PART - TWO

UNSTEADY LAMINAR STRATFIED FLOW OVER A POROUS BED UNDER THE ACTION OF BODY FORCE

Introduction

The study of flow through porous media is of principal interest due to the importance in chemical engineering for filtration and water purification purposes., petroleum engineering for studying the movement of natural gas, oil and water through the oil reservoirs and to study the underground water in river beds. Its flow behaviour of fluids in a petroleum reservoir rock depends to a large extent the viscous stratification and also on the porous properties of the rock, a technique of core study that can give a new or additional information on the characteristics of the rock which would provide better understanding of petroleum reservoir performance. Unsteady flow of viscous fluid over permeable bed has been treated by Hunt (1959). Bhattacharya (1980) considered the unsteady laminar flow in a channel with porous bed. Mukherjee et al. (1986) considered the unsteady flow of viscous stratified fluid in a rotating system. Majumder and Debnath (1978) studied the unsteady motion of an inviscid rotating stratified fluid in different geometrics. Channabassappa and Ranganna (1975) and Gupta and Sharma (1978) discussed the stratified viscous flow of variable viscosity between a porous bed and moving impermeable plate. Harikrishan and Sharma (1980) considered the stratified viscous flow of variable viscosity between a porous bed and moving impermeable plate under the action of body force.

In this paper, we have studied the unsteady flow of viscous stratified fluid over a permeable bed under the action of a body force. We have also calculated the fractional increase $|\phi|$ in mass flow rate with the assumption of the validity of Darcy law for laminar flow $R < 1$. We have seen that for very small body force, n, σ and α have no effect on $|\phi|$, but for large body force, $|\phi|$ increases with the increase of σ for each set of values of n and α and $|\phi|$ decreases with the increase of n, for each set of values σ, α. Thus it can be predicted that the stratification factor is not favourable to the fractional increase in the mass of the fluid.

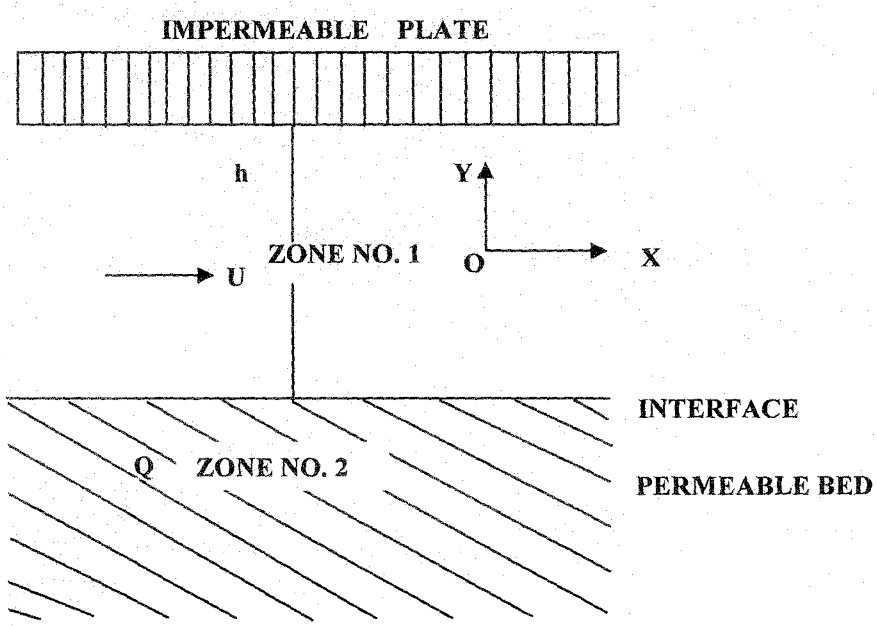


Fig. 1. PHYSICAL MODEL

Mathematical formulation of the problem and its solution :

The physical model shown in Fig. 1 consists of two zones. In Zone-1, from the impermeable upper rigid plate up to the interface, the flow called the free flow is governed by the usual ***Navier-Stokes*** equations. In the other zone, below the interface, the flow is governed by the Darcy law. In the following discussion we shall refer to these zones as 'zone 1' and 'zone 2' respectively.

The basic equations for Zone 1 are taken as

$$\rho\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) + A\,e^{-\alpha_1 y} \qquad .....(1)$$

$$\mu = \mu_0 \exp(-\beta y), \quad \rho = \rho_0 \exp(-\beta y) \qquad .....(2)$$

$$\frac{\partial p}{\partial y} = -\,g\rho \qquad .....(3)$$

Here μ_0 and ρ_0 are the co-efficient of viscosity and density respectively at the interface y=0. The term $Ae^{-\alpha_1 y}$ represents the body force. Also $\beta > 0$ represents the stratification factor. $\frac{\partial p}{\partial x}$ is the common pressure gradient by which the flow in zone-1 and zone-2, is driven in the x-direction.

The basic equations for zone-2 are

$$Q = Q_0\, e^{\beta y} \qquad .....(4)$$

where

$$Q_0 = -\frac{k}{\mu_0}\left(\frac{\partial p}{\partial x}\right) \qquad .....(5)$$

The relevant boundary conditions are

$$u = u_0 \text{ at } y = h \qquad .....(6)$$

and

$$\frac{\partial u}{\partial y} = \frac{\alpha}{\sqrt{k}}(u_B - Q_0),\ y = 0 \qquad .....(7)$$

where α is the slip parameter, k is the permeability co-efficient which has the dimension of length square. Q is the Darcy velocity at the nominal surface y = 0. u_B is the slip velocity at the nominal surface y =0.

Let us make equation (1) non-dimensional by using the quantities

$$\left.\begin{aligned} u'=\frac{u}{u_m},\ t'=\frac{u_m}{x/t},\ x'=\frac{x}{h},\ y'=\frac{y}{h}, p'=\frac{p}{\rho_0\, u^2{}_m} \\ u_0'=\frac{u_0}{u_m},\ v_B'=\frac{u_B}{u_m},\ \frac{Ah}{\rho_0\, u^2{}_m}=A' \end{aligned}\right\} \qquad \text{.....(8)}$$

Equation (1) becomes

$$\frac{\partial^2 u'}{\partial y'^2}-n\frac{\partial u'}{\partial y'}-R\frac{\partial u'}{\partial t'}=R\,e^{ny}\frac{\partial p'}{\partial x'}-R\,A'e^{(n-\alpha_1')y'} \qquad \text{.....(9)}$$

Dropping dashes equation (9) becomes

$$\frac{\partial^2 u}{\partial y^2}-n\frac{\partial u}{\partial y}-R\frac{\partial u}{\partial t}=R\,e^{ny}\frac{\partial p}{\partial x}-R\,Ae^{(n-\alpha_1)y} \qquad \text{.....(10)}$$

The boundary conditions of non-dimensional quantities become

$$u=u_0 \text{ at } y=1 \qquad \text{.....(11)}$$

$$\frac{\partial u}{\partial y}=\alpha\sigma\left(V_B+\frac{R}{\sigma^2}\frac{\partial p}{\partial x}\right), \text{ at } y=0 \qquad \text{.....(12)}$$

Taking

$$u=f(y)\,e^{-\alpha_1 t} \qquad \text{.....(13)}$$

$$V_B=u_B\,e^{-\alpha_1 t} \qquad \text{.....(14)}$$

and

$$\frac{\partial p}{\partial x}=B\,e^{-\alpha_1 t} \qquad \text{.....(15)}$$

Equations (10), (11) and (12) take the form

$$\frac{d^2 f}{dy^2}-n\frac{df}{dy}+R\,\alpha_1 f=-R\,Be^{ny}-R\,Ae^{(n-\alpha_1)y}\exp(\alpha_1 t) \qquad \text{.....(16)}$$

$$f(y)=u_0\,e^{\alpha_1 t} \text{ at } y=1 \qquad \text{.....(17)}$$

and $$\frac{df}{dy} = \alpha\sigma\left(u_B + \frac{RB}{\sigma^2}\right) \quad \text{at} \quad y = 0 \qquad .....(18)$$

Solutions of equation (16) using boundary conditions (17) and (18) become

$$f(y) = e^{ny/2}\left[\alpha_i\, e^{\alpha_2 y} + \alpha_j\, e^{-\alpha_2 y}\right] - \frac{B}{\alpha_1}\, e^{ny} - \frac{RA}{\alpha_1}\, e^{\alpha_1 t}\, \frac{e^{(n-\alpha_1)y}}{(R - n + \alpha_1)} \qquad ...(19)$$

where $$\alpha_i = \frac{T_1 - T_2\, e^{\alpha_2}\,(n/2 - \alpha_2)}{(n/2 + \alpha_2) - (n/2 - \alpha_2)e^{2\alpha_2}}$$

$$\alpha_j = \frac{T_2\, e^{\alpha_2}\,(n/2 + \alpha_2) - T_1\, e^{2\alpha_2}}{(n/2 + \alpha_2) - (n/2 - \alpha_2)e^{2\alpha_2}}$$

$$\alpha_2 = \frac{(n^2 - 4\alpha_1 R)^{1/2}}{2}$$

$$T_1 = \alpha\sigma\left(u_B - \frac{RB}{\sigma^2}\right) + u_2\, \alpha_1^{-1} + RA\, e^{\alpha_1 t}\alpha_1^{-1}(n - \alpha_1)(R - n + \alpha)^{-1}$$

$$T_2 = u_0\, e^{\alpha_1 t - n/2} + B\alpha_1^{-1}\, e^{n/2} + RAe^{\alpha_1 t - 1}\, \alpha_1^{-1}\, e^{n/2 - \alpha_1}\,(R - n + \alpha_1)^{-1}$$

and therefore $$u = e^{-\alpha_1 t}\, f(y) \qquad .....(20)$$

where f(y) is given by (19)

If M denotes the dimensionless mass flow rate per unit channel width then

$$M = \int_0^1 e^{-ny}\, u\, dy$$

$$= e^{-\alpha_1 t}\left[\left(\alpha_i \frac{e^{\alpha_2} - n/2}{(\alpha_2 - n/2)} - \alpha_j \frac{e^{-\alpha_2} - n/2}{(\alpha_2 + n/2)}\right) - \left(\frac{\alpha_i}{\alpha_2 - n/2} - \frac{\alpha_j}{\alpha_2 + n/2}\right)\right]$$

$$- \frac{B}{\alpha_1}\, e^{-\alpha_1 t} + RA\frac{(e^{-\alpha_1} - 1)}{\alpha_1^2\,(R - n + \alpha_2)} \qquad .....(21)$$

If the porous bed is replaced by an impermeable rigid plate, then $\overset{*}{M}$, the dimensionless mass flow is obtained as

$$\overset{*}{M} = e^{-\alpha_1 t}\left[e^{-n/2}\left(\overset{*}{\alpha}_i \frac{e^{\alpha_2}}{\alpha_2 - n/2} - \overset{*}{\alpha}_j \frac{e^{-\alpha_2}}{\alpha_2 + n/2} \right) - \left(\frac{\overset{*}{\alpha}_i}{\alpha_2 - n/2} - \frac{\overset{*}{\alpha}_j}{\alpha_2 + n/2} \right)\right]$$

$$- \frac{B}{\alpha_1} e^{-\alpha_1 t} + RA \frac{e^{-\alpha_1} - 1}{\alpha_1^2 (R - n + \alpha_1)} \qquad \text{.....(22)}$$

$$\overset{*}{\alpha}_i = \frac{\overset{*}{T}_1 - \overset{*}{T}_2\, e^{\alpha_2} (n/2 - \alpha_2)}{(n/2+\alpha_2) - (n/2 - \alpha_2)\, e^{2\alpha_2}},$$

$$\overset{*}{\alpha}_j = \frac{\overset{*}{T}_2\, e^{\alpha_2} (n/2 + \alpha_2) - \overset{*}{T}_1\, e^{2\alpha_2}}{(n/2+\alpha_2) - (n/2 - \alpha_2)\, e^{2\alpha_2}}$$

$$\overset{*}{T}_1 = \frac{u_2}{\alpha_1} + RAe^{\alpha_1 t} (n - \alpha_1)\, \alpha_1^{-1} (R - n + \alpha_1)^{-1}$$

$$T_2 = \overset{*}{T}_2$$

The fractional increase in the mass flow rate through the channel with a permeable lower wall over what it would be if the wall were impermeable is given by

$$\phi = (M - \overset{*}{M}) / \overset{*}{M}$$

$$= e^{-\alpha_1 t}\, \alpha\sigma \left(u_B - \frac{RB}{\sigma^2} \right) \left[\exp(\alpha_2 - n/2) - 1\right] / (\alpha_2 - n/2) \overset{*}{M} \qquad \text{.....(23)}$$

Discussion :

To study the effect of the slip parameter α, porosity factor σ and viscous stratification factor n on the flow over a permeable bed, we calculate the fractional increase in the mass flow rate $|\phi|$ which has been numerically evaluated for different sets of values of α, σ and n.

The fractional increase in the mass flow rate $|\phi|$ as a function of porosity factor (σ) is calculated numerically for different values of α and n, shown as in table 1.

From the *Table 1* we can conclude that for each set of values of α and n, $|\phi|$ is increasing with the increase of sigma which suggests that porosity factor σ is favourable to the fractional increase in the mass flow rate of the fluid.

From *Fig. 1* it is clear that for a given set of values of α *and* n, $|\phi|$ directly varies with σ when the flow is governed by a body force $Ae^{-\alpha_1 y}$.

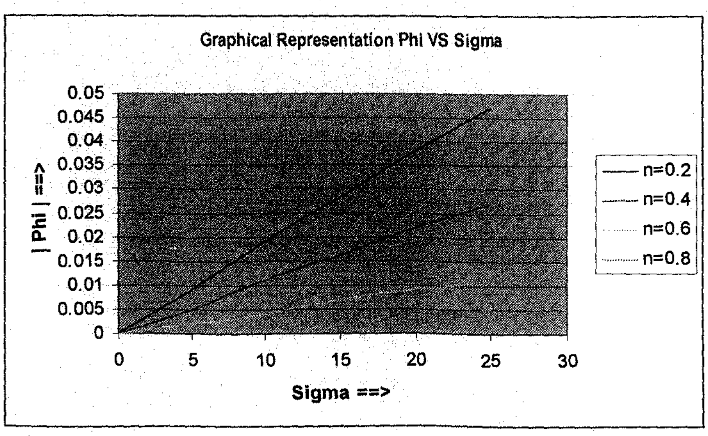
From *Fig. 2* it is seen that in presence of a body force $Ae^{-\alpha_1 y}$, $|\phi|$ decreases with the increase of n for each set of values of α, σ which shows that the stratification factor is not favourable to the fractional increase in the mass flow rate .

Now we assume $u_0 = 1$, $u_B = 2$, $u_m = 5$, $\alpha_1 = 1$, B=1, R = 0.01, A=10, t=0. For all calculations of *Fig. 1* and *Fig. 2*.

Table – 1

$\alpha = 0.01$, n = 0.2					
α	5	10	15	20	25
$\lvert\phi\rvert$	21.6×10^{-6}	46×10^{-6}	69×10^{-6}	94×10^{-6}	121×10^{-6}
$\alpha = 0.1$, n = 0.6					
α	5	10	15	20	25
$\lvert\phi\rvert$	200×10^{-6}	500×10^{-6}	700×10^{-6}	900×10^{-6}	1200×10^{-6}
$\alpha = 0.01$, n = 0.6					
α	5	10	15	20	25
$\lvert\phi\rvert$	20×10^{-6}	50×10^{-6}	70×10^{-6}	90×10^{-6}	120×10^{-6}

(assuming $u_0 = 1/2$, $u_B = 0.25$, $\alpha_1 = 0.1$, $u_m = 1$, B=1, R = 0.99, A = 1, t = 0.)



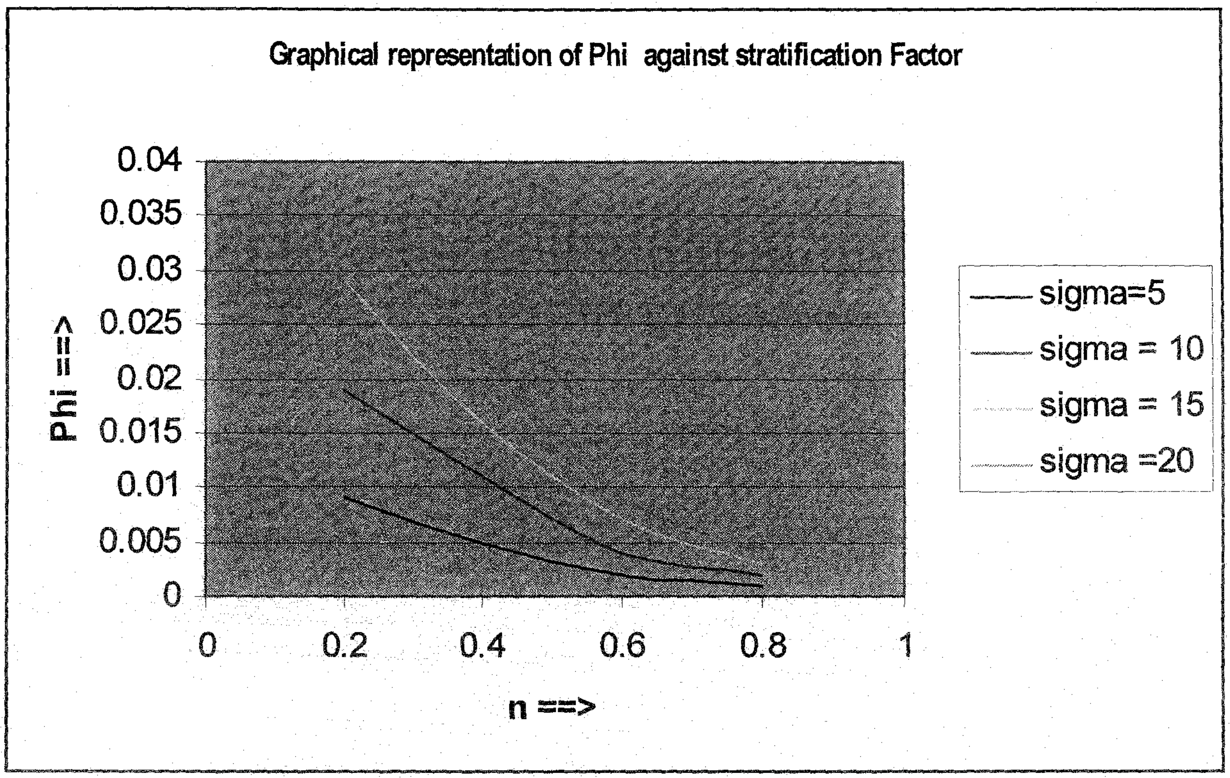

Graphical representation of Phi against stratification Factor
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
Phi ==>
0
0.2
0.4
0.6
0.8
1
n ==>
sigma=5
sigma = 10
sigma = 15
sigma =20

Fig.2