Chapter 6

Cosmological models with Holographic Dark Energy

6.1 Introduction:

As discussed earlier, the discovery of Riess *et al.* in 1998 and Perlmutter *et al.* in 1999 changed the landscape scenario of the universe that we see today. In addition to other observations it predicts that the current universe is accelerating. The origin of the acceleration is considered to be due to a mysterious component of matter with a negative pressure called dark energy (DE). In the literature it is mainly focused on theory [175, 176, 177], probes of DE [178] and on cosmological constant [179, 180]. The preferred candidate for DE is a small cosmological constant (Λ). A ΛCDM model is then constructed as an alternative approach to the DE problem. Although ΛCDM fits most observational data well, it suffers from two main shortcomings: (i) the low value of vacuum energy (ii) the cosmic coincidence problem. In order to address the above issues a constant Λ is replaced by a time varying Λ, resulting in a dynamical DE models. When cosmological observations are analyzed in Big Bang framework it predicts that 23% matter in the universe is due to dark matter (DM). DM and DE in some cases are considered independent but coupling between them may exist. DM and DE manifests through their gravitational action. From unified models in the cosmological substratum DM & DE may be considered as single entity. In the previous Chapters a MCG is considered which play a significant role in this direction in cosmological model building. Among the many different approaches to describe the dark cosmological sectors considerable attention is focused with holographic DE. According to Holographic principle, the number of degrees of freedom in a bounded system should be finite and related to the area of its boundary. Recently Holographic principle ([181]-[186]) is applied in cosmology ([187]-[198]) to track the dark energy content of the universe following the work of Cohen *et al.* [199]. Holographic principle is a speculative conjecture about quantum gravity theories proposed by G't Hooft. Fischler and Susskind [181, 182, 183] subsequently promoted the idea claiming that all the information contained in a spatial volume may be represented by a theory that lives on the boundary of that space. For a given finite region of space the volume may contain matter and energy within it. If this energy suppresses a critical density then the region collapses to a black hole. A black hole is known theoretically to have an entropy which is proportional to its surface area of its event horizon. A black hole event horizon encloses a volume, thus a more massive black hole have larger event horizon and encloses larger volumes. The most massive black hole that can fit in a given region is the one whose event horizon corresponds exactly to the boundary of the given region under consideration. The maximal limit of entropy for an ordinary region of space is directly proportional to the surface area of the region and not to its volume. The basic idea of a holographic dark energy in cosmology is that the saturation of the entropy bound may be related to an unknown ultra-violet (UV) scale Λ to some known cosmological scale in order to enable it to find a viable formula for the dark energy which may be quantum gravity in origin and it is characterized by Λ. The choice of UV-Infra Red (IR) connection from the covariant entropy bound leads to a universe dominated by black hole states. Cohen *et al.* [199] proposed that any state in Hilbert space with an energy E corresponds to Schwarzschild radius $R_s \sim E$ which is less than the IR cut off value L (where L is a cosmological scale). It may now be possible to obtain a relation between the UV cut-off $\rho_\Lambda^{1/4}$ and the IR cut off which eventually leads to a constraint $\left(\frac{8\pi G}{c^2}\right) L^3 \left(\frac{\rho_\Lambda}{3}\right) \leq L$ [200, 201] where ρ_Λ is the energy density corresponding to dark energy characterized by Λ. The holographic dark energy density is

$$\rho_\Lambda = 3c^2 M_P^2 L^{-2}, \tag{6.1}$$

where $M_P^{-2} = 8\pi G$. The present acceleration may be described in the case when $\omega_\Lambda = \frac{p_\Lambda}{\rho_\Lambda} < -\frac{1}{3}$. If one considers $L \sim \frac{1}{H}$ it leads to $\omega_\Lambda = 0$. However it is known that holographic cosmological constant model based on Hubble scale as IR cut off does not permit an accelerating universe. The holographic dark energy model based on the particle horizon as the IR cut off is unable to achieve an accelerating universe noted in Ref. [187]. Subsequently an alternative model of dark energy using particle horizon in closed model is proposed [202]. Later, Li [188] obtained an accelerating universe considering event horizon as the cosmological scale. The model is consistent with the cosmological observations. Thus a cosmological model consistent with observations may be admitted adopting the covariant entropy bound and choosing L as an event horizon.

6.2 Modified Chaplygin gas in FRW universe:

Using the metric (1.6) and the energy momentum tensor, the Einstein's field equation (1.5) can be written as

$$H^2 + \frac{k}{a^2} = \frac{1}{3M_P^2}\rho \tag{6.2}$$

where $M_P^{-2} = 8\pi G$. The energy conservation equation is given by eq. (1.12). Let us now define the following density parameters:

$$\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{cr}},\ \Omega_m = \frac{\rho_m}{\rho_{cr}},\ \Omega_k = \frac{k}{a^2H^2} \tag{6.3}$$

where $\rho_{cr} = 3M_P^2H^2$. Ω_Λ, Ω_m and Ω_k represents density parameter corresponding to Λ, matter and curvature respectively. We assume here that the origin of dark energy is due to a scalar field in order to obtain potential for the dark energy model. Using Barrow's scheme [203, 204], we obtain the following equations for homogeneous scalar field:

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) = \left(\frac{A}{B+1} + \frac{C}{a^n}\right)^{\frac{1}{\alpha+1}}, \tag{6.4}$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) = \frac{-\frac{A}{B+1} + B\frac{C}{a^n}}{\left(\frac{A}{B+1} + \frac{C}{a^n}\right)^{\frac{\alpha}{\alpha+1}}}, \tag{6.5}$$

where $n = 3(1+B)(1+\alpha)$ and C is the integration constant. The corresponding scalar field potential and its kinetic energy term are given by

$$V(\phi) = \frac{\frac{A}{B+1} + \frac{1-B}{2}\frac{C}{a^n}}{\left(\frac{A}{B+1} + \frac{C}{a^n}\right)^{\frac{\alpha}{\alpha+1}}}, \tag{6.6}$$

$$\dot{\phi}^2 = \frac{(B+1)\frac{C}{a^n}}{\left(\frac{A}{B+1} + \frac{C}{a^n}\right)^{\frac{\alpha}{\alpha+1}}}. \tag{6.7}$$

In a flat universe ($k = 0$), eq. (6.7) can be integrated which yields

$$\phi = \pm\frac{2}{\sqrt{n}}\ sinh^{-1}\left[\sqrt{\frac{C(B+1)}{A}}a^{-\frac{n}{2}}\right] \tag{6.8}$$

and the potential is given by

$$V(\phi) = \frac{\frac{A}{1+B} + \frac{A(1-B)}{2(1+B)}\ sinh^2\left(\frac{\sqrt{3(1+B)(1+\alpha)}}{2}\ \phi\right)}{\left(\frac{A}{1+B}\right)^{\frac{\alpha}{\alpha+1}}\ cosh^{\frac{2\alpha}{1+\alpha}}\left(\frac{\sqrt{3(1+B)(1+\alpha)}}{2}\ \phi\right)}. \tag{6.9}$$

It may be mentioned here that for a non flat universe it is not so simple to obtain ϕ in known form.

6.3 Holographic dark energy as MCG:

For a non-flat universe ($k \neq 0$), the holographic dark energy density given by eq. (6.1) becomes

$$\rho_\Lambda = 3c^2 M_P^2 L^{-2}, \tag{6.10}$$

where c is the speed of light and L is the cosmological length scale for tracking the field corresponding to holographic dark energy in the universe. The parameter L is

$$L = ar(t) \tag{6.11}$$

where $a(t)$ is the scale factor of the universe and $r(t)$ is relevant to the future event horizon of the universe. Using Robertson-Walker metric one gets ([189]-[197])

$$L = \frac{a(t)\ sinn\left[\frac{\sqrt{|k|}R_h(t)}{a(t)}\right]}{\sqrt{|k|}}, \tag{6.12}$$

where R_h represents the event horizon which is given by

$$R_h = a(t) \int_t^{\infty} \frac{dt'}{a(t')} = a(t) \int_o^{r_1} \frac{dr}{\sqrt{1-kr^2}}. \tag{6.13}$$

Here R_h is measured in r direction and L represents the radius of the event horizon measured on the sphere of the horizon. Using the definition of $\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{cr}}$ and $\rho_{cr} = 3M_p^2 H^2$, one can derive [198]

$$HL = \frac{c}{\sqrt{\Omega_\Lambda}}. \tag{6.14}$$

Using eqs. (6.12)-(6.13), we determine the rate of change of L which is

$$\dot{L} = \frac{c}{\sqrt{\Omega_\Lambda}} - \frac{1}{\sqrt{|k|}} cosn \left(\frac{\sqrt{|k|}\, R_h}{a(t)} \right), \tag{6.15}$$

where

$$\frac{1}{\sqrt{|k|}} cosn \left(\sqrt{|k|}\, x \right) = cos(x) \ [1, cosh(x)] \ for\ k = 1 \ [0, -1]. \tag{6.16}$$

Using eqs. (6.10)-(6.15), it is possible to construct the required equation for the holographic energy density ρ_Λ, which is given by

$$\frac{d\rho_\Lambda}{dt} = -2H \left[1 - \frac{\sqrt{\Omega_\Lambda}}{c} \frac{1}{\sqrt{|k|}} cosn \left(\frac{\sqrt{|k|}\, R_h}{a(t)} \right) \right] \rho_\Lambda. \tag{6.17}$$

The energy conservation equation is

$$\frac{d\rho_\Lambda}{dt} + 3H(1+\omega_\Lambda)\rho_\Lambda = 0 \tag{6.18}$$

which is used to determine the equation of state parameter

$$\omega_\Lambda = -\left(\frac{1}{3} + \frac{2\sqrt{\Omega_\Lambda}}{3c} \frac{1}{\sqrt{|k|}} cosn \left(\frac{\sqrt{|k|}\, R_h}{a(t)} \right) \right). \tag{6.19}$$

Now we assume that the holographic dark energy density may be replaced by a modified Chaplygin gas energy density. The corresponding energy density then obtained from eq. (4.1). The equation of state parameter becomes

$$\omega = \frac{p}{\rho} = B - \frac{A}{\rho^{\alpha+1}}. \tag{6.20}$$

To determine dark energy fields we use eqs. (6.4) and (6.5) in eqs. (6.19)-(6.20), the following expressions for A and C are obtained:

$$A = (3c^2M_P^2L^{-2})^{\alpha+1}\left[B + \frac{1}{3} + \frac{2\sqrt{\Omega_\Lambda}}{3c}\frac{1}{\sqrt{|k|}}cosn\left(\frac{\sqrt{|k|}\,R_h}{a(t)}\right)\right], \tag{6.21}$$

$$C = (3c^2M_P^2L^{-2})^{\alpha+1}\,a^n\left[1 - \frac{3B+1}{3(B+1)} - \frac{2\sqrt{\Omega_\Lambda}}{3(B+1)c}\frac{1}{\sqrt{|k|}}cosn\left(\frac{\sqrt{|k|}\,R_h}{a(t)}\right)\right]. \tag{6.22}$$

The scalar field potential becomes

$$V(\phi) = 2c^2M_P^2L^{-2}\left[1 + \frac{\sqrt{\Omega_\Lambda}}{2c}\frac{1}{\sqrt{|k|}}cosn\left(\frac{\sqrt{|k|}\,R_h}{a(t)}\right)\right], \tag{6.23}$$

and the corresponding kinetic energy of the field is given by

$$\dot{\phi}^2 = 2c^2M_P^2L^{-2}\left[1 - \frac{\sqrt{\Omega_\Lambda}}{c}\frac{1}{\sqrt{|k|}}cosn\left(\frac{\sqrt{|k|}\,R_h}{a(t)}\right)\right]. \tag{6.24}$$

Considering x $(= \ln a)$, we transform the time derivative to the derivative with logarithm of the scale factor, which is the most useful function in this case. We get

$$\phi' = M_P\sqrt{2\Omega_\Lambda\left(1 - \frac{\sqrt{\Omega_\Lambda}}{c}\frac{1}{\sqrt{|k|}}cosn\left(\frac{\sqrt{|k|}\,R_h}{a(t)}\right)\right)} \tag{6.25}$$

where $()'$ prime represents derivative with respect to x. Thus, the evolution of the scalar field is given by

$$\phi(a) - \phi(a_o) = \sqrt{2}M_P\int_o^{\ln a}\sqrt{\Omega_\Lambda\left(1 - \frac{\sqrt{\Omega_\Lambda}}{c}\frac{1}{\sqrt{|k|}}cosn\left(\frac{\sqrt{|k|}\,R_h}{a(t)}\right)\right)}\,dx. \tag{6.26}$$

6.4 Squared speed of sound:

For a closed universe model ($k = 1$), the equation of state parameter for dark energy given in eq. (6.19) reduces to

$$\omega_\Lambda = -\frac{1}{3}\left(1 + \frac{2}{c}\sqrt{\Omega_\Lambda}\, cosy\right) \tag{6.27}$$

where $y = \frac{R_H}{a(t)}$. The minimum value it can take is $\omega_{min} = -\frac{1}{3}\left(1 + 2\sqrt{\Omega_\Lambda}\right)$ and one obtains a lower bound $\omega_{min} = -0.9154$ for $\Omega_\Lambda = 0.76$ with $c = 1$. Taking variation of the state parameter with respect to $x = \ln\, a$, we get [198]

$$\omega'_\Lambda = -\frac{\sqrt{\Omega_\Lambda}}{3c}\left[\frac{1-\Omega_\Lambda}{1-\gamma a} + \frac{2\sqrt{\Omega_\Lambda}}{c}\left(1 - \Omega_\Lambda cos^2 y\right)\right], \tag{6.28}$$

where $\frac{\Omega_k}{\Omega_m} = \gamma a$. We now introduce the squared speed of holographic dark energy fluid as

$$v_\Lambda^2 = \frac{dp_\Lambda}{d\rho_\Lambda} = \frac{\dot{p}_\Lambda}{\dot{\rho}_\Lambda} = \frac{p'_\Lambda}{\rho'_\Lambda}, \tag{6.29}$$

where variation of eq. (6.20) w.r.t. x is given by

$$p'_\Lambda = \omega'_\Lambda \rho_\Lambda + \omega_\Lambda \rho'_\Lambda. \tag{6.30}$$

Using the eqs. (6.29) and (6.30) we get

$$v_\Lambda^2 = \omega'_\Lambda \frac{\rho_\Lambda}{\rho'_\Lambda} + \omega_\Lambda$$

which becomes

$$v_\Lambda^2 = -\frac{1}{3} - \frac{2}{3c}\sqrt{\Omega_\Lambda}\, cosy + \frac{1}{6c}\sqrt{\Omega_\Lambda}\left[\frac{\frac{1-\Omega_\Lambda}{1-\gamma a} + \frac{2}{c}\sqrt{\Omega_\Lambda}\left(1 - \Omega_\Lambda\, cos^2 y\right)}{1 - \frac{\Omega_\Lambda}{c}\, cosy}\right]. \tag{6.31}$$

The variation of v_Λ^2 with Ω_Λ is shown in fig. (6.1) for different y values. It is found that for a given value of c, a, γ, the model admits a positive squared speed for

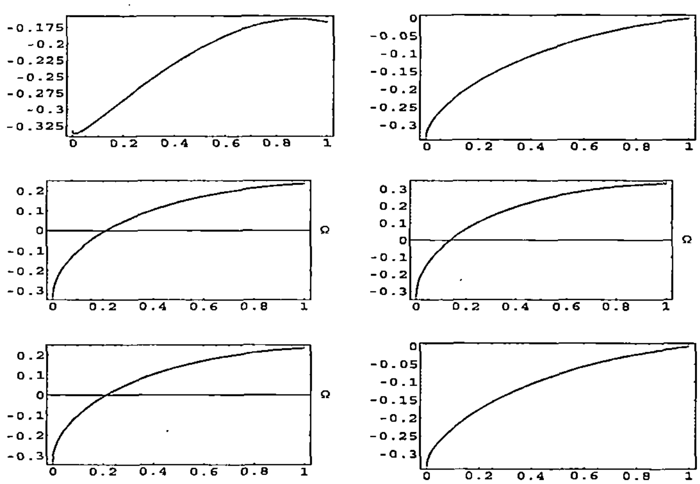

Figure 6.1: Variation of v_Λ^2 with Ω_Λ for different values of y at $c = 1$, $\gamma = \frac{1}{3}$ and $a = 1$, in the first array the figures are for $y = \frac{\pi}{3}$ and $y = \frac{\pi}{2}$, in the second array for $y = \frac{1.5\,\pi}{2}$, $y = \pi$ and in the third array for $y = \frac{2.5\,\pi}{2}$, $y = \frac{3\pi}{2}$.

$\Omega_\Lambda > 0$. However, Ω_Λ is bounded below otherwise instability develops. We note also that for $\frac{(2n+1)\pi}{2} < y < \frac{(2n+3)\pi}{2}$, (where n is an integer) no instability develops. We plot the case for $n = 0$ in fig. (6.1), it is evident that for $y \leq \frac{\pi}{2}$ and $y \geq \frac{3\pi}{2}$, the squared speed for holographic dark energy becomes negative which led to instability. But for the region $\frac{\pi}{2} < y < \frac{3\pi}{2}$ with $n = 0$ no such instability develops. It is also found that for $y = 0$ *i.e.*, in flat case the holographic dark energy model is always unstable [205].

6.5 Discussions:

The holographic dark energy model in FRW universe is studied with a scalar field equivalent to MCG. For a large energy density $\rho \to \infty$ *i.e.*, $a \to 0$ one obtains the following: (i) $V(\phi) \to \infty$ for $B \neq 1$, (ii) $V(\phi) \to 0$ for $B = 1$. However, for large size of the universe *i.e.*, $a \to \infty$ leads to $\phi \to 0$, the potential asymptotically attains a constant value $(\phi) \rightarrow \left(\frac{A}{B+1}\right)^{\frac{1}{1+\alpha}}$. We obtain the evolution of the holographic dark energy field and the corresponding potential in the framework of MCG in a non flat universe. For $B = 0$ and $\alpha = 1$, the EoS given by (1.3) reduces to the Chaplygin gas which was considered by Setare [206] to derive the fields of dark energy. It is also observed that inclusion of a barotropic fluid in addition to Chaplygin gas (which is modified CG) does not alter the form of potential and evolution of the holographic dark energy field but the parameter B in the equation of state varies as a^n where $n = 3(1 + B)(1 + \alpha)$. Thus the contribution of the holographic dark energy is more as $(B \neq 0)$ compared to the case $B = 0$ in Ref. [206]. Thus it is noted that although the form of the potential does not change due to addition of a barotropic fluid, it changes the overall holographic dark energy density. It is found that the holographic dark energy is stable for a restricted domain of the values of Ω_Λ in a closed model of the universe.

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[1]* We use EoS for modified Chaplygin gas as $p = B\rho - \frac{A}{\rho^\alpha}$ which is different from that used in the published paper $p = A\rho - \frac{B}{\rho^\alpha}$ for a consistent representation