Chapter 5

CONFORMAL GRAVITY: A DIFFERENT APPROACH TO EXPLAIN FLAT ROTATIONAL CURVE.

5.1 Introduction

We have mentioned earlier that the observed velocity profile of the spiral galaxies, flat rotational velocity, mismatches with the Newtonian velocity which decreases with distance from the centre. This mismatch have forced us to incorporate a new idea, the "Dark Matter" i.e. the flat rotational velocity is an input here and the dark matter hypothesis comes out as a result. Thus, we have imported the concept of dark matter to explain the galactic rotational curve velocity. As there is no clear evidence about the non-luminous component of the Universe, some authors trying to emphasis on the field of different modified theory rather than on 'Dark Matter'. A few author have taken the risk to suggest that the dark matter may not exists in real and modified Newtonian law may have the answer to it at the galactic scale. Milgrom's Modified Newtonian Dynamics (MOND) [1] and Moffat's Metric Skew Tensor Gravity (MSTG) [2] are some candidates in the literature of the alternative theories of DM.

The existence of dark matter is based on nothing else rather than the validity (on all distance scales) of the standard Newton-Einstein gravitational theory expressed via Einstein equation

$$R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R^{\gamma}_{\gamma} = 8\pi G T_{\alpha\beta}, \tag{5.1}$$

for the gravitational field $g_{\alpha\beta}$. Generally, we take Einstein equation as an empirical one and would like to modify $T_{\alpha\beta}$ through the introduction of necessary gravitational sources so that the observational difficulties may remove. The only apparent way for a better understanding of the galactic rotational curve and to rid off from DM problem is that we have to modify or generalize the left-hand side of Eq.(5.1) not it's right. Conformal gravity is such a candidate. We will, now, discuss about this modified theory of gravity.

5.2 Local effects on rotational velocity in conformal gravity

Conformal gravity [3] is a covariant metric theory of gravity with equivalence principle structure of standard Einstein gravity. The geometry under this theory is a standard Riemannian geometry and all the general coordinates are invariant. The action in this theory is left invariant under the local metric transformation

$$g_{\mu\nu}(x) \to e^{2\alpha(x)} g_{\mu\nu}(x), \tag{5.2}$$

where $\alpha(x)$ is any arbitrary local phase. The unique Weyl action is defined by

$$\begin{aligned} S_{\rm Weyl} &= -\alpha_g \int (-g)^{1/2} C_{\lambda\mu\nu\kappa} C^{\lambda\mu\nu\kappa} d^4x \\ &= -2\alpha_g \int (-g)^{1/2} [R_{\mu\kappa} R^{\mu\kappa} - \frac{1}{3}(R^{\alpha}_{\alpha})^2] d^4x, \end{aligned} \tag{5.3}$$

where

$$\begin{aligned} C_{\lambda\mu\nu\kappa} &= R_{\lambda\mu\nu\kappa} - \frac{1}{2}(g_{\lambda\nu}R_{\mu\kappa} - g_{\lambda\kappa}R_{\mu\nu} - g_{\mu\nu}R_{\lambda\kappa} + g_{\mu\kappa}R_{\lambda\nu}) \\ &\quad + \frac{1}{6}R^{\alpha}_{\alpha}(g_{\lambda\nu}g_{\mu\kappa} - g_{\lambda\kappa}g_{\lambda\nu}), \end{aligned} \tag{5.4}$$

is the conformal Weyl tensor and α_g is the dimensionless gravitational coupling constant. Unlike the standard Einstein theory, this theory posses the control over the cosmological constant [4, 5].

The equation of motion under the action $S_{\rm Weyl}$ is given by [3]

$$4\alpha_g W^{\mu\nu} = 4\alpha_g[2C^{\mu\lambda\nu\kappa}_{\ ;\lambda;\kappa} - C^{\mu\lambda\nu\kappa}R_{\lambda\kappa}] = 4\alpha_g[W^{\mu\nu}_{(2)} - \frac{1}{3}W^{\mu\nu}_{(1)}] = T^{\mu\nu}, \tag{5.5}$$

where

$$W^{\mu\nu}_{(1)} = 2g^{\mu\nu}(R^{\alpha}_{\alpha})^{;\beta}_{;\beta} - 2(R^{\alpha}_{\alpha})^{;\mu;\nu} - 2R^{\alpha}_{\alpha}R^{\mu\nu} + \frac{1}{2}g^{\mu\nu}(R^{\alpha}_{\alpha})^2, \tag{5.6}$$

$$W^{\mu\nu}_{(2)} = \frac{1}{2}g^{\mu\nu}(R^{\alpha}_{\alpha})^{;\beta}_{;\beta} + R^{\mu\nu;\beta}_{;\beta} - R^{\mu\beta;\nu}_{;\beta} - R^{\nu\beta;\mu}_{;\beta} - 2R^{\mu\beta}R^{\nu}_{\beta} + \frac{1}{2}g^{\mu\nu}R_{\alpha\beta}R^{\alpha\beta}. \tag{5.7}$$

In the Schwarzschild solution, all the Ricci tensor vanishes and so also it's covariant derivatives. Again, when $R_{\mu\nu} = 0$ then both $W^{\mu\nu}_{(1)}$ and $W^{\mu\nu}_{(2)}$ vanish. Thus Schwarzschild vacuum solution is also a solution to the conformal gravity. But, we are interested to some non-Schwarzschild solutions of the conformal theory. To determine the nature of such a solution, Mannheim and Kazanas (1989) have considered a metric outside of a localized static, spherically symmetric source of radius r_0 embedded in a region with $T_{\mu\nu}(r > r_0) = 0$. They have considered a standard static spherically symmetric geometry with line element

$$ds^2 = -B(r)c^2dt^2 + A(r)dr^2 + r^2 d\Omega_2, \tag{5.8}$$

where $d\Omega_2 = d\theta^2 + \sin^2(\theta)d\phi^2$.

Using the conformal symmetry, one would able to transform this line element into one that would be conformally equivalent to a line element with $A(r) = 1/B(r)$ where the exterior metric coefficient $B(r > r_0)$ is given by [6]

$$B(r > r_0) = \omega - \frac{2\beta}{r} + \gamma r - \kappa r^2, \tag{5.9}$$

and $\omega = (1 - 6\beta\gamma)^{1/2}$. As the product $\beta\gamma$ being numerically too small to neglect, we can set $\omega = 1$. So, the Eq.(5.9) recovers the Schwarzschild solution in the region where $1 - \frac{2\beta}{r} \gg \gamma r - \kappa r^2$ while it departs only at large distances and not at small ones i.e. departing from it exactly in the region where the dark matter problem is first observed.

With the help of the line element (5.8), it is straightforward to show that [6]

$$\frac{3}{B}(W^0_0 - W^r_r) = B'''' + \frac{4B'''}{r} = \frac{1}{r}(rB)'''' = \nabla^4 B, \tag{5.10}$$

(the primes here denote derivatives with respect to r).

Let us define the convenient source function $f(r)$ as

$$f(r) = \frac{3}{4\alpha_g B(r)}(T^0_0 - T^r_r). \tag{5.11}$$

The equation of motion (5.10), now, takes the form [7]

$$\nabla^4 B(r) = f(r). \tag{5.12}$$

Solving this exact differential equation of motion, one can have [6]

$$\begin{aligned} B(r) \quad &= \quad -\frac{r}{2}\int_0^r dr' r'^2 f(r') - \frac{1}{6r}\int_0^r dr' r'^4 f(r') \\ &\quad -\frac{1}{2}\int_r^\infty dr' r'^3 f(r') - \frac{r^2}{6}\int_r^\infty dr' r' f(r') + \hat{B}(r), \end{aligned} \tag{5.13}$$

where $\hat{B}(r)$ satisfies $\nabla^4 \hat{B}(r) = 0$. In Eq.(5.13), we see that $f(r)$ governs the solution of Eq.(5.12). The first two integral terms of the metric coefficient (5.13) are the contribution of the local matter inside of a source to the gravitational force while, on the other hand, the last two integral terms are the same due to the global matter exterior to it. Consideration of the local contributions coming from the luminous material inside of the galaxy and global contributions due to material outside of it (viz. the rest of the universe), is essential to determine the motions within galaxies. Look that $\nabla^4(r^2)$ vanishes identically everywhere and both of $\nabla^4(1/r)$ and $\nabla^4(r)$

evaluate to delta functions and their derivatives. Matching of the interior and exterior metrics yield [7]

$$\gamma = -\frac{1}{2}\int_0^{r_0} dr' r'^2 f(r'), \tag{5.14}$$

$$2\beta = \frac{1}{6}\int_0^{r_0} dr' r'^4 f(r'). \tag{5.15}$$

A given local gravitational source, in conformal gravity, generates a gravitational potential per unit solar mass [8]

$$V^*(r) = -\frac{\beta^* c^2}{r} + \frac{\gamma^* c^2 r}{2}, \tag{5.16}$$

where β^* is given by

$$\beta^* = \frac{GM_\odot}{c^2} = 1.48 \times 10^5 cm, \tag{5.17}$$

and the best data fitting gives the numerical value of γ^*. The first term of the potential V^* is the Newtonian potential and the second one is the linear potential. V^* will reduce to a standard Newtonian potential for the small value of 'r' where the factor $\beta^* c^2/r$ dominates over $\gamma^* c^2 r/2$. But in the region where $\gamma^* c^2 r/2$ becomes competitive with $\beta^* c^2/r$, we would observe a departure from standard Newtonian gravity. Integrating V^* over the visible galactic mass distribution in a given galaxy, the visible local material would produce local gravitational potential, $V_{LOC}(r)$.

Suppose that the galaxies are to be a thin disk consisting of stars distributed in such a way that it has the surface brightness

$$\Sigma(R) = \Sigma_0 e^{-\frac{R}{R_0}}, \tag{5.18}$$

and luminosity

$$L = 2\pi\Sigma_0 R_0^2, \tag{5.19}$$

where R_0 is the scale length such that most of the surface brightness is contained in the $R \leq 4R_0$ region of the optical disk. The total number of solar mass units N^* in the galaxy is defined with the help of the observed galactic mass-to-light ratio via

$$(M/L)L = M_{\rm lum} = N^* M_\odot. \tag{5.20}$$

Then, on integrating $V^*(r)$ over this visible matter distribution, one can obtain the net local luminous contribution [3]

$$\begin{aligned} \frac{v_{LOC}^2(R)}{R} &= \frac{N^*\beta^* c^2 R}{2R_0^3}[I_0(\frac{R}{2R_0})K_0(\frac{R}{2R_0}) - I_1(\frac{R}{2R_0})K_1(\frac{R}{2R_0})] \\ &+\frac{N^*\gamma^* c^2 R}{2R_0} I_1(\frac{R}{2R_0})K_1(\frac{R}{2R_0}), \end{aligned} \tag{5.21}$$

for the centripetal acceleration of particles in circular orbits in the plane of the galactic disk. For the limit $R \gg R_0$, Eq.(5.21) approximates to

$$\begin{aligned} \frac{v_{LOC}^2(R)}{R} &= \frac{N^*\beta^* c^2}{R^2}(1 + \frac{9R_0^2}{2R^2}) + \frac{N^*\gamma^* c^2}{2}(1 - \frac{3R_0^2}{2R^2} - \frac{45R_0^4}{8R^4}) \\ &\to \frac{N^*\beta^* c^2}{R^2} + \frac{N^*\gamma^* c^2}{2}, \end{aligned} \tag{5.22}$$

on the supposition that the entire galaxy acting as if it were a point source located at the galactic center.

In the next section, we will discuss the effects of the rest of the material in the Universe on the rotational velocity of galaxies.

5.3 Global contributions to local motions

In standard gravity, the solution to the second-order Poisson equation

$$\nabla^2\phi(r) = g(r), \tag{5.23}$$

is

$$\phi(r) = -\frac{1}{r}\int_0^r dr^{'} r^{'2} g(r^{'}) - \int_r^\infty dr^{'} r^{'} g(r^{'}), \tag{5.24}$$

with it's derivative

$$\frac{d\phi(r)}{dr} = \frac{1}{r^2}\int_0^r dr^{'} r^{'2} g(r^{'}), \tag{5.25}$$

g(r) is the general static spherically symmetric source.

The importance of Eq.(5.25) is that to determine the force at any radial point r one have to consider only the material in the local $0 < r' < r$ region though $g(r)$ could continue globally all the way to infinity. Since to explain the gravitational effect in some local region one only needs to consider the material in that region that's why Newtonian gravity is called local. Thus, to explain the behavior of galactic rotational curve in Newtonian gravity with the help of dark matter, one must locate the dark matter where the problem exactly exists and not elsewhere. Since the discrepancy problem of rotational curve in galaxies occurs primarily in the region beyond the optical disk, thus one must locate galactic dark matter precisely in the region of galaxies where there is little or no visible matter. This local character to Newtonian gravity is not a generic property of all gravitational potential.

Let us consider, for instance, the fourth-order Poisson equation

$$\nabla^4\phi(r) = h(r) = f(r)c^2/2. \tag{5.26}$$

The general solution of Eq.(5.26) is [6]

$$\begin{aligned}\phi(r) &= -\frac{r}{2}\int_0^r dr' r'^2 h(r') - \frac{1}{6r}\int_0^r dr' r'^4 h(r') \\ &\quad -\frac{1}{2}\int_r^\infty dr' r'^3 h(r') - \frac{r^2}{6}\int_r^\infty dr' r' h(r'),\end{aligned} \tag{5.27}$$

with its derivative

$$\frac{d\phi(r)}{dr} = -\frac{1}{2}\int_0^r dr' r'^2 h(r') + \frac{1}{6r^2}\int_0^r dr' r'^4 h(r') - \frac{r}{3}\int_r^\infty dr' r' h(r'). \tag{5.28}$$

The third integral in Eq.(5.28) is a potential global contribution to local motions. The material in the region $0 < r' < r$ and that beyond the radial point of interest are responsible to the force given by Eq.(5.28). Thus, a test particle in the orbit of a galaxy being able to sample not only the local field due to the matter in the galaxy but also the global field due to the material in the rest of the Universe. As the motions of particles, in conformal gravity, inside of galaxies are affected by the material outside of them, we have to take into account of it. The effects of the homogeneous background cosmology and the inhomogeneities in it that arise due to fluctuations around that background are the two effects that the material exterior to galaxies might have on it.

In the conformal theory, one have to take the account of two linear potential terms, namely, a local $N^*\gamma^*$ – dependent term associated with the matter within a galaxy and a global cosmological one $\gamma_0 c^2 r/2$ associated with the cosmological background. Therefore, in the weak gravity limit, one have to add this γ_0 dependent potential term with the local one to get total circular velocity, v_{TOT}, given by [9]

$$\begin{aligned} v^2_{TOT}(R) &= v^2_{LOC} + \frac{\gamma_0 c^2 R}{2} \\ &\rightarrow \frac{N^*\beta^* c^2}{R} + \frac{N^*\gamma^* c^2 R}{2} + \frac{\gamma_0 c^2 R}{2}. \end{aligned} \tag{5.29}$$

The third and fourth integrals in the Eq.(5.13) are the contribution due to inhomogeneities in the cosmological background. To apply Eq.(5.13) on galactic distance scales which are much smaller than any typical cluster of galaxies scale r_{clus} associated with cosmological inhomogeneities, to evaluate $v^2_{TOT}(R \ll r_{clus})$ one can replace the lower limits in each of those two integrals by r_{clus}. Thus with the quadratic term in Eq.(5.27), up to peculiar velocity effects for weak gravity and on scales $r < r_{clus}$, we can modify Eq.(5.29) to [6]

$$\begin{aligned} v^2_{TOT}(R) &= v^2_{LOC} + \frac{\gamma_0 c^2 R}{2} - \kappa c^2 R^2 \\ &\rightarrow \frac{N^*\beta^* c^2}{R} + \frac{N^*\gamma^* c^2 R}{2} + \frac{\gamma_0 c^2 R}{2} - \kappa c^2 R^2, \end{aligned} \tag{5.30}$$

where

$$\kappa = \frac{1}{3c^2}\int_{r_{clus}}^{\infty} dr' r' h(r') = \frac{1}{6}\int_{r_{clus}}^{\infty} dr' r' f(r'). \tag{5.31}$$

Thus, Eq.(5.30) gives the expectation of conformal gravity for galactic rotational velocities and one can apply it to galactic rotational curve data. v^2_{TOT} contains only one free parameter per galaxy, the total number of stars N^*. With Eq.(5.29), Mannheim and O'Brien are able to fit the galactic rotational curve data of a sample of

111 galaxies. The best fitted values for the two universal linear potential parameters being found to be [9]

$$\gamma^* = 5.42 \times 10^{-41}\ cm^{-1},$$
$$\gamma_0 = 3.06 \times 10^{-30}\ cm^{-1}. \quad (5.32)$$

The linear potential of the Sun for the value obtained for γ^* is so small that it is difficult to find any modification of it to standard solar system phenomenology. With that $N^*\gamma^*$ and γ_0, is indeed of cosmological magnitude, one has to go all the way to galactic systems before their effects can become as big as the Newtonian contribution. Despite the small effect of the term $\kappa c^2 R^2$ for the fitting of the galactic rotational curves to some galaxies [9] via Eq.(5.29), the role of that term for the fitting of the galactic rotational curve of highly extended galaxies is not a matter of utter contempt. Through the fitting of 21 highly extended galaxies as reported in [9], the extracted value for κ is given by

$$\kappa = 9.54 \times 10^{-54}\ cm^{-2} \approx (100\ \text{Mpc})^{-2}. \quad (5.33)$$

One of the important features of the Eqs.(5.29) and (5.30) is that there are situations in which departure from the luminous Newtonian prediction can be very proclaimed – one of the situation is when N^* is small (then the net Newtonian contribution will unable to compete with the fixed magnitude γ_0 and κ term) and another one is when the quantity $N^*/R_0^2 \sim \Sigma_0$ is small. As the quadratic term is surpassed by the linear one until the largest distances so in both small N^* and/or small Σ_0, galaxies rotational curves expect to start rising immediately just as is systematically seen in the data sample. In case of HSB galaxies where the luminous Newtonian contribution is not suppressed and the falling Newtonian contribution can be a competitor for the rising linear term to give a region of approximate flatness before any rise could set in. Finally for all galaxies, the quadratic term will eventually take over to then arrest the rising linear potential terms and cause all rotation velocities to fall. As v^2 is always positive, any bound circular orbit is not possible beyond radius R where

$$R \simeq (N^*\gamma^* + \gamma_0)/2\kappa$$

($\sim$ 100 kpc for $N^* = \gamma_0/\gamma^* \simeq 5.65 \times 10^{10}$) [10]. With that natural way of terminating, the allowable size of galaxies being determined by an interplay between galaxies and the global structure of the Universe. Eq.(5.30) is not simply a phenomenological or empirical formula that is extracted solely from consideration of the systematics of galactic rotational curves, rather it is explicitly derived from first principles in a fundamental, uniquely prescribed metric-based theory of gravity, the conformal gravity.

5.4 Contribution of gases and bulges

The contribution of the gas distribution with a total atomic mass $N_{gas}M_{\odot}$ to rotational velocities will follow [10]

$$\begin{aligned} v_{LOC}^2(gas) &= \frac{N_{gas}\beta^* c^2 R^2}{2R_0^3(gas)}[I_0(\frac{R}{2R_0(gas)})K_0(\frac{R}{2R_0(gas)}) \\ &-I_1(\frac{R}{2R_0(gas)})K_1(\frac{R}{2R_0(gas)})] \\ &+\frac{N_{gas}\gamma^* c^2 R^2}{2R_0(gas)}I_1(\frac{R}{2R_0(gas)})K_1(\frac{R}{2R_0(gas)}), \end{aligned} \tag{5.34}$$

similar to Eq.(5.21).

For the galaxies having a spherical bulge with a spherical mass density $\sigma(r)$ per unit solar mass and a total mass $N_{bu\lg e}M_{\odot}$, the net gravitational potential due to a spherical bulge can, then, be obtained by integrating Eq.(5.16) over the $\sigma(r)$ distribution and is given by[10]

$$\begin{aligned} v_{LOC}^2(bulge) &= \frac{4\pi\beta^* c^2}{r}\int_0^r dr' r'^2\sigma(r') + \frac{2\pi\gamma^* c^2}{3r}\int_0^r dr'(3r^2r'^2 - r'^4)\sigma(r') \\ &+\frac{4\pi\gamma^* c^2 r^2}{3}\int_r^\infty dr' r'\sigma(r'), \end{aligned} \tag{5.35}$$

with

$$N_{bu\lg e} = 4\pi\int dr' r'^2\sigma(r'). \tag{5.36}$$

So, considering the gas and bulge contributions along with the disk contributions, Eq.(5.30) modifies to[10]

$$\begin{aligned} v_{TOT}^2(R) &= v_{LOC}^2(disk) + v_{LOC}^2(gas) + v_{LOC}^2(bu\lg e) + \frac{\gamma_0 c^2 R}{2} - \kappa c^2 R^2 \\ &\rightarrow \frac{N_{TOT}\beta^* c^2}{R} + \frac{N_{TOT}\gamma^* c^2 R}{2} + \frac{\gamma_0 c^2 R}{2} - \kappa c^2 R^2, \end{aligned} \tag{5.37}$$

where $N_{TOT} = N^{*} + N_{gas} + N_{bu\lg e}$.

The only one parameter in Eq.(5.37) that can vary from one galaxy to the next is the total stellar mass $(N^{*} + N_{bu\lg e})M_{\odot}$ in each galaxy. Newtonian contribution is the most influential candidate (as we can see in the Eq.(5.37)) in the inner regions of galaxies and thereby allow us to determine $(N^{*} + N_{bu\lg e})M_{\odot}$ from inner region rotational velocity data alone. Surprisingly, the ratios of $(N^{*} + N_{bu\lg e})M_{\odot}$ to the measured luminosities are very close to the mass to light ratio i.e. are in same order of the mass to light ratio $M_{\odot}/L_{\odot}$ measured in the local solar system neighborhood. The use of Eq.(5.37) to fit rotational curves in the outer regions is effectively parameter free as $(N^{*} + N_{bu\lg e})M_{\odot}$ being determined by inner region data alone. Thus, the predictions in the outer region by the conformal gravity theory are highly constrained and the theory is fully capable of accounting for the rotational curve systematics that are observed right across the updated galaxy sample. Eq.(5.37) is thus very sensitive to distances of the galaxies as the parameters γ^{*}, γ_0 and κ are absolute quantities and any variation in R would affect the spontaneous contributions of the linear and quadratic terms in it.

5.5 Limit on the size of galaxies

We are interested to find a possible limit on the size of galaxies in terms of γ^*, γ_0 and κ. To do this, let us consider a point particle Lagrangian[10]

$$L = \frac{\dot{r}^2}{2} + \frac{r^2\dot{\phi}^2}{2} - V(r), \tag{5.38}$$

for the motion in a plane having central potential $V(r)$. For a circular orbit

$$\begin{aligned} \ddot{r} &= 0 \\ &\Rightarrow r\dot{\phi}^2 - V^{'}(r) = 0 \\ &\Rightarrow V^{'}(r) = \frac{J^2}{r^3}, \end{aligned} \tag{5.39}$$

where

$$r^2\dot{\phi} = J.$$

On defining

$$V_{eff}(r) = V(r) + \frac{J^2}{2r^2},$$

one can show that if $r\dot{\phi}$ is real then the circular orbits will satisfy $V^{'}(r) \geq 0$ and for the stability of the orbit one must have [10]

$$V^{''}_{eff}(r) = V^{''}(r) + 3V^{'}(r)/r \geq 0. \tag{5.40}$$

Now considering $V = \frac{\gamma_0 c^2 r}{2} - \frac{\kappa c^2 r^2}{2}$, $V^{'}(r)$ vanishes at $r = \gamma_0/2\kappa$ while $V^{''}_{eff}(r)$ vanishes at $r = 3\gamma_0/8\kappa$. Thus, the orbit with $r \leq 3\gamma_0/8\kappa$ would be stable [10] but not for $3\gamma_0/8\kappa < r \leq \gamma_0/2\kappa$.

The potential in conformal theory is an effect of the interplay between each local galaxy and the global cosmology, that's why this problem in cosmology is actually a many body in nature. On the other hand, the above stability analysis would be applicable if the problems were a one-body problem [10]. Perturbations in the cosmological background are the inhomogeneities and are responsible for the existence of galaxies. The fate of the galaxies generated by this way will depend on the cosmological perturbations i.e. they will be stable as long as the cosmological perturbations themselves are stable. It could thus be very didactic to investigate both theoretically and observationally where (and of course whether) galaxies might actually terminate. As the Eq.(5.37) (also Eq.(5.30)) account for the data, one would need to be able to derive it in the theory of dark matter with out any approximation if the dark matter theory to be the correct explanation of the missing mass problem. Despite its success in fitting of galactic rotational curves with only one free parameter per galaxy, namely galactic mass to light ratio for each individual galaxy and given in addition its theoretical underpinnings, the future will test the reliability of conformal gravity as a candidate for alternate theory of gravity.

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