CHAPTER V

REAL TIME FORECASTING OF DAILY FLOWS OF A NON TIDAL RIVER BY INSTRUMENT VARIABLE ALGORITHM

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5.0 Abstract

A recursive procedure is presented for real time forecasting of daily flows of a non tidal river. The dynamic model is found to simulate adequately the major variations observed in the field measurements of daily flows of the river Teesta gauged at Domohani Road Bridge near Jalpaiguri town in North Bengal.

5.1 Introduction

The paper deals with the problem of identifying the parameters described by linear stochastic difference equations. The technique includes application of recursive instrument variable algorithms. From the knowledge of the available data, a recursive procedure is presented for real time forecasting of daily flows of a non tidal river to best fit and also to track adaptively the observed data in the least squared sense to give one step ahead prediction. When tested with field data the mathematical description of the daily river flow process is found to simulate adequately the major variations observed in the field measurements collected over a 5-year period from the 1st. January, 1975 to the 31st. December, 1979 of the river Teesta in North Bengal at Domohani gauging station near Jalpaiguri town. The technique of real time forecasting discussed in this paper highlight the possibilities of implementation of on-line regulatory mechanism for controlling daily flow of a non tidal river in accordance with a predetermined decision methodology.

The recursive instrument variable algorithms have the advantages of minimum computational requirements and they have the ability to adaptively update the slowly varying parameters of the physical process.

5.2. Development of the real time mathematical description of the daily river flow process.

In recursive estimation algorithm the estimate of the output at a future instant is constantly updated on receipt of fresh information. The presence of significance co-efficient of correlation of daily river flow at past instants with the current instant clearly suggests that the process may be represented in the form of a time series with the current value of the output p(k) as a function of the previous output observations, the auto-regressive terms P(k-1), P(k-2), ..., together with the current unknown realisation of the noise process. The process thus, can be represented as

$$P(k) = \left[\sum_{i=1}^{N} \beta_i q^{-1} \right] P(k) + \eta(k) \qquad \ldots \quad (5.1)$$

where q^{-1} is the backward shift operator defined as $q^{-1} P(k) = P(k-1)$. The determination of N is known as the model order determination which is also the model order identification. The number of the significant instants in the output correlation may be intuitively considered for model order determination.

$\eta(k)$ may be expressed in a moving average sequence as

$$\eta(k) = \sum_{p=1}^{P} \gamma_p \, \eta(k-p) + \upsilon(k) \qquad \ldots \quad (5.2)$$

Where ν (.) is a white noise innovation process with

$$E\left\{\vartheta(k)\quad\vartheta(j)\right\} = 0 \quad \text{for } j \neq k \text{ and}$$

$$E\left\{\vartheta(k)\quad\vartheta(j)\right\} = \rho^2 \quad \text{for } j = k \qquad \ldots (5.3)$$

The equation (5.1) can be written as

$$P(k) = s^T(k)\alpha + \eta(k) \qquad \ldots (5.4)$$

where α is the parameter vector with property of slowly varying with time and amenable to recursive adaptiveness, and

$$s^T(k) = [\, P(k-1),\ P(k-2),\ \ldots,\ P(k-N) \,]$$

The least square estimation of the process parameters can be obtained by minimising a loss function defined as the sum of the squared errors as

$$J \triangleq \sum_{k=1}^{N1} (P(k) - s^T(k)\hat{\alpha})^2 \qquad \ldots (5.5)$$

The estimates $\hat{\alpha}$ of α that minimise J are called least square estimates and $\hat{\alpha}$ is defined as

$$\hat{\alpha} = \left[\sum_{k=1}^{N1} s(k)\, s^T(k) \right]^{-1} \left[\sum_{k=1}^{N1} s(k)\, P(k) \right] \qquad \ldots (5.6)$$

Let $R^*(k) \triangleq \left[\sum_{j=1}^{k} s(j)\ s^T(j) \right]^{-1}$

and

$$S(k) = \left[\sum_{j=1}^{k} s(j)\, P(j) \right]$$

So equation (5.6) becomes

$$\hat{\alpha} = R^*(k)\ S(k) \qquad \ldots \qquad (5.7)$$

The recursive relationship for $R^*(.)$ and $S(.)$ can be set as

$$[\ R^*(k)\ ]^{-1} = \sum_{j=1}^{k-1} z(j)\ z^T(j) + z(k)\ z^T(k)$$

$$= [\ R^*(k-1)\ ]^{-1} + z(k)\ z^T(k) \qquad \ldots \qquad (5.8)$$

Similarly,

$$S(k) = S(k-1) + z(k)\ \beta(k) \qquad \ldots \qquad (5.9)$$

Premultiplying be $R^*(k)$ we get from equation (5.8)

$$I = R^*(k)\ [\ P^*(k-1)\ ]^{-1} + R^*(k) z(k) z^T(k) \qquad \ldots \qquad (5.10)$$

Postmultiplication of equation (5.10) by $R^*(k-1)$

$$R^*(k-1) = R^*(k) + R^*(k)\ z(k)\ z^T(k)\ R^*(k-1) \qquad \ldots \qquad (5.11)$$

Postmultiplying equation (5.11) by $z(k)$.

$$R^*(k-1)\ z(k) = R^*(k) z(k) + R^*(k) z(k) z^T(k)\ R^*(k-1) z(k)$$

So we get,

$$R^*(k-1) z(k) = R^*(k) z(k)\ [\ 1+z^T(k)\ R^*(k-1)\ (k)\ ] \qquad \ldots \qquad (5.12)$$

Post multiplication by $[\ 1+z^T(k)\ R^*(k-1) z(k)\ ]^{-1}\ z^T(k)\ R^*(k-1)$ the equation (5.12) gives,

$$R^*(k-1) z(k)\ [1+z^T(k)\ R^*(k-1) z(k)\ ]^{-1} z^T(k)\ R^*(k-1) = R^*(k) z(k)\ z^T(k)\ R^*(k-1) \qquad \ldots \qquad (5.13)$$

On substitution of equation (5.11) in equation (5.13)

$$R^*(k) = R^*(k-1) - R^*(k-1)\, z(k) \left[1 + z^T(k) R^*(k-1) z(k) \right]^{-1} \times z^T(k)\; R^*(k-1) \qquad \ldots \qquad (5.14)$$

From equation (5.7) denoting the estimate $\hat{\alpha}$ at time instant k as $\hat{\alpha}(k)$

$$\hat{\alpha}(k) = R^*(k)\; S(k)$$

Combining equations (5.9) and (5.14)

$$\hat{\alpha}(k) = \left[R^*(k-1) - R^*(k-1)\, z(k) \left[1 + z^T(k)\, R^*(k-1)\, z(k) \right]^{-1} \times z^T(k)\; R^*(k-1) \right] \left\{ S(k-1) + z(k)\, P(k) \right\} \qquad \ldots \qquad (5.15)$$

since $R^*(k-1)\; S(k-1) = \hat{\alpha}(k-1)$

The equation (5.15)

$$\hat{\alpha}(k) = \hat{\alpha}(k-1) - R^*(k-1)\, z(k) \left[1 + z^T(k)\, R^*(k-1) z(k) \right]^{-1} \times z^T(k)\; \hat{\alpha}(k-1) + R^*(k-1)\, z(k)\, P(k) - R^*(k-1)\, z(k) \left[1 + z^T(k)\; R^*(k-1)\, z(k) \right]^{-1} \times \left\{ Z^T(K)\, R^*(K-1)\, Z(K)\, P(K) \right\} \qquad (5.16)$$

since $z^T(k)\; R^*(k-1)\; z(k)$ is scalar the equation (5.6) becomes

$$\hat{\alpha}(k) = \hat{\alpha}(k-1) + R^*(k-1)\, z(k) \left[1 + z^T(k)\, R^*(k-1)\, z(k) \right]^{-1} \times \left[P(k) - z^T(k)\; \hat{\alpha}(k-1) \right] \qquad \ldots \qquad (5.17)$$

The equations (5.17) , (5.14) with

$$R^*(k) \triangleq \left[\sum_{j=1}^{k} z(j)\; z^T(j) \right]^{-1}$$

are the recursive least square algorithms. But these algorithms do not overcome the problem of bias. The technique to minimise the bias is stated as follows.

From equation (5.6) the non recursive estimation of parameters at instant k is

$$\hat{\alpha}(k) = \left[\sum_{j=1}^{k} s(j)\ s^T(j) \right]^{-1} \left[\sum_{j=1}^{k} s(j)\ P(j) \right] \quad \ldots (5.18)$$

and the process equation is,

$$P(j) = s^T(j)\alpha + \eta(j) \quad \ldots \quad (5.19)$$

Upon substitution of equation (5.19) in equation (5.18)

$$\hat{\alpha}(k) = \left[\sum_{j=1}^{k} s(j)s^T(j) \right]^{-1} \left[\sum_{j=1}^{k} s(j) \left[s^T(j)\alpha + \eta(j) \right] \right]$$

$$= \left[\sum_{j=1}^{k} s(j)s^T(j) \right]^{-1} \left[\sum_{j=1}^{k} s(j)s^T(j) \right]\alpha$$

$$+ \left[\sum_{j=1}^{k} s(j)s^T(j) \right]^{-1} \left[\sum_{j=1}^{k} s(j)\ \eta(j) \right] \quad \ldots \quad (5.20)$$

So,

$$\hat{\alpha}(k) = \alpha + \left[\sum_{j=1}^{k} s(j)\ s^T(j) \right]^{-1} \times \left[\sum_{j=1}^{k} s(j)\ \eta(j) \right]$$

For the estimate of $\hat{\alpha}(k)$ to converge to

$$E\left\{ s(j)\ \eta(j) \right\} = 0 \quad \text{for all } j$$

and

$$E\left\{ \eta(j)\ \eta(k) \right\} = 0 \quad \text{for } j \neq k$$

i.e. $\eta(j)$ converge to white noise process $v(j)$. Under these conditions the estimates will be unbiased. However, these conditions are difficult to achieve in practice. And consequently the vector $z(k)$ has been modified incorporating the instrument variable $\hat{P}(.)$ as $\hat{z}(k)$ defined as

$$\hat{z}(k) = [\hat{P}(k-1), \hat{P}(k-2), \ldots, \hat{P}(k-N)]^{-1} \quad \ldots \quad (5.21)$$

where $\hat{P}(.)$ is the recursive least square estimation of $P(.)$. Since $\hat{P}(.)$ sequence in equation (5.21) are already estimates, in a sense optimum, they are almost uncorrelated with $\eta(k)$. Thus the conditions of unbiased estimates are modified as

$$E\{\hat{z}(k) \quad \eta(k)\} = 0 \quad \text{for all } k$$

Replacing $z(k)$ by $\hat{z}(k)$ rather heuristically and not $z^T(k)$ by $\hat{z}^T(k)$, because of experience in the interpolation region, in equations (5.14) and (5.17) the recursive instrument variable algorithms become,

$$\hat{\alpha}(k) = \hat{\alpha}(k-1) + \hat{R}^*(k-1)\hat{z}(k)\,[1+z^T(k)\hat{R}^*(k-1)\hat{z}(k)]^{-1} \times \{P(k) - z^T(k) \times \hat{\alpha}(k-1)\} \quad \ldots \quad (5.22a)$$

$$\hat{R}^*(k) = \hat{R}^*(k-1) - \hat{R}^*(k-1)\hat{z}(k)\,[1+z^T(k)\hat{R}^*(k-1)\hat{z}(k)]^{-1} \times z^T(k) \quad \hat{R}^*(k-1) \quad \ldots \quad (5.22b)$$

$$R^*(k) = \left[\sum_{j=1}^{k} \hat{z}(j) \quad \hat{z}(j)\right]^{-1} \quad \ldots \quad (5.22c)$$

The algorithms may be initialised with $\hat{\alpha}(.)$ as a least square estimate for a block of data in the interpolation region and $\hat{R}^*(.)$ for the same block of data.

5.3. Illustration.

A representative portion of the data set i.e. the daily flow of the river Teesta gauged at Domohani near Jalpaiguri town, from the 20th. January, 1978 to the 31st. March, 1978 is shown in Fig 5.3.1(a). The correlation co-efficient versus shift of instances of time of the data set from the 1st. January, 1978 to the 31st. March, 1978 is shown in Fig 5.3.2 . It is evident that the process has a significant prehistory interval $N = 8$. Fig 5.3.1(b) shows the errors of prediction. The values of the integral squared errors and the mean error are 0.027353 and 0.087206 cubic metre per second respectively. It is observed from the correlation co-efficient versus shift of time instances of the error sequence shown in Fig 5.3.3 that the errors of prediction are almost uncorrelated [106, 107].

5.4. Conclusion.

A dynamic model for real time simulation of daily river flows has been presented. The model gives one step ahead on-line prediction of daily flows on the basis of the currently available information. It is hoped that the real time recursive prediction of daily river flows would assist the designer and operator of the large scale water resources systems involving control, utilisation and disposition of river water.

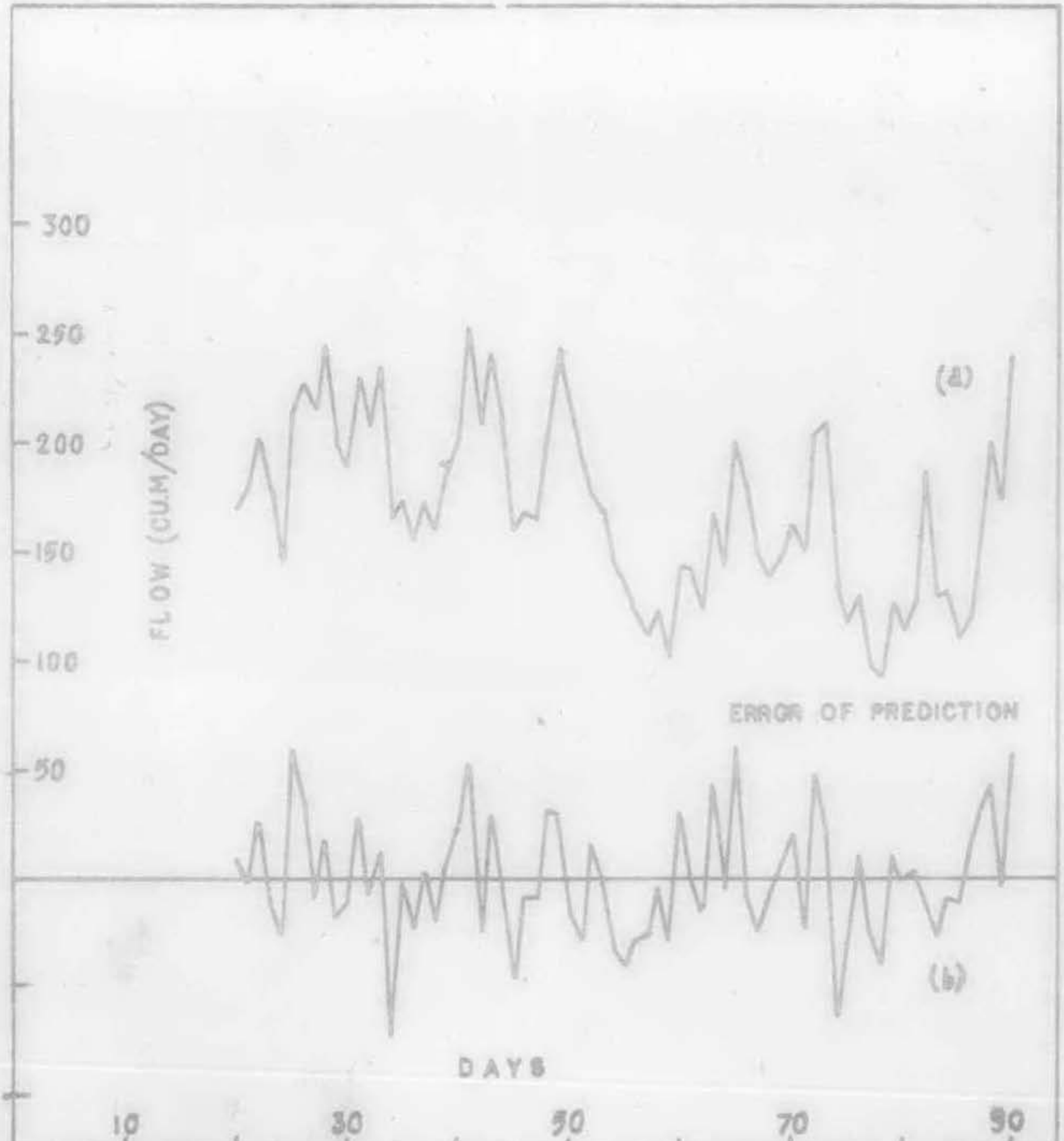

FIG 5.3.1 OBSERVED DAILY FLOW AND ERROR OF PREDICTION

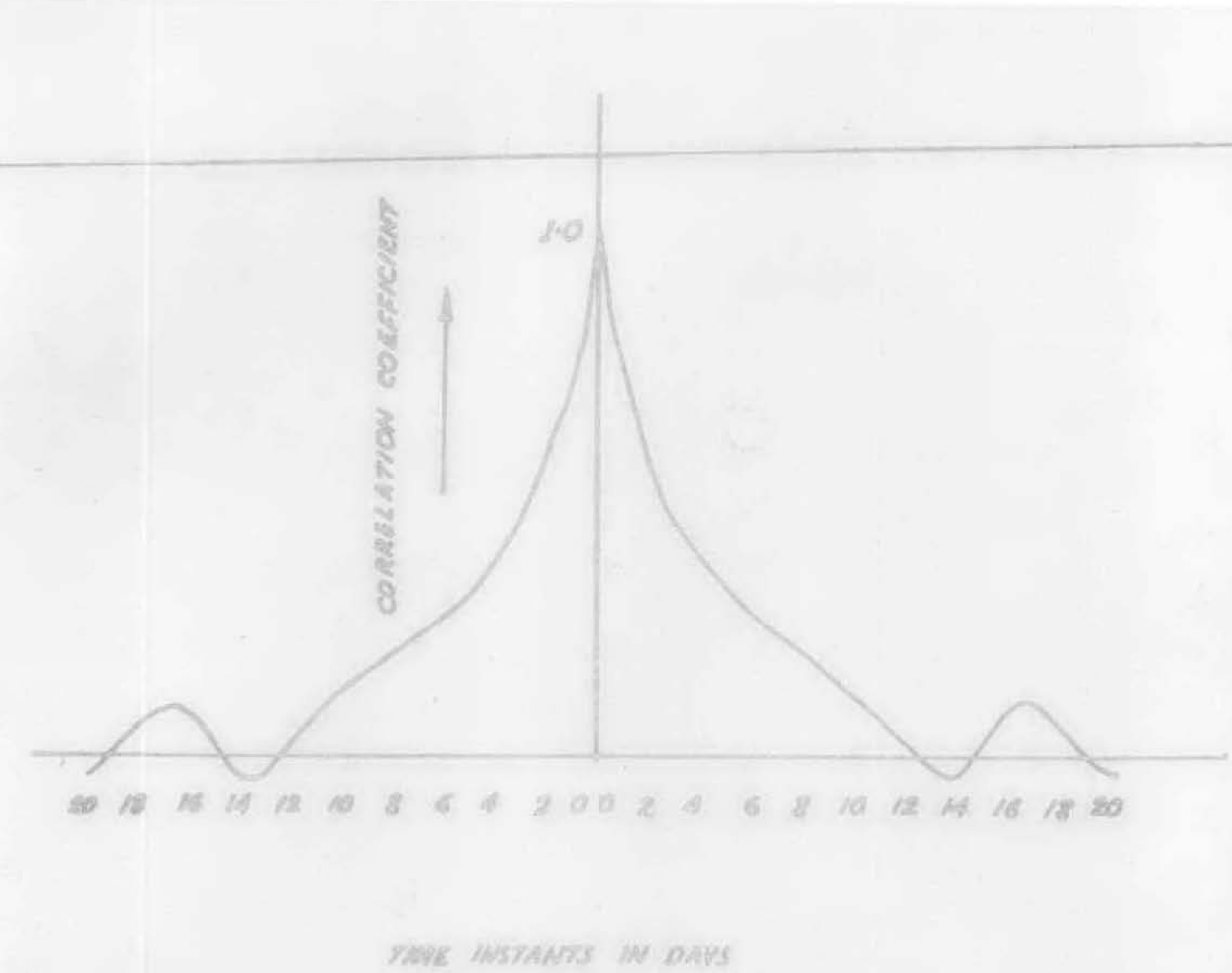

FIG. 5.3.2 CORRELATION COEFFICIENTS VS SHIFT OF INSTANTS IN DAYS FOR DATA SET

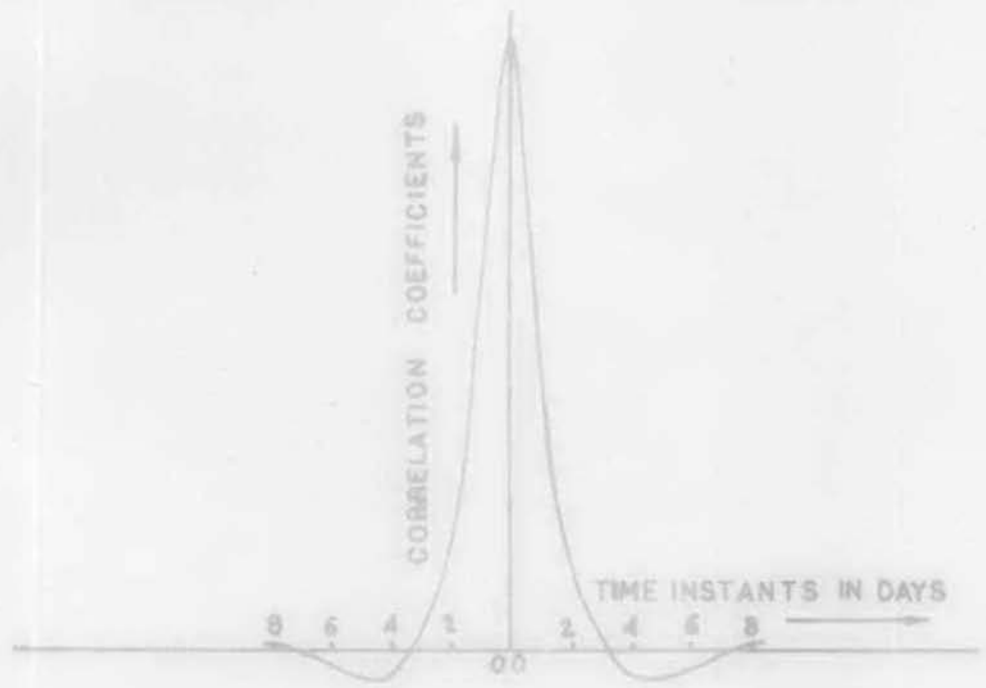

FIG 5.3.3. CORRELATION COEFFICIENT VS SHIFT OF TIME INSTANCES IN DAYS OF THE ERROR SEQUENCE