Chapter 1

INTRODUCTION.

1.1 Prologue

"The Universe is made mostly of dark matter and dark energy, and we don't know what either of them is."

- Saul Perlmutter, Astrophysicist (Nobel prize laureate, 2011).

Dark Matter (DM) problem is at the core of modern Astrophysics. Since any updated standard model particle is unable to describe dark matter, it is also a central focus of elementary particle physics. The problem of dark matter is introduced when astrophysicists have found the mismatch between how objects in the sky ought to move according to some preconception and how they are rightly observed to move.

The universe consists of galaxies, diffuse radiation, dark matter and the vacuum energy. A galaxy is a gravitationally bound collections of stars, gas, dust, planets and most probably of dark matter. The visible matter in the universe is mostly contained in galaxies. A typical galaxy can contain $10^{11} - 10^{12}$ stars and a total mass of, nearly, $10^{12} M_{\odot}$ ($M_{\odot}$ is the solar mass). Roughly, there are 10^{11} galaxies in the observable part of the universe. If the distribution of the visible matter in galaxies over the largest scales is uniformed, then it's present density would be, very nearly to, $\rho_{\rm visible}(t_0) \sim 10^{-31}$ g/cm^3 (p-372 of Ref. [1])

Amount of gas and dust in galaxies may vary from a few percent of the total stellar mass (as in lenticular galaxies) to about ten percent for the spiral galaxies which are supposed to be most gas-rich objects. According to Binney and Tremaine(1987) "galaxies are to astronomy what atoms are to physics". The other component of the universe is the radiation that contains zero rest mass particles such as photons, neutrinos and gravitational waves. This radiation travels at the speed of light and is not clustered in the gravitationally bound clumps like the matter.

Clusters of galaxies are the largest and most massive stable gravitationally bound structures raised from the process of cosmic structure formation in the universe. They generally contain a few hundred to 1000 galaxies (5%), hot X-ray emitting gas at temperature of $10 - 100$ millions degrees (15%) and huge amount of DM (80%) (p-56 of Ref. [2]). Galaxy clusters have a total mass $(10^{14} - 10^{15}) M_{\odot}$ with volume 5-30 millions light years. The velocity for the individual galaxy in such a cluster is about 800-1000 km/s. Gravitational effects due to dark matter is responsible for the equilibrium of galaxies within a cluster (p-31 of Ref. [2]). Galaxies are clumped into clusters as stars does into galaxies. The universe has a clumped structure with relatively dense and empty areas.

According to Hubble's classification system, there are mainly four types of galaxies, namely, ellipticals, lenticulars, spirals and irregulars (p-2 of Ref. [3]). The existence of DM in elliptic galaxies can be found in Danziger (1997), Binney and Merrifield (1998) and Bertin (2000). Here, we will consider only the spiral galaxies (including our own, the Milky Way) which contains a disk containing spiral arms filled with stars, gas and dust.

1.2 Evidences for the existence of Dark Matter in the galactic halo

In 1932, Jan H. Oort (p-15 of Ref. [4]) carried out to a detailed study of the amount of matter in the disk of galaxy. He made a broad analysis of the average and random motions of stars around the galaxy and also their motion perpendicular to the plane of the galaxy to reckon the amount of matter in the disk of the Milky Way. Stars protect themselves from collapsing towards the centre of the galaxy with its average and random motion while galaxy protects itself from collapsing to a structure similar to a completely flat disk with the vertical motions of the stars. Gravitational force pulls all the stars down towards the plane of the galaxy and is balanced by the random vertical motions of the stars. According to Oort, the random velocity of the stars occupying the thick disk are twice that of the stars in the thin disk. The greater vertical motions of the stars in the thick disk certainly implies the average vertical motions of the stars in the disk of the galaxy was a lot higher than it really is. From that he concluded that there had to be much more matter – perhaps twice as much as was represented by the stars of the Milky Way to hold the disk together.

After the announcement of Oort about the existence of dark matter in the disk of the Milky Way, another astronomer Fritz Zwicky declared its existence not only in individual galaxy but also in clusters of galaxies (p-21 of Ref. [4]). Zwicky, in 1937, has given the thought that the rotation curve[1] law would be an unreliable probe of the gravitational force. Horace Babcock (p-35 of Ref. [4]), in 1939, found no sign of a Keplerian decline in the rotational velocity, as one would expect from the light distribution. He explained it as – either the outer regions of the galaxies absorb more star-lights or Newton's laws would have to modify to apply it at the larger scales but there was no inclusion of dark matter by him at all.

In 1963, Finzi (p 23-24 of Ref. [5]) noted that the mass in the Milky Way directly increases with the distance i.e. $M(r) \propto r$, beyond the position of the Sun. Finzi proposed about the modification of Newton's law beyond the scale of about 1 kpc (where the forces act like $1/r^{1.5}$ instead of $1/r^2$). Influenced by Zwicky, Kenneth Freeman (in 1970) intended that the rotational curve of some of the galaxies were in wrong shape as one can see it by considering that the component of the galaxies are

[1]The rotational curve of a galaxy is the rotational velocity of the galaxy about the center, given in km/s, plotted as a function of radius usually given in kpc.

stars, gas and nothing else (p-37 of Ref. [4]) i.e. the observed rotational curves were not as expected. He noticed that the Keplerian declination, beyond the optical radii of the galaxy, are not followed by rotational curves of NGC 300 and M33.

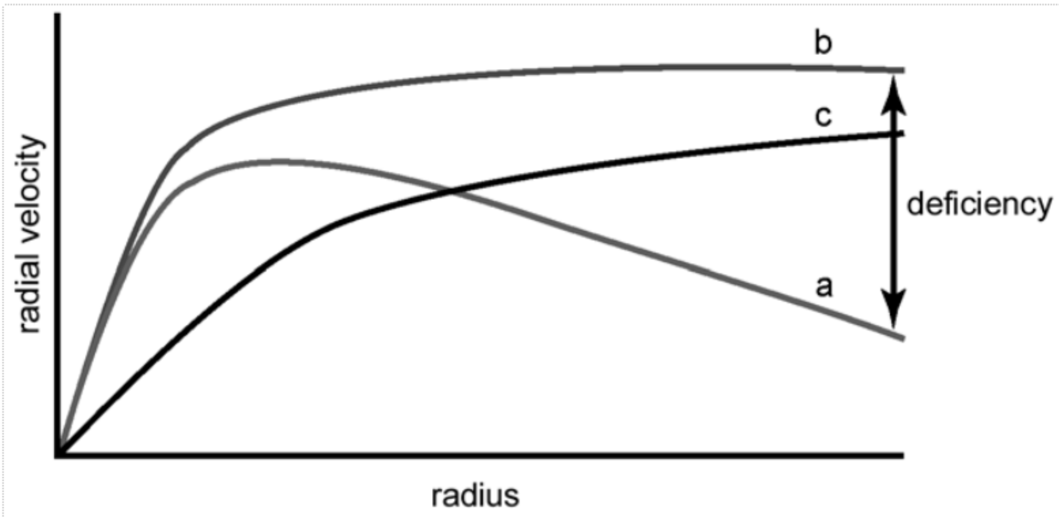

Figure 1.1. Plot of the dissension between the observed and expected rotational curve for the Milky Way. a) the expected rotation from stars and gas alone; b) the observed rotation of the Milky Way; c) additions required from the dark halo to produce the observed rotation curve [4].

We are interested to discuss the kind of evidences that astronomers have accumulated still date to ensure their presence in the halo (a hypothetical spherical region that envelops the galactic disk and extends well beyond the edge of the visible galaxy) or to characterize them.

1.2.1 Astrophysical evidence

At the present time, baryons can be found mostly in three forms – stars in galaxies, gas in clusters of galaxies, and gas in group of galaxies. In 1998, Peebles, Fukugita and Hogan [6] made a complete measure of the amount of the baryons present in the universe. In the present, the universe contains matters in the form – stars in galaxies ($\Omega = 0.004$), gas in clusters ($\Omega = 0.003$) and gas in groups ($\Omega = 0.014$) (p-87 of Ref. [4]). With that, we have a total baryonic mass $\Omega_b = 0.021 \pm 0.008$ (here all the baryonic masses are in the units of the critical density $\rho_c = \frac{3H_0^2}{8\pi G}$, where H_0 is the Hubble parameter). But, according to Wilkinson Microwave Anisotropy Probe (WMAP) launched in 2001, the accurate value of it is $\Omega_b = 0.044 \pm 0.04$ and the total matter density $\Omega_m = 0.2 - 0.3$ (p-87 of Ref. [4]).

The study of the Coma Clusters of galaxies at a redshift of $Z = 3$, by a group of Astronomers lead by Simon White (a former director of the Max Planck institute for Astrophysics, Germany) published their results in 1993. They wanted to heap up a list of the masses in this clusters as Zwicky did. They examined the stars and hot gases. With a great accuracy, they found that the total mass of stars is about $1.4 \times 10^{13} M_\odot$, while the total amount of the gas is about $13 \times 10^{13} M_\odot$ i.e. 10 times higher than that of the stars. On the other hand, the total gravitational mass found from the movements of the galaxies is about $160 \times 10^{13} M_\odot$ – ten times higher than the stars and gas (p-90 of Ref. [4]). The net amount of the gravitational mass in the form of baryons is about 10% of all the mass in the Coma Cluster. Several experiments on different types of clusters and super-clusters of galaxies agree with the results obtained by White. Thus, the only important point that comes out from the above analysis is that baryons are only a small fraction of the total mass of the universe. This discrepancy between Ω_b and Ω_m is the evidence for the existence of a large amount of non-baryonic matter in the universe.

1.2.2 Gravitational flat rotational curve

Freeman (in 1970) proved that the rotational curves of the galaxies are flat. Since then, the flatness of galactic rotation curves are a strong evidence for the existence of the dark matter in the galactic halo. According to Oort (latter by Morton Roberts, 1975), neutral Hydrogen gas can be taken as probe particle to measure the galactic rotational velocity as it extends well beyond the optical disks of the galaxy. Its rotational velocity plays a crucial role to determine the gravitational force law beyond the galaxy as the planetary motion in the solar system does for the force law beyond the Sun.

For the motion of a particle in a circular orbit, Newtonian gravitational force (F) balances the acceleration due to circular motion. So,

$$F = \frac{v_{tg}^2}{r}, \tag{1.1}$$

where v_{tg} is the rotational velocity of a particle in the circular motion at a radial distance 'r'. According to Newton's law, the gravitational force due to the mass M

to a unit mass at a distance r is

$$F = \frac{GM}{r^2}, \tag{1.2}$$

where G is the gravitational constant. Combining this two equations, we can write

$$v_{tg} = \sqrt{\frac{GM}{r}}, \tag{1.3}$$

$$\text{or, } M = \frac{v_{tg}^2 r}{G}. \tag{1.4}$$

So, theoretically, the rotational velocity should fall according to $\frac{1}{r^{1/2}}$ (Eq.(1.3)) in the outer part of the galaxy. In other words, the rotational curves should decline in a Keplerian way beyond the visible edge of the galaxy. Observations of extended rotational curve of M31 from 21-cm line made by Morton Roberts and Robert Whitehurst (1970), suggest that the rotational velocity does not obey Keplerian law ($\propto \frac{1}{r^{1/2}}$). In fact, it is not declined at all rather it is flat and completely opposes our expectation. As v_{tg} =constant, Eq.(1.4) gives $M(r) \propto r$ i.e. the mass enclosed within a radius 'r' is not constant but a function of 'r', the distance from the centre. It is increasing with the distance from the centre of the galaxy. Luminous mass distribution does not follow this behavior. So there are a large amount of non-luminous mass in the universe. This non-luminous mass is the so called 'dark matter'.

1.2.3 Galaxy clusters

A structure consists of hundreds to thousands of galaxies, bound together by gravity is called a galaxy cluster or cluster of galaxies. They are the largest, still now, gravitationally bound structures in the universe. Zwicky studied the dynamics of the galaxies, individually and in a group, in the Coma cluster (a prodigious system having size more than 200 times the size of a typical spiral galaxy) (p 27-32 of Ref. [4]). He considered the clusters as spherically symmetric structures of galaxies and heated gas, so the resultant gravitational force of all the galaxies attracts each one toward the cluster's center. Outermost galaxies compel to move following Newtonian mechanics under the influence of a mass equal to the total mass of the cluster. He determined their average velocity from the Doppler shifts of the spectral lines of these galaxies and calculated the total cluster mass require to produce the observed galactic motions. For the comparison, he also calculated the total mass based on the surface brightness of the galaxies. He found a flabbergast result – the mass require to produce the observed galactic motions is about 10 times larger than the mass based on the brightness of the galaxies. So, stars and heated gas can not be treated as the only ingredient of the intracluster medium, there are gigantic amount of unseen mass to hold the galaxies together.

1.2.4 Gravitational lensing

A model independent and totally different probe of the dark matter in the central region of clusters comes from Einstein's gravitational lensing effect. In recent years, use of gravitational lensing in astronomy have developed in a surprising manner.

The gravitational attraction of mass deflects light. We can measure the deflection angle ($\Delta\theta$) from Einstein light bending formula $\Delta\theta = \frac{4GM}{bc^2}$ where b is the impact parameter (the radial distance between the lens position and the photon in the lens plane), M is the mass of the lens and G is gravitational constant (Eq. (9.84) of Ref. [1]). The capability of a massive object to change the paths of light-rays provides an important device in the search of dark matter. Due to the bending, there can be multiple pathways for light to use in travelling from a source to an observer. An intervening mass can, therefore, produce multiple images of a distance source. Acting in this way, a concentration of mass is called a 'gravitational lens'. Gravitational lens can give informations about the source that is imaged, about the object that acting as lens and about the intervening large scale geometry of the universe when source, lens and observer are at cosmological distances from one another.

Similarly, the gravitational field of the cluster curves the space around it. Emitted light rays from the objects behind the cluster follow curved paths rather than straight paths on their way to us [7]. For strong lensing, there are multiple paths, passing through the different sides of the cluster, for the rays from the same object that arrive at our present location in the Universe. This results in multiple images of the same object which may be a galaxy or active galactic nucleus. More over, as the light from different sides of the same galaxy travel along the paths which are slightly different, the images of strongly lensed sources are bent into arcs. Weak lensing causes slightly elongated images even if they are not multiply imaged. For a lensing, deflection angle is given by (Eq. (5.3) of Ref. [8])

$$\alpha = (\frac{GM}{dc^2})^{1/2}, \tag{1.5}$$

where M is the total mass of the cluster and d is the impact parameter. Thus, if we know the deflection angle (α) and impact parameter, d (which can be measured from the redshift to the lensing cluster and source), we can measure M. It is found that the mass M obtained by lensing measurement is much larger than the observed baryonic mass M_b. As this method of determination of the mass is not affected by the factors that require assumptions and theories, it has higher accuracy. But, we have to treat the geometry (source-lens and observer-lens distances) carefully.

1.3 Broad outline of the aim and plan of the thesis

The present investigation deals mainly with the properties of the galactic halo such as structure formation of the galaxies, stability of the circular orbits, size of the galaxies as well as of the stars that it contains, inner structure of the galaxies such as its core and central density etc.

In the present work an emphasis has given on the solution of the field equations of different halo models such as Scalar Field Dark Matter(SFDM) model, Perfect Fluid model on the supposition that dark matter is a dominant component of the universe containing a major portion of the halo. The validity of the solutions not only in the Newtonian limit but also in the post-Newtonian regime is mentioned here because a post–Newtonian description of galactic halos is essential for the study of their interaction with physical effects that lacks a Newtonian equivalent.

Different features of the Cold Dark Matter (CDM) model, Navarro-Frenk-White (NFW) profile, Bose-Einstein Condensates model and the models that mentioned earlier, have studied here along with their limitations with respect to observations on different physical quantities.

Additional assumptions may bias the problem of DM. So, cautions on the determination of the physical quantities such as mass, gravitational potential etc. are also a matter of interest. Following Faber-Visser method of measuring the pseudo quantities by combining rotational curves with gravitational lensing, physical substantiability of the solutions of a space-time metrics with respect to the observed pseudo profiles has been studied.

For a better understanding of the galactic rotational curves and to avoid DM problem, modifications of Newtonian equations describing the halo are indeed important. So, the study of conformal gravity to measure the effects of local and global objects on galactic rotational velocity is a matter of utter interest. The study of Weyl conformal theory, implemented by Mannheim and O'Brien [9, 10], which can explain the flat rotational curve with out any need of dark matter, is very important to understand the role of the global and local potential on rotational velocity.

Here a study has been made to examine – the possibility of the termination of the galaxies, the role of the central density for the stability of the circular orbits in

Eddington inspired Born-Infeld (EiBI) model [11], possibility of the existence of any universal lower and upper limit on the central density.

The organization of the thesis is as follows : In Chapter 1, after giving a short review on the history of the dark matter problem, evidences for the existence of it in the galactic halo are discussed. Chapter 2, deals with the CDM and SFDM model. Solution of a spherical symmetric and static line element together with it's restrictions is discussed here, also. Following the way of Faber-Visser, how a combining measurement of rotation curves and gravitational lensing can describe the nature of the pseudo profiles, are mentioned in the same chapter. In Chapter 3, some features of the BEC model and NFW profile together with a comparison between them are studied. Some common features among BEC model, NFW profile and Pseudo Isothermal (PI) profile are mentioned there in. In Chapter 4, we have discussed solution of general static spherically symmetric space-time metric in the perfect fluid regime and it's physical viability are, also, studied there in. Cautions regarding the solution of a field equation is also mentioned there. More over, how measurements of deflection angle and rotational curve can be an important tool to discriminate different model, studied in the same chapter. Chapter 5, studied the explanation of the flat rotational curve with the help of the conformal gravity without any assumption of dark matter. It also contains the study of limit on the size of galaxies. In Chapter 6, a brief outline of the EiBI halo model and Mannheim and O'Brien model have been discussed. Stability of the circular orbits in the EiBI theory and testing the constraints on density with regard to different dark matter profile are mentioned in the same chapter. Finally, in Chapter 7, the results of this thesis are shortly summarized.

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