<u>CHAPTER - III</u>

APPLICATION OF GROWTH MODEL

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Use of structural model for forecasting purposes creates some problem (discussed in section 3.1). Hence, in this chapter different types of time-dependent models are considered for investigating their fitness to energy consumption data. After diagnostic check one of the models for each type of energy consumption is selected for forecasting. Thus section 3.1 focuses applicability of growth models to the energy consumption, section 3.2 discusses the logic of using transformation of data series, the criteria selected for diagnostic check of the predictive performance of models are given in section 3.3, the result of analysis for gas and electricity consumption are given in sections 3.4 and 3.5 respectively, choice of the variables for model building is discussed in section 3.6, and finally section 3.7 deals with the result obtained from the analysis in this chapter at a glance. Numerical results obtained are reported in the appendix in last part of the chapter.

3.1 INTRODUCTION

In the previous chapter multivariate analysis of the energy consumption data is performed. As far as the selected diagnostic criteria are concerned, it is found that the fitting performances of structural models are good. As already mentioned earlier that the purpose of the present study is to find the best fitted models which adequately predict the future observations of gas and electricity consumption in Bangladesh. Some econometricians *e.g.* Engel *et. al.* (1992), Jamal and Abdullah (1996), etc. showed that structural model sometimes gave good model for prediction for out-sample-data. But they were very difficult to use for prediction of future data. Structural models use detailed relationship based on the economic theory to describe the economic phenomena. The generation of the forecast of dependent variable from a structural model requires forecasts of the explanatory variables as well (Ma 1989 p.394). Meese and Rogoff (1983 p.3) also stated that structural models had high explanatory power, but they predicted badly because explanatory variables were themselves difficult to predict. Hence, in this study the structural models are not considered for comparison of prediction performance with other selected models due to the limitations mentioned above. Moreover, the time dependent forecasting methods have some advantages over multivariate method. Some of the advantages are cited in section 3.1.2.

3.1.1 *Definition of growth models*

The models which are used to describe the behaviour of some variables, as they vary with respect to time, are termed as growth models. This type of model needed in a specific area and in a specific problem depends on the type of growth that occurs in the time series data. In general, growth models are mechanistic, rather than empirical ones. A mechanical model usually arises as a result of making assumptions about the type of growth, writing down the differentials or difference equations that represent these assumptions and then solving the equations to obtain a growth model. An empirical model, on the other hand, is a model chosen to empirically approximate an unknown mechanistic model. Typically, the empirical model is a polynomial of some suitable order. However, in this study the growth models of different types are fitted using the software SPSSPC+.

Apart from the disadvantages of multivariate forecasting techniques discussed above, the time dependent growth model for forecasting has following advantages over multivariate techniques. i) They are quick and inexpensive to apply, and produce better forecasting model. The cost of making particular forecasting error should always be balanced against the cost of producing forecasts.

ii) Extraneous information may not require.

iii) Forecasts obtained in this manner can often be usefully combined with other forecasts in order to get superior overall forecasts.

iv) Having produced this sort of forecasting, one is in a position to assess the variation explained in terms of its own past or future behaviour, etc.

3.1.2. *Models considered in the study*

The fitness of as many as ten types of time-dependent models to the energy consumption data are investigated in this study. The models considered are

Linear $c_t = \beta_0 + \beta_1 t + \epsilon_t$ ....(3.1)

Logarithmic $c_t = \beta_0 + \beta_1 \ln t + \epsilon_t$ ....(3.2)

Inverse $c_t = \beta_0 + \beta_1/t + \epsilon_t$ ....(3.3)

Quadratic $c_t = \beta_0 + \beta_1 t + \beta_2 t^2 + \epsilon_t$ ....(3.4)

Cubic $c_t = \beta_0 + \beta_1 t + \beta_2 t^2 + \beta_3 t^3 + \epsilon_t$ ....(3.5)

Compound $c_t = \beta_0(\beta_1)^t \epsilon_t$ ....(3.6)

Power $c_t = \beta_0(t^{\beta 1})\epsilon_t$ ....(3.7)

S $c_t = \exp(\beta_0 + \beta_1/t)\epsilon_t$ ....(3.8)

Growth $c_t = \exp(\beta_0 + \beta_1 t + \epsilon_t)$ ....(3.9)

Exponential $c_t = \beta_0 \exp(\beta_1 t)\epsilon_t$ ....(3.10)

where, c_t stands for the consumption, ϵ_t is the white noise term assumed to be distributed as normal with mean zero and constant variance σ^2.

3.2. TRANSFORMATION OF VARIABLES

It is assumed here that the Box and Cox (1964) type of transformation(s) may improve the predictive performances of the models. Hence, predictive performances of the models are also studied here with transformation of variables. For defining the type of transformation used in the study, let us have a look at the Box and Cox type of transformations. Box and Cox worked with a parametric family of transformations from c to $c^{(\alpha)}$, the parameter α defining a particular transformation. The parametric family considered was the family defined for all real α by

$c^{(\alpha)} = (c^\alpha - 1)/\alpha$, for α # 0 ....(3.11)

$= \log c$, for α=0 ....(3.12) for $c>0$. Thus, in this study the transformation of the type (3.12) *i.e.* log-transformation of the variables is used. Because, one alternative suggestion Kennedy (1985 p.210) offered for the stationarity of a series was the use of this transformation which might make the series stationary in order to fit an ARIMA model. Again, Lachtermacher and Fuller (1995 p.391) stated that the use of Box and Cox transformation seemed to be justified, since in the non-stationary cases it improved the performance of neural network model by decreasing the forecasting errors and diminishing the neural networks modelling time, compared with networks trained with untransformed data. That is why in next two chapters the performances of log-transformed series are also studied for ARIMA modelling and for neural network modelling. One of the purposes of this study is to compare the predictive performances of three types (*viz.* growth type, ARIMA type and ANN type) of selected models. Same type of transformation, as in the case of ARIMA and neural network modelling, is required for comparison. So investigation toward the fitness of the growth type of models to the log-transformed series is quite justified.

Again, Sclove (1972) showed that the choice of c_t or $\log c_t$ as dependent variable, in general, depended on two statistics, *viz.* r^2, the square of sample correlation coefficient and T, where the statistic T is defined as

$$T(0) = \frac{N}{2} \ln S_0 + \sum_{t=1}^{N} \ln c_t$$

and $$T(1) = \frac{N}{2} \ln S_1$$

for log-transformed dependent variable and for original variable respectively, where, S_0 and S_1 are the residual sum of squares of the model with $\log c_t$ and c_t as dependent variable respectively.

With an illustration for a linear model Sclove (1972) shows that the model with maximum r^2 and minimum T is preferable. However, although selected models in this study are not linear, it is assumed that they are linear. So decision about the preference of the variable (untransformed or transformed) is taken in the whole study, where it is necessary, on the basis of these two criteria.

In this chapter fitness of above-mentioned ten types of growth models are also verified for log-transformed series of both types of energy consumption.

3.3. CRITERIA USED FOR SELECTION OF MODEL

In order to select the type of growth model of best fit for forecasting the energy consumption data, estimated models are compared against the values of the following criteria.

i) $\overline{R}^2$***, the adjusted coefficient of determination***: As defined in section 2.3.1.1 of chapter II. This value is computed for the estimation period only.

The greater the value of this criterion, the more is the accuracy of the model.

ii) AME, The absolute mean error : Defined as

$$\text{AME} = \sum_{t=1}^{N} \frac{|c_t(\text{obs}) - c_t(\text{pred})|}{N}$$

iii) RMSE, the root mean square error: Defined as

$$\text{RMSE} = \sqrt{\frac{\sum_{t=1}^{N} [c_t(\text{obs}) - c_t(\text{pred})]^2}{N-k}}$$

iv) MAPE, the mean absolute percent error: Defined as

$$\text{MAPE} = \frac{1}{N} \sum_{t=1}^{N} \frac{|c_t(\text{obs}) - c_t(\text{pred})|}{c_t(\text{obs})} \times 100$$

The minimum values of last three criteria are desirable for the adequacy of a model. The values of the criteria AME, RMSE and MAPE are compared for three periods, *viz.* estimation period, validation period and total period.

3.4. SELECTION OF MODEL FOR GAS CONSUMPTION

It is mentioned in chapter I that the whole period is split into two periods. The observed or estimation period contains data from 1970-71 to 1988-89 and the validation or post-sample period contains data from 1989-90 to 1992-93. The models in this chapter and subsequent chapters are fitted using the data of observed period and the validity of model is verified using data of validation period. For this purpose, at first the models are fitted using data of estimation period, then by the estimated model the consumption figures for validation period are predicted for prediction performance comparison and testing the validity of the model.

3.4.1. *For untransformed series*

Fitness of all of the ten models mentioned in section 3.1.3. are verified for untransformed series of gas consumption data of observed period. Models are fitted using the software SPSSPC+. At first three models are selected on the basis of the values of R^2, which is directly obtained from output of software. The values of the criteria mentioned in section 3.3 are computed for these three models. The computed values of the criteria are reported in table 3.1.1. in the appendix.

Form table 3.1.1, it is evident that model of type (3.4) outperforms the other two types for prediction of gas consumption by all the criteria considered for fitting and testing validity of models, except R^2 . The value of R^2 is larger in case of model (3.5), because, it is known that this value increases with the increase of the number of explanatory variables in the model. That is why the criterion adjusted R^2 is suggested. The value of this adjusted R^2 is again lesser in case of model (3.5) than (3.4). Moreover, forecasting the future figure of gas consumption is more difficult from a growth model of type (3.5) *i.e.* from a cubic model. So, the model of type (3.4) is finally considered as the best prediction model of growth type for untransformed gas consumption followed by type (3.5).

The estimated model

The estimated growth type of model selected for untransformed gas consumption is as follows:

$$c_t = 15858.61 - \underset{(15.12)}{1820.30^{*}}\, t + \underset{(4.89)}{423.57^{*}}\, t^2$$

(values in the parentheses are corresponding absolute values of t-statistics, * means significant at $P<0.01$)

The gas consumption figures for the years up to 2000-2001 are estimated with this selected model and reported in table 3.1.3.

3.4.2. *For log-transformed series*

In this case too, fitness of all of the models given in section 3.1.3. is verified to the log-transformed series of gas consumption. Values of the criteria against three better models are reported in table 3.1.2. The models at this stage are selected on the basis of the values of R^2.

It is clear from table 3.1.2. that the natures of the values of the criteria are exactly the same as that of untransformed series. So, discussion about the values of the criteria is the same as gas consumption. in untransformed consumption. So model of type (3.4) is selected as the best predictive growth model for log-transformed

The estimated model

The estimated growth type of model selected for log-transformed gas consumption is as follows:

$$\log c_t = 8.9975 + \underset{(10.42)}{0.1806^{*}}\, t - \underset{(6.55)}{0.0018^{*}}\, t^2$$

(values in the parentheses are corresponding absolute values of t-statistics, * means significant at P<0.01)

The gas consumption figures for the years up to 2000-2001 are estimated with this selected model and reported in table 3.1.3. It is mentionable here that in this case the original values of gas consumption are obtained by taking exponent of the predicted logarithmic series.

3.5. SELECTION OF GROWTH TYPE OF MODEL FOR ELECTRICITY CONSUMPTION.

As in case of gas consumption series the total period, for which electricity consumption data are used in the study, is split into two periods. The observed or estimation period, containing data from 1976-77 to 1990-91, is used for fitting the model and the validation or post-sample period, from 1991-92 to 1992-93, is used for testing the validity of selected model. In this case too, at first the models are fitted using data of estimation period, then by the estimated model the consumption figures for validation period are predicted for prediction performance comparison and testing the validity of models.

3.5.1. *For untransformed series*

All of the models given in section 3.1.3. are also fitted to the untransformed electricity consumption data. The values of the criteria for three well fitted (on the basis of R^2) models are reported in table 3.2.1. The table shows that the model of type (3.4) again fits the electricity consumption data most adequately. Although the value of R^2 is a little bit greater in case of model (3.5) than model (3.4). But model (3.4) again appears as superior with respect to adjusted values of R^2. So the growth model of type (3.4) is finally selected as the best type of growth model for electricity consumption. It is to be noted here that Paul(1994) shows that the model of type (3.5) is the best fitted model for electricity consumption, because for selection purposes he uses the R^2 criterion only.

The estimated model

The estimated growth type of model selected for untransformed electricity consumption is as follows:

$$c_t = 801.676 + \underset{(19.22)}{87.856^*}\, t + \underset{(5.63)}{11.925^*}\, t^2$$

(values in the parentheses are corresponding absolute values of t-statistics, * means significant at 0.01 level)

The electricity consumption figures for the years up to 2000-2001 are estimated with this selected model and reported in table 3.2.3.

3.5.2. *For log-transformed series*

Investigation toward the fitness of the ten types of growth models, given in section 3.1.3., to the log-transformed series of electricity consumption is also performed. As in all previous cases, at first three well-fitted models are selected on the basis of R^2. The criteria computed for three selected models are reported in table 3.2.2.

As far as the values of the criteria presented in table 3.2.2 are concerned, the model of type (3.4) is again appeared as the best fitted model for log-transformed electricity consumption. So, the model of type (3.4) is finally selected as the best growth type of model for log-transformed electricity consumption.

The estimated model

The estimated growth type of model selected for log-transformed electricity consumption is as follows:

$$\log c_t = 6.646 + \underset{(14.33)}{0.148^{*}}\, t - \underset{(8.16)}{0.0018^{*}}\, t^2$$

(values in the parentheses are corresponding absolute values of t-statistics, * means significant at 0.01 level)

The electricity consumption figures for the years up to 2000-2001 are estimated by the selected model and reported in table 3.2.3. It is mentionable here that in this case too, the original values of electricity consumption are obtained by taking exponent of the predicted logarithmic series.

3.6. PREFERENCE OF VARIABLE (UNTRANSFORMED OR TRANSFORMED)

Two statistics r^2 and T are defined in section 3.2 for taking decision about the preference of variables as dependent variable. The values of those two statistics are computed for the selected models of gas and electricity consumption of both types, transformed and untransformed. The values are computed for total period.

3.6.1. *For gas consumption*

For untransformed gas consumption series for selected quadratic model the values of r^2 and T are computed as 0.9851 and 110.7476 respectively. Similarly for log-transformed series the values are 0.9853 and 106.1503 respectively, where r^2 is maximum and T is minimum for log-transformed series. So it can be concluded that log-transformed series of gas consumption is preferable as dependent variable to untransformed one.

3.6.2. *For electricity consumption*

In case of untransformed electricity consumption for selected quadratic model the values of r^2 and T are computed as 0.9884 and 232.9721 respectively. Similarly for log-transformed series the values are 0.9885 and 224.1924 respectively. Where in this case too, r^2 is maximum and T is minimum for log-transformed series. So it can be concluded that log-transformed series of electricity consumption is preferable as dependent variable to untransformed one.

However, predicted values for both untransformed and transformed series are reported in the study, so that one can use any result.

3.7. SUMMARY

The findings in this chapter can be summarised as follows:

i) Investigation toward the fitness of as many as ten types of growth models to the gas consumption data is performed. The models are fitted to the data of observed period. The validations of models are verified using data of validation period. The performances of three selected models (on the basis of R^2) are compared on the basis of the values of criteria like Adjusted value of R^2, AME in three periods, MAPE in three periods and RMSE in three periods.

ii) Quadratic model is found to outfit all other models, with respect to the criteria, to the untransformed gas consumption.

iii) Predictive performance of quadratic growth model is the best for log-transformed gas consumption.

iv) Untransformed and log-transformed electricity consumption can also be best predicted by quadratic type of growth model.

v) Log-transformed series of both gas and electricity consumption is preferable to the untransformed series as dependent variable in order to fit quadratic models.

APPENDIX III

Table 3.1.1. Values of diagnostic criteria for selecting Growth models for untransformed gas consumption

Criteria	Period	Values of diagnostic criteria for		
		Model(3.4)	Model(3.5)	Model(3.10)
R^2	Estimation	0.9851	0.9852*	0.9760
$\overline{R}^2$	Estimation	0.9832*	0.9826	0.9746
AME	Estimation	4066.2927*	4125.5632	4229.7689
	Validation	4134.6883*	4159.4444	5236.2281
	Total	4078.1876*	4131.4555	4404.8053
MAPE	Estimation	14.3148*	14.9508	15.8679
	Validation	2.4487*	2.8132	3.1231
	Total	12.2511*	12.8399	13.6514
RMSE	Estimation	5613.1105*	5710.2380	6231.5991
	Validation	11145.0721*	11160.1027	11247.1684
	Total	5605.0194*	5684.4392	6114.8340

Note: The value of the criterion for a model with starlet shows that the model is better than other two models with respect to that criterion.

Table 3.1.2. Values of diagnostic criteria for selecting Growth type of model for log-transformed gas consumption

Criteria	Period	Values of diagnostic criteria for		
		Model(3.4)	Model(3.5)	Model(3.10)
R^2	Estimation	0.9862	0.9868*	0.9712
$\overline{R}^2$	Estimation	0.9853*	0.9851	0.9695
AME	Estimation	3269.2927*	3281.2711	3301.5252
	Validation	5890.7761*	6054.1990	5960.3127
	Total	3725.2028*	3763.5191	3763.9238
MAPE	Estimation	10.1431*	10.1435	11.3215
	Validation	3.4100*	3.4121	3.4530
	Total	8.9971*	8.9728	9.9531
RMSE	Estimation	4710.5897*	4812.3501	5069.2433
	Validation	9235.1433*	9384.0509	10126.4653
	Total	4692.1335*	4788.5277	5068.2432

Note: The value of the criterion for a model with starlet shows that the model is better than other two models with respect to that criterion.

Table 3.1.3. Observed and predicted values obtained by selected growth models for gas consumption data

Year	Observed gas consumption* in 10^6 cft	For untransformed series by quadratic growth model	For log-transformed series by quadratic growth model
ESTIMATION PERIOD			
1970-71	9400	14461.843	9665.743
1971-72	12300	13912.259	11517.800
1972-73	8800	14209.817	13676.275
1973-74	20000	15354.514	16181.919
1974-75	28313	17346.353	19079.025
1975-76	19754	20185.332	22415.389
1976-77	28871	23871.452	26242.207
1977-78	32029	28404.713	30613.879
1978-79	34131	33785.115	35587.734
1979-80	38243	40012.657	41223.632
1980-81	45032	47087.340	47583.471
1981-82	49494	55009.163	54730.567
1982-83	63717	63778.128	62728.908
1983-84	70133	73394.234	71642.289
1984-85	80257	83857.480	81533.322
1985-86	90958	95167.867	92462.318
1986-87	101138	107325.395	104486.063
1987-88	118955	120637.039	117656.494
1988-89	146309	134181.872	132019.291
VALIDATION PERIOD			
1989-90	159071	148880.827	147612.400
1990-91	164191	164426.913	164464.527
1991-92	178668	180820.145	182593.617
1992-93	194100	198060.517	202005.361
FORECAST PERIOD			
1993-94		216148.030	222690.902
1994-95		235082.684	244629.475
1995-96		254864.479	267779.725
1996-97		275493.414	292087.881
1997-98		296969.490	317473.618
1998-99		319292.707	343849.727
1999-2000		342463.065	371104.881
2000-2001		366480.563	399101.081

* Source: Different issues of Bangladesh Statistical Yearbook, Bangladesh Bureau of Statistics, Government of Bangladesh, Dhaka and Annual Report 1994, Bangladesh Oil, Gas and Mineral Resources Corporation, Dhaka.

Table 3.2.1. Values of diagnostic criteria for selecting growth model for untransformed electricity consumption

Criteria	Period	Values of diagnostic criteria for		
		Model(3.4)	Model(3.5)	Model(3.6)
R^2	Estimation	0.9884	0.9890*	0.9870
$\overline{R}^2$	Estimation	0.9866*	0.9859	0.9761
AME	Estimation	102.6636*	103.1234	110.5888
	Validation	334.8150*	335.0798	340.6345
	Total	129.9755*	130.4123	137.6530
MAPE	Estimation	4.1606*	4.1758	4.2263
	Validation	6.3264*	6.4125	7.0321
	Total	4.4154*	4.4389	4.5564
RMSE	Estimation	145.9366*	147.0038	152.5731
	Validation	479.9085*	480.3663	498.4597
	Total	186.2950*	187.0968	194.1657

Note: The value of the criterion for a model with starlet shows that the model is better than other two models with respect to that criterion.

Table 3.2.2. Values of diagnostic criteria for selecting growth models for log- transformed electricity consumption

Criteria	Period	Values of diagnostic criteria for		
		Model(3.4)	Model(3.5)	Model(3.6)
R^2	Estimation	0.9885	0.9885	0.9868
$\overline{R}^2$	Estimation	0.9865*	0.9853	0.9657
AME	Estimation	103.2713*	104.0469	112.3350
	Validation	325.5142*	325.7011	336.2487
	Total	120.4173*	130.1238	138.6779
MAPE	Estimation	4.2232*	4.2561	4.4121
	Validation	6.2061*	6.2218	6.8728
	Total	4.4565*	4.4873	4.7015
RMSE	Estimation	149.8785*	150.0042	159.2342
	Validation	474.7247*	480.3216	491.5788
	Total	188.0209*	189.1189	197.4688

Note: The value of the criterion for a model with starlet shows that the model is better than other two models with respect to that criterion.

Table 3.2.3. Observed and predicted values obtained by selected growth models for electricity consumption

Year	Observed electricity consumption* in MKWH	For untransformed series by quadratic growth model	For log-transformed series by quadratic growth model
ESTIMATION PERIOD			
1976-77	932	901.457	891.494
1977-78	1012	1025.087	1028.635
1978-79	1205	1172.566	1182.719
1979-80	1381	1343.895	1355.128
1980-81	1406	1539.073	1547.236
1981-82	1594	1758.101	1760.397
1982-83	2028	2000.977	1995.917
1983-84	2399	2267.703	2255.028
1984-85	2704	2558.278	2538.864
1985-86	2841	2872.703	2848.423
1986-87	3307	3210.977	3184.545
1987-88	3485	3573.100	3547.873
1988-89	3772	3959.073	3938.823
1989-90	4694	4368.895	4357.554
1990-91	4705	4802.566	4803.931
VALIDATION PERIOD			
1991-92	4870	5260.087	5277.503
1992-93	6021	5741.457	5777.475
FORECAST PERIOD			
1993-94		6246.676	6302.682
1994-95		6775.745	6851.577
1995-96		7328.663	7422.194
1996-97		7905.430	8012.275
1997-98		8506.046	8618.876
1998-99		9130.512	9239.102
1999-2000		9778.828	9869.258
2000-2001		10450.992	10505.455

* Source: Different issues of Bangladesh Statistical Yearbook, Bangladesh Bureau of Statistics, Government of Bangladesh, Dhaka and Annual Report 1993-94, Bangladesh Power Development Board, Dhaka.