

Chapter 4

APPLICATION OF LIGHT DEFLECTION TO MATTER DECOMPOSITION IN THE HALO

4.1 Introduction

The purpose of this chapter is to investigate how the universal rotation curve fitting parameters γ_0 , γ^* and κ in the Weyl theory can also account for the matter decomposition in the SDSS galaxies. By using the SDSS multicolor photometry and lens modeling, Grillo *et al* [1] studied the luminous and dark matter composition of the early-type lens galaxies in the sample. Early-type lens galaxies are generally dominated by old stellar populations. In addition to their luminous stellar component, significant evidence of a dark matter component (in the form of a diffuse halo) exists in those galaxies, especially in the outer regions of the most massive systems [2-5]. Although baryons are estimated to represent only a small fraction of the total mass in these galaxies, it is now commonly agreed that baryons play a crucial role in the formation and evolution of dark matter haloes. One possible process for the origin of dark matter could be the accretion of dissipational gas in the inner regions (~ 10 kpc)

of massive early-type galaxies leading to the dark-matter density profile that appears steeper than predicted by dark-matter-only cosmological simulations [6-8].

The results in Ref.[1] indicate that the total (luminous+dark) mass of the lenses is roughly linearly proportional to the luminous mass, at a confidence level of more than 99%. In addition, by assuming that the lens galaxies are homologous systems, we study their distribution of dark matter and estimate a value on the order of 30% for the dark over total projected mass fraction contained within the average Einstein ring of radius of approximately 4 kpc.

To obtain the matter decomposition within our framework, we need to recall that the MKdS solution with rotation curve fitting parameters γ_0 , γ^* and κ does *not* need dark matter at all. Using the luminous mass-to-light ratio as the only varying parameter specific to galaxies and three universal constant parameters γ_0 , γ^* and κ , the theory has been able to fit the rotation curves without the need for any dark matter whatsoever. While this success is commendable [9], a new challenge comes from the largest collection of strong gravitational lensing galaxies measured by the Sloan Digital Sky Survey (SDSS). The photometric and spectroscopic data available from the survey have been analyzed in the literature determining the luminous and dark matter compositions in the selected early-type sample of 57 lens galaxies. Therefore, we wish to investigate how far the three universal fitting parameters account for the matter decomposition of the SDSS galaxies. In the literature [1], matter decomposition is done by using different operating tools. We outline below some of the diagnostic tools used in the literature to bring forth the completely different analytical approach we are adopting, viz., matter decomposition by applying the light deflection by Weyl gravity derived in Eq.(3.46) in the previous chapter.

The organization of this chapter is as follows: In Sec.4.2, we briefly review the diagnostic tools used in the literature to highlight the very different approach we are adopting here to reach the same goal, viz., matter decomposition. In Sec.4.3, we introduce a new and crucial input needed for the decomposition. Then, in Sec.4.4, we develop our master equation for determining the luminous mass. Sec.4.5 spells out the algorithm and confronts the SDSS lens data. Then there is an Appendix in Sec.4.6, followed by conclusions in Sec.4.7.

4.2 Diagnostic tools used in the literature

Astronomers use a galaxy morphological classification dividing galaxies into groups based on their visual appearance. Despite the morphological differences, lenticular and elliptical galaxies can be considered early type lens galaxies, which also include elliptical and "lenticular" galaxies. They are round or elliptical in outline. Early-type galaxies are sometimes the hosts of quasars and powerful radio sources. There are also other schemes, the most famous being the Hubble sequence, devised by Edwin Hubble [10] and later expanded by Gérard de Vaucouleurs [11]. The Hubble sequence often known colloquially as the "Hubble tuning-fork" because of the shape in which it is traditionally represented.

Grillo *et al* [1] considered type-cD galaxy that has a large halo of stars and lenticular galaxies. The designation "cD" is frequently used to mean central Dominant galaxy. Such galaxies are similar to lenticular galaxies or elliptical galaxies, but many times larger, some having envelopes that exceed one million light years in radius. Lenticular galaxies also consist of a bright central bulge surrounded by an extended, disk-like structure but, unlike spiral galaxies, the disks of lenticular galaxies have no visible spiral structure and are not actively forming stars in any significant quantity.

We shall now point out two particular diagnostic tools usually used by astronomers. One is the spectral energy distribution (SED), which is a plot of brightness or flux density versus frequency or wavelength of light. It is used in many branches of astronomy to characterize astronomical sources. For example, in radio astronomy, an SED with a negative spectral index around -0.7 would indicate a synchrotron radiation source. In infrared astronomy, SEDs can be used to classify young stellar objects. The spectral index of a source is a measure of the dependence of radiative flux density on frequency. Given frequency ν and radiative flux S , the spectral index α is given implicitly by $S \propto \nu^\alpha$.

Another tool is the initial mass function (IMF), which is an empirical function that describes the distribution of initial masses for a population of stars. The IMF is often given as a probability distribution function (PDF) for the mass at which a star enters the main sequence (begins hydrogen fusion). The distribution function can then be used to construct the mass distribution (the histogram of stellar masses) of a population of stars. The properties and evolution of a star are closely related to its mass, so the IMF is an important diagnostic tool for astronomers studying large

quantities of stars. The IMF is relatively invariant from one group of stars to another, though some observations suggest that the IMF is different in different environments. The IMF is often stated in terms of a series of power laws, where $\xi(m)dm$ is the number of stars with masses in the range m to $m + dm$ within a specified volume of space, and is proportional to $m^{-\alpha}$, where α is a dimensionless exponent. Commonly used forms of the IMF are the Kroupa (2001) broken power law [12] and the Chabrier (2003) log-normal [13].

The IMF of stars more massive than our Sun was first quantified by Edwin Salpeter [15]. This form of the IMF is called the Salpeter function or a Salpeter IMF. It shows that the number of stars in each mass range decreases rapidly with increasing mass. The Salpeter Initial Mass Function is

$$\xi(m)\Delta m = \xi_0 \left(\frac{m}{M_\odot} \right)^{-2.35} \left(\frac{\Delta m}{M_\odot} \right). \quad (4.1)$$

It will turn out that our decompositions is consistent with the one obtained using the Salpeter Initial Mass Function (see the tables below). To accomplish our task, we would need a new input relating to the light deflection, described below.

4.3 A new input

We already calculated different contributions to light deflection given by the Mannheim-Kazanas solution of Weyl gravity. At this point, we recall a novel idea of Ederly & Paranjape [15], where they bridged two different metric theories by equating the *same* Einstein angle θ_E (caused by the luminous + dark matter) with the Weyl angle θ_W (caused by the luminous matter alone), derived useful results using the numerical equality $\theta_E = \theta_W$. We are motivated exactly by this idea, and assume $\theta_E = \theta_W$ for a light ray with the *same* impact parameter.

The rationale is as follows: It is understood that the Einstein and Weyl theories of gravity are both metric theories of gravity but otherwise very different – one with dark matter source and the other without, not to mention other differences. Hence, we are not talking here of merging or mapping the two theories into one another per se but concentrating only on a particular common observation, viz., angular radius of the observed ring. By the numerical equality $\theta_E = \theta_W$, we only mean that *observationally* the deflection must be unique, no matter whatever be

the theory. Thus, the true justification for such an equality has to come from other predictions that the equality possibly leads to. In fact, it has been shown recently that the central dark matter densities fall within the limits predicted by a similar equality [16].

Using this input, we develop an algorithm for finding N^* that gives the luminous matter component $M_*^{\text{Weyl}} = (M_*^{\text{Weyl}}/L)L = N^*M_\odot$, which is used to separate out the dark matter component $M_{\text{dm}}^{\text{Weyl}}$. With the known values of γ_0 , γ^* and κ and with *derived* values of N^* specific to galaxies, we shall find that the solution can reasonably account for the matter decompositions available in the literature by the same order of magnitudes at most up to a numerical pre-factor. We shall also calculate the average dark matter density $\langle \rho \rangle_{\text{av}}^{\text{Weyl}}$ of the sample of galaxies within their respective Einstein spheres. We now develop the master equation yielding values of N^* specific to galaxies.

4.4 The master equation

Using the observed gravitational lensing data of 57 galaxies of SDSS survey, we shall show that by using the above input, it is possible to separate out the amount of dark matter from the total matter enclosed by the Einstein sphere. We can then calculate the density of dark matter $\langle \rho \rangle$ averaged over the Einstein sphere around the galaxies. Assuming that dark matter average densities do not differ much from galaxy to galaxy, the values obtained here seem to be quite consistent with the data by order of magnitude. The two-way light deflection, obtained in Eq.(3.47), is

$$\delta = 2\psi_0 = \frac{4M}{b} + \frac{15M^2\gamma}{b} + \frac{2M\Lambda b}{3} + \gamma b. \quad (4.2)$$

Recall that the M in the above is the *luminous mass* within the sphere of radius equal to the impact parameter b , and henceforth we denote it by M_* . Then, we can separate out the contributions as follows:

$$\delta = \frac{4\widetilde{M}}{b} + \widetilde{\gamma}b, \quad (4.3)$$

$$= t_{\text{Sch}} + t_\gamma + t_{\text{Serenno}} \quad (4.4)$$

where

$$t_{\text{Sch}} = \frac{4M_*}{b} \left(1 + \frac{15M_*\gamma}{4} \right), t_\gamma = \gamma b = (N^*\gamma^* + \gamma_0)b, t_{\text{Serenno}} = \frac{2M_*\Lambda b}{3}, \quad (4.5)$$

and

$$\widetilde{M} = M_* \left(1 + \frac{15M_*\gamma}{4} \right), \quad (4.6)$$

$$M_* = N^*\beta^* \quad (4.7)$$

$$\gamma = N^*\gamma^* + \gamma_0 \quad (4.8)$$

$$\widetilde{\gamma} \equiv N^*\gamma^* + \gamma_0 + \frac{2M_*\Lambda}{3}, \quad (4.9)$$

We proceed as follows: The Einstein deflection angle is caused by $M_{\text{tot}}^{\text{lens}}$ and is given by

$$\theta_E = \left(\frac{4M_{\text{tot}}^{\text{lens}}}{D} \right)^{1/2}, \quad (4.10)$$

$$D \equiv \frac{D_1 D_s}{D_{\text{ls}}}, \quad (4.11)$$

where D_s , D_{ls} , D_1 are the angular diameter distances and $M_{\text{tot}}^{\text{lens}}$ is the total projected mass enclosed within the Einstein radius R_E defined by the impact parameter $b = R_E = D_1\theta_E$. Following Ederly and Paranjape [15], we interpret this deflection angle θ_E as δ and call it the Weyl angle caused by the luminous mass M_* alone within the radius b (i.e., same lensing geometry). Then, equating (4.5) and (4.7) gives

$$\theta_W = \left(\frac{4\widetilde{M}}{D - \widetilde{\gamma}D_{\text{ol}}^2} \right)^{1/2}. \quad (4.12)$$

where

$$\widetilde{\gamma} \equiv N^*\gamma^* + \gamma_0 + \frac{2M_*\Lambda}{3} \quad (4.13)$$

Equating the two angles, one finds

$$\widetilde{\gamma} = \frac{D_s}{D_{\text{ls}}D_1} \left(1 - \frac{\widetilde{M}}{M_{\text{tot}}^{\text{lens}}} \right), \quad (4.14)$$

$$N^*\gamma^* + \gamma_0 + \frac{2N^*\beta^*\Lambda}{3} = \frac{D_s}{D_{\text{ls}}D_1} \left[1 - \frac{N^*\beta^*}{M_{\text{tot}}^{\text{lens}}} \left\{ 1 + \frac{15N^*\beta^*(N^*\gamma^* + \gamma_0)}{4} \right\} \right]. \quad (4.15)$$

This is the master equation, which will now be used to tabulate the mass decompositions using the following algorithm.

4.5 Algorithm and SDSS data

In the above equation, the galaxy independent universal parameters ($\gamma_0, \gamma^*, \kappa$), are known from the rotation curve fitting to Weyl gravity [9], the distances D_s, D_{ls}, D_l and the total mass $M_{\text{tot}}^{\text{lens}}$ are provided by the SDSS data specific to individual galactic lensing measurements. Putting these values in Eq.(4.9), we first find from above the numerical value of N^* specific to each sample, which enables us to find the value of the luminous component M_* , which we now call M_*^{Weyl} to distinguish it from M_*^{Salpeter} . We use the latter notation because it turns out that our mass decomposition is more consistent the one based on Salpeter Initial Mass Function analyzed by Grillo *et al* [1]. We then compare the following ratios within the Einstein radius: $f_*^{\text{Salpeter}} = (M_*^{\text{Salpeter}}/M_{\text{tot}}^{\text{lens}})|_{\leq R_E}$, $f_*^{\text{Weyl}} = (M_*^{\text{Weyl}}/M_{\text{tot}}^{\text{lens}})|_{\leq R_E}$.

The mean density $\langle \rho \rangle_{\text{av}}^{\text{Weyl}}$ is obtained by averaging the dark matter mass $M_{\text{dm}}^{\text{Weyl}} = M_{\text{tot}}^{\text{lens}}(\leq R_E) - M_*^{\text{Weyl}}(\leq R_E)$ over the Einstein sphere of radius $b = R_E = D_l \theta_E$ centered at the galactic origin:

$$\langle \rho \rangle_{\text{av}}^{\text{Weyl}} = \frac{3M_{\text{dm}}^{\text{Weyl}}}{4\pi b^3}. \quad (4.16)$$

Data for distances D_s, D_l and $M_{\text{tot}}^{\text{lens}}$ are taken from [17]. We consider a typical situation with the lens lying about halfway between the observer and source such that $D_{ls} = 2D_l$. The conversions used are: 1 arcsec = (1/206265) rad, 1 Mpc = 3.085×10^{24} cm, $1 \text{ cm}^{-2} = 1.98 \times 10^{59} M_{\odot}(\text{kpc})^{-3}$. In the tables, we shall display only 15 galaxies:

Table I

Galaxy	$M_{\text{dm}}^{\text{Weyl}}$	$M_{\text{tot}}^{\text{lens}}$	$\langle \rho \rangle_{\text{av}}^{\text{Weyl}}$	f_*^{Salpeter}	f_*^{Weyl}
-	(M_{\odot})	(M_{\odot})	$M_{\odot}(\text{kpc})^{-3}$		
J0008-0004	3.50×10^{10}	3.50×10^{11}	2.90×10^7	$0.54^{+0.10}_{-0.33}$	0.90
J0029-0055	2.42×10^9	1.20×10^{11}	1.36×10^7	$0.76^{+0.35}_{-0.23}$	0.98
J0037-0942	1.03×10^{10}	2.90×10^{11}	2.02×10^7	$0.74^{+0.17}_{-0.28}$	0.96
J0044+0113	1.39×10^9	9.00×10^{10}	6.62×10^7	$0.55^{+0.27}_{-0.15}$	0.98
J0109+1500	5.01×10^9	1.30×10^{11}	4.29×10^7	$1.08^{+0.29}_{-0.22}$	0.96
J0157-0056	1.66×10^{10}	2.6×10^{11}	3.38×10^7	$1.21^{+0.20}_{-0.52}$	0.93
J0216-0813	5.99×10^{10}	4.90×10^{11}	8.42×10^7	$0.71^{+0.19}_{-0.28}$	0.88
J0252+0039	6.45×10^9	1.80×10^{11}	1.79×10^7	$0.52^{+0.07}_{-0.24}$	0.96
J0330-0020	1.51×10^{10}	2.50×10^{11}	2.23×10^7	$0.99^{+0.11}_{-0.27}$	0.94
J0405-0455	4.81×10^7	3.00×10^{10}	7.75×10^6	$0.73^{+0.43}_{-0.23}$	0.99
J0728+3835	5.56×10^9	2.00×10^{11}	1.76×10^7	$0.50^{+0.25}_{-0.08}$	0.97
J0737+3216	2.16×10^{10}	2.90×10^{11}	5.03×10^7	$0.77^{+0.09}_{-0.17}$	0.92
J0822+2652	1.03×10^{10}	2.40×10^{11}	2.78×10^7	$0.93^{+0.10}_{-0.16}$	0.96
J0903+4116	5.44×10^{10}	4.50×10^{11}	3.42×10^7	$0.87^{+0.09}_{-0.31}$	0.88
J0912+0029	2.03×10^{10}	4.00×10^{11}	5.02×10^7	$0.51^{+0.11}_{-0.09}$	0.95

Table II

Galaxy	M_*^{Weyl}	b	t_{Sch}	t_{γ}	t_{Serenio}
-	(M_{\odot})	(kpc)			
J0008-0004	3.15×10^{11}	6.59	9.16×10^{-6}	4.09×10^{-7}	1.81×10^{-14}
J0029-0055	1.17×10^{11}	3.49	6.46×10^{-6}	1.01×10^{-7}	3.58×10^{-15}
J0037-0942	2.79×10^{11}	4.97	1.08×10^{-5}	2.79×10^{-7}	1.21×10^{-14}
J0044+0113	8.86×10^{10}	1.71	9.95×10^{-6}	4.14×10^{-8}	1.32×10^{-15}
J0109+1500	1.25×10^{11}	3.03	7.91×10^{-6}	9.19×10^{-8}	3.30×10^{-15}
J0157-0056	2.43×10^{11}	4.89	9.55×10^{-6}	2.45×10^{-7}	1.04×10^{-14}
J0216-0813	4.30×10^{11}	5.53	1.50×10^{-5}	4.50×10^{-7}	2.07×10^{-14}
J0252+0039	1.73×10^{11}	4.41	7.54×10^{-6}	1.70×10^{-7}	6.67×10^{-15}
J0330-0020	2.35×10^{11}	5.44	8.28×10^{-6}	2.65×10^{-7}	1.11×10^{-14}
J0405-0455	2.99×10^{10}	1.14	1.06×10^{-6}	7.87×10^{-8}	1.42×10^{-15}
J0728+3835	1.94×10^{11}	4.22	8.84×10^{-6}	4.78×10^{-8}	1.93×10^{-15}
J0737+3216	2.68×10^{11}	4.67	1.10×10^{-5}	2.54×10^{-7}	1.09×10^{-14}
J0822+2652	2.30×10^{11}	4.45	9.90×10^{-6}	2.13×10^{-7}	8.91×10^{-15}
J0903+4116	3.96×10^{11}	7.24	1.70×10^{-5}	3.36×10^{-7}	1.53×10^{-14}
J0912+0029	3.80×10^{11}	4.59	1.00×10^{-5}	5.28×10^{-7}	2.39×10^{-14}

4.6 Appendix

The Schwarzschild deflection is

$$\alpha = \frac{4M_{\text{tot}}^{\text{lens}}}{b}$$

where $b = \theta D_1$ is the impact parameter. The lens equation is [18] is

$$\theta D_s = \beta D_s + \alpha D_{\text{ls}}.$$

The Einstein angle $\theta = \theta_{\text{Ein}}$ is defined by the case when source, lens and observer stay in a line, that is, when $\beta = 0$. Thus

$$\begin{aligned} \theta &= \alpha \left(\frac{D_{\text{ls}}}{D_s} \right) = \frac{4M_{\text{tot}}^{\text{lens}}}{b} \left(\frac{D_{\text{ls}}}{D_s} \right) = \frac{4M_{\text{tot}}^{\text{lens}}}{\theta D_1} \left(\frac{D_{\text{ls}}}{D_s} \right) \\ \Rightarrow \theta &= \theta_{\text{Ein}} = \sqrt{\frac{4M_{\text{tot}}^{\text{lens}}}{D}} \\ D &\equiv \frac{D_1 D_s}{D_{\text{ls}}}. \end{aligned}$$

The Weyl angle, for the same impact parameter $b = \theta D_1$, is

$$\begin{aligned} \theta &= \delta \left(\frac{D_{\text{ls}}}{D_s} \right) = \left[\frac{4\widetilde{M}}{\theta D_1} + \widetilde{\gamma}(\theta D_1) \right] \left(\frac{D_{\text{ls}}}{D_s} \right) \\ \Rightarrow \theta^2 &= \frac{4\widetilde{M}}{D \left(1 - \widetilde{\gamma} \frac{D_1^2}{D} \right)} = \frac{4\widetilde{M}}{D - \widetilde{\gamma} D_1^2} \\ \Rightarrow \theta &= \theta_{\text{Weyl}} = \sqrt{\frac{4\widetilde{M}}{D - \widetilde{\gamma} D_1^2}}, \end{aligned}$$

which is just the Eq.(4.12).

4.7 Conclusions

Weyl theory without dark matter should be observationally indistinguishable from Einstein theory with dark matter in the context of observable predictions such as the Einstein angle θ_E , which measures the total mass inside the sphere of Einstein radius $R_E = D_1 \theta_E$. Obviously, this total mass includes both luminous and dark matter. Our principle is that, *all* theories, with or without dark matter, should agree

with actual observations if they are to be in the reckoning at all, which led to the input $\theta_E = \theta_W$. We then calculated the Weyl angle θ_W and used the input to derive the master equation (4.15) from which we obtained the values of N^* specific to galaxies.

Using the algorithm, we then calculated the mass decomposition f_*^{Weyl} , which is surprisingly consistent with the decomposition based on Salpeter Initial Mass Function f_*^{Salpeter} within the error bars as displayed in the last two columns of Table I. Only 15 galaxies are shown, but we have verified that all the 57 samples in [1] support this behavior. As a spin-off, we also display the average dark matter density inside the Einstein sphere $\langle \rho \rangle_{\text{av}}^{\text{Weyl}}$, which are consistent with the limits derived in Ref.[16]. In Table II, we display various contributions to light bending, which predicts that the contribution t_γ is about an order or two magnitudes less than the first order t_{Sch} . This means that with a little more advancement in technology, the contribution t_γ may be amenable to measurements, which can be used to observationally support or rule out Weyl gravity. The coupling Sereno term t_{Sereno} is of course too small to be measured even in the far future.

4.8 References

- [1] C. Grillo, R. Gobat, M. Lombardi and P. Rosati, *Astron. Astrophys.* **501**, 461 (2009).
- [2] C. Grillo, R. Gobat, P. Rosati and M. Lombardi, *Astron. Astrophys.* **477**, 25 (2008).
- [3] T. Treu and L.V.E. Koopmans, *Astrophys. J.* **575**, 87 (2002); *ibid.* **611**, 739 (2004).
- [4] R. Gavazzi, T. Treu and J.D. Rhodes *et al*, *Astrophys. J.* **667**, 176 (2007).
- [5] C. Grillo, M. Lombardi and P. Rosati *et al*, *Astron. Astrophys.* **486**, 45 (2008).
- [6] S. Kazantzidis, A.V. Kravtsov, A.R. Zentner *et al*. *Astrophys. J.* **611**, 73 (2004).
- [7] R. Jesseit, A. Burkert and T. Naab, *Astrophys. J.* **571**, 89 (2002).
- [8] O.Y. Gnedin, A.V. Kravtsov, A.A. Klypin and D. Nagai, *Astrophys. J.* **616**, 16 (2004).
- [9] See: J.G. O'Brien and P.D. Mannheim, *Mon. Not. R. Astron. Soc.* **421**,

- 1273 (2012) for 138 galaxy rotation data fittings.
- [10] E.P. Hubble, *The Realm of the Nebulae* (Yale University Press, New Haven, 1936).
 - [11] G. de Vaucouleurs, *Astrophys. J. Suppl.* **8**, 31 (1963).
 - [12] P. Kroupa, *Science* **295**, 82 (2002).
 - [13] G. Chabrier, *Publ. Astron. Soc. Pacific* **115**, 763 (2003).
 - [14] E. Salpeter, *Astrophys. J.* **121**, 161 (1955).
 - [15] A. Edery and M.B. Paranjape, *Phys. Rev. D* **58**, 024011 (1998).
 - [16] A.A. Potapov *et al*, *JCAP* 07 (2015) 018.
 - [17] Z. Horváth, L. Á. Gergely, Z. Keresztes, T. Harko and F.S.N. Lobo, *Phys. Rev. D* **84**, 083006 (2011).
 - [18] J.B. Hartle, *Gravity – An introduction to Einstein’s general relativity* (Addison-Wesley, San Francisco, 2003)
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