CHAPTER IV

RECURSIVE ESTIMATION OF RIVER FLOWS DURING A SHORT SPAN STORM PERIOD

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4.0 Abstract :

A recursive non-stationary parameter estimation technique has been developed and used to forecast on on-line basis the hourly flows of a non-tidal river during a short span storm period. The model is simple and requires no elaborate initialisation. The model has been subjected to test with field data observed at different gauging stations of the river Teesta in North Bengal during a short span storm period. Predicted data are found to simulate closely the variation of flows observed during the storm period.

4.1 Introduction :

The real time monitoring of river flow during a storm period and the on-line control of river flow through different hydraulic structures require a consistent and reliable set of data representing the predicted flow on the basis of past measurements and a suitable parameter tracking algorithm for the flow process. This work introduces some of the fundamental techniques and principles of control theory broadly classified as cybernetics and indicates how they might be used for forecasting of hourly flow of non-tidal river during a short span strom period. This forecasting model would help in the subsequent development of the methodologies for on-line operation of complex water resources control systems.

In the present work the recursive non-stationary time series analysis technique has been used for on-line forecasting of hourly river flow during a storm period. The recursive parameter estimation algorithms have the advantages of minimal computational requirements with ability to track the time varying parameters. On the basis of these information processing algorithms a real time dynamic model of the river flows during a short span storm period has been obtained to forecast the hourly flows at Domohani Road put Bridge point of the river Teesta near Jalpaiguri town in North Bengal.

4.2. Development of non-stationary process model

Non-stationary process can be expressed as,

$$Y(k) = U(k) + V(k) \qquad \ldots (4.1)$$

where $U(k)$ is the deterministic input components,
$V(k)$ is the stationary stochastic components,
$k = 1, 2, \ldots,$ are time instances.

The strong correlation of down-stream flows with those at up-stream flows particularly at the confluence of tribularies suggests that the process may be represented in the form of a non-stationary time series with a probability that the current value of the output $Y(k)$ is a function of the previous output observations, the auto regressive terms $Y(k-1)$, $Y(k-2)$, ..., and the highly correlated deterministic inputs, the up-stream flows.

$U1(k-r_1)$, $U_2(k-r_2)$, ..., $U_1(k-r_1-1)$, $U_2(k-r_2-1)$, ... ,

together with the current unknown realisation of the noise process $\eta(k)$.

The process can be represented as

$$Y(k) = \sum_{i=1}^{n} \beta(k-i)Y(k-i) + \sum_{j=1}^{m} \sum_{i=0}^{n} \alpha(k-ji)U_j(k-r_j-i) + \eta(k) \quad \ldots (4.2)$$

where the deterministic input components

$$U(k) = \sum_{j=1}^{m} \sum_{i=0}^{n} \alpha(k-ji)U_j(k-r_j-i)$$

and the stationary stochastic component

$$V(k) = \sum_{i=1}^{n} \beta(k-i)\, Y(k-i) + \eta(k)$$

$\eta(k)$ is modelled as

$$\eta(k) = \sum_{p=1}^{r} \gamma(k-p)\, \eta(k-p) + \nu(k) \quad \ldots \quad (4.3)$$

Where $\nu(k)$ is a white noise innovation process with

$$E\{\nu(k) \quad \nu(j)\} = 0 \quad \text{for } j \neq k$$

$$\text{and} \quad E\{\nu(k) \quad \nu(j)\} = 1 \quad \text{for } j \neq k$$

The determination of n is known as the model order determination. The output correlation has been suggested as an intuitive consideration for model order determination which is also known as model order identification. The determination of r in autoregressive error sequence $\eta(k)$ depends on the conditions when residuals of the estimate of $\eta(k)$ i.e., $\eta(k) - \hat{\eta}(k)$ becomes a white noise process.

The model is depicted in equation (4.2) is quite flexible since it requires that the equation be linear in parameters.

The equation (4.2) may be represented in the form as

$$Y(k) = a^T (k-1)\, Z(k-1) + \nu(k) \qquad \ldots \qquad (4.4)$$

where

$$a^T(k-1) = \Big[\beta(k-1),\ \beta(k-2), \ldots, \beta(k-n)$$

$$\sum_{j=1}^{m} \Big[\alpha(k-jo),\ \alpha(k-j1), \ldots, \alpha(k-jn) \Big],\ \gamma(k-1)$$

$$\gamma(k-2), \ldots, \gamma(k-r) \Big] \qquad \ldots \qquad (4.5)$$

and

$$Z(k-1) = \Big[Y(k-1),\ Y(k2), \ldots, Y(k-n)$$

$$\sum_{j=1}^{m} \Big[U_j(k),\ U_j(k-1), \ldots, U_j(k.n) \Big]$$

$$\eta(k-1),\ \eta(k-2), \ldots, \eta(k-r) \Big]^T \qquad \ldots \qquad (4.6)$$

The estimate of Y(k) is written as

$$\hat{Y}(k) = a^T (k-1)\ Z(k-1) \qquad \ldots \qquad (4.7)$$

4.3 Development of recursive parameter estimation algorithm

The coefficient vector 'a' can be estimated by minimising the quadratic function $J_k(a)$,

$$J_k(a) = \sum_{j=1}^{k} (Y(j) - a^T Z(j-1))^2 + (a-a(o))^T S^{-1}(o)\ (a-a(o)) \qquad \ldots \qquad (4.8)$$

where a (o) is the available a priori estimate of the coefficient vector 'a' and S(o) is the positive definite weighting matrix of the order q X q, q being n + (n+1) m + r.

For minimization

$$\frac{\partial J_k(a)}{\partial a} = -2 \sum_{j=1}^{k} Z(j-1)Y(j)-a^T Z(j-1) + 2S^{-1}(o)(a-a(o)) \quad \dots (4.9)$$

since we are seeking for a minima

$$\frac{\partial J_k(a)}{\partial a} = 0 \quad \dots (4.10)$$

By matrix inversion lemma and by denoting the estimate of 'a' as 'a (k)' at the time instant k the recursive parameter estimation algorithms become,

$$a(k+1) = a(k) + S(k+1)Z(k) \left[Y(k+1) - a^T(k)Z(k) \right] \quad \dots (4.11a)$$

$$S(K+1) = S(k) - S(k)Z(k)Z^T(k)\, S(k) \left[1 + Z^T(k)\, S(k)\, Z(k) \right]^{-1}$$

The algorithms have been initialised with

$k = o$, $S(o) \hat{=} I$ (unit matrix), $a(o) \hat{=} o$ and

$Y(j) = o$, for $j = o, -1, -2, \dots$

4.4. Illustration.

Data for a short span storm period from 16.00 hours on 23.7.79 to 15.00 hours on 25.7.79 were observed at the gauging stations at Sankalan, Rongpo (Teesta), Great Rangeet (Singlabazar), Teestabazar, Coronation Bridge and Domohani Road Bridge (Jalpaiguri town) of the river Teesta in North Bengal. The gauging stations are shown in Fig 4.4.1.

The input and the output variables are rationalised in accordance with

$$x(k) = \frac{X(k) - X_{min}}{X_{max} - X_{min}} \quad \dots \quad (4.12)$$

where $x(.)$ is the observed value at the k-th instant and X_{min} and X_{max} are the minimum and maximum values of the data sequence. The input variables are selected on the basis of the strongest coefficient of correlation with the flows at Domohani. The coefficient of correlation has been defined as,

$$\phi_{yx}(\lambda) = \frac{\sum_{i=1}^{N-\lambda}\left(y(i) - \frac{1}{N}\sum_{i=1}^{N} y(i)\right)\left(x(i+\lambda) - \frac{1}{N}\sum_{i=1}^{N} x(i)\right)}{\left[\sum_{i=1}^{N-\lambda}\left(y(i) - \frac{1}{N}\sum_{i=1}^{N} y(i)\right)^2 \sum_{j=1+\lambda}^{N}\left(x(j) - \frac{1}{N}\sum_{i=1}^{N} x(i)\right)^2\right]^{\frac{1}{2}}} \quad \ldots (4.13)$$

where λ is the shift of instances of time. On the basis of the coefficients of correlation the hourly flows at Domohani Road Bridge in rationalised units may be expressed as

$$y(k) = f(y(k-1),\ y(k-2), x_1(k-6), x_2(k-7), x_3(k-9), x_4(k-8), x_5(k-10)) \quad \ldots (4.14)$$

where $x_1(.)$, $x_2(.)$, $x_3(.)$, $x_4(.)$ and $x_5(.)$ are respectively the hourly flows at Coronation Bridge, Teesta Bazar, Great Rangeet (Singlabazar), Rongpo (Teesta) and Sankalan respectively.

With output prehistory interval = 2 and the moving average terms $r = 2$, the integral square error defined as

$$ISE = \frac{\sum_{i=1}^{N}\left[y(i) - y(i) \right]^2}{\sum_{i=1}^{N} (y(i))^2} \quad \ldots (4.15)$$

has been found as 0.017584 where $y(.)$ is the modelled output. Fig 4.4.2a and Fig 4.4.2b show the observed and the modelled values. The model is found to adhere very closely to the observed data. Algorithms are simple and require no elaborate initialisation [105].

FIG- 1

TEESTA

AND ITS TRIBUTARIES.

SCALE - 1 INCH = 16 MILES

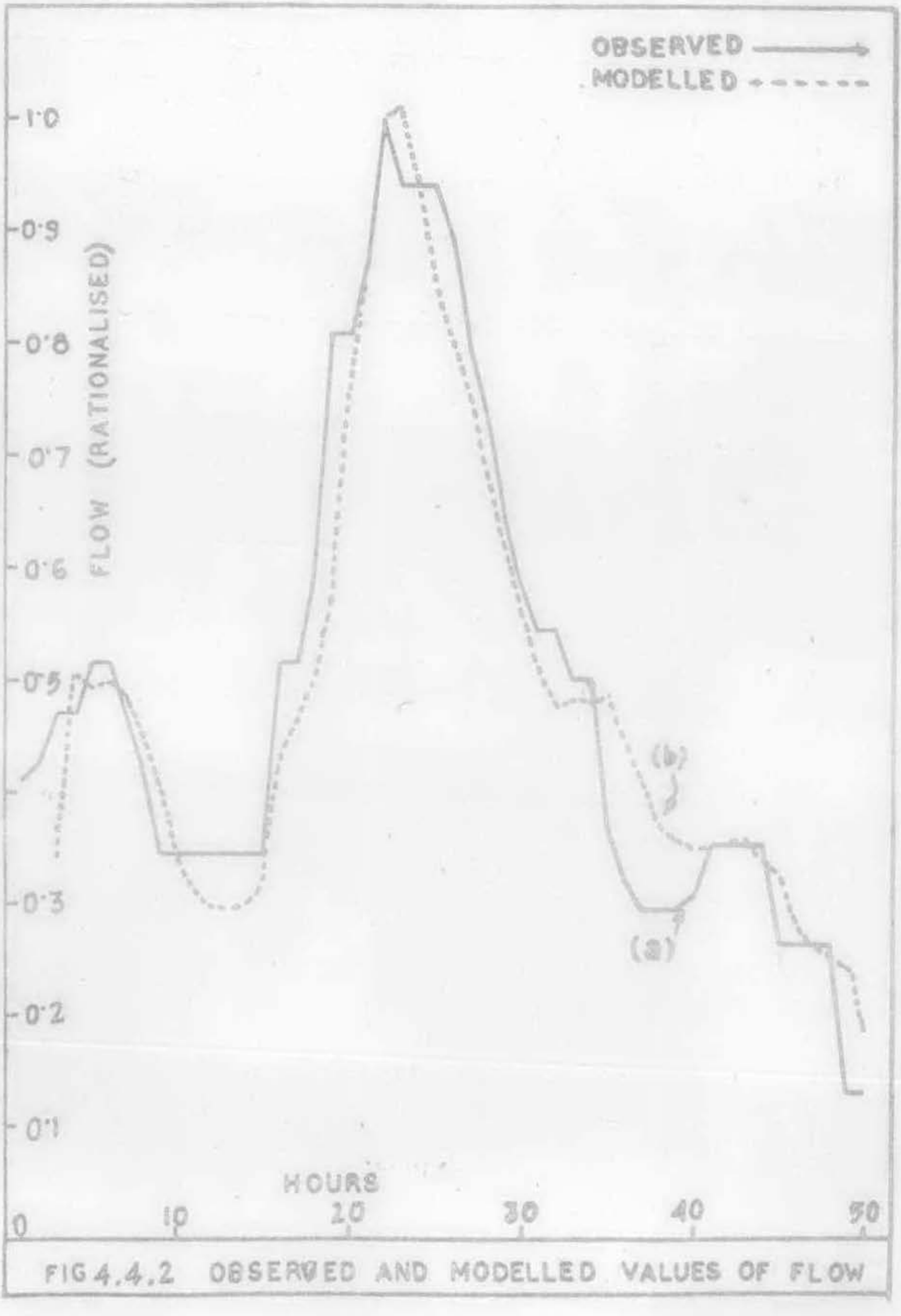

FIG 4.4.2 OBSERVED AND MODELLED VALUES OF FLOW

4.5 Conclusion

From the work presented here it is claimed that on-line flood warning and control measures can be implemented provided adequate real time information processors are installed at the gauging stations with closed loop decision regularisation for operation and control of hydraulic structures of the complex water resources system.