Chapter 8

SUMMARY OF THE WORK

The thesis presents a number of new and useful results, which we chapterwise summarize below.

In the introductory **Chapter 1**, we underlined our broad objective, viz., to study light motion (specifically, light deflection) in the galactic halo including in the wormhole spacetime and analyze in detail the consequences. We consider wormholes because it has been recently shown that galactic halo can support wormholes. Therefore, to understand light motion in the galactic halo, it is necessary to understand the peculiarities of light motion in a wormhole spacetime. In order for a wormhole to have survived till today, it is then necessary to study its stability against perturbations, a task that is also included in the present work.

The the contents of **Chapter 2** are as follows: We used the relatively new Rindler-Ishak method to calculate the required deflection. The method is a combination of the geometric invariant angle ψ and the metric itself. Originally, the method was applied to the asymptotically non-flat Schwarzschild-de Sitter spacetime that threw up a dependence of the light deflection on the cosmological constant Λ contrary to the prevailing belief. Naturally, we felt it necessary to re-check if the method truly works in more complicated spacetimes. To that end, we investigated the bending of light in the well known Janis-Newmann-Winnicour (JNW) spacetime, which could be interpreted both as a naked singularity or as a non-traversable wormhole inherited from the Brans Class I wormhole. The two way bending angle α with respect to the impact parameter b is obtained as

$$\alpha(b) = \frac{4m\zeta}{b} + \frac{(16\pi\zeta^2 - 32\zeta^2 + 48\zeta - \pi - 16)m^2}{4b^2} + O\left(\frac{m}{b}\right)^3,$$

which we have confirmed by using two other text book methods. Another interesting result is that there is a *repulsive* term $-(16+\pi)\frac{M^2}{4b^2\zeta^2}$, heretofore seemingly unnoticed in the literature, in the second order deflection. Its measurement would allow us to measure the scalar field charge ζ. The above expression leads to Schwarzschild bending upto $O(m^2)$ at $\zeta = 1$. In addition, we checked the stability of the circular orbits for JNW spacetime. An error existing in the literature is also corrected. Our central argument is that the Rindler-Ishak method is an excellent method that yields correct deflection in any spherically symmetric metric, regardless of whether the metric is asymptotically flat or non-flat.

In **Chapter 3**, we apply the Rindler-Ishak method to calculate light deflection in the Mannheim-Kazanas-de Sitter (MKdS) solution of Weyl conformal gravity because the solution has remarkable success in explaning the flat rotation curves in the galactic halo *without* dark matter, as explicitly shown by Mannheim and O'Brien. The MKdS solution has a Schwarzschild-like potential $V_{\beta^*} = N^*\beta^*c^2/R$, two linear potential terms, viz., a local $V_{\gamma^*} = N^*\gamma^*c^2R/2$ associated with the matter distribution within a galaxy and a global $V_{\gamma_0} = \gamma_0 c^2 R/2$ associated with the cosmological background, while the universal de Sitter-like quadratic potential term $V_k = -kc^2R^2$ is induced by inhomogeneities in the cosmic background. With $\gamma \equiv N^*\gamma^* + \gamma_0$ and $k \equiv \Lambda/3$, the two way bending angle in terms of the impact parameter b has been calculated to be

$$2\psi = \frac{4M}{b} + \frac{2Mb\Lambda}{3} + \frac{15M^2\gamma}{b} + \gamma b,$$

where M is the luminous galactic mass. We are considering light rays grazing the Einstein sphere with radius $b = \theta_E D_l$, where θ_E is the Einstein angle. This also means that $b >> M$. This seemingly innocuous condition is rather crucial in one respect: When it is properly implemented in the Rindler-Ishak method, one gets two positive contributions $+\frac{15M^2\gamma}{b}$ and $+\gamma b$ instead of respective negative signs, which follows if the condition is not respected. By the same token, we have also derived the exact local coupling term $\frac{2Mb\Lambda}{3}$ derived earlier by Sereno. The last three terms are new results that represent increase of Schwarzschild bending $\frac{4M}{b}$, which is physically expected since the galactic halo has attractive gravity. These results indicate that conformal gravity can potentially test well against astrophysical observations.

In **Chapter 4**, we apply the above deflection to the mass decomposition (luminous+dark) within the Einstein sphere having the luminous mass at the center and compare it with the observed SDSS lens data. To do that, we shall use an input that *observationally* the deflection angle must be unique, no matter whatever be the theory. That is, for a theory to be valid, it must numerically predict the same deflection. So, we use the input the numerical equality of the Einstein and Weyl angles: $\theta_{\mathrm{E}} = \theta_{\mathrm{W}}$. Using this,

$$N^{*}\gamma^{*} + \gamma_0 + \frac{2N^{*}\beta^{*}\Lambda}{3} = \frac{D_{\mathrm{s}}}{D_{\mathrm{ls}}D_{\mathrm{l}}}\left[1 - \frac{N^{*}\beta^{*}}{M_{\mathrm{tot}}^{\mathrm{lens}}}\left\{1 + \frac{15N^{*}\beta^{*}(N^{*}\gamma^{*}+\gamma_0)}{4}\right\}\right].$$

In the above equation, the galaxy *independent* universal parameters (γ_0, γ^{*}, κ), are known from the Mannheim-O'Brien rotation curve fitting, while the distances D_{s}, D_{ls}, D_{l} and the total mass $M_{\mathrm{tot}}^{\mathrm{lens}}$ are provided by the observed SDSS lens data specific to individual galactic lensing measurements. Putting these values in the master equation, we first find from above the numerical value of N^{*} specific to each sample, which stands for the number of solar units in the luminous mass of the galaxy. Typically, $N^{*} \sim 10^{10} - 10^{11}$. The exact value enables us to find the value of the luminous component M_{*}, which we now call M_{*}^{Weyl} to distinguish it from $M_{*}^{\mathrm{Salpeter}}$. We use the latter notation because it turns out that our mass decomposition is more consistent the one based on Salpeter Initial Mass Function than other available fittings.

We then compare the following ratios within the Einstein radius: $f_{*}^{\mathrm{Salpeter}} = (M_{*}^{\mathrm{Salpeter}}/M_{\mathrm{tot}}^{\mathrm{lens}})|_{\leq R_{\mathrm{E}}}$, $f_{*}^{\mathrm{Weyl}} = (M_{*}^{\mathrm{Weyl}}/M_{\mathrm{tot}}^{\mathrm{lens}})|_{\leq R_{\mathrm{E}}}$. The mean density $\langle\rho\rangle_{\mathrm{av}}^{\mathrm{Weyl}} \left(\equiv \frac{3M_{\mathrm{dm}}^{\mathrm{Weyl}}}{4\pi b^3}\right)$ is obtained by averaging the dark matter mass $M_{\mathrm{dm}}^{\mathrm{Weyl}} = M_{\mathrm{tot}}^{\mathrm{lens}}(\leq R_{\mathrm{E}}) - M_{*}^{\mathrm{Weyl}}(\leq R_{\mathrm{E}})$ over the Einstein sphere of radius $b = R_{\mathrm{E}} = D_{\mathrm{l}}\theta_{\mathrm{E}}$ centered at the galactic origin. For illustrative purposes, we tabulate only 15 galaxies out of the 57 lens galaxies available from the SDSS survey. Remarkably, we find that the mass ratios $f_{*}^{\mathrm{Salpeter}}$ and f_{*}^{Weyl} are very much comparable, thanks to the input we used. We also tabulated various deflection contributions: t_{Sch}, t_{γ} and t_{Sereno}. It turns out that t_{γ} is just an order of magnitude less than t_{Sch}, hence amenable to near future observations, while t_{Sereno} is too small.

Next three chapters are devoted to wormholes.

Chapter 5 is devoted to deflection and lensing observables in the Einstein Minimally coupled Scalar field (EMS) theory with a negative sign stress tensor, meaning exotic matter. The only physically important regular solution is the EMS Class II solution (which is henceforth referred to as *the* Ellis wormhole) with a free parameter η. The interest of the work in this chapter lies in exploring how the deflection and lensing observables differ in the environment of exotic matter from those in the Schwarzschild black hole. We adopted the PPN method developed by Keeton and Petters, where the lensing observables are expressed in series with respect to the small parameter $\varepsilon = \theta_{\mathrm{E}}/4D = \left(\frac{\theta_{\mathrm{E}}}{4}\right)(D_s/d_{ls})$. We obtained the deflection angle, position, magnification, magnification weighted centroid, total magnifiation and time delay of the relativistic images. Deflection angle with respect to the impact parameter b reads,

$$\alpha(b) = 4\left(\frac{M}{b}\right) + \frac{\pi}{4}\left(\frac{1+16\eta^2}{\eta^2}\right)\left(\frac{M}{b}\right)^2 + \frac{16}{3}\left(9+\frac{1}{\eta^2}\right)\left(\frac{M}{b}\right)^3 + ...$$

The peculiarity we observe is that the Schwarzschild deflection cannot be obtained for no *real* values of η! Only for $\eta^2 = -1$, i.e., for an imaginary $\eta = \pm i$ can one obtain the Schwarzschild bending upto *all* orders because at this imaginary value the Ellis wormhole exactly reduces to Schwarzschild black hole (see chapter 7). Finally, lensing observables are computed and represented graphically with respect to the source position β, which show considerable deviation from those in the Schwarzschild black hole. These deviations could help distinguish wormholes from black holes by means of lensing observables.

Chapter 6 investigates a generalized wormhole in the recently revived Eddington-inspired-Born-Infeld (EiBI) gravity, a theory that has all the successes of the Einstein gravity but has no Big Bang singularity. Currently, a lot of work is going on in this theory. One of its novel prediction is that it produces a wormhole solution with a parameter $\kappa \neq 0$ that generalizes the $\kappa = 0$ Ellis zero mass wormhole. (By zero mass we mean that the Keplerian masses of the individual mouths, $+r_0/2$ and $-r_0/2$, add to zero. Clearly, individual mouths show light deflection, gravitational lensing etc as investigated by Abe). The value of κ away from zero signifies departure from general relativistic effects and has been shown in the literature to depend on the chosen astrophysical scenarios. In the same spirit, we have found the correction terms due to κ contributing to various observables in the massless EB wormhole. One observable is

$$\epsilon = 2\delta \simeq \frac{\pi r_0^2}{4R_0^2} + \frac{3\pi\kappa r_0^2}{8R_0^4}, \tag{8.1}$$

where r_0 is the closest approach distance, and the second term explicitly reveals the effect of κ. The other one is the excess tidal force in the orthonormal geodesic frame (denoted by hat) near the throat is

$$\left|\mathbf{R}^{(\mathrm{ex})}_{\widehat{0}\widehat{2}\widehat{0}\widehat{2}}\right| = \left(\frac{1}{2\kappa + r_0^2}\right)\left(\frac{\mathbf{v}^2}{1-\mathbf{v}^2}\right). \tag{8.2}$$

Now suppose that $\kappa \to 0$ (general relativity) and of course $\mathbf{v} \neq 0$. Then, as $r_0 \to 0$, the force becomes *arbitrarily large,* $\left|\mathbf{R}^{(\mathrm{ex})}_{\widehat{0}\widehat{2}\widehat{0}\widehat{2}}\right| \to \infty$. In complete contradistinction, depending on the values of non-zero κ, the tidal forces may become *arbitrarily small*, $\left|\mathbf{R}^{(\mathrm{ex})}_{\widehat{0}\widehat{2}\widehat{0}\widehat{2}}\right| \to 0$, even when $r_0 \to 0$. This is the novelty of the generalized wormhole brought about by the presence of the parameter κ as exposed here. Another result is that the massless character is preserved also in the generalized case, where $\kappa \neq 0$. Energy condition violations are discussed in detail.

In **Chapter 7**, we have investigated the stability of the Ellis wormhole from the view point of local and asymtotic observers. To do so, we applied Tangherlini's non-deterministic, pre-quantal statistical simulation about photon motion in the real optical medium to an effective medium reformulation of motions obtained via Hamilton's optical-mechanical analogy in a gravity field. This application is possible because all the foundational equations of Tangherlini are shown to be already imbedded in the formulation of general relativity. The result is Tangherlini's reflection (R) and the transmission (T) probabilities for the ingoing light in the massive Ellis wormhole spacetime for both asymtotic and local observers. Now in general relativity, we know that observations (here of stability) depend on the location of the observers. Our analysis reveals that the probabilities may conspire in such a way that there is always the possibility that, while near-throat local observers see instability (low probability of reflection), the asymptotic observers see stability (high probability of reflection), thus leading to *ghost wormholes.* This possibility is one of our novel predictions in this thesis.

We studied the probabilities in the case of phantom wormholes ($p/\rho = \omega < -1$) recently obtained by Lobo, Parsaei and Riazi. Their existence could be a real possibility, at least as real as the phantom energy itself. The question is: Did they survive the bombardment by photons and particles? Our analysis indicates that extreme phantom wormholes ($a \to -1$) should have survived because $R \to 1$ and $T \to 0$, as happens in the case of Schwarzschild black hole.

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