CHAPTER 8

INEXACTNESS[꜔]

8.1. **Introduction**

In real world, we have the experience that sometimes either we have no knowledge about an object or we have some incomplete, vague and imprecise knowledge about an object [1, 2]. Any computer-based software / tool e.g. an expert system, to aid in human decision making needs to take into account the inexact nature of information expecting to lead to rational and correct decisions. For a problem domain, different forms of inexactness may surface. Lacking any unique theory to manage all the forms as a whole, different approaches have been proposed with their own zone of applicability. Actually, the proper design and selection of a suitable scheme should be in tune with the requirements of the application domain. In addition, the proper selection should also depend on certain important properties of a scheme like expressive power and adequacy in context to the application domain.

In section 8.2, the sources and nature of inexactness have been identified for our present child growth and development (CGD) problem domain. In the same section, we have classified the associated inexactness of the domain. Section 8.3 deals with the tools for managing those inexactness in information. Some common approaches of dealing with inexactness in expert systems have been discussed in brief. In section 8.4, a suitability analysis has been provided in context to the present problem domain. Our conclusions and discussion have been provided in the last section.

8.2. **Sources and Nature of Inexact Information**

Any software tool, in particular, a computer based consultation system / expert system developed to aid people in their decision making needs to take into account the inexact nature of information expecting to lead to rational decisions. While developing an expert system which needs to reason with inexact information, one has to identify the sources and nature of inexactness for justifying the relative suitability of any method mechanising the process of reasoning with inexactness for the problem domain at hand.

[꜔] This is based on the publications [Proc. XXX Annual Conv. of CSI, 1995, 258-267; CSI Communications, April 1996, 19-29] of the author.

We now discuss the typical physical and logical sources and nature of inexactness of information for the paediatric problem domain. In table 8.1, the possible physical sources and their nature and explanation have been presented. Combining all these possible physical sources, one can identify the following possible logical sources of inexactness of information :

- Lack of adequate data,
- Inconsistency of data,
- Inherent human fuzzy concepts,
- Matching of similar rather that identical situations,
- Differing (expert) opinions,
- Ignorance,
- Imprecision in measurements,
- Lack of available theory to describe a particular situation.

In our system, five types [3] of inexactness have been classified as follows :

8.2.1. **Uncertain information**

Fumbling answer from a child and / or from parents / guardians - an issue concerning lack of confidence.

Example :

Question : Did your baby have fever last night ?
Answer : Yes / No / May be or most probably.

The answer 'may be or most probably' does not offer absolute certainty about the piece of information. The degree of (un)certainty is usually represented by a numerical value. In the above example, if the answer is 'Yes' or 'No', the certainty factor (CF) is 1. But, if the answer is 'may be or most probably' the certainty factor may be in between 0.8 to 1.0 depending upon the stress of answer.

8.2.2. **Fuzzy information**

Fuzziness (sharplessness) occurs when the boundary of a piece of information is not clear cut.

Example :

The fever of the child is quite high.
The condition of the child is critical.

The linguistic articulations like 'quite high', 'critical' are fuzzy terms and these terms have shades of meaning.

8.2.3. **Simultaneous occurrence of uncertainty and fuzziness**

Sometimes, there are situations where uncertainty and fuzziness may occur simultaneously.

Example :

If the Rhythms (sleep and meals) of the baby is alright then the growth of the baby should be good (0.95).

Here, 0.95 is the certainty factor and 'alright', 'good' are fuzzy terms.

8.2.4. **Uncertain-fuzzy**

There are situations where uncertainty can be fuzzy.

Example :

The fever of the child is very high (around 105^{O} F). Here, 'around 105^{O} F' is the fuzzy uncertainty and 'very high' is fuzzy term.

8.2.5. **Non-monotonic nature**

Question : Does your baby take 4-5 meals/day ?
Answer : Doctor. She doesn't want to take meals.

With this reply (CF=0.9) the doctor proceeds to other investigations. After overall investigations, the doctor finds that the growth of the baby is normal. This finding, obviously, contradicts the reply from the mother whose CF may be 0.3 or 0.2. The associated facts and rules have also to be changed with this changing CF. This is how the nonmonotonic reasoning works to offer a safeguard to the subjective reply during a consultation session.

SOURCES and NATURE	EXPLANATION
1. Problem domain a) Lack of precise numeric aspiration levels; b) Lack of appropriate or available well defined algorithms.	1. In a medical diagnostic system, it is very difficult to assign precise aspiration levels to some or all the controlling physiological parameters. Moreover, human body, does not follow a strict algorithmic approach for its running.
2. Child a) Lack of adequate data; b) Inconsistency of data; c) Subjective reply; d) Reply in fuzzy terms; e) Fumbling answer; f) No information.	2. Sometimes, a child may not be able to supply any verbal relevant information where language development is not adequate. The other natures of inexactness are also frequently received by a doctor during a consultation session.
3. Parents / Guardians a) Lack of adequate data b) Inconsistency of data; c) Subjective reply; d) Reply in fuzzy terms; e) Fumbling answer indicating lack of confidence; f) Ignorance.	3. Parents / guardians sometimes fail to supply complete history of the child, may be, due to their illiteracy. The other forms of reply are also frequently as received by a doctor during a consultation session.
4. Doctors Matching of similar rather than identical situations : model inequivalance.	4. Sometimes doctors are suppossed to apply heuristic knowledge gained in a number of years of practice. Any particular heuristic may not work with similar situation.
5. Laboratory tests / technicians Imprecision in measurement during clinical / pathological / radiological tests : instrumental / technician's error; impurity with the chemicals / glasswares used.	5. It is reported, sometimes,that the findings from the clinical view of a doctor may not match the findings from laboratory reports. Doctors,then advice to repeat the test for confirmation.
6. Symptoms a) Most of the symptoms are valid for more than one adverse situation; b) Information hiding.	6. A particular symptom may be reflected from two or more adverse situations. A secondary symptom may be more prominent than a primary symptom in a situation : information hiding.
7. Laboratory results not available.	7. Laboratory investigations may not be possible either due to financial constraints or there may not be any laboratory in the nearby area.

Table 8.1. Sources and nature of inexactness

At the starting of a typical consultation session, a doctor has to interrogate the child, where possible, or the parents / guardians of the child on different issues. She / he may examine the child with his / her medical / clinical view and so on. She / he may face the above forms of inexactness of information which may be the result of the combined conspiracy of the above discussed sources and other unidentified sources. Here, proper management of inexact information is necessary which plays a pivotal role in rational decision making.

8.3. **Tools for Managing Inexact Information**

A number of methods have been proposed to deal with different aspects of inexact information management with their varying degrees of success. In essence [4], they can take one of the seven forms such as non-numerical techniques, categorical techniques, probabilistic modelling, ad-hoc techniques, Bayesian inference, fuzzy logic and Dempster-Shafer theory of evidence. Recently, Reynolds et al. [4] have emphasised the need for the graphical representation of uncertainty for an application domain due to some limitations of the above methods.

In expert systems, the common approaches in dealing with inexactness are : Bayesian probability approach, DS theory of evidence, Stanford CF calculus and Fuzzy set theoretic approach. In addition, inexact reasoning has itself non-monotonic aspect. It may be noted here that none of the methods except CF-calculus has been developed with a reference to AI and expert systems and neither has yet been universally adopted by theoreticians or practitioners.

It may be stated as a generally accepted fact that any theory designed to cover the concept of inexactness should essentially be able to provide a method for changing prior opinion in the light of new evidence : non-monotonic reasoning. For example, DS approach having a rigorous mathematical theory, handles the situation using Dempster's rule of combination.

8.3.1. **Bayesian probability theory** [5]

The Bayesian approach is based on formal probability theory and has shown up in several areas of AI research, including expert systems and pattern recognition problems. However, this particular approach can deal only with uncertainty.

One of the most important results of probability theory is Bayes' theorem. Bayes' theorem states :

$$\Pr\{E_j|F\} = \frac{P_r\{F|E_j\}}{\sum_{i=1}^{n} P_r\{F|E_i\} P_r\{E_i\}} \Pr\{E_j\} \qquad (8.1)$$

where we assume that

$$\Pr\{E_i\} > 0 \qquad \text{for all } i, \qquad (8.2)$$

$$\Pr\{E_i \cap E_j\} = 0 \qquad \text{for } i \neq j \qquad (8.3)$$

and

$$\sum_{i=1}^{n} P_r\{E_i\} = 1 \qquad (8.4)$$

Usually, $\Pr\{E_j\}$ and $\Pr\{E_j|F\}$ are called Bayesian **'Prior'** and Bayesian **'Posterior'** respectively.

This theorem has been used in a number expert systems. One such notable expert system is PROSPECTOR developed [6] as a consultation system for mineral exploration. It is based on the above assumptions (8.2), (8.3) and (8.4). We may cite here at least one PROSPECTOR - like system which has been developed based on this theorem where the above assumptions are implied [7].

8.3.2. **Dempster / Shafer theory of evidence** [8,9]

Dempster-Shafer theory has been considered as a prominant candidate to handle inexactness in expert systems. The DS theory begins with the view that problem-solving activities can be described as a process of attempting to answer questions of interest [10]. Possible answers to a question are represented as a set of interrelated propositional sentences called a frame of discernment (FOD), denoted by θ. For a given FOD, at any instant, one and only one propositional sentence is possibly true. One has to form a consensus about the truth of propositions based on opinions that bear upon the question of interest to find the correct evidences from two or more independent sources and then they are combined to get a consensus.

This theory assumes that observations from real world are usually partial, imprecise, and occassionally unreliable to varying degrees. Within DS theory, each independent sources conveys its belief via a mass distribution m which is defined as :

$$m : 2^{\theta} \longrightarrow [0,1] \qquad (8.5)$$

where

$$m(\Phi) = 0 \qquad (8.6)$$

and

$$\sum_{A \subseteq \theta} m(A) = 1 \qquad (8.7)$$

The quantity m(A) is called A's 'basic probability' or 'basic probability assignment' or simply 'basic assignment' and it is understood to be the measure of the belief that is committed exactly to A. To obtain the measure of total belief committed to A, one must use the following :

$$Bel(A) = \sum_{B \subseteq A} m(B) \qquad (8.8)$$

Evidence obtained in the same context from two independent sources which are expressed by two basic assignments m_1 and m_2 on some power set *P*(X) must be appropriately combined to obtain a joint basic assignment $m_{1,2}$. This can be achieved using Dempster's rule of combination as :

$$m_{1,2}(A) = \frac{\sum_{B \cap C = A} m_1(B).m_2(C)}{1 - k} \qquad (8.9)$$

For $A \neq \phi$,

$$K = \sum_{B \cap C = \phi} m_1(B).m_2(C) < 1 \qquad (8.10)$$

and

$$m_{1,2}(\phi) = 0$$

Multiple independent opinions can be combined in any order affecting the final result since Dempster's rule is both commutative and associative. If the initial opinions are independent, the derivative opinions are independent as long as they share no common ancestors. If normalisation is performed after each pair of opinions have been pooled, then the total degree of conflict between all the knowledge sources will be lost. However, it is possible to show that deferring normalisation until the final orthogonal sum is computed will produce the same results while extracting the total degree of conflict [2].

The concept of support pair [S, U] is the basis of Dempster-shafer theory of evidence. S, the necessary support, is the minimum support that is given to the truth value probability assignment. U, the possible support, is the maximum possible truth value probability assignment. In DS theory, a belief in hypothesis, B(H), is the sum of probabilities for all the subsets of H. Support pairs accommodate for the situations where the sum of the belief that a hypothesis is true and the belief that the negation of the hypothesis is also true, is less than one i.e.,

$$B(H) + B(\overline{H}) < 1$$

But for B(H) and $B(\overline{H})$ to be truth value probabilities, their sum must be equal to 1. Since this is not always the case, B(H) is called necessary support, and the complement of $B(\overline{H})$ is called the possible support. So,

$$S = B(H) \text{ and } U = 1 - B(\overline{H})$$

Therefore, [S, U] thus defines an evidential interval, where residual inexactness in H is given by U - S, which is sometimes referred to as the ignorance of H which is represented as Igr(H). This residual inexactness can be reduced upon learning new evidence supporting the hypothesis. Igr(H) will become zero having full evidence supporting the hypothesis; the evidential interval merges to become a point probability, the Bayesian method becomes a special case of the DS - representation.

8.2.3. **Stanford certainty factor model** [11]

This approach can deal with uncertainty only. If we have a hypothesis h based on an evidence e, then from some simple assumptions of certainty theory we may define "measure of belief (MB)" and "measure of disbelief (MD)". We write :

> $MB(h \mid e)$, the measure of belief of a hypothesis h based on an evidence e, and
> $MD(h \mid e)$, the measure of disbelief of a hypothesis h on an evidence e.

Now,

While $MD(h|e) = 0$, $\quad 1 > MB(h|e) > 0$, or
While $MB(h|e) = 0$, $\quad 1 > MD(h|e) > 0$.

We combine $MB(h|e)$ and $MD(h|e)$ to get the certainty factor as

$$CF(h|e) = MB(h|e) - MD(h|e).$$

As the certainty factor (CF) approaches 1, the evidence(e) is stronger for a hypothesis(h); as CF approaches -1, the confidence against the hypothesis gets stronger; and CF around 0 indicates that there is little evidence either for or against the hypothesis.

In a rule-based system, different certainty factors are attached to every premises. These certainty factors are combined to get the overall certainty of the inference as follows. If p1 and p2 are two premises of an inference, then the combined certainty factors are :

$$CF(p1 \text{ and } p2) = \min(CF(p1),\ CF(p2)), \quad \text{and}$$

$$CF(p1 \text{ or } p2) = \max(CF(p1),\ CF(p2)).$$

One more measure is required, i.e., how to combine multiple CFs when two or more rules support the same result R. Here, CF - theory uses the analogy of the probability theory procedure of multiplying the probability measures to combine independent evidence. By using this rule repeatedly one can combine the results of any number of rules that are used for determining result R. Suppose CF(R1) is the present certainty factor associated with result R, and a previously unused rule produces result R (again) with CF(R2); then the new CF of R is calculated by :

$$CF(R1) + CF(R2) - CF(R1) * CF(R2) \quad \text{when } CF(R1) \text{ and } CF(R2) \text{ are +ve}$$

$$CF(R1) + CF(R2) + CF(R1) * CF(R2) \quad \text{when } CF(R1) \text{ and } CF(R2) \text{ are -ve}$$

$$\frac{CF(R1) + CF(R2)}{1 - MIN(|CF(R1)|, |CF(R2)|)} \quad \text{otherwise}$$

where |X| is the absolute value of X.

The combined certainty factor (CF) of the premises obtained from the above combining rules is multiplied by the originally assumed CF of the inference to get the new CF of the inference.

8.3.4. **Fuzzy set theory** [12]

This approach has been used in a commercially notable expert system REVEAL from ICL [13,14] which is essentially a decision support system. A number of commercial knowledge based shells have also incorporated fuzzy reasoning [15-19]. As a matter of fact, fuzzy logic has previously been used successfully in a number of knowledge based systems and the trend is good enough [20-22].

The concept of fuzzy set and fuzzy logic were introduced by Zadeh [12]. Zadeh was working in the field of control engineering. His intention of introducing this fuzzy set theory was to deal with problems involving knowledge expressed in vague, linguistic terms. Classically, a set is defined by its members. An object may be either a member or a non-member : the characteristic of traditional (crisp) set. The connected logical proposition may also be true or untrue. This concept of crisp set may be extended to fuzzy set with the introduction of the idea of partial truth. Any object may be a member of a set 'to some degree'; and a logical proposition may hold true 'to some degree'. Often, we communicate with other peoples by making qualitative statements, some of which are vague because we simply do not have the precise datum at our disposal e.g., a person is tall (we have no exact numerical value at that moment) or because the datum is not measurable in any scale e.g., a beautiful girl (for beautiful, no metric exists). Here, tall and beautiful are fuzzy sets. So, fuzzy concepts are one of the important channels by which we mediate and exchange information, ideas and understanding between ourselves. Fuzzy set theory offers a precise mathematical form to describe such fuzzy terms such as tall, small, rather tall, very tall, etc. in the form of fuzzy sets of a linguistic variable such as height. The details of the topic have been provided in **chapter 9**.

8.3.5. **Non-monotonic reasoning**

Most of the available knowledge-based consultation systems / expert systems and different ES-shells use monotonic reasoning as their inference strategies which essentially assumes that axioms do not change and conclusions drawn from them remain true. In contrast to monotonic reasoning, nonmonotonic reasoning (NMR) proceeds with its reasoning as if the assumptions are true with their possible inexactness in the information. With its reasoning it reaches a conclusion. If one finds the conclusion absured, it is demanding at this stage to change an assumption and / or to change the associated (un)certainty values. NMR may be considered as an important feature of human problem solving and commonsense reasoning.

The information supplied by the parents / guardians or by the child himself / herself are subjective sometimes. To deal with this subjective reply, a kind of inexactness, NMR will be useful. NMR is also important and advantageous in connection with modifiability. We expect it useful to incorporate in our system as one of the measure of inexactness in information.

8.4. **Suitability Analysis**

Let us now investigate the suitability of the above method(s) of handling inexactness in information which seem(s) to be most sympathetic to the problem domain at our hand.

8.4.1. **Bayesian probability theory** :

This approach works with two major assumptions : (1) All the statistical data on the relationships of the evidence with the various hypothesis are known in advance of processing starts; (2) All relationships between evidence and hypothesis are independent. Despite the commercial success of PROSPECTOR, the wide applicability of this approach is restricted and sometimes infeasible in some problem domain [1]. These asumptions are the bottlenecks of using this technique for a problem domain of diagnostic nature of child growth and development. In a medical diagnostic problem domain, it is very difficult for the domain experts to collect or estimate all prior conditional and joint probabilities. This seems to contradict the reasons of using an expert systems framework when and / or where the complete logic may not be known in advance.

For the medical domain, the assumption of independence of relationships between evidence and hypothesis cannot really be justified. The last problem (may not be the least) arises in connection with modifiability, a desirable feature of an ES, of knowledge base. The knowledge base may have to be changed or updated due to different reasons. Particularly, when complete and sound knowledge may not be available in advance, a fact for the present domain, existing system should easily and quickly incorporate the required changes. In this situation, there is the need to rebuild all probability relationships which seems to be a cumbersome task. Considering all these factors, we find hardly any good reason to use this technique for the present problem domain at our hand.

8.4.2. **Dempster / Shafer theory of evidence**

This approach recognizes the distinction between uncertainty and ignorance by creating 'belief functions' - measurements of the degree of belief. The theory allows the decomposition of a set of evidence into separate, unrelated set of evidence. It allows us to use our knowledge to bound the assignment of probabilities to events without having to come up with exact probabilities when these may be unavailable; the situation where DS method may be a good candidate for applications like the integration of data from multiple radar sensors [23-25]. It is concluded by O'Neill [2] that DS theory may be considered as a promising candidate for managing inexactness, as it includes PROSPECTOR's Bayesian belief functions and MYCIN's certainty factors as special cases. It also is based on a more mathematical foundation than either PROSPECTOR or MYCIN. However, we find to date, no notable expert system in the market using this model except some research applications [26]. The reason may be due to its involvement of so many numerical computations reducing the speed of inferencing and in the case of long inference chain the structure of the resulting belief functions would be very complex. One may expect its use where the length of inference chain is of low or moderate size. Some studies are reported to reduce the computational complexity of the method using local computation technique for computing belief functions [27-29] and using some optimizing techniques [30]. However, the ways and means of using a simplification scheme seems to depend on case specific algorithms which deserves more scrutiny and thereby restricting its general use. We find no such commercially successful ES or ES-shell using this particular model. The above observations advice us, at present, not to use the technique for our present problem domain.

8.4.3. **Stanford certainty factor model**

This is a heuristic approach to the management of uncertainty. It is criticized as an ad-hoc technique. In particular, criticism from Adams may be considered worthwhile. Adams [31] concludes that the empirical success of MYCIN may be due to the fact that the chains of reasoning are short and the hypothesis involved are simple; this ideal situation may not be true for a complex system. Nevertheless, CF calculus finds its foundation among the expert system / expert system shell designers for its simplicity of use. The commercial success of MYCIN, EMYCIN, S.1, LEVEL 5 etc. encourages people to use this technique for handling uncertainty. We do expect it useful for our problem domain to handle inexact information of uncertain nature.

8.4.4. **Fuzzy set theory**

In about 30 years of its existence, fuzzy set theory has been used in many areas including engineering, business, mathematics, psychology, management, semiology, medicine, image processing and pattern recognition. It may be fair to state that it has been used at length in control applications. In Japan alone, it has been reported, 2000 patents have been issued [32]. However, its applicability and usefulness are increasing interestingly in other fields as well [20-22]. In medical domain, fuzzy logic has previously been successfully used in a number of knowledge based systems

[33-38]. For the paediatric problem domain, we find no such reported rigorous use. In connection with the management of inexact information in expert systems, the conventional approaches fail in four important respects [39] :

- They do not provide the means for dealing with the fuzziness of antecedents and consequents;
- They assume that the probabilities can be estimated as crisp numbers;
- They do not offer a for inference from rules in which the qualifying probabilities are fuzzy;
- The rules for composition of probabilities depend on unsupported assumptions about some conditional independence.

Fuzzy logic addresses some, but not all, of these problems. More specifically, fuzzy logic allows the antecedents and / or consequents and / or qualifying probabilities to be fuzzy. Furthermore, fuzzy logic makes it possible to estimate probabilities as fuzzy rather than crisp number.

Fuzzy set theory has done quite well as a formal mathematical system. Whether its theorems are interesting is a subjective opinion among mathematicians, but a large body of mathematical work exists. Where more work needs to be done is in establishing that fuzzy set theory actually captures something real in applicative fields and can make a pragmatic difference, for the right reasons [40].

It is tempting at this stage to use the technique as a measure of fuzzy concepts associated with the problem domain.

8.4.5. **Non-monotonic reasoning**

The information supplied by the parents / guardians or by the child himself / herself are subjective sometimes. To deal with this subjective reply, a kind of inexactness, NMR will be useful. NMR is also important and advantageous in connection with portability and modifiability. We expect it useful to incorporate in our system as one of the measures of inexactness in information.

8.5. **Conclusions** and **Discussion**

The future trend of expert system / expert system shell may probably emphasize more on the capability in handling both exact and inexact reasoning. There are situations in real life where one has to reason with vague, insufficient, imprecise information to come to a rational decision. Any software / software tool developed for assisting peoples in their decision making needs to take into account the inexact nature of information. In this chapter, we have identified the potential sources and nature of inexactness in information in context to the present problem domain. We have also discussed different common approaches for managing inexactness in expert systems. We have tried to analyse the relative suitability of those methods considering the problem domain of child growth and development. We have confined ourselves in considering certainty factor model and fuzzy set theoretic approach for managing inexactness in this thesis.

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