Chapter V:
Volatility Clustering:

Market Upheavals and Switching Behaviour: A GARCH Approach

5.1. Introduction:

In our study to offer behavioural explanations to Indian capital market upheavals, we have applied heterogeneous structural model. Traders are divided in the market in two heterogeneous groups on the basis of their respective expectation formulation models viz. Fundamental and technical traders. This approach departs from rationality driven behaviour in expectation formation, model decision makers as bounded rational. Prices in this model are driven by exogenous random news about fundamentals and evolutionary forces underlying the trading process itself. An evolutionary force is generated out of their interactions and generates adaptive belief favouring fitness of either of these two heterogeneous forecasting techniques. This evolutionary force switches itself between the groups conditioning upon actual price changes from fundamentals. (See equations nos.06, 07, 08, 09, 10, and 11 of chapter III).When adaptive belief is formed in favour of fundamentals, then the model expect rational solution. Otherwise when evolutionary belief is formed in favour of technical traders, then a self fulfilling prophecy is set into motion, arbitrage operation gets restricted and bubble grows. At this juncture, the model introduces socio psychological underpinnings of human behaviour as a source of generating evolutionary forces which tips balances between either fundamental or bubble solution. In such a way the model predicts changes in prices to switch itself irregularly between phases of low volatilities where price changes are small, and phases of high volatilities, where small price changes due to random news are reinforced and may become large due to trend following trading rules. Thereby, volatility clustering becomes a model endogenous phenomenon. Taking cue from this, we will be interested to study the presence of volatility clustering in Indian market. The evidence of which will reflect more structurally the influences of socio psychological underpinning of human interactions generating evolutionary forces resorting either to fundamental or bubble solution.

The phenomenon of clustering of volatility was observed and reported first by Mandelbrot in 1963 in commodity prices . Since the pioneering paper by Engle(1982) and Bollerslev (1986) on autoregressive conditional heteroskedastik (ARCH) models and their generalization to GARCH models , volatility clustering has been shown to be present in a wide variety of financial markets including stocks , exchange rates and interest rate securities markets (see Pagan A 1996, Broke 1997).Recently, a large number of heterogeneous agent models generating volatility clustering have been introduced in a wide range of literature and the trend is increasing (see Le Baron, Arthur and Palmer 1999,Lux and Marchesi 1999,2000,Kirman and Teyssiere 2002,De Grauwe and Grimaldi 2004,Gounersdorfer and Hommes 2007, Le Baron 2004, Amilon 2008,Corsi 2009).An interesting feature of these models is that , due to heterogeneity in expectations and switching between strategies , the deterministic skeleton that is the model with exogenous shocks shut off to zero, is a nonlinear dynamical system that exhibits periodic and even chaotic fluctuations in asset prices and returns. Thus, depending upon the initial state, different types of long run dynamical behaviour can occur. Particularly this group of models, furthermore, exhibit, coexistence of stable steady state and a stable limit cycle. Hence market prices , depending on initial condition, will either settle down to the locally stable fundamental steady state price, or converge to a stable cycle fluctuating in a regular pattern around the fundamental steady state. In the presence of dynamic noise, the market will then switch irregularly between close to 'fundamental steady state fluctuations' with small price changes, and 'periodic fluctuations' triggered by technical trading with large price changes. An important critique from 'rational expectation finance' upon heterogeneous agent models using simple habitual rule of thumb in forecasting is that 'irrational' traders will not survive in the market. This point has been extensively discussed by several scholars (Broke and Hommes 1997 and 1998).These papers emphasized more on the fact that, in an evolutionary framework, technical analysts are not irrational, but they are in fact bounded rational. In periods when prices deviate from rationality driven fundamental solution, chartists make better forecasts and earn higher profits. Basically these models are based on simple formalization of general ideas from behavioural finance, where markets are populated by different agents using trading strategies based on psychological heuristics. Moreover, the traditional benchmark rational expectation model is nested as a special case within this heterogeneous framework and provides a link between the traditional framework and the new behavioural approach in finance.

5.2. Volatility clustering: The Model[12]:

Let, R_t be the rate of return from a particular index from period t-1 to t.Also let ψ_{t-1} be the information set containing the realized values of all relevant variables up to the time t-1.Since investors know the information in ψ_{t-1} when they make their investment decision at time t-1, the relevant expected return and volatility to the investors are the conditional expected value of R_t, given ψ_{t-1} and conditional variance of R_t given ψ_{t-1} .These are denoted by m_t and h_t respectively. That is, $m_t = E(R_t / \psi_{t-1})$ and $h_t = \text{var}\, E(R_t / \psi_{t-1})$ .Given these definitions, the return series R_t can be defined as:

$$R_t = E(R_t / \psi_{t-1}) + \varepsilon_t = m_t + \varepsilon_t \text{ ....Eq. No (01)}$$

where, the unexpected return at time t is $\varepsilon_t = R_t - m_t$, ε_t is treated as a collective measure of news at time t. A positive ε_t (an unexpected increase in price) suggests the arrival of good news while a negative ε_t (an unexpected decrease in price) suggests the arrival of bad news. Further, a large value of $|\varepsilon_t|$ implies that the news is 'significant' or big in the sense that it produces a large unexpected change in price.

Now the conditional variance h_t can be modeled as a function of the lagged $\varepsilon's$ (Engle ,1982)where, the predictive volatility is assumed to be dependent on past news(Engel 1982). The most generalized model the author, developed is the q^{th} order *ARCH* model, below:

$$h_t = \omega + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 + \dots\dots\dots + \alpha_q \varepsilon_{t-q}^2 \text{ ....Eq. No. (02)}$$

where , $\varpi > 0$, $\alpha_1, \alpha_2 \dots\dots\dots\dots, \alpha_q \geq 0$ and $\varepsilon_t / \psi_{t-1} \sim N(0, h_t)$.The effect of a return shock i periods ago $(i \leq q)$ on current volatility is governed by the

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[12] See Karmakar. M (2005), "Modeling Conditional Volatility of Indian Stock Markets", *VIKALPA* • Vol. 30 • No. 3 • July– September.

parameter α_i. Normally, we would expect that $\alpha_i < \alpha_j$ for $i > j$. That is , the older the news , the less effect it has on current volatility. In a $ARCH(q)$ model, old news which arrived at the market more than q periods ago has no effect at all on current volatility. Alternatively, if a major market movement occurred yesterday, the day before or up to q days ago; the effect will be to increase today's conditional variance.

The $ARCH(q)$ model can be generalized the to the $GARCH(p,q)$ model as (see Bollerslev ,1986) :

$$h_t = \omega + \alpha_1\varepsilon_{t-1}^2 + \alpha_2\varepsilon_{t-2}^2 + \ldots\ldots + \alpha_q\varepsilon_{t-q}^2 + \beta_1 h_{t-1} + \beta_2 h_{t-2} + \ldots + \beta_p h_{t-p}, \ldots\ldots \text{Eq.No.(03)}$$

where $\omega > o, \alpha_1, \alpha_2 \ldots\ldots\ldots\ldots\ldots\ldots, \alpha_q \geq 0, \beta_1, \beta_2, \ldots\ldots\ldots\ldots\ldots\ldots \beta_q \geq 0$.

This $GARCH(p,q)$ process is stationary when

$(\alpha_1 + \alpha_2 + \ldots\ldots + \alpha_q) + (\beta_1 + \beta_2 + \ldots\ldots\ldots\ldots\ldots\ldots + \beta_q) < 1$.

Over the times, there have been numerous refinements of the approach in modeling conditional volatility to better capture the volatility clustering applying a variety of time periods and data around the globe. Of these models, GARCH (1,1) is considered by most studies to be an excellent model for estimating conditional volatilities for a wide range of financial data Gujrati and Bollerslev et. al 1992). This simplest but often very useful $GARCH(1,1)$ process is given by:

$$h_t = \omega + \alpha_1\varepsilon_{t-1}^2 + \beta_1 h_{t-1} \ldots\ldots\ldots\ldots \text{Eq.No.(04)}$$

where $\omega > 0, \alpha_1 \geq 0, \beta_1 \geq 0$. The stationary condition of $GARCH$ (1,1) is $\alpha_1 + \beta_1 < 1$.

In the $GARCH$ (1,1) model,the effect of a return shock on current volatility declines geometrically over time. As referred earlier , the $GARCH$ (1,1) model is found to be an excellent model for a wide range of financial data (Bollerslev et. al. 1992).

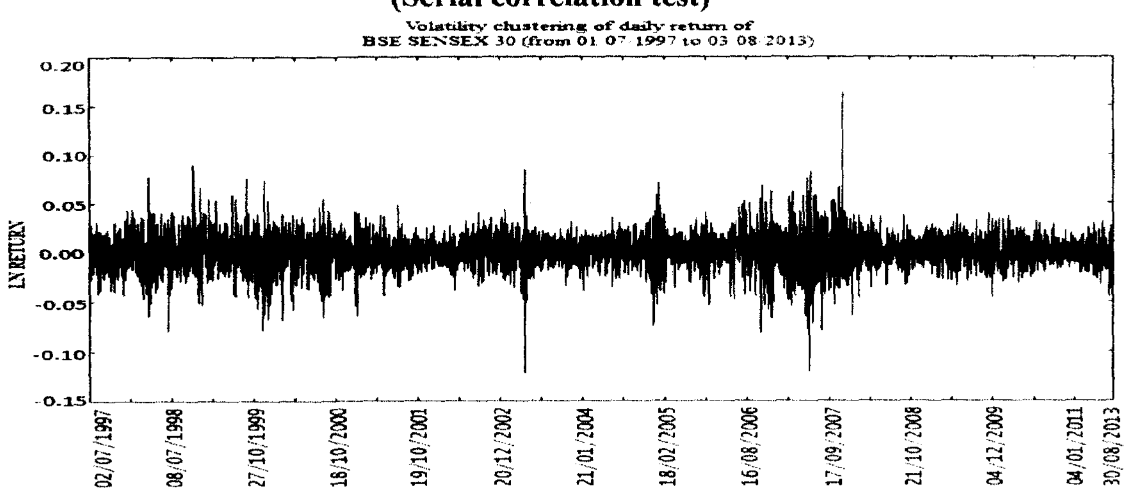
The sizes of the parameters α_1 and β_1 are the crucial determinant factors of short run dynamics of the resulting volatility time series. Large $GARCH$ lag coefficient β_1 indicate that shocks to conditional variance takes a long time to die out, so volatility is 'persistent'.Large *GARCH* error coefficient α_1 means that volatility react quite intensely to market movements and so if α_1 is relatively high and β_1 is relatively low , then volatilities tend to be more 'spiky'.In financial markets , it is common to estimate lag (or, persistence')coefficients based on daily observations in excess of 0.8 and error (or reaction) coefficient not more than 0.2.If $\alpha_1 + \beta_1$ is close to unity , then a 'shock 'at time t will persists for many future periods. A high value of $\alpha_1 + \beta_1$ therefore implies a 'long memory'. For $\alpha_1 + \beta_1 = 1$, we have what is known as an integrated *GARCH* process,. For integrated *GARCH* , the conditional variance is non stationary and the unconditional variance is unbounded.

5.3. Identification of conditional Volatility:

In this section we will try to fit appropriate GARCH model to find the presence, if any, of the time varying volatilities in Indian equity for the period ranging from 1.7.1997 to 30.8.2013

Figure 5.1 below is the graphical representation of daily return series of BSE SENSEX 30.

Figure: 5.1
(Serial correlation test)



From the above figures (figure 5.1) we can identify visually that, there are stretches of periods where the volatility is relatively low and in other periods the volatility is relatively high. This suggests an apparent volatility clustering in daily return series of BSE Sensex 30 in some periods. Statistically we would find a strong autocorrelation in squared returns (see table 5.1).

Initially here we are interested to compute the first order auto correlation coefficient in daily data on squared returns. To test joint hypothesis that all the serial correlations of the returns for lag one through k are simultaneously equal to zero, we will apply the modified Box Pierce statistics (Q), (Ljung and Box 1978).Mathematically the statistic is given as $Q = n(n+2)\sum r^2/(n-k)$, where n is the sample size and k the lag length (Ljung and Box 1978).In the large sample, the Q statistic is approximately distributed as the chi- square distribution with k degrees of freedom .In an application , if the computed Q/Q^2 exceeds the critical Q/Q^2 value from the chi-square table at the chosen level of significance, one can reject the null hypothesis that all r_k are zero; at least some of them must be non zero.

Table: 5.1
(Unit Root Test)

Statistic	Value
$Q(24)$	88.088 (P=0.0000)
$Q^2(24)$	817.735 (P=0.0000)

The values of Q/$Q^2(24)$ test statistic (Table 5.1) reject the joint hypothesis that all the serial correlations of squared returns for lags 1 through k are simultaneously equal to zero and thereby suggest the presence of serial autocorrelation in the return series under this study.

Table: 5.2

(Unit Root Test)

Series	Augmented Dickey-Fuller (t-Statistic)		Phillips-Perron (Adj. t-Statistic)		Dickey-Fuller (ERS) (t-Statistic)	
	Constant	Constant with linear trend	Constant	Constant with linear trend	Constant	Constant with linear trend
LN (At log Level)	-58.87618^	-58.87146^	-58.80847^	-58.80283^	-58.86011^	-58.58013^

(Note: critical values at 1% level are -3.431802 (constant) and -3.960336 (constant and linear trend) for Augmented Dickey-Fuller (t-Statistic) and Phillips-Perron (Adj. t-Statistic); critical values at 1% level are -2.565541 (constant) and -3.480000 (constant and linear trend) for Dickey-Fuller (ERS) (t-Statistic); ^ indicates rejection of null hypothesis at 1% level .)

Table:5.3
(Unit Root Test)

Exogenous: Constant
Lag length: 0 (Spectral GLS-detrended AR based on SIC, MAXLAG=30)
Sample: 7/01/1997 8/30/2013
Included observations: 3996

		MZa	MZt	MSB	MPT
Ng-Perron test statistics		-1987.45	-31.5214	0.01586	0.01322
Asymptotic critical values*:	1%	-13.8000	-2.58000	0.17400	1.78000
	5%	-8.10000	-1.98000	0.23300	3.17000
	10%	-5.70000	-1.62000	0.27500	4.45000

*Ng-Perron (2001)

Table: 5.4

(Unit Root Test)

Exogenous: Constant, Linear Trend
Lag length: 0 (Spectral GLS-detrended AR based on SIC, MAXLAG=30)
Sample: 7/01/1997 8/30/2013
Included observations: 3996

		MZa	MZt	MSB	MPT
Ng-Perron test statistics		-1986.05	-31.5112	0.01587	0.04695
Asymptotic critical values*:	1%	-23.8000	-3.42000	0.14300	4.03000
	5%	-17.3000	-2.91000	0.16800	5.48000
	10%	-14.2000	-2.62000	0.18500	6.67000

*Ng-Perron (2001)

We have employed ADF test (Said and Dicky ,1984) ,DF –GLS, Phillip – Peron Test(Phillips and Perron, 1988) and Ng Paren Test (Ng, and Perron, 2001) to estimate the order of integration of the return series under the study. The optimal lag order is searched and used in the Unit Root Test on the basis of either by AIC (Akaike, 1974), and SIC(Schwarz ,1978),.The results obtained confirmed that the series is I(0) at log level and hence can be used to measure volatility through GARCH in VAR framework.

5.4. Measuring the GARCH (1, 1) coefficients:

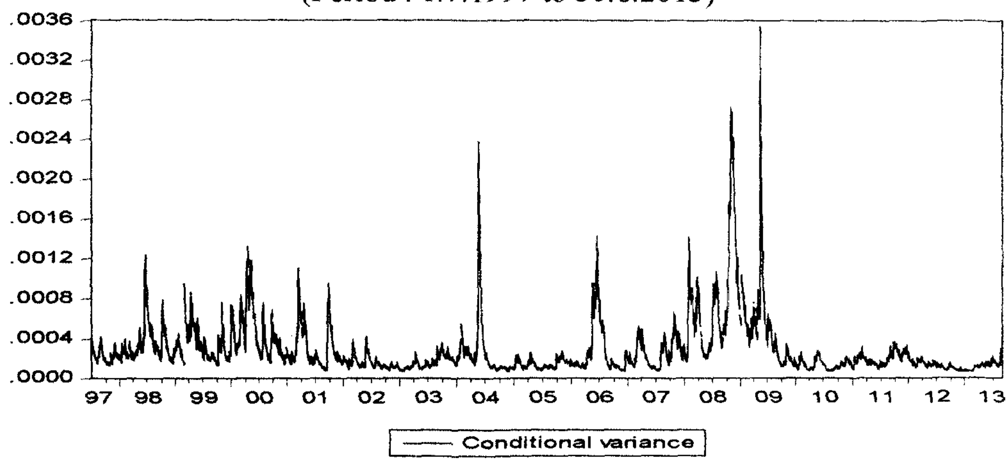
Once the presence of volatility clustering is evident in the series under the study, now, our focus is on determining the fitted GARCH model applicable to the return series. We, at the first step, run the $GARCH(1,1)$ on the daily data of return series of the period under review.The results so obtained are summarized in Table 5.5. We have estimated the parameters, viz.$\omega, \alpha_1 and \beta_1$, for the GARCH (1, 1) model and computed the series $\hat{h}_t$ for the BSE Sensex and plotted the series in figure 5.2.We have used the software, Microfit 5.0.

Table:5.5
Results on GARCH (1,1)

Dependent Variable: RETURN				
Method: ML - ARCH (Marquardt) - Normal distribution				
Sample (adjusted): 01/07/1997 30/08/2013				
Included observations: 3995				
Convergence achieved after 13 iterations				
Bollerslev-Wooldridge robust standard errors & covariance				
Presample variance: backcast (parameter = 0.7)				
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1)				
Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	1.001044	0.000215	4650.485	0.0000
AR(1)	0.023738	0.004903	4.841504	0.0000
	Variance Equation			
C	3.98E-06	1.00E-06	3.974784	0.0001
RESID(-1)^2	0.102345	0.014678	6.972796	0.0000
GARCH(-1)	0.886761	0.014787	59.96714	0.0000
R-squared	0.000901	Mean dependent var		1.000504
Adjusted R-squared	0.000651	S.D. dependent var		0.016577
S.E. of regression	0.016571	Akaike info criterion		-5.630946
Sum squared resid	1.096486	Schwarz criterion		-5.623071
Log likelihood	11252.82	Hannan-Quinn criter.		-5.628154
Durbin-Watson stat	1.904605			
Inverted AR Roots	.02			

From the results we found the presence of large GARCH lag coefficient (Table 5.5).The results also point to the fact that shocks to the conditional variance take a long time to die out, thus volatility is persistent in Indian equity market. Large value of GARCH lag coefficient along with a low value of GARCH error coefficient indicates larger conditional variances to cluster together. The study found similar inferences for the low values also.

Figure :5.2
Conditional variance of BSE daily return data
(Period : 1.7.1997 to 30.8.2013)



From the above figure 5.2 it is noticed that, estimated volatility is high for some periods and low for other periods. This attests the fact that large values of conditional volatilities are clustered together and so are the small values and shocks to the conditional volatility takes a long time to die out , hence, volatility is persistent in Indian asset market.

5.5. Diagnostics for GARCH (1,1) Model:

If the GARCH (1, 1) model perfectly describes the data, then standardized residuals should have zero mean and unit variance and be independently and identically distributed. Some diagnostic information on the estimation is presented in Table 5.6.

Table :5.6

Descriptive Statistics of standardized Residuals

Descriptive statistics	Value
Mean	-0.036717
Median	-0.008489
Maximum	7.326411
Minimum	-4.926539
Std. Deviation	0.999231
Skewness	0.008680
Kurtosis	5.332110

Figure : 5.3

Figurative Presentation of Standerised residuals from 1.7.1997 to 30.8.2013

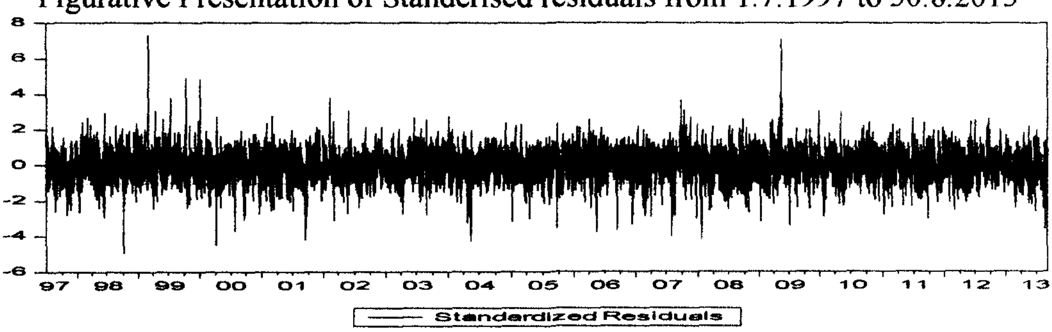


Figure: 5.4

ACF and PCF Plots of Standardized Residuals

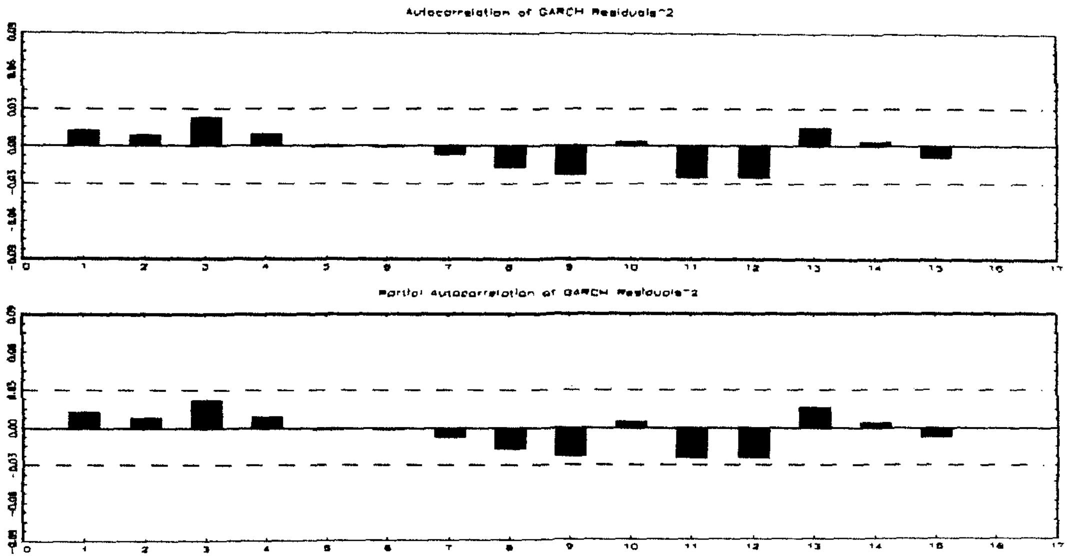


Table :5.7

ACF, PCF and Residuals

Included observations: 3996

Sample: 7/01/1997 8/30/2013

Lag	AC	PAC	Q-Stat	Prob
1	0.011	0.011	0.5144	0.473
2	0.008	0.008	0.7705	0.680
3	0.023	0.023	2.9229	0.404
4	0.009	0.008	3.2458	0.518
5	-0.001	-0.002	3.2506	0.661
6	-0.002	-0.002	3.2629	0.775
7	-0.007	-0.007	3.4351	0.842
8	-0.018	-0.018	4.7994	0.779
9	-0.025	-0.024	7.2295	0.613

10	0.003	0.005	7.2778	0.699
11	-0.026	-0.025	9.9943	0.531
12	-0.026	-0.025	12.797	0.384
13	0.015	0.016	13.735	0.393
14	0.003	0.004	13.765	0.467
15	-0.008	-0.007	14.040	0.523
16	0.001	0.001	14.048	0.595
17	0.001	-0.001	14.050	0.664
18	-0.016	-0.016	15.025	0.660
19	-0.012	-0.012	15.563	0.686
20	0.002	0.001	15.583	0.742
21	-0.001	-0.000	15.586	0.792
22	0.010	0.011	16.014	0.815
23	-0.020	-0.021	17.551	0.781
24	0.004	0.004	17.619	0.821
25	-0.036	-0.036	22.821	0.588
26	-0.007	-0.006	22.997	0.633
27	-0.013	-0.014	23.657	0.649
28	0.005	0.007	23.770	0.694
29	0.017	0.017	24.916	0.683
30	-0.008	-0.009	25.156	0.717
31	-0.009	-0.010	25.513	0.744
32	-0.003	-0.005	25.556	0.783
33	-0.023	-0.023	27.633	0.731
34	0.011	0.010	28.143	0.750
35	0.016	0.015	29.211	0.743
36	-0.003	-0.004	29.245	0.780

The mean and variance shown in the Table 5.6 are found to be -0.036717 and 0.999231 .The coefficients, Q-Statistics and the corelleogram of ACF and PCF (See table .5.7 and Fig.5.4)confirms the presence of no auto correlation in the residuals. In other words the residuals are random. This suggests that the GARCH (1, 1) model is an adequate description of the volatility process in Indian context.

5.6. Conclusions:

In the Indian equity market return series, we found that , the series is integrated at order zero which allows us to use GARCH process in the VAR framework to measure the clustering of volatility. The GARCH residuals are found to be random which validates the GARCH model applied on the series under our study. The GARCH coefficients like β_1 and α_1 are statistically significant and the values are 0.88(app.) and 0.10 (appx.) which clearly indicates the presence of 'long memory' and volatility would persist in future also. Low α_1 value describes the slow but converging reaction of the traders to the shocks or news. The results attests the results of Karmakar(2005,2013, Bhattacharyya and Ritolia, 2008). The existences of volatility clustering in Indian market fairly validate the major propositions of underlying behavioural dynamics of non linear heterogeneous model under the assumptions of bounded rationality. This pattern of price changes in Indian market also points towards "coexistence of attractors" in belief formation. Our findings also suggest that decisions of Indian investors are context specific. They are divided into heterogeneous groups in terms of their expectation formation rule and under the condition each groups are facing similar cognitive limitations. They are changing the sides definitely as we found the values of β_1 is very high which is indicative for the clustering of return volatility. Hence, the hypothesis of this study is grossly rejected.

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