CHAPTER X

FORECASTING OF DAILY DISSOLVED OXYGEN LEVELS OF A NON TIDAL RIVER BY DYNAMIC LEAST SQUARE RECURSIVE ESTIMATION TECHNIQUE

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10.0 Abstract :

This work deals with an application of the dynamic least square estimation algorithms for on-line modelling of dissolved oxygen levels of a non tidal river passing through a highly industrialised region. The mathematical description of the dissolved oxygen levels allows for the real time monitoring of water quality. Actual data are used to calibrate the real time dynamic model. Agreement between the observed and the predicted values of the levels of dissolved oxygen illustrates the potential use of the dynamic least square recursive estimation technique for on-line control of water quality in accordance with predetermined decision trategies.

10.1. Introduction :

The objective of this paper is to develop a dynamic model of the daily levels of the dissolved oxygen concentration of a non tidal river for the purpose of forecasting of the levels of dissolved oxygen on real time basis. The most important variable of interest in water quality is the concentration of dissolved oxygen, usually expressed in mg/litre, in the natural water body. Even the temporary fluctions of the concentration of the dissolved oxygen in a water body below the critical values may cause a long-term and possible irreversible damage to the aquatic ecosystem.

The improvement and consequent control of water quality of a river can be achieved by intelligent regulation of water discharges as to volume of discharge and the point of discharge. The important engineering decisions in water quality control relate to the determination of the concentration of the dissolved oxygen that is consistent with the multiple uses of the natural water bodies. This implies that the mathematical simulation models must have the ability to forecast the concentration of dissolved oxygen levels as closely as possible on real time basis.

This paper clearly examines the development of dynamic least square recursive technique for forecasting the dissolved oxygen concentration on real time basis. The model has been verified by field measurement of the concentration of dissolved oxygen collected over a 80 day period in summer of 1972 from the river Cam in Eastern England.

For the sake of clarity of presentation the development of the dynamic least square recursive estimation technique has been given as a complete exposition.

10.2. Development of the Real Time Model :

In recursive estimation algorithms the estimate of the output at a future instant is constantly updated on receipt of fresh information. The presence of significant co-efficient of correlation with the levels of dissolved oxygen at past instants with the current instant clearly suggests that the process may be represented in the form of a stationary time series with a probability that the current value of the output, $y(t_k)$ is a function of the previous output observations, the auto-regressive terms $y(t_{k-1})$, $y(t_{k-2})$, ..., together with a current unknown realisation of the noise process.

The process, thus, can be represented as

$$y(t_k) = \left[\sum_{i=1}^{N1} \delta_i q^{-i} \right] y(t_k) + \eta(t_k) \quad \ldots \quad (10.1)$$

where q^{-1} is the backward shift operator defined as $q^{-1}y(t_k) = y(t_{k-1})$. The determination of N1 is unknown as the model order determination which is also the model structure identification. The number of the significant instants in the output correlation may be intuitively considered for model order determination. The co-efficient of correlation may be defined as,

$$\phi_{yy}(\lambda) = \frac{\sum_{i=1}^{N-\lambda} \left(y(t_i) - \frac{1}{N} \sum_{i=1}^{N} y(t_i)\right)\left(y(t_{i+\lambda}) - \frac{1}{N} \sum_{i=1}^{N} y(t_i)\right)}{\left[\sum_{i=1}^{N-\lambda} \left(y(t_i) - \frac{1}{N} \sum_{i=1}^{N} y(t_i)\right)^2 \sum_{j=1+\lambda}^{N} \left(y(t_j) - \frac{1}{N} \sum_{i=1}^{N} y(t_i)\right)^2 \right]^{\frac{1}{2}}}$$

$$\ldots \quad (10.2)$$

where N is the number of data points and λ are shift of instances of time.

The equation (10.1) can be expressed as

$$y(t_k) = s^T(t_k)\alpha + \eta(t_k) \quad \ldots \quad (10.3)$$

where 'α' is the parameter vector with property of slowly varying with time and amenable to recursive adaptiveness, and

$$s^T(t_k) = \left[y(t_{k-1}), y(t_{k-2}), \ldots, y(t_{k-N1}) \right]$$

The least square estimation of the process parameters can be obtained by minimising a loss function defined as the sum of the squared errors as

$$J \triangleq \sum_{k=1}^{N} (y(t_k) - z^T(t_k)\,\alpha)^2 \quad \ldots \quad (10.4)$$

The estimates $\hat{\alpha}$ of α that minimise J are called least square estimates and $\hat{\alpha}$ is obtained as

$$\hat{\alpha} = \left[\sum_{k=1}^{N} z(t_k)\; z^T(t_k) \right]^{-1} \left[\sum_{k=1}^{N} z(t_k)\, y(t_k) \right] \ldots (10.5)$$

Let

$$P^*(t_k) \triangleq \left[\sum_{j=1}^{k} z(t_j)\; z^T(t_j) \right]^{-1}$$

and

$$b(t_k) \triangleq \left[\sum_{j=1}^{k} z(t_j)\; y(t_j) \right]$$

So the equation (10.5) becomes,

$$\hat{\alpha} = P^*(t_k)\; b(t_k) \quad \ldots \quad (10.6)$$

The recursive relationship for $P^*(.)$ and $b(.)$ can be set as

$$\left[P^*(t_k) \right]^{-1} = \sum_{j=1}^{k-1} z(t_j)\, z^T(t_j) + z(t_k) z^T(t_k)$$

$$= \left[P^*(t_{k-1}) \right]^{-1} + z(t_k)\; z^T(t_k) \quad \ldots \quad (10.7)$$

Similarly,

$$b(t_k) = b(t_{k-1}) + z(t_k)\, y(t_k) \quad \ldots \quad (10.8)$$

Premultiplying by $P^*(t_k)$ we get from equation (10.7)

$$I = P^*(t_k)\, [P^*(t_{k-1})]^{-1} + P^*(t_k)\, z(t_k)\, z^T(t_k) \quad \ldots \quad (10.9)$$

Post multiplication of equation (10.9) by $P^*(t_{k-1})$

$$P^*(t_{k-1}) = P^*(t_k) + P^*(t_k)\, z(t_k)\, z^T(t_k)\, P^*(t_{k-1}) \quad \ldots \quad (10.10)$$

Post multiplication of equation (10.10) by $z(t_k)$

$$P^*(t_{k-1})z(t_k) = P^*(t_k)z(t_k) + P^*(t_k)z(t_k)z^T(t_k)P^*(t_{k-1})z(t_k)$$

So we get,

$$P^*(t_{k-1})z(t_k) = P^*(t_k)z(t_k)\, [1+z^T(t_k)P^*(t_{k-1})z(t_k)] \quad \ldots \quad (10.11)$$

Post multiplication by $[1+z^T(t_k)P^*(t_{k-1})z(t_k)]^{-1} z^T(t_k)P^*(t_{k-1})$ the equation (10.10) becomes,

$$P^*(t_{k-1})z(t_k)\, [1+z^T(t_k)P^*(t_{k-1})z(t_k)]^{-1}\, z^T(t_k)P^*(t_{k-1})$$

$$= P^*(t_k)z(t_k)z^T(t_k)P^*(t_{k-1}) \quad \ldots \quad (10.12)$$

Substitution of equation (10.10) in equation (10.12) gives,

$$P^*(t_k) = P^*(t_{k-1}) - P^*(t_{k-1})z(t_k)\, [1+z^T(t_k)P^*(t_{k-1})z(t_k)]^{-1}$$

$$z^T(t_k)\; P^*(t_{k-1}) \quad \ldots \quad (10.13)$$

From equation (10.6)

$$\hat{\alpha}(t_k) = P^*(t_k)\; b(t_k)$$

Combining equations (10.8) and (10.13)

$$\hat{\alpha}(t_k) = \Big[P^*(t_{k-1}) - P^*(t_{k-1}) z(t_k) \big[1 + z^T(t_k) P^*(t_{k-1}) z(t_k) \big]^{-1} \times z^T(t_k) P^*(t_{k-1}) \Big] \Big\{ b(t_{k-1}) + z(t_k)\, y(t_k) \Big\} \quad \ldots \quad (10.14)$$

Since

$$P^*(t_{k-1})\, b(t_{k-1}) = \hat{\alpha}(t_{k-1})$$

The equation (10.14) becomes,

$$\hat{\alpha}(t_k) = \hat{\alpha}(t_{k-1}) - P^*(t_{k-1})\, z(t_k) \big[1 + z^T(t_k) P^*(t_{k-1}) z(t_k) \big]^{-1} \times z^T(t_k)\, \hat{\alpha}(t_{k-1}) + P^*(t_{k-1})\, z(t_k)\, y(t_k) - P^*(t_{k-1}) z(t_k) \big[1 + z^T(t_k) P^*(t_{k-1}) z(t_k) \big]^{-1} \quad \ldots \quad (10.15)$$

Since $z^T(t_k)\, P^*(t_{k-1})\, z(t_k)$ is scalar the equation (10.15) becomes,

$$\hat{\alpha}(t_k) = \hat{\alpha}(t_{k-1}) + P^*(t_{k-1}) z(t_k) \big[1 + z^T(t_k) P^*(t_{k-1}) z(t_k) \big]^{-1} \times \big[y(t_k) - z^T(t_k)\, \hat{\alpha}(t_{k-1}) \big] \quad \ldots \quad (10.16)$$

The equations (10.16) and (10.13) with

$$P^*(t_k) \triangleq \Big[\sum_{i=1}^{k} z(t_i)\, z^T(t_i) \Big]^{-1}$$

are the recursive least square algorithms.

The recursive least square algorithms do not overcome the problem of bias. The problem of bias in the parameter estimates may be defined as follows.

Recalling equation (10.5)

$$\hat{\alpha}(t_k) = \left[\sum_{j=1}^{k} z(t_j) z^T(t_j) \right]^{-1} \left[\sum_{j=1}^{k} z(t_j) y(t_j) \right]$$

Upon substitution of the process equation

$$y(t_j) = z^T(t_j)\alpha + \eta(t_j) \text{ in equation (10.5)}$$

$$\hat{\alpha}(t_k) = + \left[\sum_{j=1}^{k} z(t_j) z^T(t_j) \right]^{-1} \left[\sum_{j=1}^{k} z(t_j) \eta(t_j) \right] \quad \ldots \quad (10.17)$$

for (t_k) to be unbiased

$$E \left\{ z(t_j) \quad \eta(t_j) \right\} = 0 \text{ for all } j$$

and

$$E \left\{ \eta(t_k) \quad \eta(t_j) \right\} \quad 0 \text{ for } k \neq j$$

and

$$E \left\{ \eta(t_k) \quad \eta(t_k) \right\} = \sigma^2$$

i.e. (t_k) approaches the white noise sequence (t_k).

Let us define the covariance matrix of the least square parameter estimation as $P(t_k)$

$$P(t_k) = \text{Cov} \left\{ \tilde{\alpha} \quad (t_k) \right\} = E \left(\hat{\alpha}(t_k) - \alpha \right) \left(\hat{\alpha}(t_k) - \alpha \right)^T$$

$$= P*(t_k) \left[E \left\{ \left[\sum_{j=1}^{k} z(t_j) \quad (t_j) \right] \left[\sum_{j=1}^{k} (t_j) z^T(t_j) \right] \right\} \right] p*^T(t_k)$$

$$P(t_k) = \sigma^2 P^*(t_k) \qquad \ldots \qquad (10.18)$$

σ^2 may be estimated as the variance of the residual error sequence $B(t_k) = y(t_k) - z^T(t_k) \quad (t_k)$ after the estimates have achieved satisfactory convergence. Least square estimates of α may be considered for initialisation. Thus for almost unbiased estimate the least square recursive algorithms of (10.3) and (10.16) have been transformed by making substitution of equation (10.18) as

$$\hat{\alpha}(t_k) = \hat{\alpha}(t_{k-1}) + P(t_{k-1})z(t_k) \,/\left[\sigma^2 + z^T(t_k)P(t_{k-1})z(t_k)\right]$$

$$\left\{ y(t_k) - z^T(t_k)\hat{\alpha}(t_{k-1}) \right\} \qquad \ldots \qquad (10.19)$$

$$P(t_k) = P(t_{k-1}) - P(t_{k-1})z(t_k) \,/\left[\,^2 + z^T(t_k)P(t_{k-1})z(t_k)\right] z(t_k)P(t_{k-1}) \quad ..(10.20)$$

In case of time varying parameters the large errors between the observed and the modelled outputs obtained from the algorithms (10.19) and (10.20) are mainly due to the error for changing values of the parameters $\hat{\alpha}$ of the model. If the parameters variation laws are known a model for the parameter variation can be obtained and the algorithms of equation (10.19) and (10.20) can be updated accordingly. In the absence of any specific parameter variation law a random walk model may be postulated as

$$\alpha(t_k/t_{k-1}) = \alpha(t_{k-1}/t_{k-1}) + \zeta(t_k/t_{k-1}) \qquad \ldots \qquad (10.21)$$

Recalling equation (10.18)

$$P(t_k/t_{k-1}) = \mathrm{Cov}\left\{\tilde{\alpha}(t_k/t_{k-1})\right\}$$

$$= E\left\{\alpha(t_k/t_{k-1}) - \alpha)(\alpha(t_k/t_{k-1}) - \alpha)^T\right\}$$

Hence, from equation (10.21)

$$P(t_k/t_{k-1}) = E\left\{ \left(\alpha(t_{k-1}/t_{k-1}) + \zeta(t_k/t_{k-1}) - \alpha \right) \times \left(\alpha(t_{k-1}/t_{k-1}) + \zeta(t_k/t_{k-1}) - \alpha \right)^T \right\} \quad \ldots (10.22)$$

From equation (10.17)

$$P(t_k/t_{k-1}) = E\Big\{ \Big(\Big[\sum_{j=1}^{k-1} s(t_j)s^T(t_j) \Big]^{-1} \Big[\sum_{j=1}^{k-1} s(t_j)\ \eta(t_j) \Big] + \zeta(t_k/t_{k-1}) \Big)$$

$$\Big(\Big[\sum_{j=1}^{k-1} s(t_j)s^T(t_j) \Big]^{-1} \Big[\sum_{j=1}^{k-1} s(t_j)\ \eta(t_j) \Big] + \zeta(t_k/t_{k-1}) \Big)^T \Big\}$$

since $\zeta(t_k/t_{k-1})$ is a white noise process it is uncorrelated with $s(.)$, and $\eta(t_j)$ is scalar,

$$P(t_k/t_{k-1}) = \Big[\sum_{j=1}^{k-1} s(t_j)s^T(t_j) \Big]^{-1} \Big[\sum_{j=1}^{k-1} s(t_j)\ E\Big\{ \sum_{j=1}^{k} \eta(t_j)\ \eta^T(t_j) \Big\} \sum_{j=1}^{k-1} s^T(t_j) \Big] \times$$

$$\Big[\Big[\sum_{j=1}^{k} s(t_j)s^T(t_j) \Big]^{-1} \Big]^T + E\left\{ \zeta(t_k/t_{k-1})\ \zeta^T(t_k/t_{k-1}) \right\} \quad \ldots (10.23)$$

Considering $D = E\left\{ \zeta(t_k/t_{k-1})\ \zeta^T(t_k/t_{k-1}) \right\}$ ... (10.24)

$$P(t_k/t_{k-1}) = \sigma^2 P^*(t_{k-1}/t_{k-1}) + D \quad \ldots \quad (10.25)$$

$$P(t_k/t_{k-1}) = P(t_{k-1}/t_{k-1}) + D \quad \ldots \quad (10.26)$$

and

$$\hat{\alpha}(t_k/t_{k+1}) = \hat{\alpha}(t_{k-1}/t_{k-1}) \quad \ldots \quad (10.27)$$

Algorithms (10.19) and (10.20) are updated as

$$\hat{\alpha}(t_k/t_k) = \hat{\alpha}(t_k/t_{k-1}) + P(t_k/t_{k-1}) z(t_k) \left[\sigma^2 + z^T(t_k) P(t_{k-1}) z(t_k) \right]^{-1}$$

$$\times \left\{ y(t_k) - z^T(t_k)\ \hat{\alpha}(t_k/t_{k-1}) \right\} \quad \ldots \quad (10.28)$$

$$P(t_k/t_k) = P(t_k/t_{k-1}) - P(t_k/t_{k-1}) z(t_k) \left[\sigma^2 + z^T(t_k) P(t_k/t_{k-1}) z(t_k) \right]^{-1}$$

$$\times\ z^T(t_k)\ P(t_k/t_{k-1}) \quad \ldots \quad (10.29)$$

Combining equations (10.26) and (10.27) with (10.28) and (10.29) the dynamic recursive least square algorithms are

$$\hat{\alpha}(t_k/t_k) = \hat{\alpha}(t_{k-1}/t_{k-1}) + \left\{ P(t_{k-1}/t_{k-1}) + D \right\} z(t_k)$$

$$\left[\sigma^2 + z^T(t_k) \left\{ P(t_{k-1}/t_{k-1}) + D \right\} z(t_k) \right]^{-1}$$

$$\times \left\{ y(t_k) - z^T(t_k)\ \hat{\alpha}(t_{k-1}/t_{k-1}) \right\} \quad \ldots \quad (10.30)$$

$$P(t_k/t_k) = \left\{ P(t_{k-1}/t_{k-1}) + D \right\} - \left\{ P(t_{k-1}/t_{k-1}) + D \right\} z(t_k)$$

$$\times \left[\sigma^2 + z^T(t_k) \left\{ P(t_{k-1}/t_{k-1}) + D \right\} z(t_k) \right]^{-1} z^T(t_k)$$

$$\left\{ P(t_{k-1}/t_{k-1}) + D \right\} \quad \ldots \quad (10.31)$$

Retaining the simpler notations the equations (10.30) and (10.31) become

$$\hat{\alpha}(t_k) = \hat{\alpha}(t_{k-1}) + \left\{P(t_{k-1}) + D\right\} s(t_k) \left[\sigma^2 + s^T(t_k) \left\{P(t_{k-1}) + D\right\} s(t_k)\right]^{-1} \times \left\{y(t_k) - s^T(t_k)\,\hat{\alpha}(t_{k-1})\right\} \quad \ldots (10.32)$$

$$P(t_k) = \left\{P(t_{k-1}) + D\right\} - \left\{P(t_{k-1}) + D\right\} s(t_k) \left[\sigma^2 + s^T(t_k) \left\{P(t_{k-1}) + D\right\} s(t_k)\right]^{-1} \times s^T(t_k) \left\{P(t_{k-1}) + D\right\} \quad \ldots (10.33)$$

The dynamic recursive least square algorithms are initialised with least square estimate of parameter vector $\hat{\alpha}(.)$.

10.3 Illustration :

Fig 10.3.1(a) shows the dissolved oxygen level at down stream end of the river Cam in Eastern England, collected over a 80 day period in summer of 1972. Fig 10.3.1(b) shows the predicted values of D.O. levels (dissolved oxygen levels) obtained with dynamic least square recursive estimation algorithms for optimum prehistory interval N1=7. The corresponding integral square error defined as

$$ISE = \frac{\sum_{i=1}^{N} (y(t_i) - y_0(t_i))^2}{\sum_{i=1}^{N} (y(t_i))^2}$$

is 8.179E-03 and the mean error is 0.0179, where $y_0(.)$ are the predicted values of dissolved oxygen levels. Fig 10.3.2. shows the correlation co-efficient versus shift of instances of time for the dissolved oxygen levels.

Fig 10.3.3. shows the correlation co-efficient versus shift of instances of time for the errors of prediction. It is found that the errors are almost uncorrelated.[112].

10.4 Conclusion :

The close agreement between the observed and the computed values of the levels of dissolved oxygen clearly demonstrates the potential application of dynamic least square recursive estimation algorithms for real time control of water quality of a non tidal river.

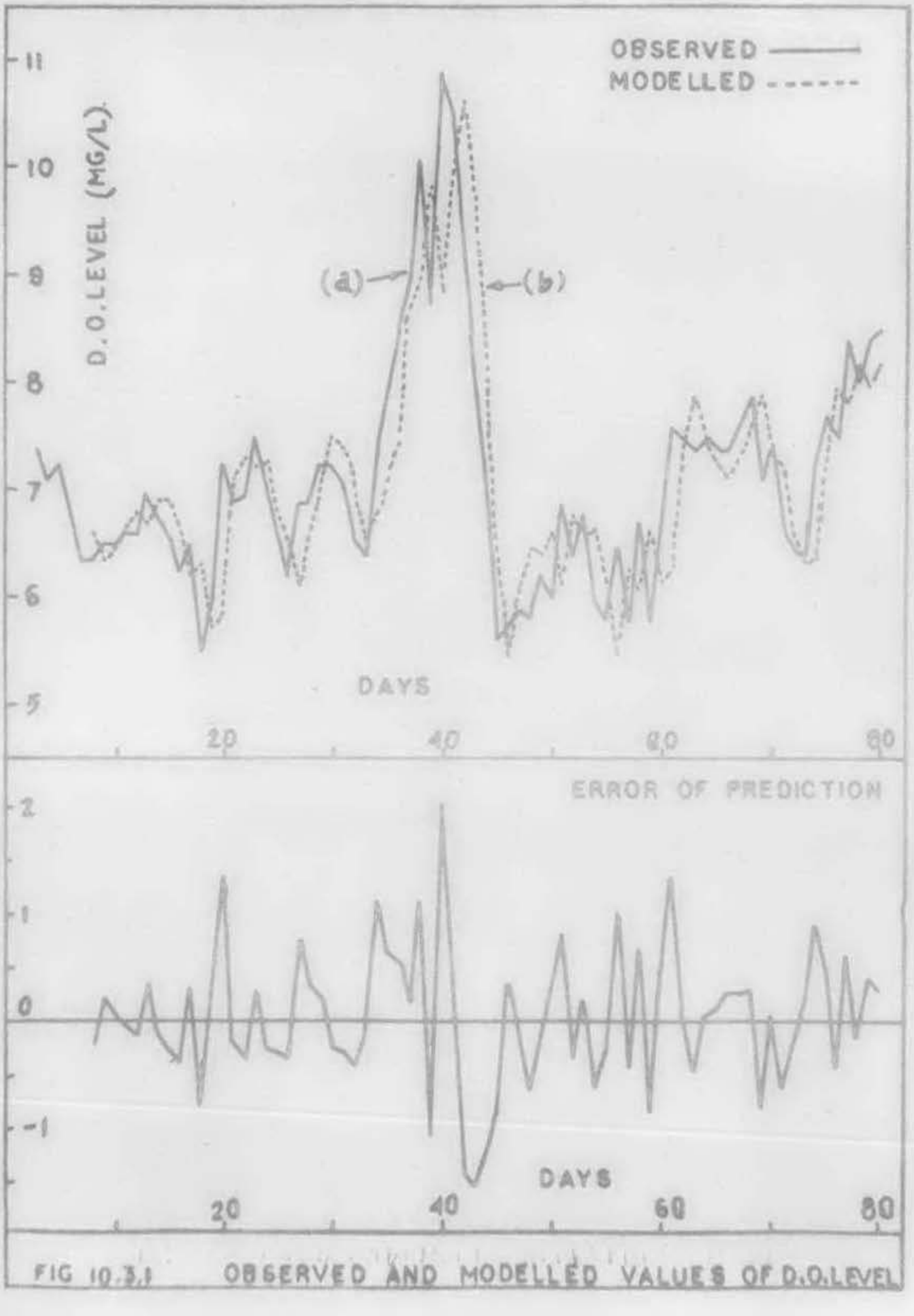

FIG 10.3.1 OBSERVED AND MODELLED VALUES OF D.O.LEVEL

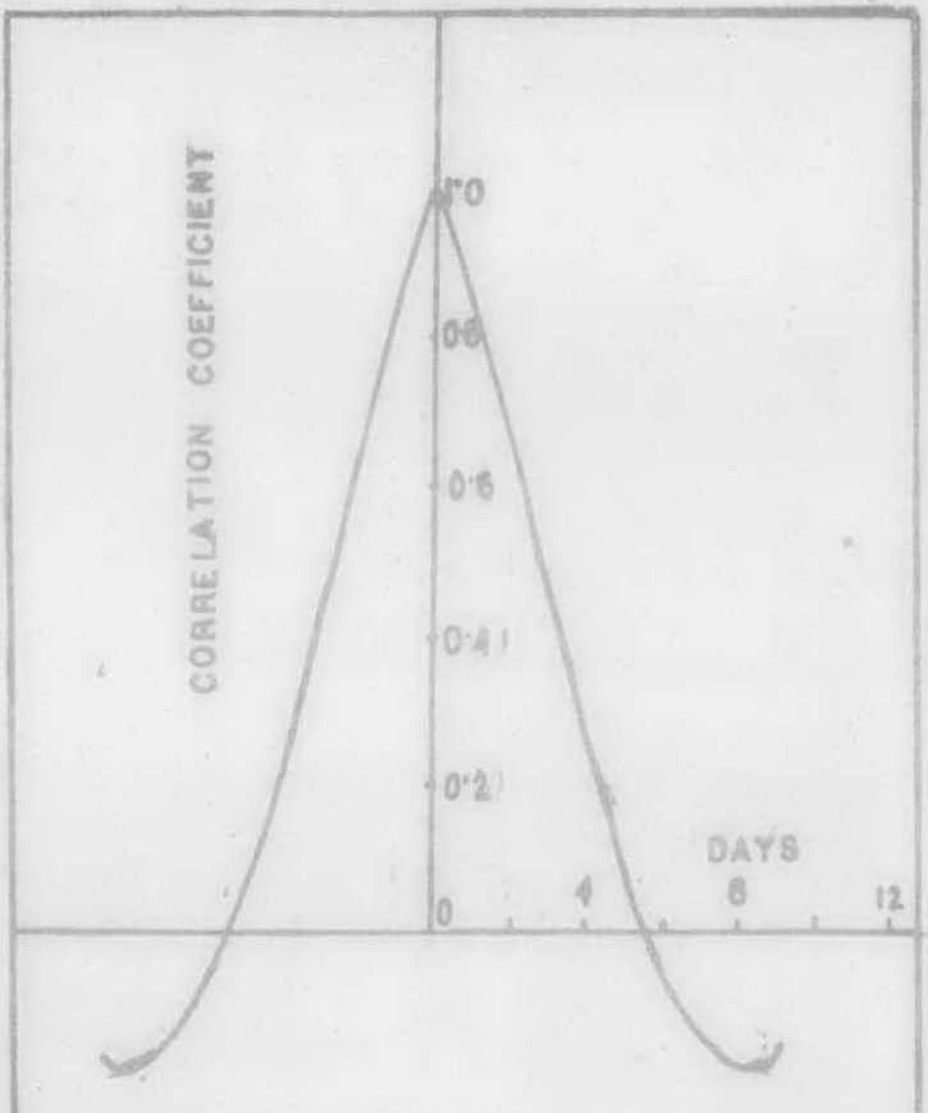

FIG 10.3.2 CORRELATION COEFFICIENT. D.O. LEVEL

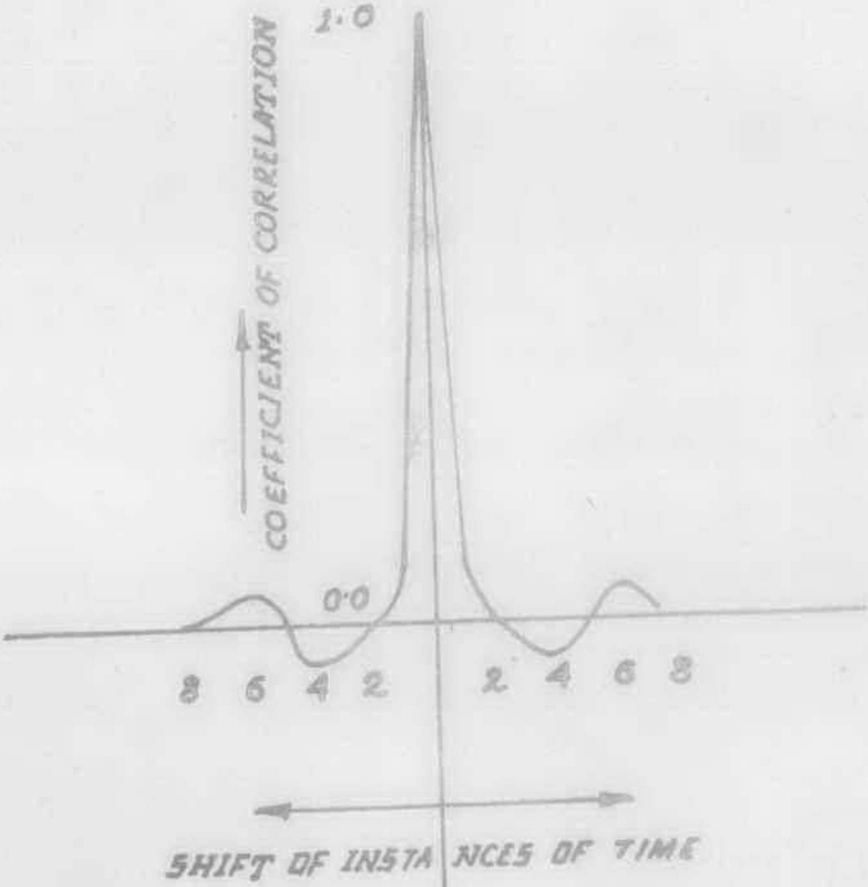

FIG. 10.3.3

COEFFICIENT OF CORRELATION VS SHIFT OF INSTANCES OF TIME FOR THE ERROR OF PREDICTION.