Chapter 8

Demand Analysis

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8.1 Introduction

Demand theory and analysis can be a source of many useful insights for business decision making. The fundamental objective of demand theory is to identify and analyze the basic determinants of consumer needs and wants. An understanding of the forces behind demand is a powerful tool for managers. The demand analysis has been included in the GIS based integrated tea management system to facilitate the management decision of tea garden. Such knowledge provides the background needed to make pricing decisions, forecast sales and formulate marketing strategies. Now individual demand and market demand are discussed in section 8.2 and 8.3. Three methods have been used for the present analysis and implementations are (a) Demand estimation and least squares regression method (b) Economic forecasting and Time series analysis and (c) Forecasting of Tea Production using Neural Network based on fuzzy data. These methods are described in section 8.4, 8.5 and 8.8 respectively [6, 28, 29].

8.2 Individual Demand

Consumer choice can be a difficult task in a modern economic system. But the purchases of most consumers are constrained by their income. In determining what to purchase, individual consumers face a constrained optimization problem. That is , given their income, they select that combination of goods and services that maximizes their personal satisfaction. Implicitly, these choices involve a comparison of the satisfaction associated with having a goods or service and its opportunity cost. In a market economy, opportunity costs are reflected by prices. Thus prices act as signals to guide consumer decisions.

One of the most basic concepts of demand theory is the law of demand. This law indicates that there is an inverse relationship between price and quantity demanded – as price increases quantity demanded substitution and income effects resulting from price changes. The substitution effect reflects changing opportunity costs. When the price of a goods increases its opportunity cost in term of other goods also increases. Consequently, consumers may substitute other goods for the good that has become more expensive. For example , garden A and garden B produces orthodox tea T-A and T-B respectively. The prices of T-A and T-B are almost same. Now if garden A increase the price of T-A then most of the consumer will consume T-B. This is an example of substitution effect.

Now consider income effect. When the price of a goods increases the consumer's purchasing power decreases. The change in purchasing power is called an income effect because the price increase is equivalent to a reduction in the consumer's income decrease. The law of demand can be explained in terms of substitution and income effects resulting from price changes.

8.3 Market Demand

Market demand is the sum of all individual demands. Although choices by individuals are the basis of the theory of demand, it is total or market demand that is of primary interest to managers. The market demand function can be expressed mathematically. If the price of the product, income, consumer preferences and the prices of other goods and services, the demand equation can be written as

$$Q_D = f(P, I, P_o, T)$$

Where P is the price of the good or service, I is income, P_0 represents the prices of other goods and T is the measure of consumer preferences. This equation implies only that there are general relationships. The following equation is the functional form of the market demand

$$Q_D = B + a_P P + a_I I + a_o P_o + a_T T$$

The coefficients a_P, a_I, a_o and a_T indicate the change in quantity demanded of one unit changes in the associated variables. For example, a_P is the coefficient of price. Its interpretation is that, holding the other three variables constant, quantity demanded changes by a_P units for each one unit change in price. If I, P_o and T are not allowed to vary, then demand is a function only of P. Hence the linear form of the demand equation can be written as

$$Q_D = B + a_P P$$

Where B is the combined influence of all the other determinants of demand and $a_P <= 0$.

8.4 Demand estimation and least squares regression method

Here regression analysis, a statistical method for fitting a demand equation to a data set is considered. It is used for demand estimation and also to estimate production and cost equation. Consider the simple demand equation $\mathbf{Q_d = B + aP}$. In making pricing decisions, it may not be sufficient to know that quantity demanded and price are inversely related. An estimate of numerical value of **a** and the coefficient **B** may be required for decision making. To fully understand the correspondence between inputs and outputs in production functions and between costs and output in cost functions, it is often necessary to quantify relationships between the variables. The most widely used technique for estimating these relationships is the least squares regression method.

Now consider the equation $Y= a + bX$, where the dependent variable Y is total cost and the independent variable X is total output. If this function is plotted on a graph, the parameter a would be the vertical intercept and b would be slope of the function. The best estimate of the coefficients of a linear function is to fit the line through the data points so that the sum of squared vertical distances from each point to the line is minimized. This technique is called least squares regression estimate.

Now, $\bar{Y} = \sum Yi / n$ and $\bar{X} = \sum Xi / n$ are the means of Y and X variables.

The equations for least square estimators are

$$b^p = \sum(Xi - \bar{X})(Yi - \bar{Y}) / \sum(Xi - \bar{X})^2 \qquad \text{and} \quad a^p = \bar{Y} - b^p\bar{X}$$

Hence the estimated demand equation for the total cost function is

$$Y^p = a^p + b^pX$$

The deviation of the actual Y value from the predicted value (the vertical distance of the point from the line), $Yi - Y^p$ is called as residual or the prediction error.

8.5 Economic forecasting and Time series analysis

Forecasting involves predicting future economic conditions and assessing their effect on the operations of the firm. The objective of forecasting is to predict demand. Forecasting is an important management activity. Major decisions in large businesses are almost based on forecasted data. Forecasting requires the development of a good set of data on which to base the analysis. A forecast can not be better than the data from which it is derived. The time series analysis is a method for forecasting future values.

The focus of time-series analysis is to identify the components of change in the data. Normally these components are divided into following categories:

1. Trend
2. Seasonality
3. Cyclical patterns
4. Random fluctuations

The trend is a long term increase or decrease in the variable. The seasonal component represents changes that occur at regular intervals. Analysis of a time series may suggest that there are cyclical patterns, defined as sustained period of high values followed by low values. Various methods can be used to determined trends, seasonality, and any cyclical patterns in time series data. One of the most commonly used forecasting techniques is trend projection. This approach is based on the assumption that there is an identifiable trend in a time series of data.

Forecasts based on the assumption of a constant rate of change can be made by fitting time-series data to an equation of the form

$$S_t = S_0 + bt$$

Where S denotes output production and t indicates the time period. The two parameters to be estimated are S_0 and b. The value S_0 corresponds to the output (vertical) axis intercept of the line and parameter b is the constant rate of change and corresponds to the slope of the line.

Now suppose that the individual responsible for the forecast wants to estimate a percentage rate of change in production. So here the forecasting equation is

$$S_t = S_0\,(1 + g)^t$$

Where g is the constant percentage rate of change, or the growth rate. This equation is not linear. By simple transformation the above equation is to be estimated using ordinary least squares.

Taking the natural logarithm of the above equation, we get

$$\ln S_t = \ln[S_0(1 + g)^t] = \ln S_0 + t[\ln(1 + g)]$$

The above equation is linear in form. Now make the following substitutions

$$Y_t = \ln S_t$$

$$Y_0 = \ln S_0$$

$$b = \ln(1 + g)$$

Thus the new linear equation is

$$Y_t = Y_0 + bt.$$

8.6 Fuzzy Logic

In standard set theory, an object is either a member of a set or it is not. This brings to the use of characteristic function f for a set A, where $fA(x)=1$ if x is in A; otherwise it is 0.Thus, f is defined on the universe U and " *xÎ U*, $f: U \rightarrow \{0,1\}$. The notion of a set has been generalized by allowing characteristic functions to assume other than 0 and 1. For example, the notion of a fuzzy set has been defined with the characteristic function u which maps from U to a number in the interval [0,1]; that is $u: U \rightarrow [0,1]$. Thus, the definition of the fuzzy set A is as follows:

Let U be a set, denumerable or not , and let x be an element of U. A fuzzy subset A of U is a set of ordered pairs $\{(x, uA(x))\}$, for all x in U, where $uA(x)$ is a membership characteristic function with values in [0,1], and which indicates the degree or level of membership of x in A. A value of $uA(x) = 0$ has the same meaning as $fA(x) = 0$, that means that x is not a member of A. If a value of $uA(x) = 1$ signifies that x is completely contained in A. Values of $0 < uA(x) < 1$ signify that x is partial member of A.

8.7 Neural Network

Artificial Neural Network (ANN) is a model that emulates a biological neural network. The neurode(artificial neuron) is analogous to the biological neuron, receiving inputs that represent electrical impulses that dendrites of biological neurons receive from other neurons. The output of the neurode corresponds to a signal sent out from a biological neuron over its axon. The axon of the biological neuron branches to the dendrites of other neurons and the impulses are transmitted over synapses. A synapse is able to increase or decrease its strength, thus affecting the level of signal propagation and is said to cause excitation or inhabitation of a subsequent neuron.

An ANN (Artificial Neural Network) is composed of collection of interconnected neurons that are often grouped in layers; however in general no specific architecture should be assumed. In terms of layered architecture, two basic structures are considered. In one type, two layers are seen: input and output. In other type there are three layers: input, intermediate (called hidden) and output. An input layer receives data from the outside world and sends signals to subsequent layers. The outside layer interprets signals from the previous layer to produce a result that is transmitted to the outside world. In three layers ANN ((Artificial Neural Network) architecture, the concept of hidden layer is assumed in order to control the weights between input to hidden layer and hidden layer to output layer.

The Feedforward Neural Network does not have feedback connections, but errors are back propagated during training. Errors in the output determine the measures of hidden layer output errors, which are used as a basis for adjustment of connection weights between the input and hidden layers. Adjusting the two sets of weights between the pair of layers and recalculating the outputs is an iterative process that is carried on until the errors fall below a tolerance level. Learning rate parameters are scaling the adjustments to weights. A momentum parameter can be used in scaling the adjustments from a previous iteration and adding to the adjustments in the current iteration.

8.8 Forecasting of Tea Production using Neural Network based on fuzzy data

The universe U is partitioned into four equal length intervals. The intervals are chosen as:

U1=[456,480], U2=[480,504], U3=[504,528], U4=[528,552], U5=[552,576]

Fuzzy sets are defined on the universe. First, some linguistic values are determined. For linguistic variables number of students employed, Let, A1 = (many) A2 = (many, many) A3 = (very many) A 4= (too many) be the possible value.

The available data are fuzzified based on the gaussian function, which are furnished in table 8.1.

Table 8.1: Actual Data and Fuzzy Set

Actual Data	A1	A2	A3	A4	A5	Fuzzy Set
560	0	0	0	0.6	1	A5
562	0	0	0	0.6	1	A5
564	0	0	0	0.6	1	A5
566	0	0	0	0.4	1	A5
568	0	0	0	0.4	1	A5
570	0	0	0	0.4	1	A5
563	0	0	0	0.6	1	A5
556	0	0	0	0.8	1	A5
549	0	0	0.2	1	0.8	A4
542	0	0	0.4	1	0.6	A4
535	0	0	0.6	1	0.4	A4
520	0	0.4	1	0.6	0	A3
505	0	0.8	1	0.2	0	A3
490	0.6	1	0.4	0	0	A2
475	1	0.8	0	0	0	A1
460	1	0.4	0	0	0	A1
471	1	0.6	0	0	0	A1
482	0.8	1	0.2	0	0	A2
493	0.4	1	0.6	0	0	A2
504	0.2	1	0.8	0	0	A2
515	0	0.6	1	0.4	0	A3
508	0	0.8	1	0.2	0	A3
501	0.2	1	0.8	0	0	A2
494	0.4	1	0.6	0	0	A2

8.8.1 Fuzzy Logic(computation)

All the fuzzy logical relationships are obtained as follows:-

A5 ----> A5, A5 ----> A4, A4 ----> A4, A4 ----> A3, A3 ----> A3, A3 ----> A2, A2 -----> A1, A1 -----> A1, A1 -----> A2, A2 -----> A2, A2 -----> A3.

It is to note that the repeated relationships are counted only once.

Let us define an operator 'X' of two matrices. Suppose C and B are row matrices of dimension m and D = (dij) = CT X B . Then the element dij of matrix D at row i and column j is defined as dij = min (Ci, Bj) (i, j = 1, ..., m) where Ci and Bj are the ith. and the jth. element of C and B respectively.

Let R1 = A5T X A5, R2 = A5T X A4, R3 = A4T X A4, R4 = A4T X A3, R5 = A3T X A3, R6 = A3T X A2, R7 = A2T X A1, R8 = A1T X A1, R9 = A1T X A2, R10 = A2T X A2, R11 = A2T X A3.

The computation has been made as follows :- W = U Ri

Where W is a 5 × 5 matrix termed as weight matrix and U is the union operator. Through certain calculation W is obtained as follows:

$$W = \begin{matrix} 1 & 1 & 0.6 & 0 & 0 \\ 1 & 1 & 1 & 0.4 & 0 \\ 0.6 & 1 & 1 & 0.6 & 0.2 \\ 0.2 & 0.6 & 1 & 1 & 0.8 \\ 0 & 0.4 & 0.8 & 1 & 0.8 \end{matrix}$$

Using the Weight Matrix W, the forecasting model is defined as,

Ai = Ai - 1 . W, where Ai - 1 is the yield of year i - 1 and Ai is the forecasted yield of year i in terms of fuzzy sets and '.' is the max - min operator.

The forecasted output is interpreted which are actually all fuzzy sets. Now it is necessary to translate the fuzzy output into a regular number (equivalent scalar). This step is called defuzzification.

(1) If the membership of the output has only one maximum, the mid point of the interval corresponding to the maximum is selected as the forecasted value.

(2) If the membership of an output has two or more consecutive maximums, the midpoint of the corresponding conjunct intervals as the forecasted value.

(3) Otherwise, the fuzzy output is standardized and the midpoint of each interval is used to calculate the centroid of the fuzzy set as the forecasted value.

Table 8.2: Forecasting Error based on fuzzy logic

Actual Data	Forecasted Data	Forecasting Error(%)
562	540	3.9
564	540	4.2
566	540	4.5
568	540	4.9
570	540	5.2
563	540	4.0
556	540	2.8
549	540	1.6
542	528	2.5
535	528	1.3
520	528	1.5
505	504	2.9
490	504	2.8
475	492	3.5
460	480	4.3
471	480	1.9
482	480	0.4
493	492	0.2
504	492	2.3
515	492	4.4
508	504	0.7
501	504	0.5
494	492	0.4
487	492	1

8.8.2 Neural Network Implementation

Under neural network, a feed forward back propagation neural network is used which comprises of a 5 noded input layer, 5 noded output layer and 2 noded hidden layer.

Programs are developed to generate the estimated fuzzy output vector. After 10 iterations the said fuzzy output vectors are calculated. The value of the parameters is furnished in Table 2. It is to mention the M1 array be the matrix of weights from input to the hidden layer, M2 array be the matrix of weights from hidden to output layer, A array be the threshold value for jth hidden layer, K array be the threshold or bias to jth output layer as furnished in Table 2. The value of learning rate Bl is taken as 1.5 and Bh as 2.0. The momentum parameter α is taken as 0.7. The average error is 1.306 %

8.9 Conclusion

The average errors are 3.25% and 4.76% for linear and exponential equations using least square techniques respectively. That of fuzzy logic is 2.3 %.

The average error of neural network is 1.306 %. The estimated data based on neural network can be used for future prediction.