CHAPTER – III

BOUNDARY LAYER EFFECTS ON STRATIFIED FLUID FLOWS

PART - ONE

TO STUDY THE BOUNDARY LAYER FOR MHD STRATIFIED FLUID THROUGH A POROUS MEDIUM

Introduction : The motion of viscous stratified fluid largely depends on the magnitude of stratification factor, porosity factor, slip parameter and Hartmann number, so they must have an effect on the boundary layer also. Density and stratification factor play an important role on many atmospheric and oceanic geophysical phenomena. Owing to the increasing geophysical applications, the study of stratified fluid flow has received a considerable attention by a number of authors.

Bathaiah (1980) has discussed the flow of a viscous incompressible slightly conducting fluid through a porous straight channel under a uniform transverse magnetic field. Bathaiah and Bhaskara Reddy (1982) have studied the effect of hall current on the flow of viscous incompressible slightly conducting fluid through a porous straight channel under a uniform transverse magnetic field. Again, Bathaiah and Bhaskara Reddy (1987) have studied the combined effects of free and forced free convection on the flow of an incompressible viscous conducting fluid between two horizontal insulated parallel walls, one of which is at rest and the other moving parallel to itself with a linear axial temperature variation under the uniform transverse magnetic field. K. Sreenivasan and Bathaiah (1993) considered the flow of a viscous conducting fluid between two parallel plates of uniform length, lower plate being stationary and upper plate moving with constant velocity under a periodic pressure gradient superimposed on a constant pressure gradient under the influence of a uniform transverse magnetic field. Manju Gupta and Sharma (1993) considered the unsteady flow of an electrically conducting elastico-viscous dusty liquid through a rectilinear pipe having its cross-section as a hyperbolic sector in the presence of a transverse magnetic field under the influence of an arbitrary time varying pressure gradient. Kumar, Prasad and Gupta (1990) considered the MHD flow of stratified fluid through a porous medium between two oscillating plates. Chawla (1972) has investigated the boundary layer flow of a micro polar fluid along an infinite plate when the plate performs impulsive motion in its own plane. Chiu (1962), Soo (1961) and Singleton (1965) investigated the problem of boundary layer flow of dusty fluid over a semi finite flat plate. Srivastava and Maiti (1966) have solved the boundary layer equation for two dimensional flow of a second order fluid. Mathur and Nandanan (1972) have studied the laminar boundary layer flow of an oldroyed fluid under the influence of pressure gradient with and without suction through a wedge. Gersten and Gross (1974) have studied the three dimensional incompressible boundary layer flow past a porous flat plate. Lighthill (1954) studied the two dimensional boundary layer to the fluctuations in the oncoming stream. Sharma and Singh (1992) investigated three dimensional laminar boundary layer free convection flow past (i) a porous flat plate and (ii) a porous vertical plate. Jain and Singh (1992) considered the unsteady magneto hydrodynamics boundary layer flow past a porous flat plate with sudden change in suction. Rath and Parida (1982) considered the oscillating free convection boundary layer flow of a viscous fluid near an infinite vertical wall and investigated that (i) by increasing injection velocity the fluid layer very near the wall may be made to oscillate with an amplitude larger than that of the wall velocity (ii) the boundary layer temperature fluctuates with a phase which changes only with the magnitude but not the sign of fluid influx parameter.

The motion of viscous stratified fluid largely depends on the magnitude of stratification factor, porosity factor, slip parameter and Hartmann number, so they must have an effect on the boundary layer also.

In this paper, we have studied the effect of stratification factor, porosity factor and slip parameter on the boundary layer thickness of the flow of viscous stratified fluid under the influence of transverse magnetic field. The effects of various parameters on the boundary layer have been shown with the aid of graph.

Mathematical formulation of the problem and its solution

To study the viscous stratified flow of variable viscosity between a porous bed and an impermeable plate in presence of transverse magnetic field. It is seen there exists a thin boundary just beneath the interface. It is of interest to find the expression for this boundary layer thickness.

A physical model for this problem considered here have been depicted in figure-1 which consists of two zones. Zone one is the region occupied by the viscous fluid from moving impermeable wall rigid plate to the interface when the flow is characterised by free flow and is governed by Navier-Stokes equations. The motion being taken uni-directional we choose u the velocity in the x-direction and y-axis perpendicular to direction of motion, ρ density, μ the co-efficient of viscosity and p the pressure at any point the space occupied by the fluid. Zone 2 lies below the interface where the flow is governed by modified Darcy law.

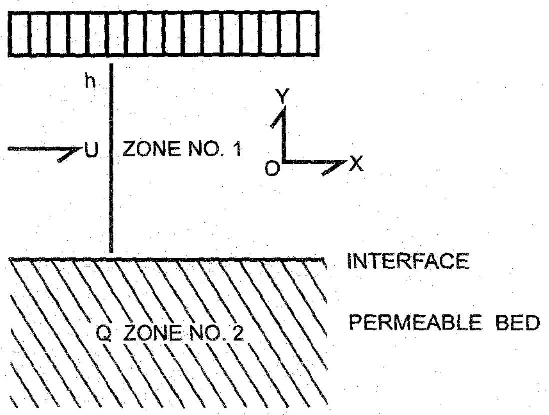

Fig. 1 PHYSICAL MODEL

The modified governing equation of motion is (Zone-1)

$$\rho\frac{\partial u}{\partial t}=-\frac{\partial p}{\partial x}+\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)-\frac{\mu u}{k}-\sigma_1{}^2B^2u\ ....(1)$$

where $\rho=\rho_0 e^{-\beta y/2}, \mu=\mu_0 e^{-\beta y}$ and

$$B=B_0 e^{-\beta y}\ ....(2)$$

and $Q=Q_0 e^{\beta y}\ ....(3)$

where $Q_0=-k/\mu\left(\frac{\partial p}{\partial x}\right)$ is the basic equation for the Zone-2

Here μ_0 and ρ_0 are the co-efficient of viscosity and density respectively at the interface y=0. $\beta>0$ represents the stratification factor.

The boundary conditions for the problem are

$$u = u_B \text{ at } y = 0 \text{..........(4)}$$

$$u = Q_0 e^{-\beta\partial} = -k/\mu_0 e^{-\beta\partial}\left(\frac{\partial p}{\partial x}\right) \text{at } y = -\delta \text{ ....(5)}$$

u_B is the velocity at the nominal surface y=0 and Q is given by (3) , k, the permeability co-efficient, σ_1, the electrical conductivity of the fluid, B_0 the induced magnetic field, δ, the boundary layer. Now introducing the following non-dimensional quantities

$$x' = \frac{x}{h},\ y' = \frac{y}{h},\ t' = \frac{ht}{u_m}$$

$$\delta' = \frac{\delta}{h},\ p' = \frac{p}{\rho_0 \mu_m^2},\ \frac{u_2}{u_m} = V_2,$$

$$M = \frac{h\mathbf{B}_0^2}{\sqrt{\mu_0/\sigma_1^2}},\ u' = \frac{u}{u_m} \text{ ....(6)}$$

Equation (1) now reduces to

$$\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x}e^{ny} + \frac{1}{R}\left\{\left(\frac{\partial^2 u}{\partial y^2} - n\frac{\partial u}{\partial y}\right) - \left(M^2 + \sigma^2\right)u\right\} \text{............(7)}$$

Corresponding boundary conditions are

$$u = V_B \text{ at } y = 0 \text{ ....(8)}$$

$$u = -\frac{R}{\sigma^2}e^{-n\delta}\frac{\partial p}{\partial x} \text{ at } y = -\delta \text{ ....(9)}$$

Here n is the non-dimensional stratification factor, σ, is the porosity factor, R the Reynolds number, V_2 is the dimensional slip velocity at the nominal surface y=0, u_m the maximum velocity of the flow, M the Hartmann number.

Let us now assume that the flow is driven by an unsteady pressure gradient given by

$$-\frac{\partial p}{\partial x} = Ae^{-\alpha_1 t} \ldots\ldots\ldots\ldots\ldots(10)$$

where A & α_1 are known as real constants.

We assume that the velocity is given by

$$u = ve^{-\alpha_1 t} \ldots\ldots\ldots\ldots\ldots(11)$$

$$v_B = u_B e^{-\alpha_1 t} \ldots\ldots\ldots(12)$$

Equation (7) now becomes

$$\frac{d^2 v}{dy^2} - n\frac{dv}{dy} - \left(M^2 + \sigma^2 - \alpha_1 R\right)V = A\,Re^{ny} \ldots\ldots\ldots\ldots\ldots\ldots (13)$$

Solution of Equation (13) is

$$V = e^{-ny/2}\left[C \ \cosh\alpha_2 y + D\,\mathrm{Sinh}\,\alpha_2 y\right] + \frac{A\,Re^{ny}}{M^2 + \sigma^2 - \alpha_1 R} \ldots\ldots\ldots\ldots\ldots (14)$$

Where C and D are functions independent of y.

Using boundary conditions (8) & (9) in (14), we get the velocity fields as

$$u = e^{ny/2}\left\{\left(u_* - \frac{AR}{M^2+\sigma^2-\alpha_1 R}\right)Cosh\alpha_2 y + \left(u_n - \frac{AR}{M^2+\sigma^2-\alpha_1 R}\right)Coth\alpha_2\delta\,Sinh\alpha_2 y + \frac{\left(\alpha_1 R - M^2\right)ARe^{-n\delta/2}}{\sigma^2\left(M^2+\sigma^2-\alpha_1 R\right)Sinh\alpha_2\delta}Sinh\alpha_2 y\right\}$$

$$\ldots\ldots\ldots\ldots\ldots (15)$$

We know that at the edge of the boundary layer, the shear has to be zero. In other words

$$\frac{\partial u}{\partial y} = 0 \ \text{ at } y = -\delta \ \ldots\ldots\ldots\ldots (16)$$

Then from equation (15) using (16) we get,

$$\mathrm{Sinh}\,\alpha_2\delta e^{-n\delta/2}+\frac{\alpha_2\left(\alpha_1 R-M^2\right)}{\left(M^2-\alpha_1 R+2\sigma^2\right)n}\mathrm{Cosh}\,\alpha_2\delta e^{-n\delta/2}$$

$$+\frac{\alpha_2\sigma^2}{nAR\left(M^2-\alpha_1 R+2\delta^2\right)}\left\{u_B\left(M^2+\sigma^2-\alpha_1 R\right)-1\right\}=0 \quad \text{..............} \quad (17)$$

This equation for δ is transcendental and it is difficult to obtain an analytical solution. However, we feel that since the boundary layer and stratification factor are very small, we can neglect cubes and higher powers of δ and obtain.

$$a\delta^2+b\delta\ +c=0 \quad \text{..............} \quad (18)$$

Where $a=AR\left(3n^2\alpha_1 R-3n^2M^2-\alpha_2^2M^2-\alpha_2^2\alpha_1 R-4\sigma^2\right)$

$$b=8nAR\left(M^2+\sigma^2-\alpha_1 R\right)$$

$$c=8\left(M^2+\sigma^2-\alpha_1 R\right)\left(u_B\sigma^2-AR\right)$$

DISCUSSION :

We represent δ graphically and it is observed from fig.-2 that for a fixed n and σ the boundary layer δ decreases with increase of M (Hartmann number) and for fixed n and M, the boundary layer δ increases with the increase of σ(fig.-3) and for fixed σ and M, the boundary layer increases with the increase of n. (fig.-4).

			$\sigma = 5$			
M $\rightarrow$	0	1	2	2.5	3	4
δ $\rightarrow$	8.636	7.727	6.364	5	4.091	2.727
			$\sigma = 10$			
M $\rightarrow$	0	1	2	2.5	3	4
δ $\rightarrow$	17.727	15.455	11.818	10.455	9.091	5.903
			$\sigma = 15$			
M $\rightarrow$	0	1	2	2.5	3	4
δ $\rightarrow$	25.909	22.273	17.273	15	12.727	8.636
			$\sigma = 20$			
M $\rightarrow$	0	1	2	2.5	3	4
δ $\rightarrow$	34.545	31.364	24.545	20.909	17.273	12.273

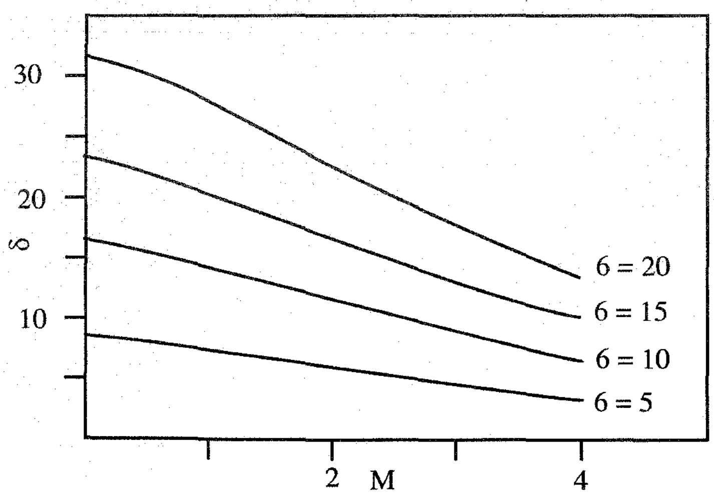

Fig.2 Plot of δ against M for different values of 6.
when n = 0.2

M = 1					
$\sigma \rightarrow 0$	5	10	15	20	30
$\delta \rightarrow 0$	7.2	14	20.8	27.2	34.8

M = 2					
$\sigma \rightarrow 0$	5	10	15	20	30
$\delta \rightarrow 0$	5.6	10.8	16	21.6	26.4

M = 3					
$\sigma \rightarrow 0$	5	10	15	20	30
$\delta \rightarrow 0$	4	8.4	12	16	20.4

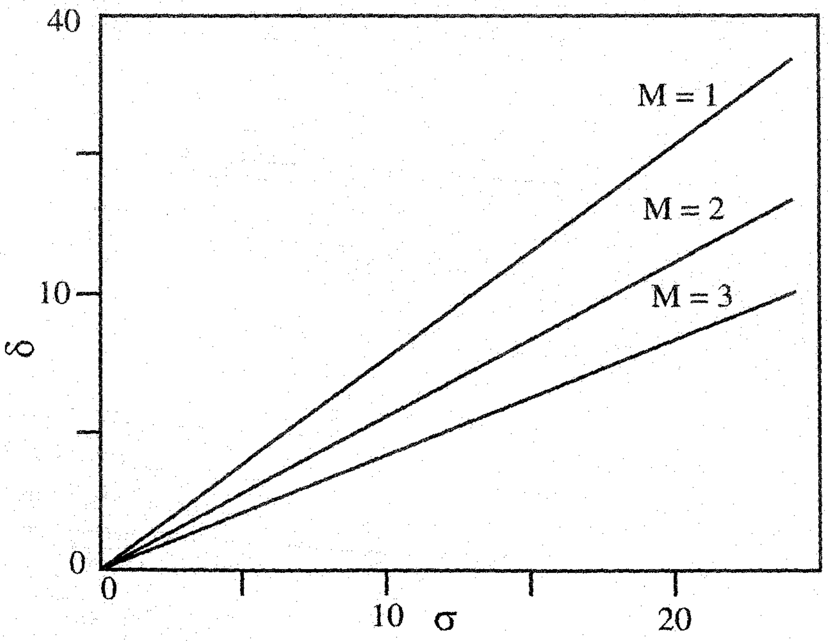

Fig.3 Boundary layer (δ) of the flow of visous stratified fluid for different values of σ when N=0.2

PLOT OF δ against M FOR DIFFERENT VALUES OF σ WHEN n=0.2

M = 0				
n → 0	.4	.6	.8	1
δ → 7.826	8.696	9.130	9.565	10

M = 1				
n → 0	.4	.6	.8	1
δ → 6.957	7.391	7.826	8.043	8.261

M = 2				
n → 0	.4	.6	.8	1
δ → 5.217	5.652	5.870	6.087	6.304

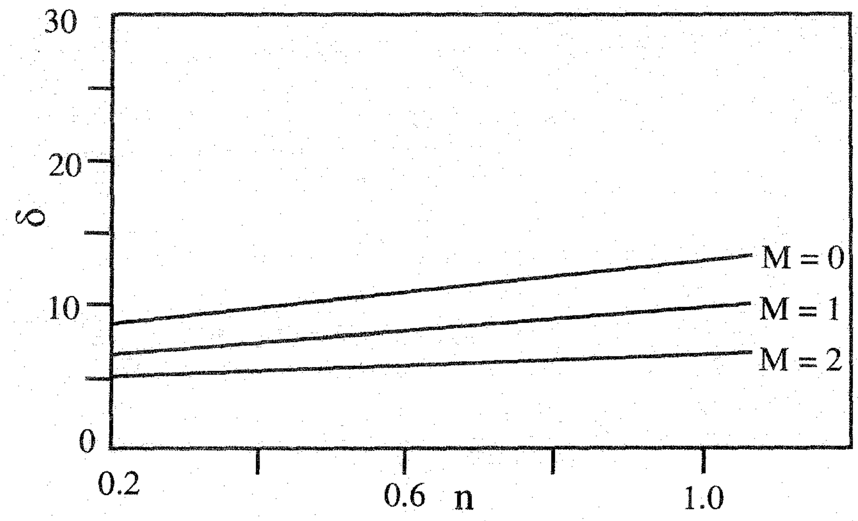

Fig. 4 : Plot of δ against n for different values of M when σ =5