Chapter 3

Observational Constraints on EoS parameters for a class of Emergent Universe

3.1 Introduction

Emergent universe scenario in the GTR which can be realized in the presence of a non-linear equation of state (in short, EoS) is discussed in the previous Chapter. There are two unknowns in the EoS namely A and B which are arbitrary constants. It is interesting to note that the non-linear EoS admits a cosmological model effectively with a composition of three different fluids for a given B. Mukherjee *et al.* [68] tabulated the composition of fluids for discrete values of B^* namely, $-\frac{1}{3}, 0, \frac{1}{3}, 1$. Cosmological model with $B = 0$ is very interesting as it can accommodate dust, exotic matter and dark energy.

In this Chapter EU is analyzed using observational data namely, Observed Hubble data (OHD) [99], SDSS data measuring a model independent BAO peak parameter [88] and WMAP 7 measurement of CMB shift parameter. The permissible range of values of A and B are determined from the observations. The evolution of density in each of the model from early to late era is also studied.

3.2 Field equations

The Hubble parameter H corresponding to a flat universe which may be obtained from the Einstein's field equation (1.9). The matter conservation equation is given by eq. (1.12). Using the EoS given by eq. (1.18) in eq. (1.12) one obtains:

$$\rho\left(z\right)=\left(\frac{A}{B+1}\right)^{2}+\frac{2AK}{\left(B+1\right)^{2}}\left(1+z\right)^{\frac{3(B+1)}{2}}+\left(\frac{K}{B+1}\right)^{2}\left(1+z\right)^{3(B+1)} \tag{3.1}$$

where z represents the cosmological redshift. The first term in the right hand side of eq. (3.1) is a constant which can be interpreted as cosmological constant and useful for describing dark energy. Eq. (3.1) can be re-written as:

$$\rho\left(z\right)=\rho_0+\rho_1\left(1+z\right)^{\frac{3(B+1)}{2}}+\rho_2\left(1+z\right)^{3(B+1)} \tag{3.2}$$

where $\rho_0=\left(\frac{A}{B+1}\right)^2$, $\rho_1=\frac{2AK}{(B+1)^2}$ and $\rho_2=\left(\frac{K}{B+1}\right)^2$ are the densities of the fluid components at the present epoch. The Friedmann equation (1.9) can be re-written in terms of redshift and density parameters as follows:

$$H^2\left(z\right)=H_0^2\left[\Omega_0+\Omega_1\left(1+z\right)^{\frac{3(B+1)}{2}}+\Omega_2\left(1+z\right)^{3(B+1)}\right] \tag{3.3}$$

where we define density parameter: $\Omega=\frac{8\pi G\rho}{3H_0^2}=\Omega\left(A,B,K\right)$. Fixing B one can re-write eq. (3.3) as:

$$H^2\left(H_0,A,K,z\right)=H_0^2E^2\left(A,K,z\right) \tag{3.4}$$

with $B = 0$, $B = \frac{1}{3}$, $B = 1$:
68.3%, 95.4% and 99.7% confidence regions are shown

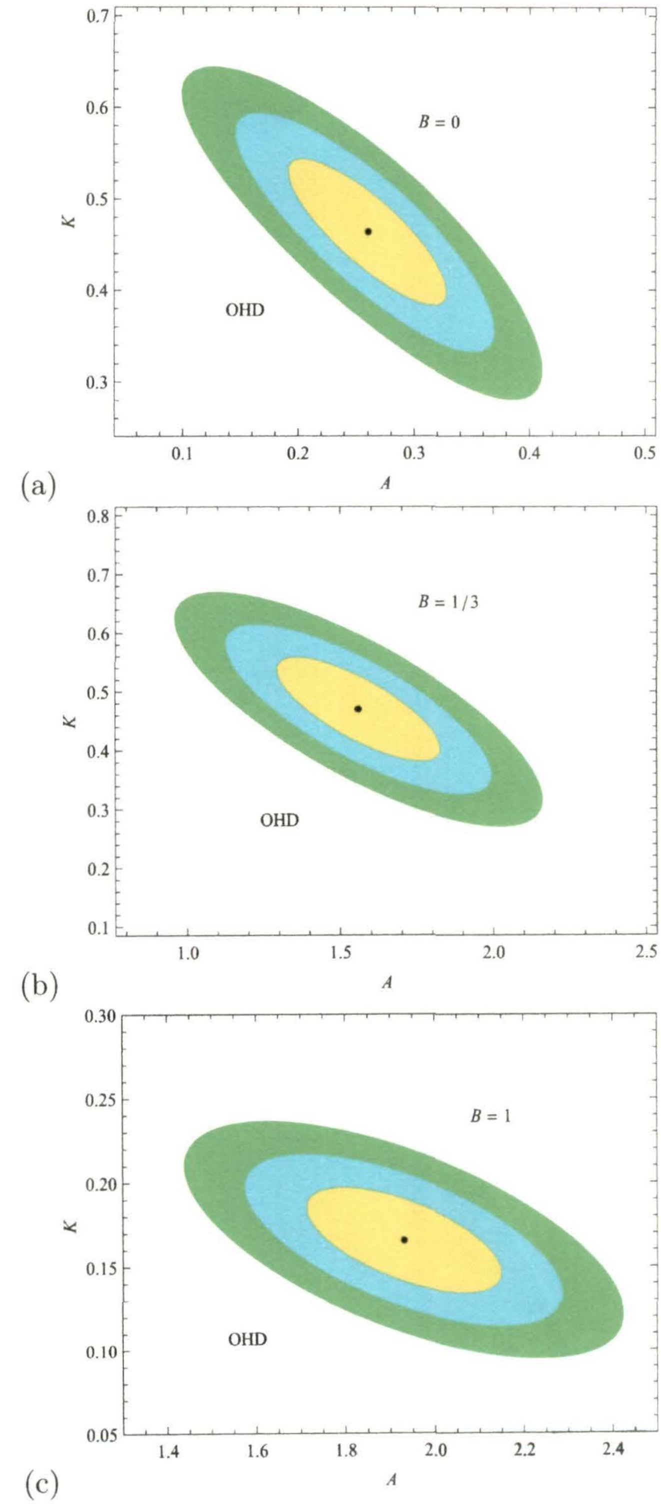

Figure 3.1: $K - A$ contours using OHD data for EU with $B = 0$, $B = \frac{1}{3}$, $B = 1$: 68.3%, 95.4% and 99.7% confidence regions are shown

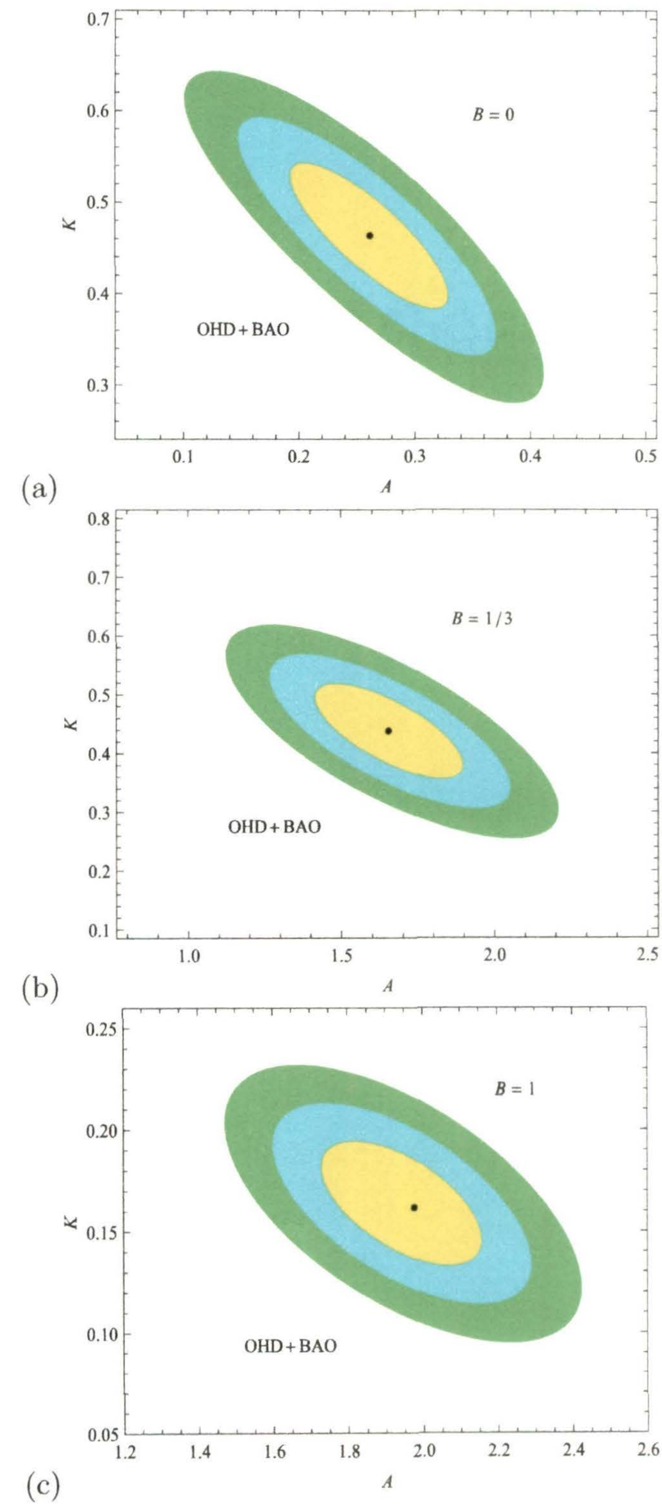

Figure 3.2: $K - A$ contours using OHD and SDSS (BAO) data for EU with $B = 0$, $B = \frac{1}{3}$ and $B = 1$: 68.3%, 95.4% and 99.7% confidence regions are shown

where,

$$E^2(A,K,z) = \Omega_\Lambda + \Omega_1(1+z)^{\frac{3(B+1)}{2}} + \Omega_2(1+z)^{3(B+1)}. \tag{3.5}$$

The constant part of the density parameter is represented by Ω_Λ, which corresponds to dark energy.

3.3 Analysis with observational data

In the next section we use data from OHD, BAO peak parameter, CMB shift parameter to analyze cosmological models.

3.3.1 Observed Hubble data (OHD)

Using observed value of Hubble parameter at different redshifts (*twelve data points listed in Observed Hubble data [99] shown in Table-(2.1)*) we analyze the Emergent Universe model in this section. Let us define a *Chi-square* function as follows:

$$\chi^2_{OHD} = \sum \frac{(H_{Th}(H_0,A,K,z) - H_{Ob})^2}{\sigma^2} \tag{3.6}$$

where H_{Th} and H_{Ob} are theoretical and observational values of Hubble parameter at different redshifts respectively and σ is the corresponding error. Here, H_0 is a nuisance parameter and can be safely marginalized. We consider $H_0 = 72 \pm 8$ [100] with a fixed prior distribution. A reduced *Chi-square* function (χ^2_{red}) can be defined as follows:

$$\chi^2_{red} = -2ln \int \left[e^{-\frac{\chi^2_{OHD}}{2}} P(H_0) \right] dH_0 \tag{3.7}$$

where $P(H_0)$ is the prior distribution.

For the numerical analysis we consider cosmologies for three different values of B

Model	A	K	χ^2_{min} (d.o.f)
$B = 0$	0.2604	0.4640	1.026
$B = \frac{1}{3}$	1.559	0.4702	0.737
$B = 1$	1.931	0.1656	0.818

Table 3.1: Best-fit values using OHD data

Model	CL	A	K
$B = 0$	68.3%	(0.1907, 0.3263)	(0.3807, 0.5461)
	95.4%	(0.1445, 0.3696)	(0.3287, 0.5980)
	99.7%	(0.0983, 0.410)	(0.2787, 0.6461)
$B = \frac{1}{3}$	68.3%	(1.299, 1.817)	(0.3781, 0.5606)
	95.4%	(1.124, 1.991)	(0.3212, 0.6175)
	99.7%	(0.9608, 2.160)	(0.2674, 0.6713)
$B = 1$	68.3%	(1.713, 2.144)	(0.1343, 0.1968)
	95.4%	(1.581, 2.290)	(0.1135, 0.2187)
	99.7%	(1.443, 2.425)	(0.0947, 0.2364)

Table 3.2: Range of values of the EoS parameters using OHD data

which are $0, \frac{1}{3}, 1$. We plot contours of A with K for different B. In drawing the above contours we consider positive values of A and K. The regions of 68.3%, 95.4% and 99.7% confidence level are shown in fig. (3.1 a), fig. (3.1 b) and in fig. (3.1 c) for $B = 0$, $B = \frac{1}{3}$ and $B = 1$ respectively. The best-fit values and the range of values of the parameters in this case corresponds to the OHD data which are tabulated in Table-(3.1) and Table-(3.2) respectively.

3.3.2 Joint analysis with BAO peak parameter

In this section we consider analysis that is independent of the measurement of H_0 and does not consider any particular dark energy model. For this a method proposed by Eisenstein *et al.* [88] is considered here. A model independent BAO

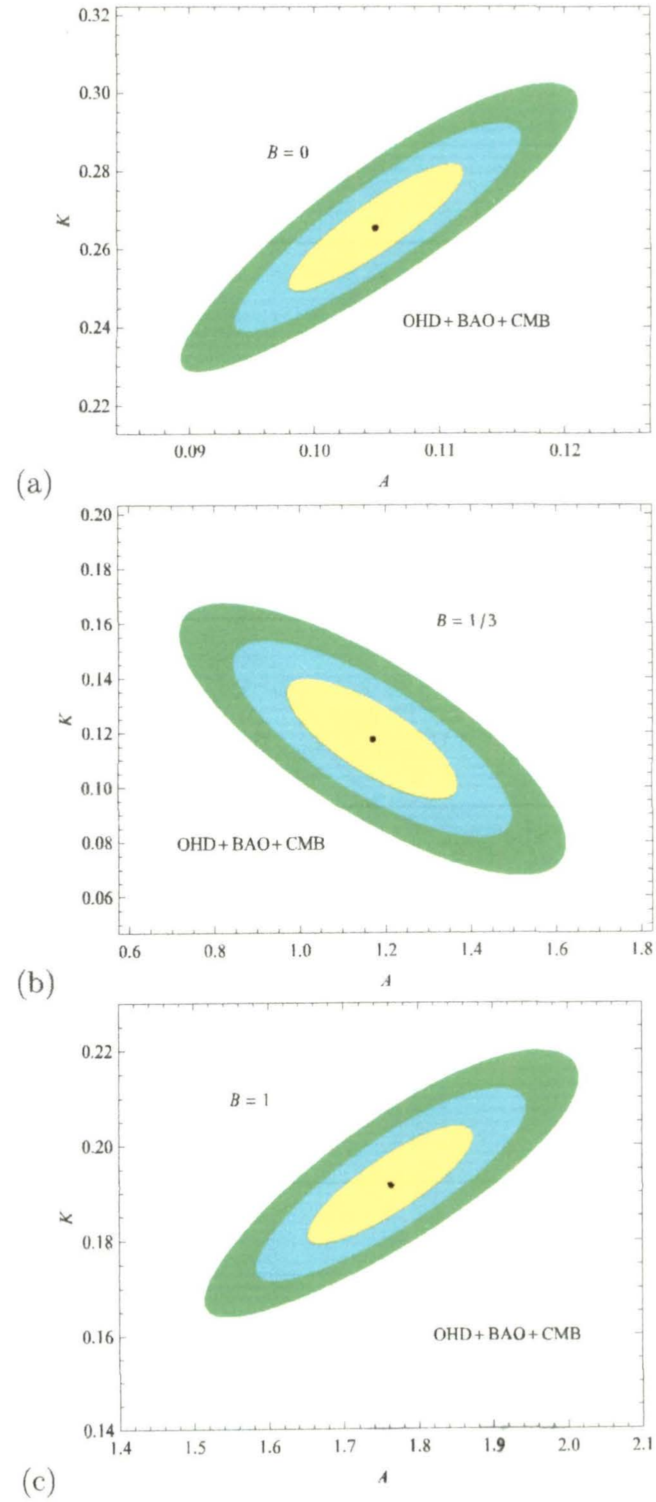

Figure 3.3: $K - A$ contours using OHD, SDSS (BAO) and WMAP 7 (CMB shift) data for EU with $B = 0$, $B = \frac{1}{3}$ and $B = 1$: 68.3%, 95.4% and 99.7% confidence regions are shown

Model	A	K	χ^2_{min} (d.o.f)
$B = 0$	0.2609	0.4636	1.168
$B = \frac{1}{3}$	1.6525	0.4383	0.707
$B = 1$	1.9768	0.1617	0.875

Table 3.3: Best-fit values using OHD + SDSS (BAO) data

(Baryon Acoustic Oscillation) peak parameter can be defined for low redshift (z_1) measurements in a flat universe as in eq. (1.30), where Ω_m is the matter density parameter for the Universe. The definition of *Chi-square* function is same as given in eq. (1.31). The measured value for $\mathcal{A}$ (0.469 ± 0.017) as was obtained in Ref. [88] from the SDSS data for LRG (*Luminous Red Galaxies*) survey is considered here. The total *Chi-square* function for joint analysis is defined as:

$$\chi^2_{tot} = \chi^2_{red} + \chi^2_{BAO}. \tag{3.8}$$

The 68.3%, 95.4% and 99.7% regions obtained from this joint analysis are shown in figs. (3.2 a), (3.2 b), (3.2 c) for $B = 0$, $B = \frac{1}{3}$ and $B = 1$ respectively. The best-fit values and the range of values of the parameters of importance for $B = 0$, $B = \frac{1}{3}$, $B = 1$ from the OHD+BAO data are tabulated in Table-(3.3) and Table-(3.4) respectively.

3.3.3 Joint analysis with OHD, BAO peak parameter and CMB shift parameter ($\mathcal{R}$)

In addition to OHD and BAO peak parameter we analyze cosmological models using CMB shift parameter ($\mathcal{R}$) which is given by eq. (1.34). The WMAP 7 data gives us $\mathcal{R} = 1.726 \pm 0.018$ at $z = 1091.3$ [93]. Thus the new *Chi-square* function

Model	CL	A	K
$B=0$	68.3%	(0.1921, 0.3277)	(0.3826, 0.5442)
	95.4%	(0.1459, 0.3710)	(0.3307, 0.5961)
	99.7%	(0.0998, 0.4114)	(0.2806, 0.6442)
$B=\frac{1}{3}$	68.3%	(1.413, 1.893)	(0.3541, 0.5187)
	95.4%	(1.266, 2.051)	(0.3003, 0.5726)
	99.7%	(1.119, 2.203)	(0.2524, 0.6175)
$B=1$	68.3%	(1.727, 2.156)	(0.1343, 0.1968)
	95.4%	(1.606, 2.285)	(0.1136, 0.2145)
	99.7%	(1.472, 2.419)	(0.0948, 0.2324)

Table 3.4: Range of values of the EoS parameters using OHD + BAO data

Model	A	K	χ^2_{min} (d.o.f)
$B=0$	0.1048	0.2654	1.341
$B=\frac{1}{3}$	1.169	0.1175	0.925
$B=1$	1.762	0.192	0.925

Table 3.5: Best-fit values using $OHD+SDSS$ $(BAO)+WMAP$ 7 (CMB shift) data

is defined as $\chi^2_{tot} = \chi^2_{(H-z)} + \chi^2_{BAO} + \chi^2_{CMB}$ which imposes additional constraints on the model parameters. The statistical analysis with the new *Chi-square* function χ^2_{tot} puts further tight bounds on the range of permissible values of A and K. In figs. (3.3 a), (3.3 b) and (3.3 c) we plot contours with $B=0$, $B=\frac{1}{3}$ and $B=1$ respectively at different confidence level. The best-fit values and the range of values of the parameters corresponding to $B=0$, $B=\frac{1}{3}$, $B=1$ obtained from the OHD+BAO+CMB data analysis are tabulated in Table-(3.5) and Table-(3.6) respectively .

3.3.4 Goodness of fit

In the above numerical analysis the values of χ^2 per degrees of freedom are determined. Generally, numerical value of χ^2 per degree of freedom should be 1. However, a better qualitative assessment may be obtained determining χ^2-probability as dis-

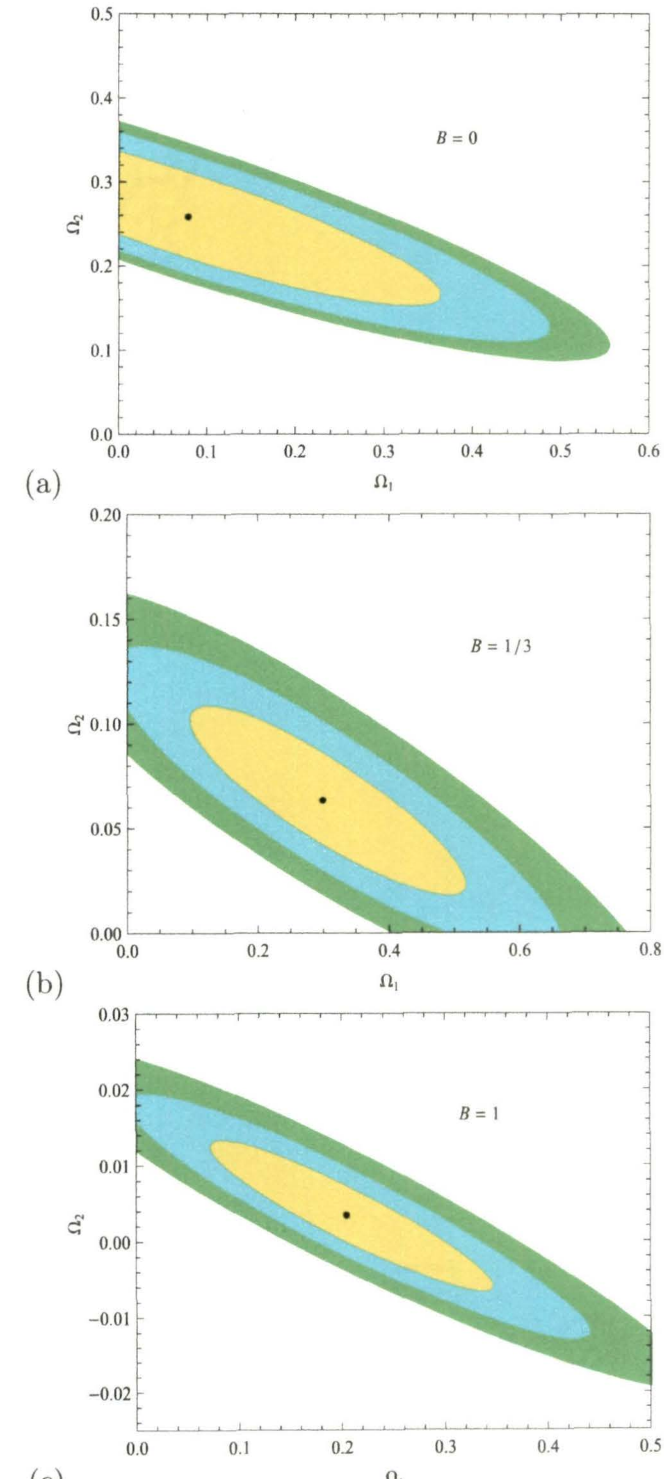

Figure 3.4: $\Omega_1 - \Omega_2$ contours for (a) $B = 0$, (b) $B = \frac{1}{3}$ and (c) $B = 1$: 68.3%, 95.4% and 99.7% confidence regions are shown.

Model	CL	A	K
$B=0$	68.3%	(0.0979, 0.1118)	(0.2488, 0.2818)
	95.4%	(0.0934, 0.1164)	(0.2384, 0.2922)
	99.7%	(0.0892, 0.1210)	(0.2284, 0.3021)
$B=\frac{1}{3}$	68.3%	(0.9657, 1.368)	(0.0952, 0.1401)
	95.4%	(0.8368, 1.497)	(0.0808, 0.1544)
	99.7%	(0.7157, 1.618)	(0.0672, 0.1688)
$B=1$	68.3%	(1.655, 1.873)	(0.1791, 0.204)
	95.4%	(1.581, 1.947)	(0.1715, 0.212)
	99.7%	(1.514, 2.016)	(0.1638, 0.2201)

Table 3.6: Range of values of the EoS parameters using $OHD+BAO+CMB$ data

Model	$P(OHD)$	$P(BAO)$	$P(CMB)$
$B=-\frac{1}{3}$	0.715	0.004	$0 << .001$
$B=0$	0.698	0.664	0.194
$B=\frac{1}{3}$	0.689	0.733	0.004
$B=1$	0.5721	0.562	0.520

Table 3.7: Goodness of fit

cussed in Ref. [103]. In the case of a model where χ^2 is a function of one variable say, x for n degrees of freedom, the probability distribution is given by:

$$P(n,x)=\frac{1}{\Gamma\left(\frac{n}{2}\right)}\int_{x/2}^{\infty}e^{-u}u^{n/2-1}du. \tag{3.9}$$

However, this strictly holds good for normally distributed errors. In the case of non-Gaussian distribution the probability P decreases. Generally, models with $P>0.001$ is considered acceptable. For different EU models, we tabulated P in Table-(3.7). Note that the EU model with $B=-\frac{1}{3}$ yields very poor fit with WMAP 7 data. So the above model fails the credibility test and cosmological model with $B=-\frac{1}{3}$ is ruled out by observations.

Model	Ω_1	Ω_2	Ω_Λ
$B=0$	0.079	0.259	0.662
$B=\frac{1}{3}$	0.299	0.063	0.638
$B=1$	0.205	0.004	0.791

Table 3.8: Best-fit values of density parameters

3.4 Density parameters in different EU model

In the previous section the parameters A and K are determined for $B = 0, \frac{1}{3}, 1$ respectively. In this section we study the evolution of density parameters. In fig. (3.4) contours are drawn on $\Omega_1 - \Omega_2$ plane. The different contours for 68.3%, 95.4% and 99.7% confidence level are also shown. Here Ω_2, Ω_1 represent dust and exotic matter for $B = 0$ which is shown in fig. (3.4 a). It is noted that at best-fit value $\Omega_1 + \Omega_2 = 0.338$ leads to $\Omega_\Lambda = 0.662$. It is also observed that $\Omega_\Lambda \approx 0.72$ and $\Omega_2 \approx 0.04$ are not ruled out within 68.3% confidence level. For $B = \frac{1}{3}$ in fig. (3.4 b), Ω_1 represents density parameter (DP) for cosmic strings and Ω_2 represents DP for radiation. EU model with $B = 1$ leads to a universe with a composition of dark energy (Ω_Λ), dust (Ω_1) and stiff matter (Ω_2) [68] which is plotted in fig. (3.4 c). The best-fit values for Ω_1 and Ω_2 for different models are obtained which in turn determines the best-fit values for Ω_Λ in the corresponding model as:

$$\Omega_\Lambda = 1 - \Omega_1 - \Omega_2. \tag{3.10}$$

The best-fit values for the density parameters of EU are tabulated in Table-(3.8).

3.5 Discussion

A class of EU models are permitted in GTR with a composition of three different types of fluid where one of the constituents is dark energy as shown by [68] for $B = -\frac{1}{3}, 0, \frac{1}{3}, 1$ respectively. The EoS parameters of the EU are constrained using the observed Hubble data (OHD) as well as using a joint analysis with the measurement of a BAO peak parameter. Using the definition of BAO peak parameter as proposed in [88] we analyze cosmological models. Also we obtain observational constraints on EoS parameter from the measurement of CMB shift parameter ($\mathcal{R}$) as predicted by WMAP 7. It is found from the numerical analysis that the case $B = -\frac{1}{3}$ cannot be fitted well with WMAP 7 data and hence the EU model corresponding to the above B value is ruled out. In the other cases for $B = 0$, $B = \frac{1}{3}$ and $B = 1$ one obtains cosmological models with physically realistic density parameters. Consequently density in $\Omega_1 - \Omega_2$ plane are plotted at 68.3%, 95.4% and 99.7% confidence level which are shown in fig. (3.4). The best-fit values for the model parameters A and K are determined and the corresponding contours are drawn at 68.3%, 95.4% and 99.7% confidence level for $B = 0$, $B = \frac{1}{3}$ and $B = 1$. Using OHD, $OHD + BAO$, $OHD + BAO + CMB$ data we plot contours between A and K, which are shown in the figs. (3.1), (3.2), (3.3) respectively. It is found that the model admits dark energy density close to that predicted by observations in ΛCDM cosmology.

[1]

[1]* Please note that the EoS for emergent universe expressed here as $p = B\rho - A\rho^{\frac{1}{2}}$ which is different from the convention used in the published paper $p = A\rho - B\rho^{\frac{1}{2}}$, this is done to keep similarity with modified Chaplygin gas EoS.