

Chapter: IV

Feedback Behaviour, Market Upheavals and Extreme Value Theory

4.1. Introduction:

In heterogeneous structural model the essence of a speculative bubble is a sort of feedback mechanism, whereby rise in prices generate increased investor's enthusiasm and demand, shaping expectation for further rise. The high demand for the asset is a result of public memory of high past returns, and the optimism that those high returns would also generate in the future. This sort of feedback can amplify positive forces affecting the market, making the market to reach higher levels not explainable with fundamentals. Moreover, a bubble is not indefinitely sustainable. Prices cannot go up forever, and when price increases end, then increased demand for shares resulting from price increases also ends. Now, a downward feedback can replace the upward feedback. This kind of bubble theory requires that only past price changes produce an inconstancy to investor's judgments, not that they foolishly believe past increases must continue. The theory does not require that investors forecasting future price changes by some mechanical extrapolation rule, or those they are placing rulers to chart paper to forecast. It only requires that investor's observations of the past price changes alter the way they resolve the confusing array of conflicting information that they must all sift through in judging the market. More the prices consistently converges to the prediction of a type of forecasting rule, more it appears as intuitively fit in formulating further opinion and lesser the perception for risk in using it (see equation nos. 09, 10 and 11 of chapter III). Better the prediction of a particular rule, more its evolutionary force in generating adaptive belief in its favour and higher its weight in determination of price. In this way, if fundamentalists dominate then, market converges to rational solution. On the contrary, if technical traders dominate then more and more traders with their more wealth becomes inclined toward using technical trading rules and its weightage in determination of further price increases. In this situation evolutionary forces in the market set in favour of price to price feedback and ultimately results into bubble formation either positive or negative. These phenomenons tend to be reflected in the presence of extreme increases and decreases in market prices constituting return distribution having fat tail which ultimately produces leptokurtic shape of the distribution. In this backdrop, we

will attempt to use Extreme Value Theory to identify and measure the presence of extreme movements in Indian stock market for the period under review. According to the prediction of heterogeneous market model under rationality, if we ignore the fundamental solution of the asset price, the other may be the bubble solution. Bubbles are in effect collection of infinite number of bounded rationality models which may help to explain non random movement of asset price and its impact on correlated errors on long memory properties of the market (Sims 1980;Lillo and Farmer, 2004;Alfarano and Lux,2005;Farmer et.al.2006, Farmer and Geanakoplos,2008).

Though extreme value theory is widely used in climatology and hydrology [de Hann1990,] currently financial economists interested in banking, insurance [Mcneil 1997, Embrechts and Klupperbag, Mikosch, 1997], stock and exchange markets that are exposed to catastrophic loss are using extensively this tool to estimate and manage tail related risk. Earlier studies of Mandelbrot [1963] and Fama [1965] based on commodity and stock markets revealed that the logarithmic returns are far from normal and suggested that they might be drawn from Levy distribution⁸. Later Engle [1982] proposed an alternative autoregressive model, dubbed ARCH , that could both exhibit the predictability in volatility while retaining the zero mean for the returns reflecting the absence of arbitrage opportunities. The studies by Lillo et.al.[2005], Lillo and Farmer [2004], Farmer et.al.[2004] also attest the hypothesis that probability of extreme movements is more frequent relative to if the distributions were normal. Hence, fat tails and temporal dependence of second moment leading to “clustering of volatility” particularly to be analyzed in detail by those who are not exclusively interested on minimizing a quadratic loss around the mean to the neglect of possibility and consequences of extreme events occurring. Some commendable research showing relevance of EVT in risk management based on experiences of different markets around the world are:the studies of Gencay et.al.(2003),Danielson and Morimoto(2003),LeBaron and Samanta (2004), Tolikas and Barron (2005), Gettingby et.al.(2006) etc. Available information suggests that, till date, very few

⁸ In probability theory and statistics, the Levy distribution, named after Paul Lévy, is a continuous probability distribution for a non-negative random variable. Paul Pierre Lévy (15 September 1886 – 15 December 1971) was a French mathematician who was active especially in probability theory, introducing martingale and Lévy flight.

studies using Extreme Value Theory are available on Indian experiences. Most of the remarkable studies carried on Indian context were targeted towards estimation and management of extreme losses based on EVT (Karmakar,2013; Bhattacharyya and Ritolia, 2008 ;Sarma ,2002). But no such studies have been carried so far to offer behavioural interpretations of extreme losses and gains in Indian context. Hence, findings of the present thesis may add to our knowledge about the behavior of this emerging market that enjoys tremendous focus of foreign portfolio investors.

4.2. Data and its properties:

The study is based on daily return data of Indian stock market for a period ranging from 1.7.1997 to 31.08.2013 We considered BSE SENSEX that consists of 30 most popular shares and account for nearly forty percent of market capitalization. Inadequate data may pose some problem as only few points may qualify for extreme observations thus any meaningful study on long memory may not be possible. Altogether our data series consists of 3995 observations and covers a period more than 12 years that may be considered suitable to study the behavior of the Indian market. We analyze the continuously compounded rates of return:

$$r_t = \ln\left(\frac{S_t}{S_{t-1}}\right), \text{ where } S_t \text{ denotes the stock index in day } t.$$

4.3. Exploratory Statistical Analysis:

The Table 4.1 summarizes the results on distributional pattern of return in Indian context for the total period under study. Especially we will emphasise on the form of the distribution of price changes since it provides descriptive information concerning the nature of the process generating price changes

Table :4.1**Summary Statistics of Indian Capital market(daily log return 1.7.1997 to 30.8.2013)**

Statistics	values
Mean	0.00036
Median	0.00099
Minimum	-0.11
Maximum	0.15
Standard deviation	0.016
C.V.	45.18
Skewness	-0.09
Ex. kurtosis	5.53
Standard Error of skewness	0.039
Standard error of kurtosis	0.077
Jarque bera test	test statistic: 5107.4901 p-Value: 0.0000

The single strongest feature that emerges out of this preliminary analysis is the non normality of return distribution. The series is clearly leptokurtic indicating presence of fat tail in empirical distribution. The above results tend to indicate towards the presence of a distribution which is more peaked in the centre with longer tails than normal distribution. If the tails of the empirical frequency distribution is longer than those of the normal distribution, the slopes of extreme tail areas of normal probability graphs should be lower than those in the central parts of the graphs. The graph in general take the shape of an elongated 'S' with the curvature at the top and the bottom varying directly with excess relative frequency in the tails of empirical

distribution. This tendency for the extreme tails to show lower slopes than the main portion of the distribution will be accentuated by the fact that the central bell of empirical frequency distribution are higher than those of normal distribution. In this situation central portion of the normal probability graph should be steeper than the one which would be in the case if underlying distribution was strictly normal. The findings are thus in contrary to the proposition of ongoing paradigm where normality of return distribution is widely accepted underlying phenomenon of market behaviour and which in turn confirms the notion of random movement in speculative asset prices. In an efficient market setting, however, leptokurtosis of returns could result only from similar leptokurtosis in the news arrival process and is therefore, explained by the statistical distribution of the news. According to the existing paradigm in market equilibrium; volatility should be caused by new information while it is difficult to measure new information. As the arrival of new information cannot be predicted successive price changes ought to be random. Information arrives in the market infrequently and thus large movements in prices will be rare. Ordinarily, price will move within a narrow band due to investor's liquidity needs or portfolio rebalancing consideration. However many recent studies based on both long and short time intervals suggest that the correlation between volatility and news is weak [Cutler 1989, R.Engle and J.Rangel 2005, R.F.Engle et.al 2006].

Admittedly above analysis is a weak measure of departures from Gaussian statistics and we will look for a somewhat sharper characterization of empirical distribution that has emerged from recent applied literature.

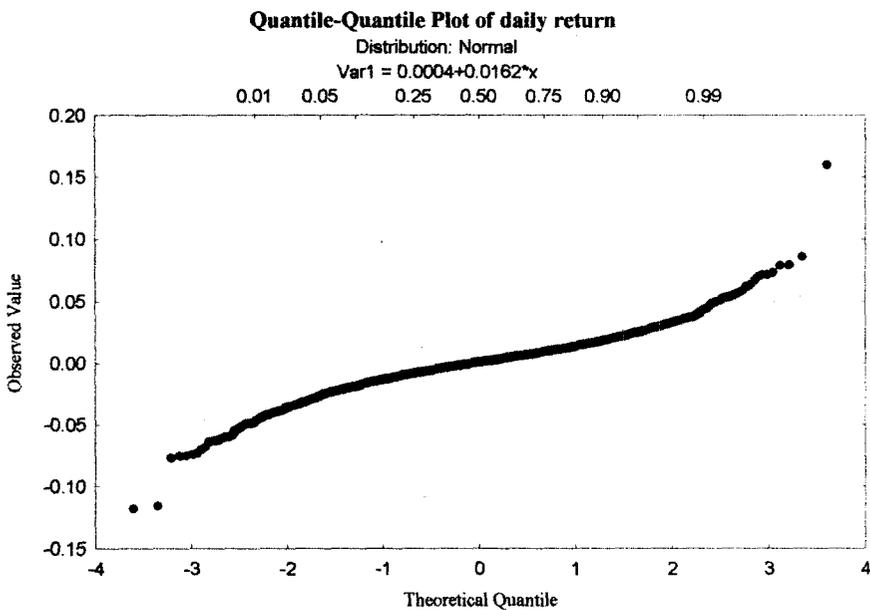
A more sensitive tool for examining deviations from normality is Q-Q graph. The QQ-plot against the Normal distribution is a widely used technique to measure heavy – tailed ness of a series. It examines visually the hypothesis that the returns come from Normal distributions, i.e. from a distribution with medium sized tail. The quantiles of the empirical distribution function on the X axis are plotted against the quantiles of distribution function on the Y-axis. The plot is

$$\left\{ X_{k,n}, F^{-1} \left(\frac{n-k+1}{n+1} \right), k = 1 \dots n \right\} \dots \dots \dots \text{Eq. No(01)}$$

where, X_1, X_2, \dots, X_n be a succession of random variables that are independent and identically distributed (iid), and $X_{(1)}, \dots, X_{(n)}$ the order statistics, F_n being the empirical distribution. Note that $F_n(X_{(k)}, n) = (k-1)/n$ and F is the estimated parametric distribution of the data.

If the parametric model fits the data well, this graph must have a linear form. Thus, the graph helps to compare various estimated models and choose the best. The more linear the Q-Q plots, the more appropriate the model in terms of goodness of fit. Also, if the original distribution of the data is more or less known, the Q-Q plots can help to detect outliers; (Embrechts, Kluppelberg, and Mikosch, 1997). Finally, this tool makes it possible to assess how well the selected model fits the tail of the empirical distribution. For example, if the series is approximated by a normal distribution and if the empirical data are fat-tailed, the graph will show a curve on the top at the right end or to the bottom at the left end.

Figure: 4.1



A concave departure from the ideal shape as in the case of India [See Figure-4.1] indicates a heavier tailed distribution. Central part of the distribution (Fig-4.1) aligns well with our expectations of normal distribution; however outside this area the curve in the tail indicates departure from normality i.e. a stronger concentration around mean, more probability mass in the tails of the distribution and

thinner shoulders. Specifically, we will try to identify and measure these extreme observations in Indian market which ultimately vitiates the return distribution from normality, that is, departs from the traditional “Mean - Variance” framework in expectation formation.

4.4. Extreme Value Theory [EVT]⁹.

Extreme Value Theory considers extreme events, provides a classification of continuous distributions according to the behavior of the tail region or their extreme realizations. The theory distinguishes three limiting stable distributions for the maximum values of a random variable, called Generalized Extreme Value Distributions [GEV], and the three associated Generalized Pareto Distributions [GPD] which are the limiting distributions for the tail region. The central limit theorem suggests that the limiting distribution of the sample mean is normally distributed. Whereas EVT proposes that the limiting distribution of sample maximum is an extreme value distribution and for a wide class of severity distribution which exceeds high enough threshold the GPD holds true [Balkema & de Haan theorem 1974, Pickands 1975].

4.4.1. GEV: Limiting Distributions for Extrema:

Let us consider a stationary sequence of i.i.d. variables¹⁰ $\{x_i\}_{i=1}^N$ with a common distribution function $F(x)$. By dividing the entire data-set into L non-overlapping sub-samples, and taking the maximum M_j from every sub-sample, we will end up with a subset of maxima $\{M_j\}_{j=1}^L$ (the so-called block maxima). It turns out that the distribution of maxima converges to one of the three distributions known as the extreme value distributions as suggested by Fischer and

⁹ See Alfarano, S. and Lux, T. (2010) Extreme Value Theory as a Theoretical Background for Power Law Behavior. Kiel Institute for the World Economy, Working Paper No. 1648, September 2010.

¹⁰ The same limiting distribution is obtained if the i.i.d. hypothesis is relaxed. Bermei (1963) shows the same result stand if the variables are correlated and if the series of squared correlation coefficients is finite. Assumption of independence is less important for extreme values than it would seem at first sight (Longin 2005, McNeil and Frey 1999)

Triplett[1928].According to their suggestion, let X_n be a sequence of independent and identically distributed random variables and let $M_n = \max(X_1, X_2, X_3, \dots, X_n)$ be the maximum of the first n terms. If there exists constants $a_n > 0$ and b_n and some non-degenerate distribution function H such that

$$\frac{M(L) - b_L}{a_L} \xrightarrow{d} H$$

where the subscript 'd' indicates convergence in distribution, then H belongs to one of the following extreme value distributions:

Frechet : $G_{1,\alpha}(x) = \begin{cases} 0 & x < 0 \\ \exp[-x^{-\alpha}] & x \geq 0 \end{cases}$ Type -I.....Eq.No.(02)

Weibull: $G_{2,\alpha(x)} = \begin{cases} \exp[-x^{-\alpha}] & x \leq 0 \\ 1 & x > 0 \end{cases}$ Type - II.....Eq.No.(03)

Gumbell: $G_3(x) = \exp[-e^{-x}]$, $x \in \mathfrak{R}$ Type - III.....Eq.No.(04)

where α is the shape parameter.

Hence, distributions are categorized into three groups: (i) heavy-tailed distributions, whose extremes follow the first type of law:Extreme positive or negative returns resulting out of strong increase or decrease of prices on account of price to price channel or feedback loop may be characterized under this category¹¹. (ii) short-tailed distributions with finite end-point, whose extremes follow the Weibull's type; and (iii) medium tailed distributions, whose extremes are governed by the distributions of the type III above. In case (i) and (ii) we have a one-parameter family of distributions, parameterized by the shape coefficient α . Representative members of the three groups are respectively: the Student-t, the uniform and the Normal . distribution The Von Mises [1936] representation of the GEV provides a unified formula for the previous three limiting distributions (I), (II) and (III):

¹¹ Thus it captures class to mass theory, representativeness heuristics etc.

$$G_\gamma = \exp\left[-(1 + \gamma x)^{-\frac{1}{\gamma}}\right] \dots\dots\dots \text{Eq. No. (5)}$$

where, positive γ represents the Frechet distribution (Frechet 1927), negative γ corresponds to the Weibull type (Weibull 1939), and the limit case $\gamma \rightarrow 0$ suggests Gumbel distribution (Gumbel 1958). The shape parameters of the two representations

are related to each other by the formula $\alpha = \frac{1}{\gamma}$ for the distribution Type-I, and $\alpha = -$

$\frac{1}{\gamma}$ for the type (II). Von Mises [1936] approach turns out to be very useful in the sense

it nests all these types of limiting behavior in a unified framework through estimation of γ , and allow to infer about the characteristics of limit laws.

4.4.2. GPD: Limiting Distributions for the Tail:

Investors, policy makers most presumably are concerned about any loss or gain that exceeds a predetermined threshold level often referred as attachment point. Let us suppose that X_1, X_2, \dots, X_n represent the ground-up losses or gains over a given period. Again let u be the predetermined threshold and $Y = [X - u | X \geq u]$ be the excess of X over u given that the ground-up loss exceeds the threshold. The risk managers will be interested in the distribution of the exceedances; that is, in the conditional distribution of $Y = X - u$ given that X exceeds the threshold (u).

Let F denote the distribution of the random variable X ,

$$F(x) = \text{Prob}(X < x),$$

and let F_u denote the conditional distribution of the exceedance $Y = X - u$ given that X exceeds the threshold

$$F_u(y) = \frac{F(y+u) - F(u)}{1 - F(u)} \dots\dots\dots \text{Eq.No. (06)}$$

The exceedances for a high enough threshold always converge in generalized Pareto distribution [Pickands 1975 and Balkema & de Haan 1974].

Hence, result of the GPD focuses on the tails of the distributions instead of maxima; the selected events, in this case, are those events that exceed a given threshold u . Using the so-called α parameterization the Generalized Pareto Distributions can be represented as:

$$W_{1,\alpha} = 1 - x^{-\alpha}, \quad x \geq 1, \dots\dots\dots\text{Eq. No.}(07)$$

$$W_{2,\alpha} = 1 - (-x)^\alpha, \quad -1 \leq x \leq 0, \dots\dots\dots\text{Eq. No.}(08)$$

$$W_3 = 1 - \exp(-x), \quad x \geq 0, \dots\dots\dots\text{Eq. No.}(09)$$

All the three distributions assume the value zero outside the pertinent intervals. For the GPD a similar one-parameter representation exists as with the extreme value distributions, symbolically:

$$W_\gamma = 1 - (1 + \gamma x)^{\frac{1}{\gamma}} \dots\dots\dots\text{Eq. No. (10)}$$

where for $\gamma > 0, \gamma < 0$ and $\gamma \rightarrow 0$ we recover the first, second and third group, respectively where γ is the shape parameter. The relations between α and γ are again $\alpha = 1/\gamma$ for the first type, and $\alpha = -1/\gamma$ for the second type. The GPD formalization is very flexible in describing the tail behavior, although it depends on one parameter only, index α , after accounting for location and scale parameter.

While in applying the above theorem the most difficult task is selection of an appropriate threshold. To point a threshold, we have to trade off between bias and variance. If we choose a low threshold the number of observations increase and that includes some events from the centre of the distribution and the estimation becomes biased. Similarly choosing too high a threshold will result in an inadequate fit. Therefore, a careful combination of several techniques such as QQ Plot, Mean Excess Function (MEF), and Hill Estimation in general, are considered in determination of the threshold.

4.4.3. Mean Excess Function:

Mean Excess Function may be defined as:

$$e(u) = E(X - u | X > u) \quad 0 \leq u \leq x_F \dots\dots\dots\text{Eq.No(11)}$$

where $X_{1:n}$ and $X_{n:n}$ are the 1st and n-th order statistics and $e_n(u)$ is the sample mean excess function defined by McNeil [1997] as:

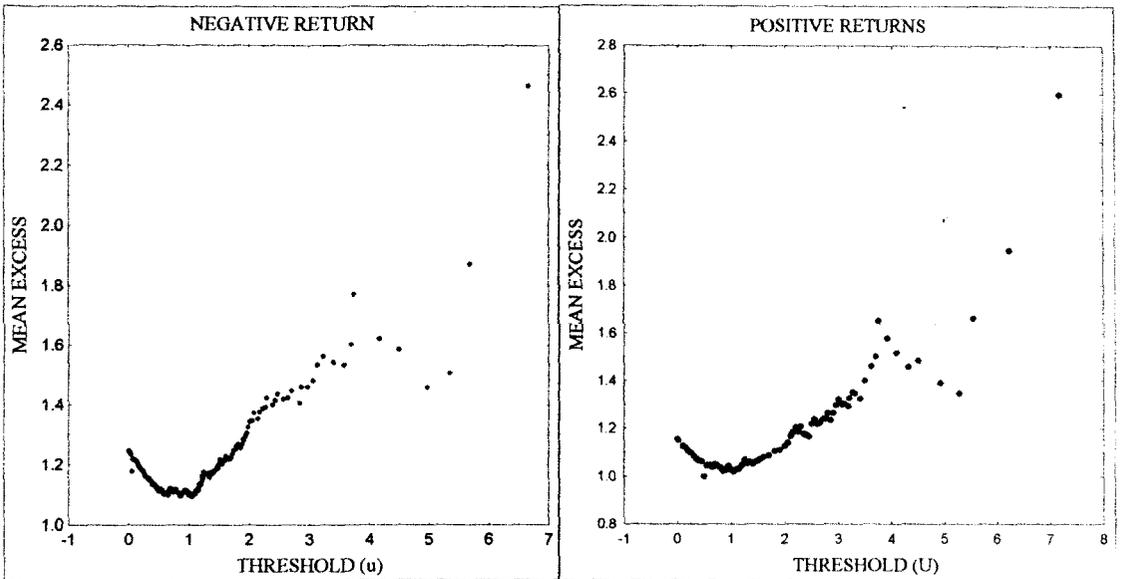
Thus Mean Excess Function is the sum of the excesses over the threshold u divided by the number of data points which exceed the threshold u .

$$e_n(u) = \frac{\sum_{i=1}^n (X_i - u)}{\sum_{i=1}^n 1_{\{X_i > u\}}} \dots\dots\dots\text{Eq.No.(12)}$$

The detailed interpretation of the mean excess plot is available in the studies of Embrechts et al. [1997], Beirlant et al, [2004]. If the points show an upward trend, then this is a sign of heavy tailed behavior [See Fig.3.2]. Exponentially distributed data would result in an approximately horizontal line and data from a short tailed distribution would show a downward trend. In particular, if the empirical plot seems to follow a reasonably straight line with positive gradient above a certain value of u , then this is an indication that data follow a generalized Pareto distribution with positive shape parameter in the tail area above u . “S” shape of the curve suggests that tails of the density function have higher probabilities than with the normal distribution thus the distribution under the study has fat tails.

Since the mean excess function for the generalized Pareto distribution is a straight line with positive slope, we are looking for the threshold points from which the mean excess plot follows a straight line.

Figure – 4.2
(Mean Excess Return of Indian capital market)
(Period from 1.7.1997 to 31.08.2013)



Above mean excess plots (Fig.4.2) help us to get an insight regarding tail behavior of the series, nonetheless they fail to define objectively the threshold values, quantify density function and suggest distribution to which it belongs. The “sketchy” estimation offers an impression that threshold in Indian asset market lies at nearly 1.10 and 1 percentage for right and left tail, respectively, beyond which any observation may be treated as extreme that deserve attention of investors and policy makers. However we go beyond this “sketchy” estimation and apply some stringent test to the series under our study so that we can be sure that it belongs to long memory with high degree of confidence.

4.4.4. Hill Estimation:

Estimation of the index α is the central issue of our empirical research dealing with extreme events and we relied on nonparametric Hill Index (Hill 1975) to estimate tail behavior of market return. Hill index is the conditional maximum likelihood estimator for heavy-tailed distributions. If we assume that the data points

exceeding a given threshold u follow a Pareto distribution with index α , the distribution of realizations exceeding u reads (Alfarano and Lux 2010):

$$F(x \geq u) = 1 - \left(\frac{u}{x}\right)^\alpha; u \geq 0 \dots \dots \dots \text{Eq.No.}(13)$$

Virtually there are two approaches for estimating “excess distribution”, first, semi-parametric model based on the Hill estimator and another, fully parametric model based on the Generalized Pareto Distribution (GPD). We relied on Hill estimator for two reasons: (i) its simplicity over maximum likelihood method and (ii) ability of the model to define precisely the threshold point beyond which any observation may be treated as extreme.

For our estimation, at the first step, we obtain the order statistics $X_{(1)}, X_{(t-1)}, \dots, X_{(1)}$ from our sample, where $X_i > X_{i-1} > \dots > X_1$. Then ,the following Hill index is estimated by :

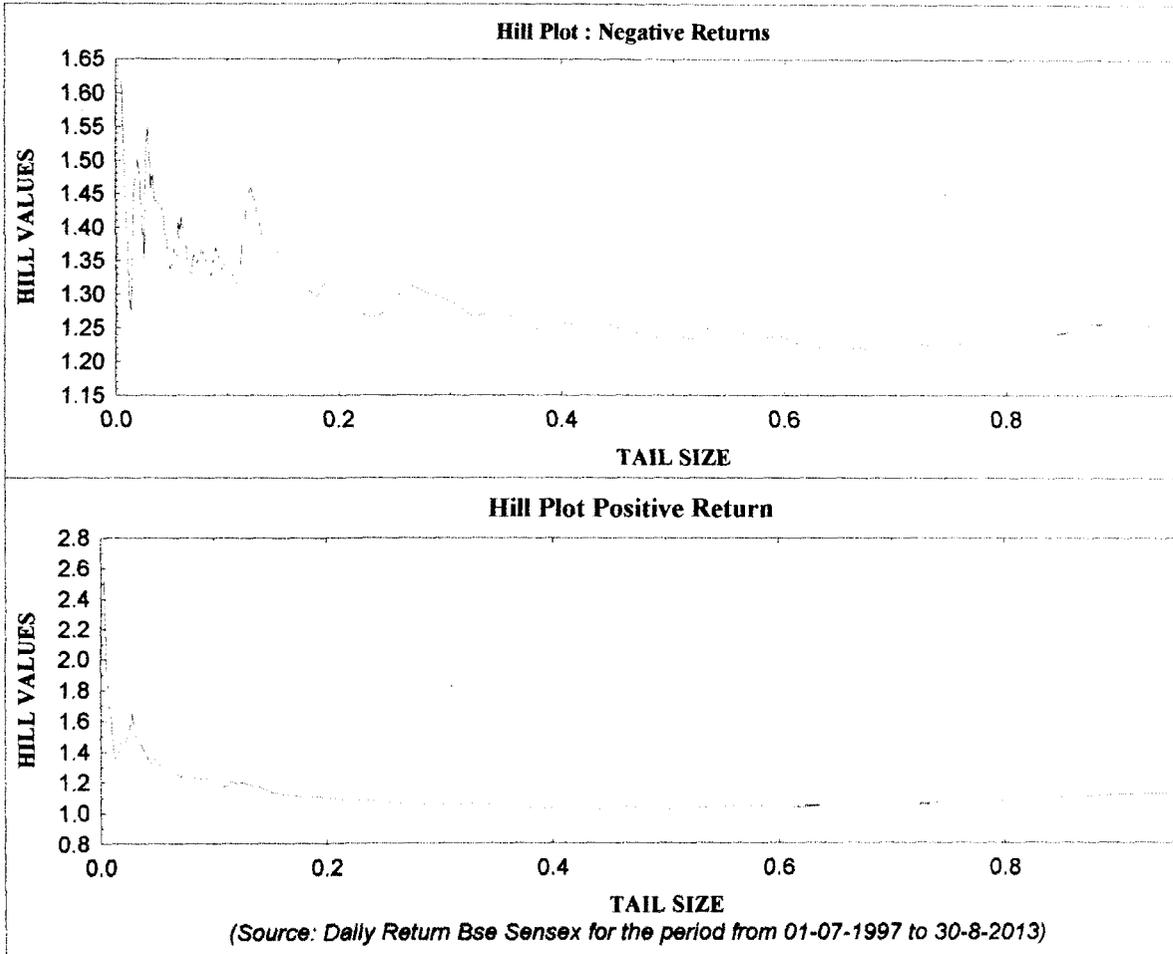
$$\hat{\gamma}_{k,n} = (\hat{\alpha}_{k,n})^{-1} = \frac{1}{k} \sum_{i=1}^k [\ln x_{(n-i+1)} - \ln x_{(n-k)}] \dots \dots \dots \text{Eq.No.}(14)$$

with x_i the order statistics of the series x , $x_{(N)} > x_{(N-1)} > \dots > x_{(1)}$, i.e. $x_{(N)}$ is the maximum of x , $x_{(N-1)}$ is the second largest value etc. As it is assumed that above equation only applies to a fraction k/N of the largest values, we only consider the x_i above the threshold u , $x_{(N-k)} = u > x_{(N-k-1)}$, where k is the number of selected large realizations, from the entire sample of N observations. It has been shown that under some mild additional restrictions on the behavior of the underlying distribution function, $\hat{\gamma}_{k,N}$ is asymptotically Gaussian with mean γ (i.e. the inverse of the true index) and variance $(\gamma^2 k)^{-1}$. Where k is the number of upper order statistics included, N is the sample size , and $\alpha = 1/\gamma$ is the tail index. While the concept of Hill estimator is straight forward, the choice of k is not. The problem may be defined as

threshold selection problem. One has to decide which events from the complete set of data points, belong to the subset relevant for the estimation of α .

Taking the daily return values of 3995 observations starting from July 1997 to August 2013, Hill estimation of the shape parameter alpha (α) has been obtained using the above mentioned methodology and the Hill values are plotted as below: (Fig. 4.3)

Figure- 4.3
Hill Plot of return of Indian Capital Market Return
(Time period : 01.07.1997 to 31.08.2014)



It is evident that Hill Values of the Shape Parameter (α) is inversely related with the corresponding tail size based on different values of threshold (u). For this reason, it is not immediately obvious what the appropriate tail fraction would suggest the best estimator for the 'true' parameter. A possible practical approach for identification of threshold point could be an 'eyeball method', searching for a region in the Hill plot where the estimated values are approximately constant (Alfarano and Lux, 2010). However, the Hill's estimator is most effective when the underlying distribution is Pareto type or approximate to Pareto (Chin Wen Cheong et al.2008).Undeniably, this approach has all the drawbacks of a subjective graphical

data analysis. This estimation along with others can effectively help to trade off between bias and variance while estimating threshold value. Our findings suggest some intrinsic features of Indian capital market. We mention some unique feature of Indian market that may help investors to decide upon strategy to maximize gains and to minimize loss when market is essentially turbulent. One interesting feature is threshold level varies in our market. It is one percentage for left tail and nearly (1.10%) for positive return. Furthermore, forty two percentages of negative returns and nearly forty percent of positive return are found as extreme. Findings about threshold, the focal point of discussion of this section are in consonance with the results of so called sketchy estimation of Mean Excess Function. Altogether approximately forty two percent of total observation falls in the extreme region either positive or negative. Thus our finding in this chapter sharply contradicts the basic propositions of Gaussian distribution underlying the efficient market hypothesis.

Table: 4A. Frequency Distribution of Extreme Gains:

(Observations 1-849, Number of bins = 29, Mean = 2.14907, S.D. = 1.22695)

Interval	Mid point	Frequency	Relative frequency	Cumulative Frequency
< 1.3916	1.1016	214	25.21%	25.21%
1.3916 - 1.9715	1.6816	276	32.51%	57.71%
1.9715 - 2.5514	2.2615	155	18.26%	75.97%
2.5514 - 3.1313	2.8414	89	10.48%	86.45%
3.1313 - 3.7113	3.4213	52	6.12%	92.58%
3.7113 - 4.2912	4.0012	21	2.47%	95.05%
4.2912 - 4.8711	4.5811	9	1.06%	96.11%
4.8711 - 5.4510	5.1611	11	1.30%	97.41%
5.4510 - 6.0309	5.7410	9	1.06%	98.47%

6.0309 - 6.6109	6.3209	3	0.35%	98.82%
6.6109 - 7.1908	6.9008	2	0.24%	99.06%
7.1908 - 7.7707	7.4807	4	0.47%	99.53%
7.7707 - 8.3506	8.0606	2	0.24%	99.76%
8.3506 - 8.9305	8.6406	0	0.00%	99.76%
8.9305 - 9.5104	9.2205	1	0.12%	99.88%
9.5104 - 10.090	9.8004	0	0.00%	99.88%
10.090 - 10.670	10.380	0	0.00%	99.88%
10.670 - 11.250	10.960	0	0.00%	99.88%
11.250 - 11.830	11.540	0	0.00%	99.88%
11.830 - 12.410	12.120	0	0.00%	99.88%
12.410 - 12.990	12.700	0	0.00%	99.88%
12.990 - 13.570	13.280	0	0.00%	99.88%
13.570 - 14.150	13.860	0	0.00%	99.88%
14.150 - 14.730	14.440	0	0.00%	99.88%
14.730 - 15.310	15.020	0	0.00%	99.88%
15.310 - 15.890	15.600	0	0.00%	99.88%
15.890 - 16.469	16.179	0	0.00%	99.88%
16.469 - 17.049	16.759	0	0.00%	99.88%
>= 17.049	17.339	1	0.12%	100.00%

Table: 4B. Frequency distribution of Extreme Losses:

(Observations 1-849 Number of bins = 19, Mean = -2.18807, S.D. = 1.27396)

Interval	Midpoint	Frequency	Relative Frequency	Cumulative Frequency
< -9.6359	-9.8898	2	0.59%	0.59%
-9.6359 - -9.1283	-9.3821	0	0.00%	0.59%
-9.1283 - -8.6206	-8.8744	0	0.00%	0.59%
-8.6206 - -8.1129	-8.3667	0	0.00%	0.59%
-8.1129 - -7.6052	-7.8590	0	0.00%	0.59%
-7.6052 - -7.0975	-7.3514	0	0.00%	0.59%
-7.0975 - -6.5898	-6.8437	0	0.00%	0.59%
-6.5898 - -6.0822	-6.3360	4	1.18%	1.78%
-6.0822 - -5.5745	-5.8283	3	0.89%	2.66%
-5.5745 - -5.0668	-5.3206	3	0.89%	3.55%
-5.0668 - -4.5591	-4.8129	9	2.66%	6.21%
-4.5591 - -4.0514	-4.3053	5	1.48%	7.69%
-4.0514 - -3.5437	-3.7976	13	3.85%	11.54%
-3.5437 - -3.0361	-3.2899	20	5.92%	17.46%
-3.0361 - -2.5284	-2.7822	33	9.76%	27.22%
-2.5284 - -2.0207	-2.2745	54	15.98%	43.20%
-2.0207 - -1.5130	-1.7668	55	16.27%	59.47%
-1.5130 - -1.0053	-1.2592	136	40.24%	99.70%
>= -1.0053	-0.75147	1	0.30%	100.00%

Frequency distribution of extreme gains and losses are shown in Table 4A and 4B. Information available from the tables would help portfolio managers to assess the pattern of asset returns of Indian market at its extreme. For example nearly cent percent (99.76% app.) of positive extreme return fall below 8.3%. Beyond this, any large movement is virtually nonexistent. Majority of extreme returns (95% approx.) cluster up to four percentages approximately. While in the negative domain most of the extreme return clusters around 1.5% to 4.5%, approximately.

4.5. Conclusion:

Our analysis based on BSE SENSEX 30 as a proxy for the equity market indicates some serious departure from those obtained using the assumption of normal return distribution. The normal distribution gives us a fair idea of return distributions for every day events; alternatively EVT gives an impression about best or worst case of returns and the frequency thereof. Fat tail is not an exclusive syndrome of India instead it pervasively dominates worldwide financial markets. These extreme movements in share prices with leptokurtic distribution of return cannot be captured under the dictum of rational expectations. Rather it is a potential threat to the theory of equilibrium and a formidable challenge for investors interested in risk reduction. These findings however confirm the social and psychological dynamic forces predicted under heterogeneous market model with bounded rationality. The model predicts the price changes to be driven by a combination of exogenous random news and an evolutionary force generate endogenously in favour of either fundamental or technical trading rule. The presence of prolonged rise and fall in prices in Indian market confirms the influence of persisting evolutionary forces operating in the market in favour of using technical trading rules and it is increasing over time. These sort of evolutionary forces are explained in the heterogeneous model as an influence of various social and psychological attributes in decision making. A self fulfilling prophecy has therefore developed with this psychological bias in human decision in favour of evolutionary fitness in using technical rules ignoring the early dogma of rationality. Perceived profitability in using technical trading rules coupled with considerable undermining of risks conditional on subsequent movement in prices thus

makes the decision making context specific. Influences of socio psychological forces in decision making are thereby confirmed to be the factor behind in generating convergence in opinion, undermining of risk, limiting the arbitrage operation and bubble formations.

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