CHAPTER : II

STRATIFIED FLUID FLOWS

PART - ONE

TO STUDY THE SLIP VELOCITY FOR THE FLOW OF STRATIFIED FLUID OF VARIABLE VISCOSITY PAST A POROUS BED UNDER THE ACTION OF PRESSURE GRADIENT

INTRODUCTION : The motion of viscous stratified fluid largely depends on the magnitude of stratification factor, porosity factor and slip parameter. So they must have an effect on the slip velocity also. To study the flow of the viscous fluid past a porous medium without stratification has been studied by Beavers and Joseph (1967), Beaver *et.al* (1970) and Rudraiah *et.al* (1973). The study of stratified fluid is of great importance in the field of petroleum industry because the density of oil varies with temperature. Channabassappa and Ranganna (1975) considered the flow of viscous stratified fluid past a porous bed with anticipation that stratification may provide a technique for studying the pore size in a porous medium. Hari Kishan and Sharma (1980) have studied the stratified viscous flow of variable viscosity between a porous and moving impermeable plate under the action of a body force. Gupta and Babu (1987) studied the flow of a viscous incompressible fluid through a porous medium near an oscillating infinite porous flat plate in slip flow regime. Mukesh Gupta and Shalini Sharma (1991) investigated the flow of viscous incompressible and electrically conducting fluid through a porous medium bounded by an oscillating porous infinite flat plate in slip flow regime under the influence of transverse magnetic field fixed relative to the fluid.

In this paper, we have studied the effect of stratification factor, porosity factor and slip parameter on the slip velocity of the flow of viscous stratified fluid.

It is known that when a Newtonian fluid flows between two impermeable surfaces the usual boundary condition with non slip condition on the boundary leads in a parabolic type of motion in the channel. Beaver and Josheph have shown that the flow between porous bed is governed by Darcy law and no slip condition is replaced by a streamwise slip velocity at the nominal surface.

Using the slip boundary condition we divided the entire flow region into two zones. Zone I relates the free flow above the bed which is governed by Navier-Stokes equations and in zone 2 below the interface the flow is governed by Darcy law. The velocity distribution under the pressure gradient in these zones are separately obtained and matched at the interface to get continuous velocity distribution.

MATHEMATICAL FORMULATION OF THE PROBLEM : Here we have considered a physical model illustrating the problem of consideration shown in figure (1). It consists of a parallel plate channel of height *h* where the lower bounding wall is permeable while the upper is rigid. Laminar unidirectinal flow is assumed to be set up by a time varying longitudinal pressure gradient in the channel and in the porous medium. To study this problem of flow region in the channel and in the porous medium is divided into two zones. Zone 1 consists of the region from the impermeable plate to the interface where the flow called the free flow is governed by the Navier-Stokes equations. In the other zone, below the interface, the flow is governed by Darcy Law. Walls of the channel are horizontal and infinitely long to allow the physical quantities to the independent of the axial co-ordinate. Velocity field is assumed to have only one component in the direction of x-axis. The velocity *u* and pressure *p* are functions of *y* and *t, t* denoting the time.

The basic equation for Zone 1 are :

$$\rho\frac{\partial u}{\partial t}=-\frac{\partial p}{\partial x}+\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) \qquad \text{...........} \ (1)$$

where

$$\mu=\mu_0 e^{-\beta y}, \rho=\rho_0 e^{-\beta y} \qquad \text{...........} \ (2)$$

and $$\frac{\partial p}{\partial y}=-g\rho \qquad \text{............ (3)}$$

Here μ_0 and ρ_0 are the coefficient of viscosity and density respectively at the interface $y=0, \beta>0$ represents the stratification factor.
The basic equations for zone-2 are

$$Q=Q_0 e^{-\beta y} \qquad \text{............ (4)}$$

where $$Q_0=-K/\mu_0 \frac{\partial p}{\partial x} \qquad \text{............ (5)}$$

The relevant boundary conditions are $u=0$ at y = h .... (6)

and $$\frac{\partial u}{\partial y}=\frac{\alpha}{\sqrt{K}}(u_B-Q_0) \text{ at y=0} \qquad \text{............ (7)}$$

where α =slip parameter, K=the permeability co-efficient, Q=Darcy velocity and u_B is the slip velocity at the interface y = 0 (nominal surface).

Solution of the Problem : Equations (1), (6), (7) are made dimensionless using the quantities

$$u^{/}=\frac{u}{u_m},\ t^{/}=u_m t/h,\ x^{/}=x/h,\ y^{/}=y/h,\ k^{/}=k/h^2$$

$$p^{/}=\frac{p}{\rho_0 u_m^2},\ u^{/}{}_B=u_B/u_m$$

Omitting dashes we get

$$\frac{\partial u}{\partial t}=-e^{ny}\frac{\partial p}{\partial x}+R^{/}\frac{\partial^2 u}{\partial y^2}-nR^{/}\frac{\partial u}{\partial y} \qquad \text{............ (8)}$$

where $$n=\beta h,\ R^{/}=\frac{1}{R}$$

with the boundary conditions

$$u=0 \text{ at } y=1 \qquad \text{............ (9)}$$

and $$\frac{\partial y}{\partial y}=\alpha\sigma\left(u_B+\frac{1}{\sigma^2 R^{/}}\frac{\partial p}{\partial x}\right) \text{ at y = 0} \qquad \text{............ (10)}$$

$R=\frac{1}{R^{/}}=\frac{u_m h}{\lambda_o}$ represents the Reynolds Number and $\sigma=\frac{1}{\sqrt{k}}$.

Let us assume that the flow is due to pressure gradient, where

$$-\frac{\partial p}{\partial x} = A\exp\left(-c^2\, t\right) \qquad \text{.......... (11)}$$

where A and c^2 are real constants.
We assume that the velocity is given by

$$u = Au_y \exp\left(-c^2 t\right) \qquad \text{.......... (12)}$$

and the slip velocity is given by

$$u_B = Au_* \exp\left(-c^2 t\right) \qquad \text{.......... (13)}$$

Equation (8) now becomes

$$\frac{d^2 u_y}{dy^2} - n\frac{du_y}{dy} + \frac{c^2}{R'}u_y = -\frac{1}{R'}e^{ny} \qquad \text{.......... (14)}$$

Boundary condition becomes

$u_y=0$ at $y = 1$

.... (15)

Solution of (14) subject to the above condition is

$$u_y = \frac{e^{ny/2}\left[2\alpha_2 c^2 \sin\alpha_1(1-y) - e^{n/2}\left(2\alpha_1 \cos\alpha_1 y - n\, Sin\,\alpha_1 y\right)\right]}{c^2\left(n\sin\alpha_1 - 2\alpha_1 \cos\alpha_1\right)} - \exp(ny)/c^2 \qquad \text{...... (16)}$$

where $\frac{4c^2}{n^2} > R'$

and

$$u_y = e^{ny/2}\left[\frac{\alpha_2 c^2 - \left(\frac{n}{2}-\alpha_1\right)e^{n/2+\alpha_1}}{c^2\left\{\left(\frac{n}{2}+\alpha_1\right)-\left(\frac{n}{2}-\alpha_1\right)e^{2\alpha_1}\right\}}e^{\alpha_1 y} + \frac{\alpha_1}{c^2}\left(e^{n/2} - \alpha_1\frac{\alpha_2 c^2 - \left(\frac{n}{2}-\alpha_1\right)e^{n/2+\alpha_1}}{\left\{\left(\frac{n}{2}+\alpha_1\right)-\left(\frac{n}{2}-\alpha_1\right)e^{2\alpha_1}\right\}}\right)e^{-\alpha_1 y}\right] - \frac{e^{ny}}{c^2}$$

$$u = \frac{2\alpha_1 \exp\left(\alpha_1 + \frac{n}{2}\right) + \left(1-e^{2\alpha_1}\right)\left(n - \frac{c^2\alpha}{\sigma R'}\right)}{c^2\left\{\left(\frac{n}{2}+\alpha_1\right)-\left(\frac{n}{2}-\alpha_1\right)e^{2\alpha_1}\right\} - \alpha\sigma\left(1-e^{2\alpha_1}\right)} \qquad \text{.......... (17)}$$

where $\frac{4c^2}{n^2} < R'$

and $$2\alpha_1 = \left(\frac{4c^2}{R'} - n^2\right)^{1/2}, \quad \alpha_2 = \alpha\sigma\left(u_* - \frac{1}{\sigma^2 R'}\right) + \frac{n}{c^2}$$

The velocity distribution is given by

$$u = Au_y \exp(-c^2 t), \quad \frac{4c^2}{R'} > n^2 \qquad \text{.......... (18)}$$

$$= Au_y \exp(-c^2 t), \quad \frac{4c^2}{R'} < n^2$$

u_y is given by (16) and (17).

The slip velocity is given by

$$u_B = Au_* \exp(-c^2 t) \qquad \text{.......... (19)}$$

where $u_* = \dfrac{2\alpha_1\left(e^{n/2} - \cos\alpha_1\right) + n\sin\alpha_1\left(1 - \dfrac{2c^2\alpha}{n\sigma R'} - 2\alpha\sigma\right)}{c^2(2\sigma\alpha - n)\sin\alpha_1 + 2\alpha_1 c^2 \cos\alpha_1}$

where $\dfrac{4c^2}{n^2} > R^1$

$$u_* = \frac{2\alpha_1 \exp\left(\alpha_1 + \dfrac{n}{2}\right) + \left(1 - e^{2\alpha_1}\right)\left(n - \dfrac{c^2\alpha}{\sigma R'}\right)}{c^2\left\{\left(\dfrac{n}{2} + \alpha_1\right) - \left(\dfrac{n}{2} - \alpha_1\right) e^{2\alpha_1}\right\} - \alpha\sigma\left(1 - e^{2\alpha_1}\right)} \qquad \text{........ (20)}$$

where $\dfrac{4c^2}{n^2} < R'$

If the bed of the channel had been impermeable the velocity profile would be given by

$$\mathring{u} = A\mathring{u}_y \exp(-c^2 t) \qquad \text{.......... (21)}$$

where

$$\mathring{u}_y = e^{ny/2}\left[\frac{\sin\alpha_1(1-y)}{\sin\alpha_1}\left\{2\alpha_1\left(e^{n/2} - \cos\alpha_1\right)\right\} + e^{n/2}\left(n\sin\alpha_1 y - 2\alpha_1\cos\alpha_1 y\right)\right] - \frac{e^{ny}}{c^2}$$

if $\dfrac{4c^2}{n^2} > R^1$ .... (22)

and

$$\overset{*}{u}_y = \frac{e^{ny/2}\left(\alpha_1 - \frac{n}{2}\right)}{c^2\left\{\left(\frac{n}{2}+\alpha_1\right)-\left(\frac{n}{2}-\alpha_1\right)e^{2\alpha_1}\right\}}\left(e^{\alpha_1 y} - e^{2\alpha_1-\alpha_1 y}\right) - \frac{e^{ny}}{c^2}\ if\ \frac{4c^2}{n^2} < R' \qquad \text{........ (23)}$$

In absence of stratification factor the velocity distribution (16) and (22) reduce to

$$u_y = \frac{1}{c^2}\left[\frac{\alpha_1 \cos\alpha_1 y - \alpha_2 c^2 \sin\alpha_1(1-y)}{\alpha_1 \cos\alpha_1} - 1\right] \qquad \text{........... (24)}$$

and $$\overset{*}{u}_y = \frac{1}{c^2}\left[\frac{\sin\frac{cy}{\sqrt{R'}} + \sin\frac{c}{\sqrt{R'}}(1-y)}{\sin\frac{c}{\sqrt{R'}}} - 1\right] \qquad \text{........... (25)}$$

which is the same as the result obtained by P. Bhattacharya (1980). When *n*, the stratification factor, is zero, from equation (20) the slip velocity reduces to

$$u_* = \frac{1}{c^2}\left[\frac{\left(\frac{c}{\sqrt{R'}} + \alpha\sigma\left(1+\frac{c}{R'\sigma^2}\right)\right)\sin\frac{c}{\sqrt{R'}}}{\frac{c}{\sqrt{R'}}\cos\frac{c}{\sqrt{R'}} + \alpha\sigma\sin\frac{c}{\sqrt{R'}}} - 1\right] \qquad \text{........... (26)}$$

which is the same as the result obtained by P.Bhattacharya (1980). When $\sigma \to \infty$ the slip velocity given by equations (19) and (20) reduce to

$$u_*\Big]_{\sigma\to\infty} = -\frac{n}{c^2} \quad \text{when } \frac{4c^2}{R'} > n^2 \qquad \text{........... (27)}$$

$$= 0 \quad \text{when } \frac{4c^2}{R'} < n^2 \qquad \text{.......... (28)}$$

From (27) it is seen that the magnitude of slip velocity for large porosity factor depends only on *n*, the stratification factor, when $\frac{4c^2}{R'} > n^2$ and from (28) it is seen that no slip condition is maintained at the interface when $\frac{4c^2}{R'} < n^2$.

DISCUSSION : From table 1 and table 2, a comparative study of the slip velocity in presence of stratification factor (β) and in absence of stratification factor (β) with variable porosity factor (σ) under constant slip parameter (α) is made.

Table -1

$\alpha = 0.01,\ n = 0$					
$\sigma \to$	5	10	15	20	25
$u_* \to$	.0048	.0046	.0044	.0042	.00403
$\alpha = 0.1,\ n = 0$					
$\sigma \to$	5	10	15	20	25
$u_* \to$	.0037	.00225	.00203	.00168	.00144
$\alpha = 1,\ n = 0$					
$\sigma \to$	5	10	15	20	25
$u_* \to$	.00116	.00054	.00035	.00026	.000207

For all calculations $c^2 = 1.44,\ R^{/} = 100$

Table-2

$\alpha = 0.01, n = 1$					
$\sigma \to$	5	10	15	20	25
$u_* \to$	.0559	.0462	.0389	.0333	.0288
$\alpha = 0.1, n = 1$					
$\sigma \to$	5	10	15	20	25
$u_* \to$	.0153	.0051	.0008	-.0016	-.0031
$\alpha = 1, n = 1$					
$\sigma \to$	5	10	15	20	25
$u_* \to$	-.0068	-.0083	-.0088	-.0091	-.0093
$\alpha = 1.5, n = 1$					
$\sigma \to$	5	10	15	20	25
$u_* \to$	-.00796	-.00887	-.00922	-.00940	-.00952

For all calculations $c^2 = 1.49,\ R^{/} = 100$

It reveals from the table that stratification factor accelerates the slip velocity when the slip parameter α<1 but it retards the velocity when the slip parameter α=1. Thus we may conclude that stratification factor (n)may accelerate or retard the motion depending on the values of slip parameter (α). Also it appears from the table that slip velocity decreases with increase of the porosity factor (σ) for every pair of n and α.

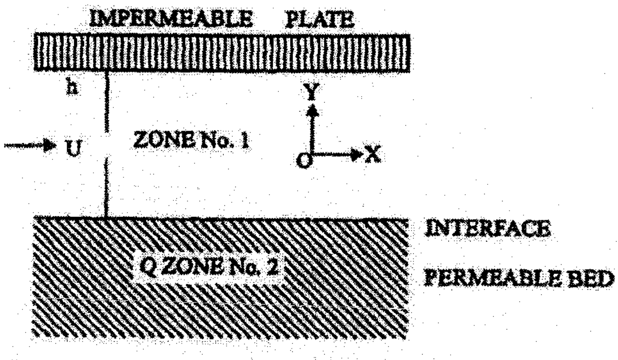

Fig. 1. Physical Model

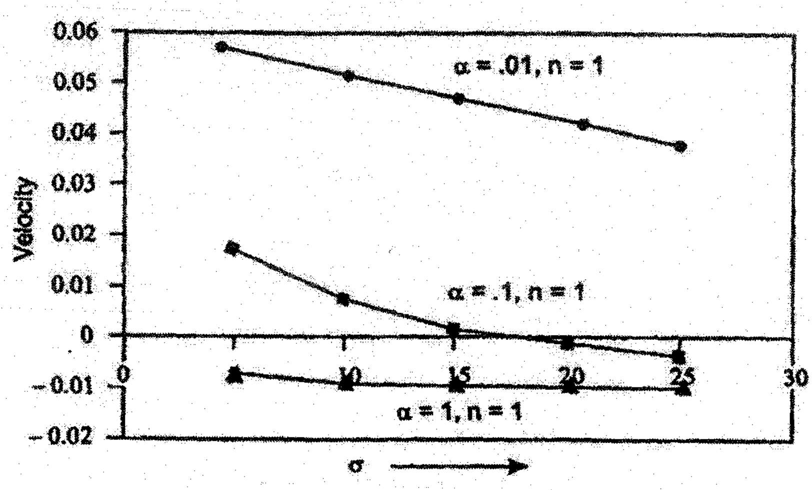

Fig. 2. Slip velocity against slip parameter.