Chapter 8

Future Scope

In the first part of this thesis, sufficient conditions for existence of the unique or multiple limit cycles for a class of Lienard systems are presented based on a simple extension of the classical Lienard theorem. The problem of developing powerful methods, leading to the proof of the exact number of limit cycles, for more wider class of Lienard systems is an active field of current research [47,51,52], with many authors, for instances, Lopez, Abbasbandy, Lopez-Ruiz [18, 67], Llibre [47] are making important new advances in this field. The limitation of our simple geometric approach becomes noticeable in our inability in obtaining a proof leading to exact number of limit cycles for the class of non-symmetric Lienard system. An extension of our approach in the light of current literature is necessary to achieve the goal. This is an important problem we would like to take up in near future.

Reconstruction of differential systems from incomplete information has been studied in Chapter 3 in the context of symmetric Lienard equation with a given number of limit cycles. Similar study for other kind of differential systems remains an open problem, especially with a given shape and size of a limit cycle. Another important class of problems in the Part I of the thesis is the possible extensions of the present approach to piece-wise smooth/continuous or even discontinuous (Lienard-like) systems [54, 79, 80], because non-smooth systems are more interesting, from the point of view of applications, rather than smooth systems.

In the second part of the thesis, a new Improved Renormalization Group Method is developed using new formalism of nonlinear time.

Use of multiple nonlinear scales enabled us to simulate cooperative behaviour among the asymptotic properties of different physical/dynamical parameters of a nonlinear system, viz. Rayleigh and Van der Pol systems in a much more efficient manner than that was possible in the context of conventional RGM. The formalism presented in the thesis, though appears to be novel and appealing, needs to be further substantiated by many more such applications in different classes of nonlinear systems, both autonomous as well as non-autonomous. One of our immediate goal in this area of research is to investigate non-autonomous VdP equation with a periodic forcing, in the context of nonlinear time formalism and to try to understand the period doubling bifurcation route to the chaotic attractor in the light of the associated hierarchy of nonlinear scales.