Chapter 4

Observational Constraints on the B parameter of Modified Chaplygin gas as DE

4.1 Introduction

The discovery of late accelerating universe from different cosmological observations namely, high redshift surveys of SNe Ia [49, 50, 51, 52, 104], CMBR ([105]-[109]), WMAP ([110]-[114]) etc. posed a challenge to the theoretical arena of physics. The fields available in the standard model of particle physics fails to accommodate such a phase of acceleration in late universe in the framework of GTR. A modified matter sector of the Einstein gravity with exotic matter are taken up in the literature to understand this issue. The main problem with usual matter and scalar field is that it cannot permit a phase of the matter with pressure $p < 0$ and EoS parameter $\omega < -\frac{1}{3}$.

The new kind of matter/energy required to describe the accelerating expansion of the late universe is called dark energy. When cosmological observations are analyzed in the framework of Big Bang cosmology it is estimated that dark energy constitutes about 73% of the matter/energy of the universe.

In the literature, cosmological constant is considered as one of the candidates for dark energy which is uniformly distributed in the form of vacuum energy density. But cosmological constant is not suitable for describing late universe as it leads to cosmic coincidence problem [115, 116, 117]. A number of unusual fields namely, phantoms [118, 119, 120], tachyons [121, 122, 123], quintessence [124, 125, 126], K-essence [127, 128, 129], exotic matter etc. are considered in the literature. One of the promising candidates for exotic matter namely, Chaplygin gas (CG) which has positive energy density in the early universe with a negative pressure is considered recently to construct model of the universe accommodating late acceleration. But CG is ruled out by observations. Subsequently GCG and MCG are proposed to address the cosmological issues. Here cosmological models are discussed with MCG that are relevant from observational aspects. There are three parameters for MCG which are to be constrained from observations. Considering a dimensionless age parameter H_0t_0 [97] and $(H(z)-z)$ data [130], we analyze cosmological models with MCG and thereafter determine the EoS parameters.

The age parameter (H_0t_0) is a dimensionless quantity which is constant irrespective of the cosmological models. For simplicity we choose its standard value to be 0.95 (ignoring error). Using the constant age parameter we determine the effective ranges of values of the free parameters in this model. Subsequently using $(H(z)-z)$ data

Table 4.1: $(H - z)$ data from Wu P *et al.* [130]

z *Data*	H(z)	σ
0.09	69	± 12.0
0.17	83	± 8.3
0.27	70	± 14.0
0.40	87	± 17.4
0.88	117	± 23.4
1.30	168	± 13.4
1.43	177	± 14.2
1.53	140	± 14.0
1.75	202	± 40.4

we further determine the constraints on the parameters in terms of age parameter analysis. Using Hubble parameter *vs.* redshift data given in Table-(4.1) we analyze cosmological models. The χ^2 minimization technique is used here. There are nine data points of $H(z)$ at different redshift z which are used to constrain the EoS parameters of the MCG. Both Cold Dark Matter (CDM) and Unified Dark Matter Energy (UDME) models are considered in next sections. UDME model refers to the model in which the modified Chaplygin gas (MCG) may be regarded as dark matter and dark energy as a whole, where the total energy density comprises of radiation, baryon and MCG energy density. In the case of CDM model, the constituents of the universe are considered to be radiation, CDM and MCG.

4.2 Analysis of cosmological models

Using eq. (1.3) in eq. (1.12), the expression for the energy density of MCG can be expressed in terms of the scale factor of the universe $a(t)$, which is given by

$$\rho = \left[\frac{A}{1+B} + \frac{C}{a^{3n}}\right]^{\frac{1}{1+\alpha}} \tag{4.1}$$

where C is an arbitrary constant and we denote $(1+B)(1+\alpha) = n$. Equation (4.1) can be further re-written as

$$\rho = \rho_o \left[A_s + \frac{1 - A_s}{a^{3n}} \right]^{\frac{1}{1+\alpha}}, \tag{4.2}$$

where

$$A_s = \frac{A}{1+B} \frac{1}{\rho_o^{\alpha+1}}, \tag{4.3}$$

$$\frac{a}{a_0} = \frac{1}{1+z} \tag{4.4}$$

a_0 being the scale factor of the universe at the present epoch, we choose $a_0 = 1$ for convenience. It reduces to GCG model when we set $B = 0$. The Friedmann's equation obtained from eq. (1.9) and eqs. (4.2-4.4), can be expressed as

$$H(z) = H_0 \Big[\Omega_{r0}(1+z)^4 + \Omega_{j0}(1+z)^3 + (1 - \Omega_{r0} - \Omega_{j0})[A_s + (1 - A_s)(1+z)^{3n}]^{\frac{1}{1+\alpha}} \Big]^{\frac{1}{2}} \tag{4.5}$$

where H_0 is the present Hubble parameter. The above equation in terms of a is given by

$$H(a) = H_0 \left[\frac{\Omega_{r0}}{a^4} + \frac{\Omega_{j0}}{a^3} + (1 - \Omega_{r0} - \Omega_{j0})[A_s + \frac{1 - A_s}{a^{3n}}]^{\frac{1}{1+\alpha}} \right]^{\frac{1}{2}} \tag{4.6}$$

where $j = m$ for CDM model and $j = b$ for UDME model. The above equations corresponds to GCG model for $B = 0$. The deceleration parameter $(q_0 = -(\frac{a\ddot{a}}{\dot{a}^2})_{t_0})$ at the present time can be written as

$$q_0 = \frac{3}{2} \left[\frac{\Omega_{j0} + \frac{4}{3}\Omega_{r0} + (1+B)(1 - \Omega_{j0} - \Omega_{r0})(1 - A_s)}{\Omega_{j0} + \Omega_{Cg_0} + \Omega_{r0}} \right] - 1 \tag{4.7}$$

where $j = m$ for CDM model and $j = b$ for UDME model. The deceleration parameter can be estimated both in CDM and UDME model. For a flat universe we have

$\Omega_{j0} + \Omega_{Cg_0} + \Omega_{r0} = 1$ which will be used to measure the parameters in the next section. In the above Ω_{Cg_0} represents the present day modified Chaplygin gas energy density, Ω_{j0} is the present energy density of either Cold Dark Matter (in CDM model) or baryon energy density (in UDME model) and Ω_{r0} represents the present radiation energy density of our universe.

4.3 Age of the universe as a constraining tool

Let us consider the age parameter [97] given by

$$t_0 = \int_0^1 \left[\frac{da}{aH(a)}\right] \tag{4.8}$$

where $H(a)$ is given by eq. (4.6). The age of the universe in MCG model becomes

$$t_0 = \frac{1}{H_0}\int_0^1 \left[\frac{da}{aE\left(a, \Omega_{j0}, \Omega_{r0}, A_s, B, \alpha\right)}\right] \tag{4.9}$$

with

$$E(a, \Omega_{j0}, \Omega_{r0}, A_s, B, \alpha) = \frac{H(a)}{H_0} \tag{4.10}$$

and $H_0 t_0 = 0.95$. For a given value of α we plot the variation of A_s with B. We note the following: Fig. (4.1): shows variation of B with A_s for $\alpha = 0.01, 0.20, 0.39$ by dotted, dashed and thin lines respectively in CDM model. It is evident that as the value of A_s approaches 1 (0.97 to 1) for $0 \leq \alpha \leq 0.39$, the B parameter picks up positive values with a maximum 0.20.

Fig. (4.2): shows variation of B with A_s for α=0.01, 0.50 and 0.99 with thin, dotted and dashed lines respectively in UDME model. In this case as the value of A_s is increased from 0.7 to 1 it is evident that the B parameter picks up positive value up

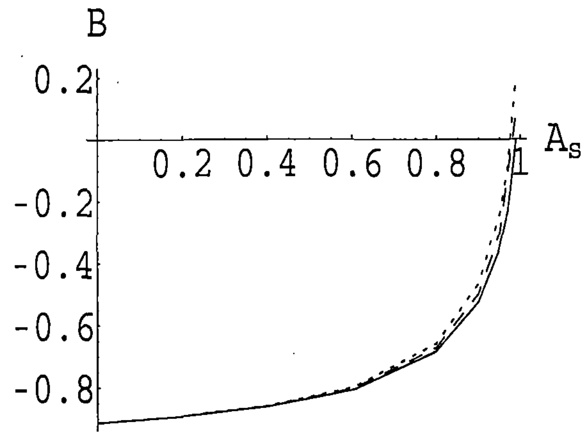

Figure 4.1: Variation of B with A_s for $\alpha = 0.01$ (Dotted line), $\alpha = 0.20$ (Dashed line) and $\alpha = 0.39$ (Thin line) in CDM model

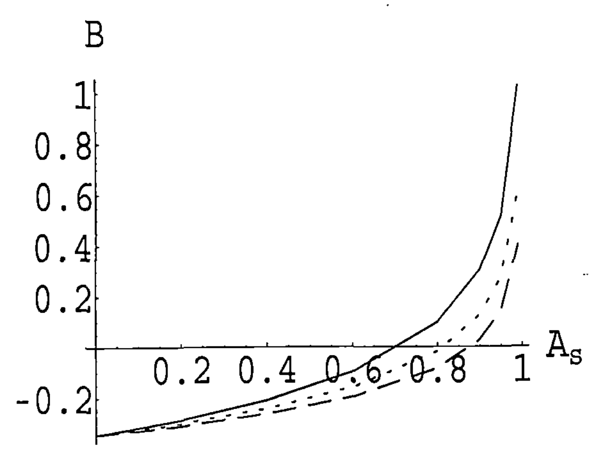

Figure 4.2: Variation of B with A_s for $\alpha = 0.01$ (Thin line), $\alpha = 0.50$ (Dotted line) and $\alpha = 0.99$ (Dashed line) in UDME model

to a maximum 1.02 for $0 \leq \alpha \leq 1$ in UDME model. The set of curves shown in fig. (4.1) and fig. (4.2) are useful to determine the range of values of B for both CDM and UDME models respectively. We note that in CDM model B lies between 0 to 0.20, whereas in UDME model B lies between 0 to 1.02. Moreover, in CDM model B is positive when $0 \leq \alpha \leq 0.39$ and A_s between 0.97 to 1. In UDME model we note that B is positive for $0 \leq \alpha \leq 1$ and A_s between 0.7 to 1.

4.4 $(H(z) - z)$ data as a constraining tool

For a flat universe containing only radiation, cold dark matter (or baryon) and the MCG, the Friedmann equation can be expressed as

$$H^2(H_0, A_s, B, \alpha, z) = H_0^2 E^2(A_s, B, \alpha, z) \tag{4.11}$$

where,

$$E = \left[\Omega_{r0}(1+z)^4 + \Omega_{j0}(1+z)^3 + (1 - \Omega_{r0} - \Omega_{j0})[A_s + (1 - A_s)(1+z)^{3n}]^{\frac{1}{1+\alpha}}\right]^{\frac{1}{2}} \tag{4.12}$$

with $j = m$ for CDM model and $j = b$ for UDME model. The best-fit values for model parameters A_s, B, α and H_0 can be determined by minimizing the following *Chi-square* function

$$\chi^2(H_0, A_s, B, \alpha, z) = \sum \frac{[H(H_0, A_s, B, \alpha, z) - H_{obs}(z)]^2}{\sigma_z^2}. \tag{4.13}$$

Since we are interested in determining the model parameters, H_0 is not an important parameter here. So we marginalize over H_0 to evaluate the probability distribution function for A_s, B, α as

$$L(A_s, B, \alpha) = \int \left[dH_0 P(H_0) \exp\left(\frac{-\chi^2(H_0, A_s, B, \alpha, z)}{2}\right)\right] \tag{4.14}$$

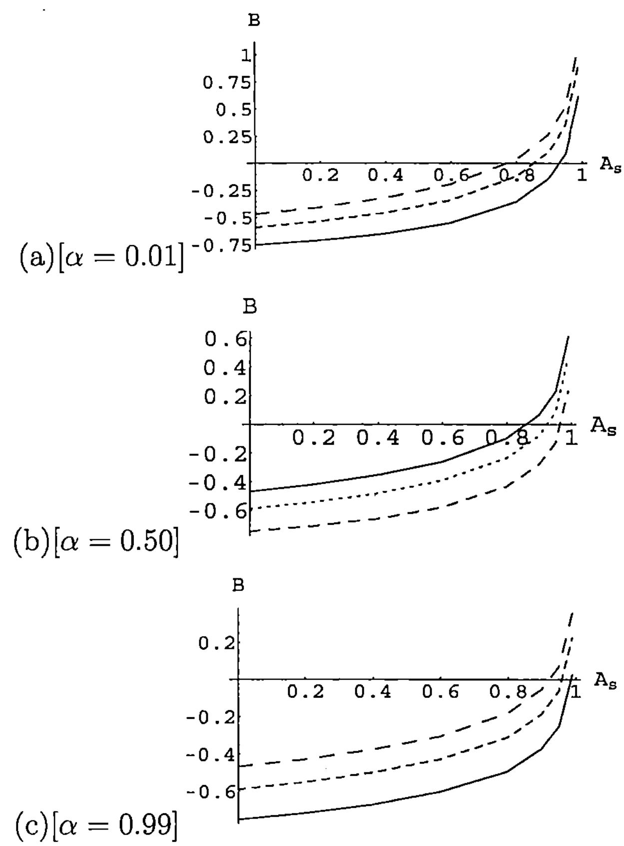

Figure 4.3: Constraints on EoS parameters in CDM model for (a) $\alpha = 0.01$, (b) $\alpha = 0.50$ and (c) $\alpha = 0.99$ using $H(z)$ *vs.* z data: 1σ, 2σ and 3σ levels are shown.

where $P(H_0)$ is the prior distribution function for the present Hubble constant. We consider Gaussian prior here with $H_0 = 72 \pm 8$ [100]. Minimizing χ^2 determines the maximum $L(A_s, B, \alpha)$ value. We determine the maximum value of the function $L(A_s, B, \alpha)$ at three different values of α to obtain a relation between B and A_s. Thus a relation between B and A_s for various α can be established by minimizing χ^2.

In CDM model, variation of B with A_s (related to A) for $\alpha = 0.01,\ 0.50,\ 0.99$ at 1σ, 2σ and 3σ levels respectively are shown in figs. (4.3 a - 4.3 c). It is observed that as the value of A_s tends to 1 we see that the B parameter picks up positive values (i) up

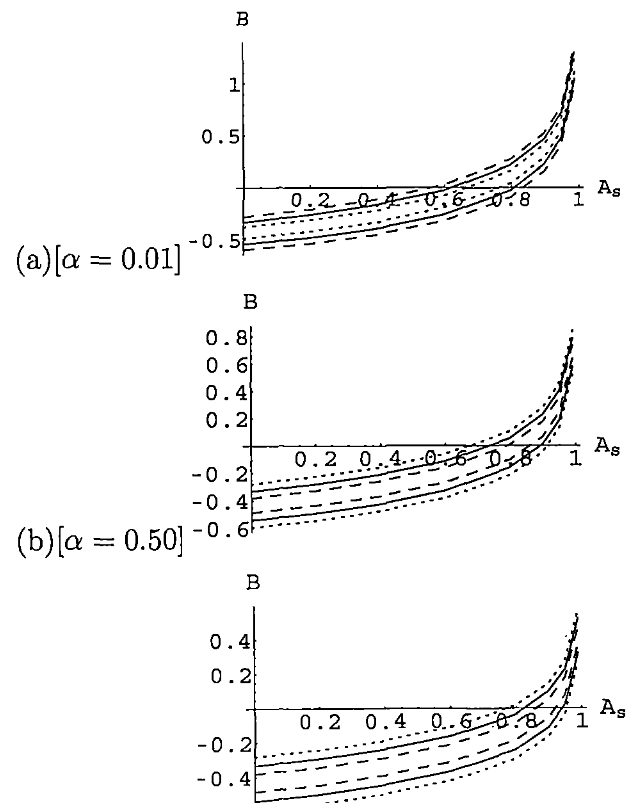

Figure 4.4: Constraints on EoS parameters in UDME model for (a) $\alpha = 0.01$, (b) $\alpha = 0.50$ and (c) $\alpha = 0.99$ using $H(z)$ *vs.* z data: 1σ, 2σ and 3σ levels are shown.

$Model$	$Data$	A_s	B	α
CDM	$Age - constraint$	(0.97, 1.0)	(0, 0.20)	(0, 0.39)
$UDME$	$Age - constraint$	(0.70, 1.0)	(0, 1.02)	(0, 1.0)

Table 4.2: Range of values of the EoS parameters in CDM and UDME model using age constraint

to 1.07 (fig. 4.3 a), (ii) up to 0.62 (fig. 4.3 b), (iii) up to 0.36 (fig. 4.3 c) at 3σ level in accordance with the $(H(z) - z)$ data [130]. Thus it is evident that as α increases, B decreases. In UDME model variation of B with A_s for $\alpha = 0.01,\ 0.50,\ 0.99$ at 1σ, 2σ and 3σ levels respectively are shown in figs. (4.4 a - 4.4 c). We note that as the value of A_s tends to 1, it is observed that the B parameter picks up positive values (i) up to 1.35 (fig. 4.4 a), (ii) up to 0.84 (fig. 4.4 b), (iii) up to 0.58 (fig. 4.4 c) at 3σ level in accordance with the $(H(z) - z)$ data [130]. Thus it is evident that as α increases, B decreases but compared to CDM model the variation of B parameter values is more for a given α and A_s in UDME model.

Thus in CDM model the range for B lies between 0 and 1.07 and in UDME model the range lies between 0 and 1.35 up to 3σ (*i.e.* 99.7% confidence) level. In CDM model B is positive only when A_s is within 0.76 to 1 for α between 0 to 1 and in UDME model B is positive (so, permissible) only when A_s is within 0.57 to 1 for α lying between 0 to 1.

4.5 Test of viability of the model

The best-fit values of the parameters for the cosmological models with MCG are determined using $(H(z) - z)$ data. The best-fit values of the parameters in the CDM model are $A_s = 0.99$, $B = 0.01$, $\alpha = 0.01$ and in UDME model $A_s = 0.80$, $B = 0.06$,

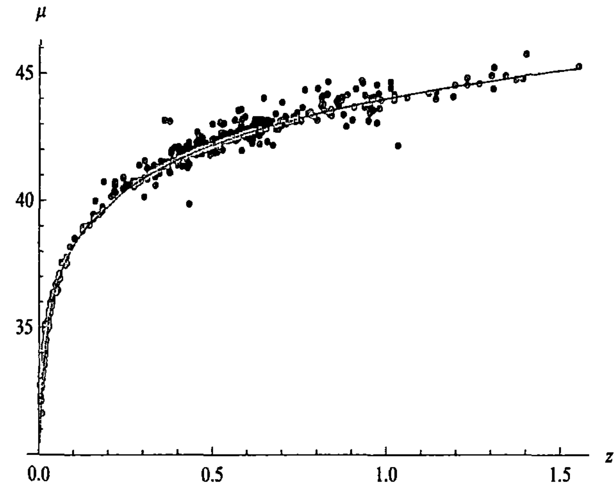

Figure 4.5: $\mu(z)$ *vs.* z curves for CDM model and union compilation data

$\alpha = 0.11$. In order to test the reliability we use the best-fit values to draw supernovae magnitudes (μ) at different redshift (z) in the two cases. We compare μ *vs.* z curves for the models with that of the original curve from union compilation data [102] (between those two parameters). Fig. (4.5) shows a plot of $\mu(z)$ *vs.* z obtained from the CDM model (the continuous line) with that obtained from union compilation data (the dots). Similar curves are drawn for the UDME model in Fig. (4.6) (continuous line for UDME model and dots for union compilation data). It is evident from the plots that both the CDM and UDME models are in excellent agreement with union compilation data.

4.6 Discussion

Cosmological models with modified Chaplygin gas contains three EoS parameters defined as: A_s, B and α are arbitrary. We determine the range of values of B

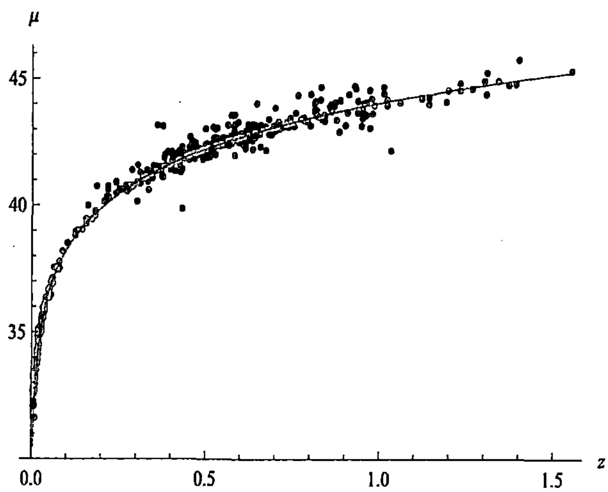

Figure 4.6: $\mu(z)$ *vs.* z curves for UDME model and union compilation data

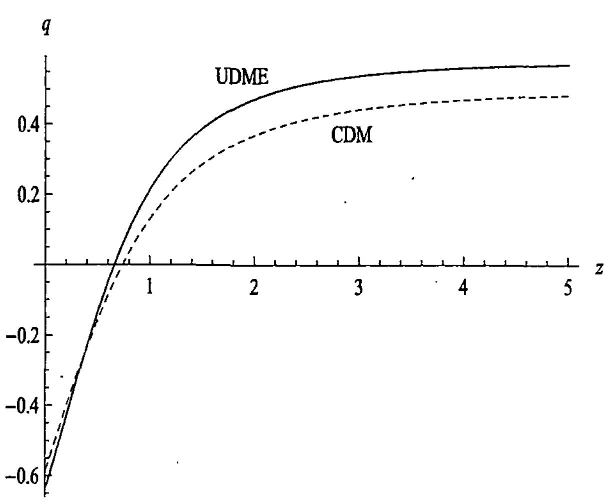

Figure 4.7: q *vs.* z curves for CDM & UDME model

$Model$	$Data$	CL	A_s	B	α
CDM	$(H-z)$	99.7%	(0.76, 1.0)	(0, 1.07)	(0, 1.0)
$UDME$	$(H-z)$	99.7%	(0.56, 1.0)	(0, 1.35)	(0, 1.0)

Table 4.3: Range of values of the EoS parameters in CDM and UDME model using $(H-z)$ data

Model	A_s	B	α
CDM	0.99	0.01	0.01
UDME	0.80	0.06	0.11

Table 4.4: Best-fit values of the EoS parameters in CDM and UDME model

parameter from the age constancy. In section (4.3) we plot B *vs.* A_s for different values of α in figs. (4.1) and (4.2). The figures are plotted for both positive and negative values of B. In the case of CDM model we note that B can pick up positive values up to 0.20 for the range of values: $0.97 \le A_s < 1$, $0 \le \alpha \le 0.39$. However, for UDME model we note that B can pick up positive values up to 1.02 for the range of values $0.7 \le A_s < 1$, $0 \le \alpha \le 1$ shown in Table-(4.2).

In section (4.4) we define *Chi-square* function and determine the constraints on B by minimizing the *Chi-square* function for the Hubble parameter *vs.* redshift data. For positive values of B a viable cosmology with MCG may be obtained. The constraints on B are: (i) $0 \le B \le 1.07$ for $0.76 \le A_s < 1$, $0 \le \alpha \le 1$ in CDM model and (ii) $0 \le B \le 1.35$ for $0.56 \le A_s < 1$, $0 \le \alpha \le 1$ in UDME model at 99.7% confidence limit as shown in Table-(4.3). For UDME model the range of values of B is found to be more than that of CDM model. If the age constant parameter is decreased then we note that the values of B permitted by CDM and UDME models are in agreement with that obtained from *Chi-square* minimization of the observed $H(z)$ *vs.* z data [130]. Consequently the limiting value of the age of our universe is pushed to lower values ($t < 13.6$ *Billion years*). The best-fit values of the parameters obtained here for CDM and UDME models are in agreement with union compilation data. We note that the best-fit values of our models are $A_s = 0.99$, $B = 0.01$, $\alpha = 0.01$ for CDM model and $A_s = 0.80$, $B = 0.06$, $\alpha = 0.11$ for UDME model as shown in Table-(4.4). Fig. (4.7) shows a plot of deceleration parameter q *vs.* z for the CDM and UDME model at the best-fit values. Both the model shows a transition from decelerating phase to accelerating phase at recent past. In the UDME model this transition occurs at lower redshift than CDM model ($z_{CDM} = 0.73$, $z_{UDME} = 0.66$). The magnitude of present acceleration is more in UDME model than CDM model ($q_{CDM}(0) = -0.57$, $q_{UDME}(0) = -0.65$).