----|----|----|----|----|----|----|
  1   .826   .218              .       |********.********
  2   .012   .218              .       *       .
  3  -.181   .218              .   ****|       .
  4   .025   .218              .       |*      .
  5   .078   .218              .       |**     .
  6  -.217   .218              .   ****|       .
  7  -.058   .218              .      *|       .
  8   .008   .218              .       *       .
  9  -.130   .218              .    ***|       .
 10  -.114   .218              .     **|       .
 11   .005   .218              .       *       .
 12  -.087   .218              .     **|       .
 13  -.069   .218              .      *|       .
 14  -.029   .218              .      *|       .
 15  -.059   .218              .      *|       .
 16   .001   .218              .       *       .

Plot Symbols:      Autocorrelations *     Two Standard Error Limits .
```

Autocorrelations:

Transformations: difference (2)

Lag	Auto-Corr.	Stand. Err.	-1 -.75 -.5 -.25 0 .25 .5 .75 1	Box-Ljung	Prob.
1	-.531	.212	*******	6.257	.012
2	-.233	.206	*****	7.530	.023
3	.468	.200	*******	12.985	.005
4	-.236	.194	*****	14.472	.006
5	-.010	.187	*	14.474	.013
6	.091	.181	**	14.728	.022
7	-.061	.173	*	14.853	.038
8	-.011	.166	*	14.857	.062
9	.084	.158	**	15.142	.087
10	-.105	.150	**	15.631	.111
11	.059	.142	*	15.807	.148
12	.000	.132	*	15.807	.200
13	-.049	.123	*	15.967	.251
14	.034	.112	*	16.057	.310
15	.033	.100	*	16.169	.371
16	-.046	.087	*	16.455	.422

Figure 4.2.2. ACF and PACF of differenced series of gas consumption (transformed)

Partial Autocorrelations:

Transformations: difference (2)

Lag	Pr-Aut- Corr.	Stand. Err.	-1 -.75 -.5 -.25 0 .25 .5 .75 1
1	-.531	.229	********
2	-.718	.229	********
3	-.363	.229	*******
4	-.413	.229	********
5	-.183	.229	****
6	-.152	.229	***
7	-.014	.229	*
8	-.093	.229	**
9	.072	.229	*
10	-.034	.229	*
11	.083	.229	**
12	-.033	.229	*
13	-.051	.229	*
14	-.258	.229	*****
15	-.197	.229	****
16	-.199	.229	****

```
Autocorrelations:

     Auto- Stand.
Lag  Corr.   Err. -1  -.75  -.5 -.25   0   .25  .5   .75   1  Box-Ljung  Prob.
                   +----+----+---+----+----+----+----+----+
  1  -.291   .212              . ******|     .                 1.878   .171
  2  -.450   .206              .*******|     .                 6.639   .036
  3   .314   .200              .       |****** .               9.095   .028
  4  -.035   .194              .      *|     .                 9.127   .058
  5  -.081   .187               .    **|    .                  9.312   .097
  6   .046   .181               .      |*   .                  9.378   .153
  7  -.022   .173               .      *    .                  9.394   .226
  8  -.003   .166               .      *    .                  9.394   .310
  9   .068   .158                .     |*  .                   9.578   .386
 10  -.058   .150                .    *|   .                   9.724   .465
 11   .010   .142                .     *   .                   9.730   .555
 12  -.006   .132                 .    *  .                    9.731   .640
 13  -.042   .123                 .   *| .                     9.848   .706
 14   .051   .112                  .   |* .                   10.054   .758
 15   .073   .100                  .   |* .                   10.584   .781
 16  -.066   .087                   . *|*.                    11.168   .799

Plot Symbols:      Autocorrelations *     Two Standard Error Limits .
```

Figure 4.2.3. ACF and PACF of error series of gas consumption (transformed) for ARIMA(1,2,1) model

```
Partial Autocorrelations:

   Pr-Aut- Stand.
Lag  Corr.   Err. -1  -.75  -.5 -.25   0   .25  .5   .75   1
                   +----+----+---+----+----+----+----+----+
  1  -.291   .229             .  ******|      .
  2  -.585   .229             .********|      .
  3  -.106   .229             .      **|      .
  4  -.314   .229             .  ******|      .
  5  -.097   .229             .      **|      .
  6  -.215   .229             .    ****|      .
  7  -.145   .229             .     ***|      .
  8  -.203   .229             .    ****|      .
  9  -.066   .229             .       *|      .
 10  -.158   .229             .     ***|      .
 11  -.027   .229             .       *|      .
 12  -.164   .229             .     ***|      .
 13  -.137   .229             .     ***|      .
 14  -.193   .229             .    ****|      .
 15  -.011   .229             .        *      .
 16  -.064   .229             .       *|      .

Plot Symbols:      Autocorrelations *     Two Standard Error Limits .
```

```
Autocorrelations:                      ELECTRICITY CONSUMPTION

     Auto- Stand.
Lag  Corr.   Err. -1   -.75   -.5 -.25   0   .25  .5   .75   1   Box-Ljung  Prob.
                   +------+------+-----+------+-----+------+-----+
  1   .803   .234               .       |********.*******          11.731   .001
  2   .571   .226               .       |********.**               18.129   .000
  3   .407   .217               .       |********.                 21.657   .000
  4   .243   .208                .      |*****   .                 23.020   .000
  5   .067   .198                .      |*       .                 23.133   .000
  6  -.083   .188                .    **|        .                 23.329   .001
  7  -.217   .177                 . ****|       .                  24.832   .001
  8  -.323   .166                 .*****|       .                  28.622   .000
  9  -.385   .153                **.****|      .                   34.929   .000
 10  -.403   .140                **.****|      .                   43.220   .000
 11  -.381   .125               ***.****|     .                    52.492   .000
 12  -.354   .108                ***.***|    .                     63.133   .000
 13  -.293   .089                 **.***|    .                     74.081   .000

Plot Symbols:      Autocorrelations *     Two Standard Error Limits .

Total cases:  15     Computable first lags:  14
```

Figure 4.3.1. ACF and PACF of electricity consumption (untransformed original series)

```
Partial Autocorrelations:                       ELECTRICITY CONSUMPTION

    Pr-Aut- Stand.
Lag  Corr.   Err. -1   -.75   -.5 -.25   0   .25  .5   .75   1
                   +------+------+-----+------+-----+------+-----+
  1   .803   .258               .       |*********.******
  2  -.205   .258               .   ****|         .
  3   .058   .258               .       |*        .
  4  -.158   .258               .    ***|         .
  5  -.134   .258               .    ***|         .
  6  -.089   .258               .     **|         .
  7  -.142   .258               .    ***|         .
  8  -.079   .258               .     **|         .
  9  -.063   .258               .      *|         .
 10  -.029   .258               .      *|         .
 11  -.008   .258               .       |         .
 12  -.085   .258               .     **|         .
 13   .028   .258               .       |*        .

Plot Symbols:      Autocorrelations *     Two Standard Error Limits .

Total cases:  15     Computable first lags:  14
```

```
Autocorrelations:                        ELECTRICITY CONSUMPTION

Transformations:  difference (2)

     Auto- Stand.
Lag  Corr.   Err. -1  -.75  -.5 -.25   0   .25  .5   .75   1   Box-Ljung  Prob.
                   +-----+----+----+----+----+----+----+----+
  1  -.421   .248             . ********|         .                2.874   .090
  2  -.183   .238             .     ****|         .                3.468   .177
  3   .266   .226              .        |*****   .                 4.843   .184
  4  -.189   .215              .    ****|        .                 5.618   .230
  5   .066   .203               .       |*      .                  5.723   .334
  6  -.034   .189               .      *|       .                  5.756   .451
  7   .157   .175                .      |***   .                   6.557   .476
  8  -.059   .160                 .    *|     .                    6.691   .570
  9  -.176   .143                 . ****|     .                    8.199   .514
 10   .086   .124                  .    |**  .                     8.685   .562
 11   .054   .101                   .   |*  .                      8.970   .625

Plot Symbols:      Autocorrelations *     Two Standard Error Limits .

Total cases:  15     Computable first lags after differencing:  12
```

Figure 4.3.2. ACF and PACF of differenced series of electricity consumption (untransformed)

```
Pa.   . Aut.   .elations:                            ELECTRICITY CONSUMPTION

Transformations:  difference (2)

   Pr-Aut- Stand.
Lag  Corr.   Err. -1  -.75  -.5 -.25   0   .25  .5   .75   1
                   +-----+----+----+----+----+----+----+----+
  1  -.421   .277           .  ********|          .
  2  -.437   .277           . *********|          .
  3  -.044   .277           .         *|          .
  4  -.191   .277           .      ****|          .
  5  -.027   .277           .         *|          .
  6  -.156   .277           .       ***|          .
  7   .199   .277           .          |****      .
  8   .100   .277           .          |**        .
  9  -.053   .277           .         *|          .
 10  -.182   .277           .      ****|          .
 11  -.025   .277           .         *|          .

Plot Symbols:      Autocorrelations *     Two Standard Error Limits .

Total cases:  15     Computable first lags after differencing:  12
```

```
Autocorrelations:

     Auto- Stand.
Lag  Corr.   Err. -1  -.75  -.5 -.25   0   .25  .5   .75   1  Box-Ljung  Prob.
                   +----+----+----+----+----+----+----+----+
  1  -.211   .248              .  ****|         .               .721   .396
  2  -.176   .238              .  ****|         .              1.267   .531
  3   .055   .226               .     |*       .               1.326   .723
  4  -.301   .215               . ******       .               3.285   .511
  5   .086   .203                .    |**     .                3.467   .628
  6   .029   .189                .    |*      .                3.490   .745
  7   .130   .175                 .   |***   .                 4.035   .776
  8  -.066   .160                  . *|      .                 4.203   .838
  9  -.149   .143                  .***|     .                 5.282   .809
 10   .077   .124                   . |**  .                   5.666   .842
 11   .070   .101                    .|*  .                    6.147   .863

Plot Symbols:      Autocorrelations *     Two Standard Error Limits .

Total cases:  15     Computable first lags:  12
```

Figure 4.3.3. ACF and PACF of error series of electricity consumption (untransformed) for the ARIMA(1,2,1) model

```
Partial Autocorrelations:

   Pr-Aut- Stand.
Lag  Corr.   Err. -1  -.75  -.5 -.25   0   .25  .5   .75   1
                   +----+----+----+----+----+----+----+----+
  1  -.211   .277             .   ****|        .
  2  -.230   .277             .  *****|        .
  3  -.043   .277             .      *|        .
  4  -.373   .277             . *******        .
  5  -.109   .277             .     **|        .
  6  -.171   .277             .    ***|        .
  7   .097   .277             .       |**      .
  8  -.173   .277             .    ***|        .
  9  -.178   .277             .   ****|        .
 10  -.120   .277             .     **|        .
 11   .086   .277             .       |**      .

Plot Symbols:      Autocorrelations *     Two Standard Error Limits .

Total cases:  15     Computable first lags:  12
```

```
Autocorrelations:

     Auto- Stand.
Lag  Corr.   Err. -1  -.75  -.5 -.25   0   .25  .5   .75   1  Box-Ljung  Prob.
                   |----|----|----|----|----|----|----|----|
  1   .812   .234              .       |********.*******         12.000   .001
  2   .598   .226              .       |********.***             19.018   .000
  3   .427   .217              .       |*********                22.899   .000
  4   .262   .208               .      |*****  .                 24.487   .000
  5   .074   .198               .      |*      .                 24.625   .000
  6  -.094   .188               .    **|       .                 24.876   .000
  7  -.229   .177                . *****|      .                 26.545   .000
  8  -.332   .166                *******|      .                 30.562   .000
  9  -.396   .153               **.*****|     .                  37.225   .000
 10  -.414   .140               **.*****|     .                  45.979   .000
 11  -.390   .125               ***.****|    .                   55.675   .000
 12  -.362   .108                ***.***|   .                    66.840   .000
 13  -.298   .089                 **.***|   .                    78.166   .000

Plot Symbols:      Autocorrelations *     Two Standard Error Limits .

Total cases:  15     Computable first lags:  14
```

Figure 4.4.1. ACF and PACF of electricity consumption (transformed original series)

```
Partial Autocorrelations:

   Pr-Aut- Stand.
Lag  Corr.   Err. -1  -.75  -.5 -.25   0   .25  .5   .75   1
                   |----|----|----|----|----|----|----|----|
  1   .812   .258             .        |*********.******
  2  -.178   .258             .    ****|       .
  3  -.001   .258             .        *       .
  4  -.122   .258             .      **|       .
  5  -.191   .258             .    ****|       .
  6  -.099   .258             .      **|       .
  7  -.103   .258             .      **|       .
  8  -.086   .258             .      **|       .
  9  -.049   .258             .       *|       .
 10  -.025   .258             .       *|       .
 11  -.003   .258             .        *       .
 12  -.093   .258             .      **|       .
 13   .032   .258             .        *       .

Plot Symbols:      Autocorrelations *     Two Standard Error Limits .

Total cases:  15     Computable first lags:  14
```

```
Autocorrelations:

Transformations:  difference (1)

     Auto- Stand.
Lag  Corr.   Err. -1  -.75  -.5 -.25   0   .25  .5   .75   1   Box-Ljung  Prob.
                  +----+-----+----+----+----+----+----+----+
  1  -.205  .241           .      ****|         .                  .721   .396
  2  -.300  .231            .   ******|        .                  2.403   .301
  3   .119  .222            .         |**      .                  2.690   .442
  4  -.041  .211             .       *|       .                   2.728   .604
  5   .058  .200             .        |*      .                   2.812   .729
  6  -.099  .189             .      **|       .                   3.085   .798
  7   .141  .177              .       |***   .                    3.717   .812
  8  -.097  .164              .     **|      .                    4.070   .851
  9  -.250  .149               .*****|     .                     6.863   .651
 10   .195  .134                .     |****.                      8.987   .533
 11   .073  .116                .     |*   .                      9.386   .586
 12  -.150  .094                 .***|   .                      11.892   .454

Plot Symbols:      Autocorrelations *     Two Standard Error Limits .

Total cases:  15     Computable first lags after differencing:  13
```

Figure 4.4.2. ACF and PACF of differenced series of electricity consumption (transformed)

```
Partial Autocorrelations:

Transformations:  difference (1)

    Pr-Aut- Stand.
Lag  Corr.   Err. -1  -.75  -.5 -.25   0   .25  .5   .75   1
                  +----+-----+----+----+----+----+----+----+
  1  -.205  .267           .      ****|         .
  2  -.357  .267           .   *******|         .
  3  -.045  .267           .         *|         .
  4  -.151  .267           .       ***|         .
  5   .047  .267           .          |*        .
  6  -.150  .267           .       ***|         .
  7   .162  .267           .          |***      .
  8  -.150  .267           .       ***|         .
  9  -.225  .267           .     *****|         .
 10  -.058  .267           .         *|         .
 11  -.004  .267           .          *         .
 12  -.144  .267           .       ***|         .

Plot Symbols:      Autocorrelations *     Two Standard Error Limits .

Total cases:  15     Computable first lags after differencing:  13
```

```
Autocorrelations:

     Auto-  Stand.
Lag  Corr.   Err. -1  -.75  -.5 -.25   0   .25  .5   .75   1  Box-Ljung  Prob.
                   +----+----+----+----+----+----+----+----+
  1  -.015   .241                .     *    .                     .004   .951
  2  -.152   .231                .  ***|   .                      .434   .805
  3  -.048   .222                .    *|   .                      .480   .923
  4  -.139   .211                 . ***|  .                       .910   .923
  5   .071   .200                 .    |* .                      1.036   .960
  6  -.080   .189                 .  **|  .                      1.215   .976
  7  -.032   .177                  .  *| .                       1.248   .990
  8  -.105   .164                  . **| .                       1.661   .990
  9  -.149   .149                   .***|.                       2.660   .976
 10   .190   .134                   .  |****.                    4.690   .911
 11   .004   .116                   .  *  .                      4.691   .945
 12  -.103   .094                    .**|.                       5.875   .922

Plot Symbols:      Autocorrelations *     Two Standard Error Limits .

Total cases:  15     Computable first lags:  13
```

Figure 4.4.3. ACF and PACF of error series of electricity consumption (transformed) for ARIMA(1,1,1) model

```
Partial Autocorrelations:

   Pr-Aut- Stand.
Lag  Corr.   Err. -1  -.75  -.5 -.25   0   .25  .5   .75   1
                   +----+----+----+----+----+----+----+----+
  1  -.015   .267              .       *       .
  2  -.152   .267              .    ***|       .
  3  -.054   .267              .      *|       .
  4  -.168   .267              .    ***|       .
  5   .050   .267              .       |*      .
  6  -.137   .267              .    ***|       .
  7  -.034   .267              .      *|       .
  8  -.177   .267              .   ****|       .
  9  -.179   .267              .   ****|       .
 10   .091   .267              .       |**     .
 11  -.076   .267              .     **|       .
 12  -.144   .267              .    ***|       .

Plot Symbols:      Autocorrelations *     Two Standard Error Limits .

Total cases:  15     Computable first lags:  13
```